# Power Network Analysis

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**Week-06**

**Lecture-29**

## Lecture 29: Transmission line models and performance – long line model, lossless long line

Hello everyone, welcome to lecture 4 of week 6 of the course Power Network Analysis, in which we continue our discussion on transmission line models and the use of these models for performance assessment of transmission lines under different operating conditions. In today's discussion, we will continue with our long line model and we will discuss what is this lossless long line and what special attributes or features come out of this lossless long line model and what are the notions of traveling waves, wavelength, etc., etc., from the lossless perspective. How it can lead to benefits in terms of analyzing the transmission network when it is struck by lightning, or for a DC network, how we do fault analysis; that idea or notion I would try to convey. The previous lecture we had our discussion on long line model wherein we understood what the distributed parameters, how they are represented in the long line model.

Every small minuscule section has a series impedance that is dictated by the line resistance and inductance and has a shunt admittance dictated by the line conductance and capacitance. And we also understood two hyperbolic functions: those functions were the hyperbolic cosine and hyperbolic sine functions. So cos hyperbolic of let us say argument theta is nothing but e to the power theta plus e to the power minus theta by 2 where e is the exponential function whereas sin hyperbolic of theta is e to the power theta minus e to the power minus theta by 2. And from here, can we get some beautiful equations because if we compare the actual trigonometric analogous functions, then cos squared theta plus sin squared theta is universally known to be equal to 1.

$$\sinh\theta = \frac{e^{\theta}+e^{-\theta}}{2}$$

$$\cosh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$

$$\cos^2\theta + \sin^2\theta = 1$$

Why is that? Again, there could be a straightforward reason for it. So let us say that if I have to take the exponential of j times theta, where j is the square root of minus 1, the complex operator. Then e to the power j theta is nothing but cos theta plus j sin theta. And similarly, if I take e to the power of minus j theta, it would be cos theta. minus j sin theta.

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Now if I multiply both these equations I get e to the power j theta minus j theta which is essentially e to the power 0 is also equal to cos square theta plus sine square theta the imaginary terms they get cancelled because cos into sine has a negative sign here whereas cos into sine has a positive sign here so the imaginary terms they get cancelled and we also know that j square is nothing but minus 1 so that's how we get our sine square theta here. Since e to the power 0 is 1, so cos square theta irrespective of the value of theta, it is always equal to, the sum is always equal to 1. So, on similar lines, can there be some relationship among hyperbolic functions? So let's try it out. So if we find the hyperbolic cosine squared of theta, it would be e to the power 2 theta plus e to the power minus 2 theta. into 2 times of e theta and e minus theta which results into only 2 divided by divided by 4 and similarly sin hyperbolic square theta would result into e to the power 2 theta plus e to the power minus 2 theta minus 2 by 4.

$$\cosh^2\theta = \frac{e^{2\theta} + e^{-2\theta} + 2}{4}$$

$$\sinh^2\theta = \frac{e^{2\theta} + e^{-2\theta} - 2}{4}$$

$$\cosh^2\theta - \sinh^2\theta = 1$$

If we now subtract sin hyperbolic square from sin cosine hyperbolic square we have 2 minus minus 2 by 4 which is eventually equal to 1. And the way sec theta is equal to cos theta inverse on similar line hyperbolic sec theta is equal to inverse of cos hyperbolic theta also. So we have sin square cos square hyperbolic square cos hyperbolic square theta minus sin hyperbolic square theta is equal to 1 unlike cos square theta plus sin square theta is equal to 1. So for the long line model, we have seen our V of X and I of X, where the Vx and Ix are per-phase voltage and current measured from the sending end. The sending end at X is equal to 0, and we have seen this relationship in the previous slide.

If we divide this expression by cos hyperbolic square gamma X throughout and rearrange terms, we would also get this term, the second equation.

$$V(x) = V_S \cosh(\gamma x) - I_S Z_C \sinh(\gamma x)$$

$$I(x) = -\frac{V_S}{Z_C} \sinh(\gamma x) + I_S \cosh(\gamma x)$$

$$\cosh^2(\gamma x) - \sinh^2(\gamma x) = 1$$

$$\operatorname{sech}^2(\gamma x) + \tanh^2(\gamma x) = 1$$

And in terms of boundary conditions when they are placed, that is at x equal to LL, we have VR and IR in terms of VS, IS, and ZC. ZC is called the characteristic impedance, which is also... square root of small z and y, small z is per unit series impedance, small y is per unit series admittance and gamma is the propagation constant which is usually a complex number, alpha being the attenuation constant, beta being the phase constant, it is equal to root over of small z and y, the unit of zc is omega which is ohm and the unit of gamma is kilometer inverse. So, with this now let us go deep into the rest of the part of our discussion. So, we have our VRIR receiving end voltage and current in terms of sending end quantities; the characteristic impedance and the propagation constant, LL, is the line length measured from the sending end. So, basically, LL is the distance from the sending end to the receiving end. We can rearrange these two simultaneous equations to express VS and IS in terms of VR and IR, and the reason I have mentioned this is that I wish to find or represent this long line model as a two-port network, wherein I can use the ABCD parameters, Z parameters, Y parameters, or H parameters as required.

