Power Network Analysis

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Lecture-26

Hello everyone, welcome to the first lecture of week 6 of the course Power Network Analysis, in which we will continue with our fifth main module, which is transmission line models and the use of these models to understand the steady-state performance of these transmission lines under different operating conditions. And in this discussion today, we will specifically understand what this short line model is and how it can be useful for understanding the behavior or performance of the transmission lines under different loading conditions. In the previous lecture, we talked at length about the properties of two-port networks, the property of symmetry, reciprocity, what the Z parameters, Y, A, B, C, D, and H parameters are, and what relationships hold true for these Z , Y, A, B, C, D, and H parameters for a symmetric reciprocal two-port network. So, we will extend this discussion of two-port networks, or essentially we will make use of this discussion of two-port networks to understand what this short line model is and how we can evaluate the corresponding equivalent per-phase short line model of a three-phase transmission line. So if you recollect, the models that are typically common in transmission line network literature are three: short line, medium line, and long line.

And as discussed in the previous lecture, short, medium, and long do not always refer to the categorization of models in terms of line length. They also depend on what type of operating voltage the line is using. So, in today's discussion, we would specifically focus on the short line model where our focus would be on line lengths of not more than 80 kilometers and an operating voltage of the line being less than 69 kilovolts in a line-to-line voltage sense. So, here is just a summary of the two-port network.

Two-port networks usually have the attribute of being linear and lumped, essentially because the two-port network often represents a power network or transmission line network perspective; they often represent passive networks, so linear and lumped representations are true. Networks are often symmetric in 2-point networks, and in terms of ABCD parameters, it mathematically correlates to the values of A and D being the same. Networks are also often reciprocal, and for the reciprocal condition in terms of ABCD , AD minus BC should be equal to 1 , which we had seen in the previous lecture. And for a fixed set of inputs, the network or parameter behavior doesn't change because the network is essentially a time-invariant network with passive properties or passive representations. So for the short line model, we have often seen in our transmission line parameter evaluation that the line capacitance per unit length is often small compared to the per unit length line resistance and inductance.

So, for short lines, the line capacitance can be considered negligible, and it is often ignored for the sake of simplicity in order to have a better trade-off in terms of complexity and the associated accuracy of the line model that we are going to understand in terms of the transmission short line model for transmission lines. Line parameters are also considered lumped. In essence, they're actually distributed, but for the short line model, these network parameters can be considered to be lumped. So, essentially, two-port networks can be used very efficiently. The reason for all these assumptions or conditions is that for short line models, these aspects do not significantly affect the overall accuracy of the analysis, with results being more or less similar to the actual numbers we would have obtained, assuming the line parameters to be distributed and capacitance being considered for a short line model.

So essentially, if line capacitance is out of the window and is ignored, the short line model is nothing but a series lumped RL circuit on a per-phase or a single-phase equivalent basis, which looks something like the figure shown here. We have our sending end marked over here with V_S and I_S, which are the per-phase sending end voltage and sending end current. V_S and V_R are being measured with respect to a neutral or ground, so that is where the negative terminal essentially makes sense. The negative terminal actually doesn't make sense in an AC network because the polarity in terms of time keeps changing, but we are choosing some reference for measuring these potentials or voltages. So, that is where this negative sign, neutral, or ground point comes in.

So, this is basically the ground or the neutral point through which, with respect to which, V_S and V_R are being measured. The line itself does not have any capacitance, or if it does, it has very negligible capacitance. So, what is important is the overall resistance and the corresponding reactance that is coming in because of the line inductance small l. And on the receiving end, again, we have V_R, I_R, the receiving end voltage, and receiving end current, which is essentially feeding in some sort of load whose apparent power is indicated or complex power is represented in terms of S_R. You would often find small r and capital R being used interchangeably in the context of receiving end quantities.

So please don't get confused in that regard. So in terms of S_R and V_R, where S_R is the complex power at the receiving end, and V_R is the receiving end voltage. If we have some load present on the receiving end, then we can evaluate the per-phase equivalent receiving end current, which is S_R conjugate by V_R conjugate; recollect that complex power is nothing but the product of the voltage phasor and the conjugate of the current phasor.

$$S = VI^*$$

So, if we rewrite "I conjugate" as S by V and take the conjugate of both sides. So, I conjugate; conjugate becomes only I , and S becomes S ; conjugate V becomes V ; conjugate that is how this relationship is defined.

And under no-load conditions, that means there is no load connected to the receiving end, which would mean that the receiving end current itself is zero. So if the current is zero, it would mean that the receiving end voltage does not have any complex power at all. So it also means that S_R is zero. The voltage, however, need not be zero because the line is charged and is receiving some input from the source end. So potential need not be zero for the no-load condition of the receiving end.

