Power Network Analysis

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Lecture-27

Hello everyone, welcome to lecture two of week six of the course Power Network Analysis, in which we will continue the discussion on the fifth main module of this course, which is transmission line models and the use of these models for understanding the performance of these transmission lines under different operating conditions. In this specific discussion or lecture, we will take up the topic of medium line model representation for transmission lines. In the previous lecture, we had seen the discussion on the short line model in which we understood that under different operating conditions, specifically for no-load conditions where the receiving end current is 0 . For the short line model, the Z parameters become undefined because it is as if the short line does not exist under no-load conditions. The sending end voltage is the same as the receiving end voltage, and the corresponding sending and receiving end currents on a per-phase basis are zero. It appears as if the short line has been perfectly shorted at the sending and receiving ends, and hence there is no role of any impedance at all, and that's the reason why Z parameters are undefined for the short line model, not only for no load conditions but for all operating conditions.

Whereas, in terms of other parameters for a two-port network, such as Y parameters, A, B, C, D parameters, or H parameters, they are well-defined for the short line model under different operating conditions. That is one aspect. The other aspect is that we also talked about percentage voltage regulation for the short line model. In fact, percentage voltage regulation is a very general concept, and it is not only applicable to transmission lines but also to any other electrical equipment.

In fact, for the upcoming models that we will discuss, medium line or long line, the same percentage voltage regulation notion would be true, which in a way is like if the percentage voltage regulation of a line has to be evaluated; this is nothing but proportional to the receiving end voltage at no load condition, the magnitude minus the receiving end voltage under full load condition with respect to some receiving end voltage under full load or no load condition. So that denominator is not so important, but what is important is that percentage voltage regulation is a measure of how much the voltage drop or difference in voltage magnitude compares to the line being loaded from no load to full load. And we saw for the short line model that specifically under the case where the short line model is feeding in a leading power factor load, where the leading power factor load means that it is mostly a capacitive load capable of delivering reactive power. The percentage voltage regulation under such conditions can become negative, which would mean that the receiving end voltage at full load or at full leading power factor load is more than the receiving end voltage under no load conditions, which in a way is nothing but also known as the Ferranti effect. We still have not seen the Ferranti effect; we will talk about it in the next few modules to come.

I probably made a mistake in the previous lecture by stating that the short line model never encounters the Ferranti effect. It was not entirely incorrect, but it was not fully correct either. The correct aspect that is true for the short line model is that the short line model can also experience the Ferranti effect, and it is probably due to the nature of the load being solved by the short line model. The short line model itself does not consider any capacitive modeling. So, the capacitance aspect is neglected or not considered in the short line, especially since the line capacitance is not considered.

So there is no presence of charging current, and even if the line is energized, the line won't have any extra charging current obtained from line capacitance, which is being neglected because line capacitance is absent. My idea or intent was that the Ferranti effect in the short line model does not occur because this line charging is not being considered, but it can still happen depending on the type of load that is connected, specifically for leading power factor loads or capacitive loads. For lagging power factor loads, the percentage voltage regulation will still generally remain positive. With that, let us now see what our medium line model is. So, as we all understand by now, as the line length increases, the total line capacitance, although still smaller than the total line resistance and inductance, its effect probably cannot be considered negligible; the presence of total line capacitance becomes appreciable, and there comes a point in line length or voltage level where this capacitance is no longer negligible.

And as a result, as I mentioned in the previous slide in the context of the short line model, the associated line charging current or capacitive current, which we discussed in the previous module while line capacitance was being discussed, becomes high and appreciable, which can also affect the nature of reactive power flow in the transmission line, specifically in the medium line model. So, as the transmission line is affected by its reactive power flow pattern, the associated receiving end voltage can also be significantly different compared to the sending end voltage, which is the source voltage, and that is the reason why the medium line model comes into the picture, which is generally applicable for lines of length between 80 and 250 kilometers and operating voltage line to line being between 69 kV and 400 kV , as indicated over here. We have also seen in the previous lectures what the medium line model is. So basically, the story or the concept of the nutshell is that line capacitance is not negligible for the medium line model, and those medium line models qualify under these conditions. There's a difference in how line capacitance, if at all it has to be considered, is no longer negligible.