$$V_R = V_S \cosh(\gamma ll) - I_S Z_C \sinh(\gamma ll)$$

$$I_R = -\frac{V_S}{Z_C} \sinh(\gamma ll) + I_S \cosh(\gamma ll)$$

$$\Rightarrow V_S = V_R \cosh(\gamma ll) + I_R Z_C \sinh(\gamma ll)$$

$$\Rightarrow I_S = \frac{V_R}{Z_C} \sinh(\gamma ll) + I_R \cosh(\gamma ll)$$

So if we recollect our ABCD parameters, then the ABCD parameters are nothing but VS plus VS, which is equal to AVR plus BIR, and IS is equal to CVR plus DIR.

$$V_s = AV_R + BI_R$$

$$I_s = CV_R + DI_R$$

If we compare these equations one-to-one in analogous form, we have AD equal to each other, and B and C can be expressed in terms of hyperbolic functions of sine, ZC, and etcetera. So, the conditions of symmetry and reciprocity still hold true. The condition of symmetry is pretty obvious because a and d are equal, and the condition of reciprocity makes it obvious. thanks to our previous discussion we had where we are making use of this particular relationship.

So here we have A into D, which is nothing but cos hyperbolic square gamma L L, which is nothing but A D and B C. If we multiply Z C, Z C gets cancelled. We still have sin hyperbolic square gamma L L cos hyperbolic square gamma L L minus sin hyperbolic square gamma L L is equal to 1, and that is how. AD minus BC is always equal to 1, which reflects the condition of reciprocity. Similarly, the ABCD parameters can be used to find the other two-port parameters, wherein, again, the conditions of symmetricity and reciprocity are still valid for Z parameters.

$$V_R = V_S \cosh(\gamma ll) - I_S Z_C \sinh(\gamma ll)$$

$$I_R = -\frac{V_S}{Z_C}\sinh(\gamma ll) + I_S \cosh(\gamma ll)$$

$$\Rightarrow V_S = V_R \cosh(\gamma ll) + I_R Z_C \sinh(\gamma ll)$$

$$\Rightarrow I_S = \frac{V_R}{Z_C}\sinh(\gamma ll) + I_R \cosh(\gamma ll)$$

$$A = D = \cosh(\gamma ll), B = Z_c \sinh(\gamma ll), C = \frac{\sinh(\gamma ll)}{Z_c}$$

$$A = D \text{ (condition of symmetry)}$$

$$AD - BC = 1 \text{ (condition of reciprocity)}$$

Similarly, we have our Y parameters, wherein the discussions we had in terms of median and model are still applicable. The corresponding hyperbolic functions can be evaluated. Here, cot hyperbolic refers to the inverse of.

The inverse of the cotangent hyperbolic of theta is nothing but 1 over the tangent hyperbolic of theta, which is also equal to the cosine hyperbolic of theta divided by the sine hyperbolic of theta because the tangent hyperbolic of theta is the sine hyperbolic divided by the cosine hyperbolic, and the negative cosecant hyperbolic, also written as CSCH theta, is nothing but 1 over the sine hyperbolic of theta. Similarly, the H parameters can also be evaluated where sec hyperbolic, we have already discussed what actually it is, which is nothing but inverse of cos hyperbolic. So the conditions of symmetricity and reciprocity are all valid for all two-port network parameters.

$$\begin{bmatrix} V_s \\ I_r \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ -h_{21} & -h_{22} \end{bmatrix} \begin{bmatrix} I_s \\ V_r \end{bmatrix}$$

$$h_{11} = B/D = Z_c \tanh(\gamma ll), h_{22} = \tanh(\gamma ll)/Z_c$$

$$h_{12} = -h_{21} = -1/D = -\text{sech}(\gamma ll) \text{ (condition of reciprocity)}$$

$$h_{11}h_{22} - h_{12}h_{21} = 1 \text{ (condition of symmetry)}$$

And these parameters are likely to be complex numbers, no matter whether they are H, Y, Z, or A, B, C, D, because Z, C, and gamma are also complex numbers, so the corresponding hyperbolic functions would also turn out to be complex numbers. So what we have here is that we have our complex ABCD Z Y or H parameters obtained for a long line.

$$\begin{bmatrix} I_S \\ I_R \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ -Y_{21} & -Y_{22} \end{bmatrix} \begin{bmatrix} V_S \\ V_R \end{bmatrix}$$

$$Y_{11} = Y_{22} = D/B = \coth(\gamma ll)/Z_c \text{ (condition of symmetry)}$$

$$Y_{12} = Y_{21} = -1/B = -\text{csch}(\gamma ll)/Z_c \text{ (condition of reciprocity)}$$