Similarly, under short circuit conditions, even if the line is charged at the sending end, a short circuit at the receiving end means that this end is completely shorted from end to end, which means that V_R would be zero. And if V_R is again zero in this product context, S would again become zero, so S_R is again zero. Remember, I_R or receiving end current need not be zero, depending on what sort of energization has come from the sending end. So under both no-load and short-circuit conditions, the complex power at the receiving end is going to be zero; the corresponding current and voltages are respectively zero for no-load or short-circuit conditions. If we apply KCL at the midpoint of this line, it appears that at this node A , or midpoint, the input current is I_S.

The output current is I_R. There is no other path for any current to leak out from anywhere. And that's the reason why sending in current is the same as receiving in current in the short line model. I_R and I_S, the receiving current and sending current, are the same because line capacitance is neglected. Line capacitance is not present.

$$I_S = I_R$$

If line capacitance had been present, unlike R and X, line capacitance would have been represented as a shunt element in a parallel mode circuit, I_S would not have been equal to I_R because some current would also flow through the shunt capacitances if they were present. Since the capacitance effect is neglected in the short line model, the sending and receiving currents are the same. If we apply KVL across this line impedance, which is the total impedance Z, then V_S is equal to V_R plus I_S times Z, where I_S is the same as I_R.

$$V_S = V_R + I_S Z$$

And with these equations, overall equations for our short line model, can we not represent them in terms of Z parameters, Y parameters, ABCD parameters, and such parameters? Yes.

If I were to replace this entire line impedance with a black box, I would call it a two-port network where I don't know exactly what is inside this two-port network; the terminals very well represent the sending terminal and the receiving terminal. So, if this is the overall equation for a short line model, then I can also represent or re-represent it in terms of a two-port network using z, y, a, b, c, d , and h parameters. Now that's where something interesting happens. The interesting part is that if I were to use a two-port network in terms of impedance parameters or Z parameters, then there might be certain issues that will come in the next few slides. Since ABCD parameters are common parameters to represent transmission line models in terms of ABCD parameters, if we correlate or recollect our ABCD parameters, these are the two equations that define the ABCD parameters.

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

If we compare equation by equation, that means this equation with the first equation and the equation sitting over here with this equation, then it's pretty obvious that a is equal to $1, b$ is equal to z, c is equal to 0 because there is no term of V_R present over here, so c is 0 and d is equal to 1 .

$$A = 1, B = Z, C = 0, D = 1$$

The conditions of symmetricity and reciprocity are eventually also true because A is equal to D and AD minus BC is equal to 1 .

$$AD - BC = (1)(1) - (Z)(0) = 1$$

So, the short line model has conditions of symmetry and reciprocity. It is linear, has a lumped representation, and its performance is going to be time-invariant because the network properties essentially consist of all passive elements, so they are not going to change if the inputs remain fixed. By input, I mean that if we change our source frequency from 50 Hz to 60 Hz , definitely the value of X is going to change, so in that condition, a new set of parameters will be defined for this two-port network, and hence time invariance would come in as long as the inputs remain fixed R and X; they are not going to change, and that's how the short line model for a given source frequency is going to have a time-invariant response.

Regarding the aspect that I was talking about in terms of Z parameters, what we see here is that if there is no load at the receiving end, it means I_R is 0 . That would also mean that

I_S is 0 , and having no load at the receiving end doesn't mean that the sending end and receiving end voltages are 0 . The source might still be present on the sending end. There need not be a load present at the receiving end. So, V_S, as per the equation that we saw here, V_S is equal to V_R plus I_S times Z. I_S is equal to 0 for no-load condition. So, V_S is equal to V_R, which need not be equal to 0 depending on the type of source that is present. And if we recollect that when we try to evaluate Z parameters, which are impedance parameters, the impedance parameters are the ratio of some voltage with respect to some current. And Z_{12} is specifically the ratio of receiving end voltage to sending end current for no-load conditions at the receiving end. That means under no load conditions for a short line, if I_R is zero, it also becomes zero; that means V_R by I_S an undefined quantity.

If V_R by I_S undefined, it would also mean that Z_{21} will also be undefined, which gives a clue that probably the short line model. If we try to represent it as a two-port network in terms of Z parameters, let's say I have Z parameters here for this two-port network, then for the short line model, this representation is not valid because, as I mentioned, the two-port networks are timeinvariant networks as long as the source frequency doesn't change, which is not changing here at all because we are only playing around with the load, whether there is a load present or not. Having a load or no load doesn't change the source. So, if the source remains the same, Z_{21} should always be uniquely defined. It is becoming undefined under no load conditions, which means there is some problem in the network.