If it has to be considered, then its representation is going to be a little different because generally, line capacitance is treated as a shunt element, while line resistance and inductance appear as series elements in the short line model. So, in the medium line model, we will still have the total resistance of the line, the total inductance of the line, which were also present in the short line, and in addition to this, we will also have the total line capacitance, which would be considered as part of the medium line model. And this capacitance cannot serve as a series element. So it has to be there as a shunt element in addition to the existing short line model parameters. And that gives rise to two typical models, two common models, I would say.

One is called the pie model. In which half of the line shunt, which is basically the line capacitance, half of it appears as a shunt element between the phase and the neutral of the ground on line ends, and that is what results in the pi model. The other model is the T model, where the entire line capacitance as a shunt appears at the midpoint of the line, whereas on either half of the line, half of the series impedance, which is the total resistance and the corresponding reactance due to inductance, appears as half on either end of the shunt element, which is the line capacitance. So we will talk at length about these pie models and T models. Usually in the literature, the pi model is quite common because it provides certain conveniences in terms of analysis, which we will see eventually in the next few lectures or slides to come.

The difference, however, is in terms of capacitance being considered, but the similarity between the short-line and medium-line models is that line parameters can still be assumed to be lumped, which are actually distributed. And as a result, the overall analysis with lumped representation and capacitance being considered doesn't lead to very significant differences in terms of actual analysis or perfect analysis, which will eventually be part of the long-line model, which we will discuss in the next lecture. And the overall assumptions that are involved in considering the medium-line model. So, the differences here in terms of accuracy are not very different provided capacitance is considered in the medium line model and parameters still remain lumped and not distributed. So to start with our first model, the first common model for medium line, we call it the pie model.

It looks something like this, as shown here in this figure. And we have the same notion of sending end, with V_S and I_S being the sending end voltage and sending end current on a per-phase basis. On the receiving end, we have V_R and I_R, which are the per-phase receiving end current and receiving end voltage. The neutral point here, or the reference here, is with respect to neutral or ground through which the potential is being measured, whereas the top wire is actually the per-phase wire or per-phase line of the transmission line. And as I mentioned earlier in the previous slide, in the pie model, half of the line capacitance or shunt appears at either end of the line, whereas the total line series impedance, which is $R + jX$, includes this X coming in because of line inductance, while these Y s come in because of line capacitance.

So $R + jX$, the entire lumped representation appears at the midpoint of the line. From an overview perspective, if we were to see how Z and R plus jX look, they represent something very similar to the pie symbol, which is present over here. And that's the reason why this model is also called the nominal pie model. So I have hopefully given the explanation of why it is called the pie model, because the architecture, structure, or circuit looks like a pie model or a pie circuit between the sending and receiving ends. Often the term nominal is not used, but the nominal pie model is the most common terminology for defining these pie models.

Often, the term "nominal" is not very clearly explained in regular textbooks. The reason why the term nominal comes in is that this nominal indicates that the model we are considering for the medium line is still not an exact representation of the actual line. The actual line still has distributed parameters. The line capacitances are not present at the line ends. They are still being distributed.

So, in a way, the nominal word is referring to a notion that the model we are considering is capturing only the essential features of the line. It is still not an exact representation. So, to segregate this feature, the term "nominal" is important in that context. And as I mentioned, the shunt admittance, which is the $Y/2, Y/2$ terms, is coming in because of line capacitance, and possibly if conductance modeling were to be perfect, then half of line shunt admittance is capable of monitoring or representing line capacitance and the conductance. So Z is equal to, as I mentioned, $r + j\omega l$, where small r and small l are the per unit line resistance and inductance on a per meter or per kilometer basis.

$$Z = r + j\omega l$$

ω is the source frequency, which is $2\pi f$. f can be 50 hertz or 60 hertz. Capital L is the line length. And similarly, Y, which is the admittance, accounts for line conductance and line capacitance. g and c, small g and small c, still represent per unit length line conductance and capacitance; ω remains $2\pi f$, and overall Y can become capital $G + j\omega c$, where j is the complex operator, which is the square root of minus 1 .

$$Y = G + j\omega C$$

So, if conductance is not to be considered, then we would consider this factor small g to be 0. And in that context, capital G would also be 0. Y would only account for overall line capacitance.

If we have to figure out certain mathematical equations that define this pi model, then we basically have two nodes. Let us say we talk about node S , which is the sending end, and we talk about node R, which is the receiving end.