Then is it not possible to come up with the nominal pi model the way we did for the medium line? Nominal PI model, nominal T model rather than always solving differential equations or trying to evaluate these complex hyperbolic functions. Is there not a simpler representation or a nominal representation of a long line similar to the medium line model? Yes, it's possible. Inspired by the medium line model, it is possible to draw a per-phase circuit equivalent representation of a long line model in terms of the nominal pi model or the nominal T model. The nominal pi model is pretty convenient and commonly used because of the way the shunt elements appear at the line ends, whereas in the T model, the shunt element appears at the midpoint of the line. the pi model would have a fictitious series branch or a series impedance whereas this impedance when we are marking remember in medium line for pi or T model we had assumed lumped representation in long line the representation of parameters is distributed it is not lumped.

So, when we are trying to find the equivalent nominal pi model or T model, the corresponding models refer to a lumped representation. So the lumped parameters would come in; they would not be the actual line impedance or admittance; they would be the fictitious quantities that nominally represent or capture the effect of the actual line; they are not the actual representation. So we would have a fictitious series impedance; we would have a fictitious series admittance or impedance that would take care of the capacitance effect: the equivalent lumped parameters. How can they be evaluated? Since we know the ABCD parameters for the actual long line, we also know what the ABCD parameters would be for a nominal pie model. So, we can do reverse engineering, compare the ABCD parameters, and find the equivalent fictitious series impedance and shunt impedance.

Such a representation would actually be very useful for steady state and dynamic performance analysis, specifically for power flow analysis or the fault analysis or stability analysis, which would be the next part or the next few modules after this current module. And that's the reason why the nominal pi model is to be evaluated for a long line as well. So, in order to do that, we go back to our actual nominal pi model, where we mark Z dash and Y dash as those fictitious series impedances and shunt admittances. In the nominal pi model, half of the shunt admittance appears at the line end. The terminals, however, refer to the same terminals of the actual long line.

We are not making any approximations or modifications of the terminals. We are only trying to represent the distributed form of line parameters in long line by using lumped representation of nominal pi model, where Z dash and Y dash are yet to be evaluated. So Z dash is the actual equivalent series impedance, and Y dash is the actual equivalent fictitious line admittance. In terms of ABCD parameters, those of you who are viewing this for the first time, please refer to the previous video and previous lecture where you will be able to understand how we get these equations of VSIS in terms of VRIR. In fact, these VSIS equations are derived by applying KVL across the sending and receiving ends and by applying individual KCLs at these nodes.

$$V_S = \left(1 + \frac{Z'Y'}{2}\right)V_R + Z'I_R = AV_R + BI_R$$

$$I_S = Y'\left(1 + \frac{Z'Y'}{4}\right)V_R + \left(1 + \frac{Z'Y'}{2}\right)I_R = CV_R + DI_R$$

That's how we obtain three equations. Those three equations, when rearranged and rewritten, can be expressed in terms of the parameters a, b, c, and d, which are functions of z' and y'. So from here we easily know that a and d are nothing but 1 plus z dash y dash divided by 2, b is simply z dash, and c is y dash plus 1 plus z dash y dash divided by 4.

$$A = D = 1 + \frac{Z'Y'}{2}$$

$$B = Z', C = Y'\left(1 + \frac{Z'Y'}{4}\right)$$

We also know what these ABCD are for the actual long line. So, if we put those numbers, we would have a sufficient number of equations to find our two unknowns, Z dash and Y dash.

So, if we do that by a one-to-one comparison of results, we would get ABCD or Z dash Y dash to be in the form where Y dash by 2 is tan hyperbolic gamma LL by 2 by ZC, and similarly, Z dash could also be evaluated here using the B parameter, which is ZC into sin hyperbolic gamma L. The actual distributed representation gets represented as a nominal pi model equivalent fictitious nominal long line model with Z dash and Y dash as the fictitious series impedance and admittance, respectively.

- One to one comparison results in ($A = \cosh(\gamma ll)$)

$$B = Z' = Z_c \sinh(\gamma ll)$$

$$A = 1 + \frac{Z'Y'}{2}$$

$$\Rightarrow \frac{Y'}{2} = \frac{A-1}{Z'} = \frac{\cosh(\gamma ll) - 1}{Z_c \sinh(\gamma ll)}$$

$$\Rightarrow \frac{Y'}{2} = \frac{2\sinh^2(\gamma ll/2)}{Z_c 2\sinh(\gamma ll/2)\cosh(\gamma ll/2)}$$

$$\Rightarrow \frac{Y'}{2} = \frac{\sinh(\gamma ll/2)}{Z_c \cosh(\gamma ll/2)}$$

$$\Rightarrow \frac{Y'}{2} = \frac{\tanh(\gamma ll/2)}{Z_c}$$

This same representation would be extensively used in power flow analysis and fault analysis as well as stability analysis. So, coming to the other aspect of understanding or rethinking what these equations look like, recollect that for a long line model, we have V of x as 1 times e to the power of gamma x, where gamma is substituted as alpha plus j beta, and the general form of the equation also had the term a 2 times e to the power of minus gamma x, where gamma is alpha plus j beta. J, again, I would reiterate, is the complex operator which is the square root of minus 1, and using boundary conditions, we can also find what are V, S, V, X, I, X, etcetera, etcetera.