So the reason why the problem arises is, in fact, that for Z , the parameters for the short line model are absolutely undefined. Because if we recollect or see our ABCD parameters where C is 0 and try to evaluate Z parameters in terms of ABCD parameters based on the discussion we had in the previous lecture, then it would appear that Z_{11}, Z_{12}, Z_{21}, and Z_{22} would all have the inverse of C , where C itself is zero for the short line model. That means none of the Z parameters, not just Z_{12}, but none of the Z parameters are defined for a short line model for a two-port network representation. What is wrong with that? Why is it coming up? The answer is very simple. When we try to talk about evaluating impedances in terms of Thevenin's theorem, it essentially means we are trying to do some open circuit or short circuit arrangement; it's mostly an open circuit arrangement where we are trying to find a ratio of voltage with respect to current.

So essentially, we are trying to find the effective Thevenin impedance from the sending perspective or the receiving perspective while opening out either end. If the impedances themselves are undefined, that means the Thevenin impedance is also undefined; it cannot be evaluated for a short line under no-load conditions. Why is that happening? So the Thevenin impedance is valid. In fact, these ratios are all equal to Thevenin impedance or effective Thevenin impedance. What we see here is why these impedances are not defined.

In case the short line model doesn't have any load, it means that essentially this representation of Z hasn't got any role. It is as if this entire Z, even if it is to be removed or cut off, would still have an analogous condition where we have V_S on one end and V_R on the other end. And here is my ground or terminal where the reference is being measured, whether there is impedance between V_S and V_R or whether there is no impedance between V_S and V_R; under no load conditions, the current in the circuit is always zero. If the current itself is always zero, what is the need for having a short line model at all? What I mentioned is that if I cut off this entire section and only connect these two terminals, V_S and V_R, the way it is shown here by a short circuit path, the necessity of the short line model is completely gone. There is no need to have a short line model, and that is probably the reason why the short line model cannot be represented in terms of impedance parameters, because impedance parameters are not important or valuable for the short line model under no-load conditions, and that is the reason why the Z parameters for the short line model cannot be defined or evaluated.

The other parameters, however, meaning Y and H parameters, can very well be obtained because in terms of Y parameters, admittances, we often do short circuit studies to find the corresponding admittances, as these are ratios of current with respect to voltages. For a short circuit at the receiving end, the currents need not be zero. The receiving end voltage, however, would be zero, and V_S would be equal to I_S multiplied by Z. Effective line admittance can very well be available in certain conditions. So Y parameters are very well defined in terms of $\mathrm{A, B, C, D}$, and the conditions of symmetricity and reciprocity are true even for a Y parameter-based two-port network.

The same analysis or the same logic is also true for H parameters for a short line model. The terms can be well defined, and the conditions of symmetry and reciprocity are also applicable according to the mathematical properties. H parameters are basically combinations of partial ABCD parameters and admittance parameters. Why do I call that? Because in H parameters, we essentially represent V_S and I_R in terms of the remaining voltages and currents on the other side, and that's where it's a combination of ABCD and admittance parameters. The first equation relates to the ABCD equation, whereas the second current represents the admittance parameter equation.

Coming to the performance of the short line model under different conditions, usually the sendingend voltages, they remain frozen. So for fixed V_S, the receiving end voltage would change depending on what the load connected to the receiving end is. Under no load condition, the currents are zero, so V_S is the same as V_R; we have seen that in the previous few slides as well. At full load conditions or partial load conditions, the receiving end current need not be zero, and usually, the sending end voltage magnitude would be greater than the receiving end voltage magnitude. Essentially indicating that the aspect of the

Ferranti effect, which we will discuss in length in the next few lectures, is likely not going to happen often because line capacitance itself is not considered in any short line model.

And since the voltages are changing according to the loading condition, we can also define the percentage voltage regulation similar to the percentage voltage regulation defined for conventional transformers and generators. The percentage voltage regulation is nothing but a measure of line voltage drop from no-load condition to full-load condition, and this percentage voltage regulation would very significantly depend on what type of load is connected, specifically what type of power factor the load has, whether it is a leading load or a lagging load that would dictate the percentage voltage regulation to be positive or negative as well. The slide here shows the typical phasor diagram for the short line model; remember, the phasor diagrams all come from the aspect that V_S is equal to V_R plus I_S into Z, and I_R is equal to I_S. Here we have chosen V_R, which is the receiving end voltage, as the reference for all phasor quantities in this particular equation. It is also being measured with respect to receiving end voltage.

So depending on the load that is present, basically V_S is leading; I mean I just assumed some angle delta. Delta need not always be leading; it could also be different for different loading conditions and the corresponding phase angle theta of the receiving end current, because this is also equal to the I_S phasor, which can also change. For a lagging power factor, that means the receiving end current is lagging the receiving end voltage by an angle theta. So theta is essentially marked as this purple-colored line, which is the one you see over here. I_R is lagging behind the receiving end voltage.