At node S , by KCL , we observe that the incoming current is I_S, and here we have the outgoing current as I_L. Since there is a capacitance assumed to be present, which is a lumped representation along the source voltage V_S, there would also exist a current equal to $V_S Y/2$. So, if we apply KCL, we have our first equation, which states that the incoming current is equal to the outgoing current, which is I_L plus $V_S Y/2$.

$$I_S = I_L + \frac{V_S Y}{2}$$

Similarly, on the receiving end, if we apply KCL , then we have I_L as the input current to this node, I_R as the receiving end output current, and across this shunt branch, we again have a current because of shunt admittance, which is $V_R Y/2$. So basically, if we relate I_L, I_R, and V_R by $Y/2$ in terms of KCL, we get our second equation.

$$I_L = I_R + \frac{V_R Y}{2}$$

Also, along S and R nodes, which are sending and receiving nodes, we can apply or write KVL, and by KVL, we would get V_S equal to V_R plus I_L into Z. So that's what this relationship is, again mentioned here.

$$V_S = V_R + I_L Z$$

So this is our third defining equation. We have all our equations in place. V_S, V_R, I_S, I_R, and I_L are all per-phase quantities for a per-phase equivalent representation.

And as I mentioned earlier, the reference point or potential measurement point is being measured with respect to neutral or ground. So, we can always rewrite these equations in terms of well-known parameters for two-port networks; for transmission lines, the ABCD parameters are common. So, if we rewrite or rearrange them. So, what is done here is that we have replaced I_L in terms of I_R and Y with $2V_R$. So, we get our first ABCD equation where, remember, in terms of ABCD parameters, V_S is equal to $AV_R + BI_R$, and the second ABCD parameter equation is I_S is equal to $CV_R + DI_R$.

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

So, what we are trying to do is re-represent these three equations in terms of these ABCD parameter equations so that we can understand our ABCD parameters. So, essentially, if you make a one-toone co-relationship here, then basically this term is nothing but capital

A, and this term is nothing but capital B. Similarly, if we substitute I_L equal to $I_R + V_R Y/2$ in the first expression over here and present V_S also again in terms of V_R and I_R in this term, we would get I_S as this larger term which, again, when simplified, results in our second equation of ABCD parameters. So, essentially, if we compare this equation with the equation given over here. We have C as one term, whereas we have D as another term.

And that's how the ABCD parameters can be evaluated. Coincidentally, A and D are equal to $1 + ZY/2$, whereas B and C are unique numbers.

$$A = D = 1 + \frac{ZY}{2}$$
$$B = Z$$
$$C = Y\left(1 + \frac{ZY}{4}\right)$$

And as per the condition of symmetry, A should be equal to D. It is not equal to 1 . There's a mistake here. It is only A equal to D, which is equal to $1 + ZY/2$, which is obvious from the medium line pi model, and if we evaluate $AD - BC$, let us do that: A is $1 + ZY/2$, which is also equal to D, so basically we have a square here. Minus B is equal to Z times C plus $Y(1 + ZY/4)$. So, if we expand the first term, we have $1 + Z^2Y^2/4 + ZY$. Then we have minus ZY opening up the bracket inside the term.

$$AD - BC = \left(1 + \frac{ZY}{2}\right)^2 - Z \cdot Y\left(1 + \frac{ZY}{4}\right)$$

Again, minus $Z^2Y^2/4$. So our terms get cancelled, and essentially we are only left with 1 .

$$AD - BC = 1 + ZY + \frac{Z^2Y^2}{4} - ZY - \frac{Z^2Y^2}{4} = 1$$

Which is essentially what is also mentioned or which is also the essential condition for reciprocity.

So basically, our nominal pi model for the medium line is also a linear time-invariant passive symmetric reciprocal network. That is essentially what I am trying to explain. Please note that the ABCD constants we are evaluating are because Z and Y are complex numbers; remember Z is equal to capital $R + jX$, and Y is equal to capital $G + j\omega C$. Z and Y themselves are complex numbers, and A, B, C, D are functions of Z and Y. So, it is obvious that the parameters A, B, C, D would be complex numbers and represent appropriate sensitivities. Now, what are these sensitivities? These sensitivities are essentially if I write the equations of A, B, C, D here: $V_S = AV_R + BI_R$ and $I_S = CV_R + DI_R$. So, under the condition when the receiving end current is 0 , it means the line is at no load.