$$V(x) = A_1 e^{(\alpha + j\beta)x} + A_2 e^{-(\alpha + j\beta)x} \qquad j = \sqrt{-1}$$

$$\Rightarrow V(x) = V_S \cosh(\gamma x) - I_S Z_C \sinh(\gamma x) \ \text{(using bound. cond.)}$$

Given this, we are just trying to rethink: okay, is there any other interpretation involved behind these equations or solutions? Yes. Recollect that gamma is alpha plus j beta, which is the square root of z of y. Z and y are made up of series resistance, inductance, conductance, and capacitance, which are usually positive numbers. So, it is expected that the individual complex numbers, the real and imaginary parts, would have their separate parts of positive numbers. So, usually, alpha would be positive in an actual transmission line, which is the usual case.

So, if alpha is positive, then e to the power alpha x would increase as x increases, whereas e to the power minus alpha x, which is appearing over here, would decrease as x increases. So, that is the notion because alpha remains positive. Similarly, beta is also usually a positive number and remember e to the power j beta x where j is square root of minus 1 can be written as cos of beta x. Plus j sin beta x, and if we want to take the modulus of this complex number, the modulus of e to the power j theta is always equal to 1, because cos squared theta plus sin squared theta is equal to 1, where e to the power j theta is cos theta plus j sin theta, which we discussed in the second slide of today's discussion. So as beta increases which is usually I mean beta is positive which is the usual case e to the power j beta x and e to the power minus j beta x which are sitting over here in this context they would have the same magnitude which is equal to 1.

$$\left|e^{j\beta x}\right| \ or \ \left|e^{-j\beta x}\right| = |\csc \beta x \pm j \sin \beta x| = 1$$

What would they do otherwise? They would create phase shifts in terms of beta x radiance as x increases, or they would retard the corresponding voltage signal or voltage wave by j beta x as x increases for the other term. So, if this is appreciable and acceptable, the way alpha and beta are usually positive, and e to the power of alpha x increases with an increase in x, e to the power of j beta x increases the phase angle, not the magnitude of the signal strength. If we now look at the individual terms, let's say we focus on the second term where V of X had this as its second term, which was A2 e to the power of minus gamma X, then as X increases, which means that we are moving away from the sending end and trying to reach the receiving end, since with an increase in X, e to the power of minus alpha X would decrease, that means the signal strength. Or the value of a to the power e to the power value of a to e to the power minus gamma x it would start reducing it would diminish because alpha being positive the signal strength is being attenuated and its phase would also start retarding. Similarly, for a1 e to the power alpha plus j beta x, which is a1 e to the power gamma x, we would have that as we move away from the sending end to the receiving end, the first term will have an increase in signal strength, and the phase angle would advance rather than retard.

Alternatively, we can also correlate these functions or interpretations with the phenomena involved in traveling waves in still water. If a pebble is thrown into a pond with still water, then the point from where the pebble is thrown causes waves to start appearing, and as these waves move away from the point of disturbance or the point where the pebble hit the surface of the water, the wave strength starts diminishing as they move away from the center where the pebble had. sort of struck the surface of the water. This is very exactly to the phenomena involved behind this wave A to the power A2 into the power gamma minus gamma x. So in a way if the second term of V of x if it can be called as the incident or the forward wave then the first term of V of x can very well be called as the reflected or the backward wave.

Incident and reflected waves, or these terminologies, become very common in the lightning analysis of transmission networks, whereas forward and backward waves are also pretty common but are often not used; I mean, they are used in the context of lossless phenomena but are not often used in the context of DC fault analysis, etc. The way we understood V of X terms A1 and A2 terms having forward notion and backward notion the way they are representing similar to the travelling waves in still water. Similar notion can also be applicable for current waves and that is what It gives the indication that the power which is transmitted from sending end to receiving end through long lines, it doesn't always have to be a current signal or a voltage signal in time domain. They can also be interpreted as waves flowing through the transmission line, and that's how power reaches the receiving end or the load end. So if we come to the focus of today's discussion, which is the lossless long line, how can a lossless long line exist? Yes, usually transmission lines are designed to amplify or act as corridors for high voltage power from the source end to the load end so that the corresponding I squared losses are as low as possible.