The corresponding sending-end voltage magnitude is typically greater than the receiving-end voltage magnitude. That's what the comparison of this property means. So this is V_R. I think I have marked other quantities here.

So, the red line is V_R. The blue line is the voltage drop across the resistance. The purple line is the voltage drop across the line reactance. So, in terms of magnitude, V_R is typically less than the sending-end voltage magnitude. So usually, it will remain generally positive for a lagging power factor load. But for a leading power factor load, which is the case shown here where the current is leading the corresponding receiving end voltage, it might happen that the no-load condition voltage can become greater than the full load voltage condition, and hence the positive voltage regulation can become negative in certain cases.

So, on similar lines, we can also define the apparent power, or complex power, at the sending end, which is if we know what V_S and I_S are, and we can find S_S as the product of V_S and the conjugate of I_S.

$$S_S = V_S I_S^*$$

The example that is discussed here would essentially replicate the fact that voltage regulation can be positive or negative. Usually, for lagging power factor loads, the VR is positive, as we have seen, but for leading power factor loads, VR can also become negative. And remember, recollect, VR is nothing but the measure of line voltage drop from no-load condition to full-load condition with sending and receiving voltage remaining fixed. There is no notion of the Ferranti effect in this particular context.

So what we have here is a transmission line whose operating voltage is 69 kV . Remember, it is less than 69 kV . And the line length is 40 kilometers, which is again less than 80 kilometers. So, if we have to model this line, we can very well use the short-line model. And that is the reason why only the line resistance per kilometer and line inductance on a per kilometer basis are defined.

Shunt capacitance is negligible. We have to find the voltage power at the sending end and voltage regulation when the receiving end of the line has this voltage or this load at 66 kV under different power factors, both lagging and leading. The other numbers, if you compare, are precisely the same; the power factor quantum is the same, the apparent power specification of the receiving and load is also the same; only the power factor is changing from lagging to leading. What would we do? We find the corresponding net Z of the entire line length of 40 kilometers, which is close to 6 +j 20 ohms. We assume the receiving end voltage to be our reference for all phasors. Since it is line-to-line voltage, we divide it by root 3 .

$$V_{R,\text{ phase}} = \frac{V_{R,\text{ line}}}{\sqrt{3}} = \frac{66}{\sqrt{3}} = 38.105\text{kV}$$

The phasor is 0 , acting as a reference. Dividing by the square root of 3 gives us the corresponding phase voltage. So it is 38.105 and an angle of 0 kilovolt. The power factor is lagging, and the receiving end voltage is fixed at 66 kV . So we can definitely find the quantum of the phasor of the corresponding receiving end current; the power factor lagging means the corresponding power factor angle is 36.87 degrees, and since current is lagging the receiving end voltage, we have a negative sign here. Also, the quantum of three-phase power or apparent power is given, so we can divide it by three to get the corresponding per-phase apparent power magnitude and apparent power value. And when we divide this apparent power by the per-phase voltage, we can get the corresponding per-phase current. So essentially, I_R is 1000 at an angle of minus 36.87 degrees ampere. We know what V_R is, we know what I_R is, and we know what Z is, which is given here. So, essentially, we can find the value of V_S, and we can also find the corresponding sending end per phase complex power in terms of V_S and I_R conjugate.

So basically, the phase voltage magnitude is 56.288 kV . If we find the line-to-line voltage, we multiply it by the square root of 3 ; then we find the total three-phase apparent power, multiply it by 3 , and the percentage voltage regulation is positive, at 47.72

So 36.87 degrees comes in instead of -36.87 degrees. Remaining quantities or magnitudes remain more or less the same. So the receiving end apparent power per phase is the same. The magnitude of the current also remains the same. What changes drastically is the magnitude of the sending-end voltage, as well as the magnitude of the complex power or the apparent power, with the phase angle being drastically different. So in terms of the line-to-line voltage and three-phase power, the quantities are different.

So here we have 109.788 MVA as the sending-end power, whereas in the other case we had 168.864 MVA. Now the reason why this total apparent power has been reduced from lagging load to leading load is that with a leading load, reactive power can still be generated, and the line need not consume that power from the sending end.

Probably that is the reason why 168.864 MVA is going down to 109.78 MVA, because excess reactive power at the load end, which is the leading power factor load, is being utilized by the line to charge itself. So we will also talk about charging current when we discuss a few more topics in the next few discussions. In this particular condition with a leading power factor load, the overall voltage regulation is a negative quantity. So with that, I conclude this discussion, and in the next lecture, we will start with the notion of the medium line model again, building upon how two-port networks can be used to model the medium line model, and we will see how the medium line differs from the short line model. Thank you.