The value of A is equal to V_S divided by V_R, which is as per the definition of A under the condition that I_R is equal to 0 .

$$A=\frac{V_S}{V_R} \text{ when } I_R=0$$

And similarly, the value of C is equal to I_S divided by V_R, again under the condition that I_R is equal to 0 .

$$C=\frac{I_S}{V_R} \text{ when } I_R=0$$

So, C and A, if we see, they are essentially sensitivities between the sending-end voltage and current with respect to the receiving-end voltage. And that is the reason or notion for defining sensitivities. A similar notion is also true for constants B and D when the receiving end part of the median model is shorted, where V_R becomes 0 .

$$B=\frac{V_S}{I_R} \quad \text{when } V_R=0$$
$$D=\frac{I_S}{I_R} \quad \text{when } V_R=0$$

And just to highlight that the ABCD constants which we evaluated for the short line are also, in general, complex numbers because, in the short line model, the series line impedance is still a function of line resistance and line inductance. So in terms of re-representing the receiving end quantities in terms of sending end quantities, we can take the inverse of the ABCD matrix, and coincidentally, this ABCD matrix does have an inverse because it represents a condition of reciprocity. Basically, $AD-BC$ also happens to be the determinant of this ABCD matrix. So if $AD-BC$ is equal to 1 , then the $AD-BC$ term is not equal to 0 . The ABCD matrix is factorizable, and if we do that, we get this as our inverse term.

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix}=\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}\begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

So, from there, we can also try to find the receiving end quantities as functions of the sending end quantities. So, if we know the sending-end quantities, we can put them in this equation and get our receiving-end quantities. Once we know the ABCD parameters, we can also make use of those ABCD parameters and convert this model into a relevant Z -parameter-based two-port network, Y -parameter-based network, or H-parameter-based network. So, in terms of Z parameters, this is the basic equation for defining the Z parameter-based two-port network, which in terms of A, B, C, and D can be defined or obtained as per these conditions. Please refer to our previous slide or the previous lecture discussion where these interchanges or exchanges were defined.

Similarly, for Y parameters, the conditions of symmetry and reciprocity are again defined, and in terms of ABCD , we can again find these numbers for Y parameters. The same is also true for H parameters; as long as we understand what H parameter representation is, we can correlate or re-represent our ABCD parameters in terms of H parameters, which are shown here, where the conditions of reciprocity and symmetry are still valid. So, moving on to our nominal T model, we have talked about the nominal pi model. In terms of the nominal T model, the term nominal still refers to the fact that the model we are considering is not an exact representation; it is only capturing the essential features of the transmission line. And in the T model, as I mentioned, the entire line capacitance is present at the midpoint of the line, where half of the series impedance Z is present on either half of the lines, where capital Z is again $r + jx$, which is what we have defined over here.

This W is actually ω, which is equal to $2\pi f$, not the small w. And if we try to understand the mathematical equations that define this model, then at the midpoint of the line we apply KCL. So, let us say if we take this node, then the input current is I_S, the output current is I_R, and we are assuming a fictitious voltage V. So, the current across this shunt element is V into Y. We apply KCL, we get this equation, and then we also have three nodes which we can talk about: one is the sending end, one is the receiving end, and the midpoint.

$$I_S = I_R + VY$$

So, if I apply KVL across the sending end and the midpoint of the voltage, we get V_S is equal to $V + I_S Z/2$, and for KVL across node V to the receiving end R, we have V is equal to $V_R + I_R Z/2$.

$$V_S = V + \frac{I_S Z}{2}$$
$$V = V_R + \frac{I_R Z}{2}$$

So, again we have three equations in a way similar to the three equations we got for the nominal pi model. Using these equations, we can again rewrite them. One straightforward way is to substitute V from V_R and I_R; over here, we would get our second abcd equation, which is I_S equal to $CV_R + DI_R$. So, if we combine these two equations, we can get the C and D parameters for the nominal T model, and likewise, if you substitute V here and I_S from here, we can get our first equations.

If we do that term-by-term correlation of our ABCD parameters again, they can be defined in this term; again, there is a mistake: it is not equal to 1 ; it is actually A is equal to D is equal to $1 + ZY/2$.

$$A = D = 1 + \frac{ZY}{2}$$
$$B = Z\left(1 + \frac{ZY}{4}\right)$$
$$C = Y$$

Where the condition of symmetry is still true, and the B and C terms are exchanging their values in terms of the nominal pie model. So the roles of B and C have now changed. Earlier Y for nominal pie model B was simply equal to C, whereas C was equal to Y times $(1 + ZY/2)$ plus $(Z + 1 + ZY/4)$. Now here are the values of B and C as they are interchanging with respect to the nominal pie model compared to the nominal T model, and again, $AD - BC$ is equal to 1 , which indicates that the condition of reciprocity is still true.