So, by proper design or by good design, transmission lines do have very small resistance, and for the sake of analysis simplicity discussion, if we were to consider for the time being that this resistance is very negligible, which is actually the case or not actually, but very close to the real, the actual resistance is very small compared to line inductance, and we also know that conductance is difficult to model. So, if we were to consider it negligible for practical purposes, then small z. which was actually r plus j omega l where r tends to 0 would simply result in z being j omega l and similarly y which was g plus j omega c g was the conductance if this tends to 0 we have y as j omega c. With this, J, Z, and Y now being defined, if we find our constants like the characteristic impedance and propagation constant, they simply become functions of small l and small c, where small l is the per unit line inductance and small c is the per unit line capacitance. The attenuation constant becomes zero because there is no loss in the network, so there would be no diminishing of the corresponding waves, in fact.

If Z and Y are pure imaginary numbers or pure complex numbers, then their product of Z and Y would be J squared omega squared LC. The product itself does not have any real component, so the square root would also not have any real component; that's why alpha is 0, which means there is no attenuation, so it is actually referring to a lossless condition. Beta becomes omega times the square root of LC. Our hyperbolic functions, which are these forms, simplify to pure transcendental functions, the cosine and sine functions. Again, remember that j is the square root of minus 1.

So if we write the Vx and Ix expressions for a lossless long line, yes, gamma is replaced by simply j beta; cos and sin hyperbolic are reduced to cos and simply j sin beta.

$$z = j\omega l, y = j\omega c$$

$$\Rightarrow Z_c = \sqrt{z/y} = \sqrt{l/c}\,\Omega \text{ (characteristics imp.)}$$

Propagation constant $\gamma = \sqrt{zy} = j\sqrt{\omega^2 lc} \Rightarrow \alpha = 0, \beta = \omega\sqrt{lc}$

- Thus,

$$\cosh(\gamma x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} = \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x)$$

$$\sinh(\gamma x) = \frac{e^{\gamma x} - e^{-\gamma x}}{2} = \frac{e^{j\beta x} - e^{-j\beta x}}{2} = j\sin(\beta x)$$

So our V of x and I of x functions can be written from the usual long line model that we observed, and for this lossless long line, again we can find the corresponding ABCD parameters; we can find the corresponding nominal PIME model resulting from using the second boundary condition where at X is equal to LL, V of X is VR and I of X is IR. In fact, interestingly, if we put X equal to 0. Into the first equation, then it becomes v of x is equal to v s cosine of beta into 0 minus j i s z c sine of beta into 0. Beta into 0 is nothing but 0; sine of 0 is always 0, so essentially the second term goes away, and cosine of 0 is 1, so we have v of x is equal to v of s.

Similarly, if you put x is equal to 0 in this equation, your i of x at X is equal to 0; it would simply be IFS. Please verify that. So, on similar lines, we can find the ABCD parameters, Z, Y, H, and the nominal pi model for the lossless long line as well. That is not our point of focus for now. So we have our Vx and Ix equations for a lossless long line where the terms present here are still in the phasor domain or frequency domain.

$$V(x) = V_S \cos(\beta x) - jI_S Z_C \sin(\beta x)$$
$$I(x) = -j\frac{V_S}{Z_C}\sin(\beta x) + I_S \cos(\beta x)$$

These equations or closed-form solutions can be represented, analyzed, or stated in the time domain, where they would pertain to certain traveling wave equations similar to traveling waves in still water. It would appear that the associated time domain voltages and currents are in a lossless long line. They appear to be waves starting from the sending end. They are reaching the receiving end.

From there, some reflection is happening. So again, we have a reflected wave. The way waves originate and propagate in the ocean or sound in the air, etc. etc. So such a model is actually the basis of the premise behind ultra-fast transient analysis, which is a separate module of discussion in itself. Those of you who are interested in or eager to learn about ultra-fast transient analysis, please refer to higher-level postgraduate courses where you will find details of this.

But the premise behind that analysis starts from this lossless long line where the waves, or the voltage and currents, are treated in terms of waves. So let's see how this traveling wave model looks for a lossless long end. I'm just going to briefly state how it appears and why it appears. So what I have here is a similarly long line where the resistive components or conductive components are 0. So, our small z is simply reduced down to j omega L, and here we are trying to find the solutions of the corresponding equations not only in terms of x but also as a function of time.

So, here the representation that I have here is a time domain representation  not the frequency domain representation VS, VR, IS, IR they refer to the time domain quantities they could be instantaneous quantities as well and before I go deep into this discussion if I have a inductor whose inductance is let's say L and across this inductor I apply a voltage source. Let us say the voltage source is V of T, and I want to find the expression for the current through this inductor in the time domain. Then the basic property of inductor governance states that the voltage drop across this inductor is nothing but V. The first derivative of the current through the inductor multiplied by the corresponding inductance.

$$V(t) = L\frac{di(t)}{dt}$$

Similarly for capacitance we have current through the capacitor is nothing but first derivative of voltage across the capacitor multiplied by the corresponding capacitance.

$$i(t) = C\frac{dV(t)}{dt}$$

These first derivatives dI by dt and dV by dt I have I will represent them as L L of I dash T and this here I will represent as C of V dash T. Those V dash and I dash terms mentioned over here essentially refer to these derivatives only. And since these derivatives can exist in terms of time and exist in terms of distance from the sending end, if I take a cross-section of.

The line section at a distance x from the sending end, with width being delta x, and I choose similar nodes, node a and node b. At node a, I have to find what v is in terms of x and t. I want to find what i is again in terms of x and t. In the long line model, where we had phasor representation, time was not there.

Now the time signal or time variable is also coming in. And I'm choosing A and B as my nodes. So can I derive an appropriate model for this lossless long line in terms of traveling waves? So yes, we have our single-phase lossless representation with distance x measured from the sending end. And if I apply KVL across node A and B, where capital L here is nothing but small l which is the permanent line inductance then I have at node A voltage as V with a voltage drop of the drop across this inductor so small l is the actual permanent line inductance the width of section is delta x so I multiply l into delta x that gives me the overall inductance and by the property which I mentioned over here The voltage drop across this inductor would be L times delta X, the partial derivative of current with respect to T. That is what the term present over here is.

$$V - l\Delta x\frac{\partial I}{\partial t} = V + \Delta x\frac{\partial V}{\partial x}$$
$$\Rightarrow \frac{\partial V}{\partial x} = -l\frac{\partial I}{\partial t}$$

I am starting from node A, trying to move along node B, flowing in the direction of the current, so the corresponding voltage drop is negative. And eventually, I reach this particular new voltage, where again V dash is the partial derivative with respect to X. Since this is the derivative of V with respect to X, if I multiply it by the section width, I would get the new voltage. So if I rearrange terms here, I have the equation which is resulting over here; V gets canceled, and I have this equation. Similarly, if I apply KCL at node A. I'll get my second equation, which is again obtained from similar lines.

And if you correlate or oversee these two equations, they look more or less analogous to the long-line model in the phasor domain. The difference here is that we now have the variable for x, the distance, as well as the variable for time t. So, let us look at it in a bit more detailed manner. So, if we reorient these equations and rearrange them we get two unique differential equations partial differential equations one in terms of voltage and the other in terms of current at the outset these two equations look very similar to each other And the way we had general form of expression or solution for long line model on similar lines, we can have our general form of expressions or equations for these two partial derivative equations also.

$$\frac{\partial^2 V}{\partial x^2} = -l\frac{\partial^2 I}{\partial t \partial x} \text{ and } \frac{\partial^2 I}{\partial t \partial x} = -c\frac{\partial^2 V}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = lc\frac{\partial^2 V}{\partial t^2} \text{ and } \frac{\partial^2 I}{\partial x^2} = lc\frac{\partial^2 I}{\partial t^2}$$

So the above equations actually pertain to traveling wave equations for voltage and current in a lossless long line.A generic form of solution for a voltage traveling wave is the same for current. Can be written in terms of v of x of t, which is equal to f1 of y, where phi is u t minus x. I still have to define what small u is. Please have patience.

$$V(x,t) = f_1(\phi)$$

$$\phi = u(t) - x$$

I will get back to you and get into those details in the next few slides. So, phi is ut minus x, and the general form of the solution can be in terms of f1(phi). I can have also a separate form where I have f 2 phi where phi is some function of this signal where phi in this f 2 case becomes u t plus x, ok.

$$f_2(\phi)$$

$$\phi = u(t) + x$$

So, I have a general form of solution for this voltage traveling wave, and consequently, if I substitute these expressions into this analogous equation. Let's do that. So what I have is if I choose v of xt as f1 of y and I want to take the first derivative with respect to x, then I would have del f1 phi with respect to x, which is also equal to del f1 phi with respect to phi using chain reaction or chain products of partial derivatives into del phi by del x since del phi.