Similar to representing Z parameters, Y parameters, and H parameters for the nominal T model, we can again use the same interchange relationships between two different two-port network parameters and obtain other values in terms of Z, Y, and H parameters. So with that, let's understand which one of these models, whether the nominal T model or the pie model, is convenient. Usually, the nominal pie model is very convenient for the upcoming discussion, specifically for power flow or fault analysis, due to easy calculation and easy interpretation of the network. And even if we were to argue that, oh, let's choose the nominal T model or the nominal pie model, the differences in terms of actual analysis will be marginally different, not very different. So from the operator's perspective, from the network owner's perspective, and from the engineering perspective, the nominal pie model is generally considered.

And once we do all our analyses, we can again get back to our complex analysis equations where we can find out the corresponding per-phase sending and complex power, receiving and complex power. The choice of using the pie model or the T model is to be used. It depends on what the operating voltage is, the line length, and what trade-off we are looking at in terms of accuracy. So I'll conclude today's discussion with two examples. The first example we have is a medium line model or a medium line that is 130 kilometers long; the values of small r, small l, and small c on a per kilometer basis are defined.

The receiving end load is a lagging load of 270 MVA with a 0.8 power factor. The receiving end voltage is mentioned at 325 kV line to line, and we have to find the corresponding sending end quantities and the voltage regulation using the nominal pie medium line model. So, the first objective is to find what our capital Z is and what the capital Y is; Z is the series impedance, which is the value over here, and capital Y is the effective line capacitance, which is also evaluated in terms of ω inverse. We have understood what the nominal pie model ABCD parameters are.

So, we can plug in those values of Z and Y as per the requirement and get all these numbers in terms of ABCD parameters. Once we know the ABCD parameters, it's time to find out what the receiving end quantities are. We are choosing the receiving end voltage as the reference for all phasors. So from there, we can find the corresponding receiving-end current. So I_R is the modulus or magnitude of I_R since the power factor is 0.8 lagging. The receiving end current is lagging the receiving end voltage by 36.87 degrees. In a three-phase power system of 270 MVA, the per-phase power is 90 MVA. From the 90 MVA fact, we can find the corresponding current, which is this per-phase current here. Once we know our V_R and I_R, we also know our ABCD parameters, so we can plug in those numbers to get our corresponding sending end voltage, which is this number.

We can also find our three-phase line-to-line voltage and the corresponding sending end current and sending end complex power. In terms of power factor, if we were to see, the receiving end load actually has a power factor of 0.8 lagging, and if we try to observe the power factor at the sending end, it is not exactly the same as 0.8 . It is different from 0.8 . In fact, it is better than the power factor at the receiving end. It tends to be closer to 1 . And the reason for this power factor improvement is that the line capacitance is now coming into the picture. So the majority of reactive power is not being fed from the sending end. The line capacitance itself is capable of feeding this lagging power factor load.

So the sending end power factor is improving. There is a lesser requirement for reactive power from the load perspective, which is to be fed from the sending end source. The line can take care of the reactive power hunger or need for the load, and that's what is leading to an improvement in the power factor at the sending end. In terms of voltage regulation, the usual expressions can be used, and we see that the voltage regulation is close to 6.155%. The same example, if we were to solve using the nominal T model, the values of Z and Y, capital Z and Y, remain the same. The ABCD parameters might be different; specifically, the B and C parameters are different. And we do a similar exercise; we find the receiving end voltage, we choose that as a phasor, we find the receiving end current, and the quantums or the values still here remain the same. Now, with the new ABCD parameters, if we find V_S, the value is a little different; earlier, it was equal to 199.1889 volts, and the sending end apparent power was 345 kV . Sorry, this is the difference: 345 kV was the line-to-line voltage, and the sending end power was somewhere around $218.815 + \mathrm{j}124$ MVA. The same number, if we see here, shows that the differences are not very significant; earlier, we had 345 kV as the value. The sending and voltages are still more or less the same. The sending and currents have changed. And the power factor has also improved because the differences have now increased in terms of the nominal T model.

The voltage regulation is also more or less the same. Earlier, it was 6.15%. The differences in the notions here in terms of red colors are not very different from the nominal power model. That's the bottom-line story. So in the next lecture, we will start with our discussion on the long line model, where we will understand how actual transmission lines can be modeled and see the comparison between short lines, medium lines, and long lines in the upcoming discussions. Thank you.