Since phi is ut minus x, the term which I have defined here, del phi by del x will become equal to minus 1. So, I have minus f1 phi over del phi, okay. Similarly, if I now take the double derivative which is del square v by del x square what I would get I would get the same steps they are shown over here del v by del x is minus del f1 by del phi if I take one more derivative then I would have minus del f 1 phi by del phi with respect to entire del x which can be rewritten as minus del square f 1 phi by del x square into del phi by del x del phi by del x is equal to minus 1 minus minus gets cancelled and that is how I get the second equation. On similar lines, I can take the time derivatives, arrive at the final time derivatives, and now, if I compare these two equations, what do I get? Del square V by del X squared is LC times of del square V by del T squared. That means U squared should be equal to LC, similar to the propagation constant that we had in terms of Z and Y; now we are getting a U constant.

$$\frac{\partial V}{\partial x} = \frac{\partial f_1}{\partial \phi}\frac{\partial \phi}{\partial x} = -\frac{\partial f_1}{\partial \phi}, \quad \frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 f_1}{\partial \phi^2}$$

$$\frac{\partial V}{\partial t} = \frac{\partial f_1}{\partial \phi}\frac{\partial \phi}{\partial t} = u\frac{\partial f_1}{\partial \phi}, \quad \frac{\partial^2 V}{\partial t^2} = u^2\frac{\partial^2 f_1}{\partial \phi^2} \quad \left(\frac{\partial^2 V}{\partial x^2} = lc\frac{\partial^2 V}{\partial t^2}\right)$$

This U is called the electromagnetic wave speed constant, traveling wave constant, and its unit is usually meters per second, where beta is the actual phase constant. Remember beta was nothing but the root over of; it was actually omega root over of LC. So if I divide beta by omega on both sides, I have the square root of LC, which is nothing but U squared.

Sorry, sorry. So this is nothing but equal to U. So that's how I get U as omega over beta. Omega is 2 pi f, the phase angle, angular frequency divided by beta, which is the phase constant, and the unit becomes meters per second. Remember, omega is 2 pi f; the unit of f is hertz. So the actual unit of omega is the inverse second. Beta doesn't have a unit in itself. Its unit is meter inverse or kilometer inverse. So omega by beta is the second inverse defined by meter inverse, which is meter per second. That's how U is referred to as the speed or velocity of some form. It's actually the electromagnetic wave propagation constant.

$$u^2 = \frac{1}{lc} \Rightarrow u = \frac{1}{\sqrt{lc}} = \frac{\omega}{\beta} m/s$$

Similarly, if I take the second wave f2 phi, where phi is ut plus x, that can also serve as a solution for this closed-term solution. The beauty here is that the f1 wave is actually called the forward wave, whereas f2 is called the backward wave.

$$V(x,t) = R_1 e^{j(\omega t+\beta x)} + R_2 e^{j(\omega t-\beta x)}$$

$$\Rightarrow V(x,t) = R_1 e^{j\beta(ut+x)} + R_2 e^{j\beta(ut-x)}$$

- 2nd term on RHS is similar to $f_1(ut-x)$, i.e., forward wave
- 1st term on RHS is similar to $f_2(ut+x)$, i.e., backward wave

For the forward wave, or in f1, if t increases, then... For f1 to remain constant where f1 phi has to remain constant, the corresponding x should also increase so that overall phi remains constant.

Remember, phi was ut minus x. If t has to increase for phi to remain constant, x also has to increase. As time progresses, x increases. That means we are moving away from sending n, reaching towards the receiving n, and that's why we call it. F1 waves are the forward waves; similarly, for F2, as time increases, x has to decrease so that ut plus x remains constant, and hence the F2 value remains constant. Therefore, it is moving away from the receiving end to the sending end, called the backward wave on similar lines.

As per slide 12, which we have seen in the previous few sets, the gamma propagation constant is a pure imaginary number. So if you plug in those values of V of X into the corresponding numbers here, we get e to the power j omega t in terms of time domain form; we get these as our closed form solutions for b in terms of x and t, where r1 and r2 are two analogous constants similar to those obtained from a1 and a2 using boundary conditions since the ut minus x wave corresponds to the forward wave, so the second term is called the forward wave, similar to f1 ut minus x. Whereas the first term for it to remain constant in terms of magnitude, if t increases, x has to go down, so the first term is called the backward wave. So that's the reason why these voltage expressions and current expressions can be considered similar to traveling waves in transmission lines. Since voltage and current in a lossless line can be interpreted as waves, if a wave can exist, then the notion of wavelength can arise, the notion of velocity can arise, and so on.

In a typical layman's perspective, wavelength is nothing but the distance traversed by a sine wave between two adjacent zero crossings or points. which is separated by a phase difference of 2 pi radians. Since we know what the phase constant beta is, we know what the angular frequency is, we know what the velocity of the wave is, which is omega by beta, and we can also define its corresponding wavelength, which is 2 pi by beta; the 2 pi comes in because we need a zero crossing difference of 2 pi radians. Beta is a phase constant, so we get lambda as 2 pi by beta.

Which is nothing but 1 over F times the square root of L over L times C. Similarly, U is called the velocity of electromagnetic wave propagation.

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{lc}}$$

$$u = \frac{1}{\sqrt{lc}}$$

If we were to evaluate this for an actual transmission line, this value of U would appear to be very close to the speed of light in a vacuum, and lambda would be on the order of a few thousand kilometers. That means, If there is a transmission line and it is energized from the sending end it would hardly take any time to reach that energy or power from sending end to receiving end because as we know the waves are traveling close to the speed of light. So that's the reason why when we turn on or off a particular switch, the corresponding appliance gets turned on or off immediately, which is hardly perceivable to the human eye.

The delay involved is not present. So the energy or power, it flows through transmission lines at the speed of light, through the speed of light. On the other hand, we also say that current can exist in a transmission line because electrons are flowing. So that means if power or energy has to be transferred at the speed of light, does it mean that the electrons are also flowing at the speed of light? Definitely not. Einstein's theory of relativity would indicate that if a matter which is an electron is also a material matter, if it is moving at the speed of light, then the electron will no longer remain as an electron; it would become energy. So, actual electrons don't flow at the speed of light; they don't travel at the speed of light.

What they actually do is dissipate or they... sort of vibrate around their mean position in the transmission and that's how the waves are created that's how the electric field and magnetic fields are created and actual speed of electrons which is also known as the drift velocity it's hardly few meters per second whereas u is in order of 3 to the power 8 meter per second which is close to the speed of light. So actual electrons don't move; they only dissipate at the rate of drift velocity, which results in the creation of electric fields and magnetic fields, which lead to resulting inductance and capacitance. And thanks to this inductance and capacitance, we get our waves traveling at close to the speed of light. And that's why human eye cannot perceive any delay in transfer of energy or power from sending end to receiving end. Likewise, we can have our attenuation constant, which we have already seen to be 0 for a Rosser line.

We can have our beta in terms of the square root of Lc to pi f. We'll conclude this discussion with an example where I have considered a long line with defined series impedance. Total line length is not provided. The total capacitance or shunt admittance is also provided.

And the line is delivering 40 megawatts of power at 220 kV on the receiving end. at 0.9 power factor lagging we have to find the sending in voltage current we have to find the sending current for 220 kV when there is no load at the receiving end so we know what is Z and Y capital Z and capital Y these are not the lumped parameters these are the actual distributed parameters multiplied by the line length from there also we can find our corresponding characteristic impedance where we would have capital L as LL the line length which is not mentioned here from there we can find ZC L is a total line length that is not given, and it is also not required because, essentially, for hyperbolic functions, we need gamma L, not just gamma. So we can directly find gamma L by taking the square root of capital Z and capital Y, where capital Z and Y are the total fictitious. Series impedance and admittance in the distributed form. So if you find gamma L, Z, and Y, which are complex numbers, then gamma L is also a complex number. This is the rectangular form of representation, whereas this is the polar form of representation.

Please cross-check these numbers, okay? And we can also find the corresponding exponential form of gamma L. Those of you who have calculators which can directly find the cos and hyperbolic functions, they need not do this step. But for others where hyperbolic functionality is not available, you have to evaluate e to the power of gamma L separately, e to the power of minus gamma L separately, and remember that in gamma L we have a real part and an imaginary part. So e to the power is actually 1. 0458; the term you are seeing here is nothing but e to the power 0.0448. Whereas 20.8327 degrees is nothing but 0.3636 radians converted into degrees by multiplying by 180 and dividing by pi.

Please cross-check that. So, you have to carefully evaluate this. This is again in polar form. We can rewrite it in rectangular form. We can also take the inverse of the first term and get it by minus gamma L. Take the means of the gamma l and the minus gamma l terms you get cos hyperbolic function the corresponding sin hyperbolic function you have to be careful in evaluating that it depends on what setting you have chosen in your calculator whether it is in radian mode or degree mode this number would drastically change you have to be very careful. Once you know the corresponding hyperbolic functions, characteristic impedance, find ABCD parameters, receiving end voltage we choose as a reference, so voltage is known, power is given, power factor is given, so we can find the corresponding complex power, then we can find the corresponding receiving end current, plug in those ABCD with VR, IR, we get the sending end voltage, the corresponding sending end current, and The no-load sending and current also for IR equal to let me cross-check. So, our second bit of the question was to find the sending and receiving current for no load, and when there is no load, and the receiving current IR becomes 0, that means there is no load, so the voltage is still 220 by root 3 at the receiving end.

Here, we have C as this term multiplied by VR, and D remains the same. The receiving end current I is 0, so IS is this particular term. Now that indicates that even if there is no load at the receiving end, the sending end current is non-zero and it is almost leading the receiving end voltage by 90 degrees, which indicates that the line is consuming reactive power or playing a lot with reactive power, which is the reason why the line is still demanding sending end current although there is no load at the receiving end. The next lecture, we will take up the discussion based on today's discussion for surge impedance loading, which is related to the characteristic impedance, and also understand how complex powerful expressions can be evaluated in lossless, long line, long line, or lossless or lossy long line. And these complex, powerful expressions that we would find or evaluate also have a lot of similarity with the capability curves of synchronous generators.

Thank you.