Power Network Analysis

Dr. Abheejeet Mohapatra

Department of Electrical Engineering

IIT Kanpur

Week - 05

Lecture-25

Hello everyone, welcome to the last lecture of week 5 of the course Power Network Analysis, in which we will start with a new module, the fifth main module, which is going to be on transmission line models and the use of these models to understand the performance or behavior of these transmission lines under different operating conditions. Specifically, in this first discussion of this module, we will start with the basics and understand at length what the two-port networks are and how these two-port networks can be used to develop the relevant line models that we are talking about. Till the last lecture, we had finished up with our discussion on the evaluation of transmission line parameters, in which we saw that a typical transmission line attributes some properties, which are line resistance, line inductance, and line capacitance. How these attributes or parameters come in, be it AC resistance, DC resistance, line inductance due to magnetic flux lines, line capacitance due to electric flux line interactions, etc. So all of those were discussed in the last module. Line conductance, specifically, is difficult to model; hence, a detailed discussion was not done, and we considered it to be a negligible quantity.

Starting with this module, our aim is to understand the steady-state performance or characteristics of these three-phase transmission lines, which tend to have non-negligible attributes, and we will try to develop certain mathematical analytical relationships between the terminal voltages at the ends of the line and the current through the line. We will also understand how to replicate or evaluate the associated single-phase line models, which will be very crucial for the upcoming modules on power flow analysis, fault analysis, and lastly, stability analysis. As I mentioned, these modules would be very useful for understanding the steady-state performance as well as dynamic performance. Steady-state performance is mostly from the perspective of power flow analysis, whereas some aspects of dynamic performance would be useful in understanding the fault analysis and stability analysis modules for a given power network with these transmission lines.

We won't talk specifically about the certain models that we would develop or discuss, but definitely certain models that are very useful for lightning analysis or lightning strike analysis and for fault analysis in DC networks. Those modules or those models could also be developed based on the models that we will discuss. So all of those would be discussed as and when required. For the fault perspective where I mentioned that dynamic performance would be useful for fault analysis, I specifically mean it is only for AC networks and not for DC networks. And I'm sure all of us understand the difference between AC networks and DC networks.

AC networks are where there is a source frequency of 50 Hz or 60 Hz , whereas DC networks do not have a typical zero crossing, specifically from the perspective of voltage signals and current signals, because they are DC networks. So, coming to transmission line models, what is shown here in this figure is a typical three-phase 500 kilovolt double-circuit transmission line. Why is it called a double circuit? Because one circuit pertains to each of these individual towers. Circuit number one, this is circuit number two, and in each of these circuits, we can see that for each of the towers there are three dedicated phases which can be easily clearly marked, and along these three phases, there are also double circuits present along each transmission tower. So, if these two phases, let us say, correspond to phase A, these correspond to phase B , and these correspond to phase C .

Essentially, it is a four-wire circuit, or a four-wire circuit, each tower having two circuits. And each of these phases, if we see specifically over here, we can see a few sets of conductors running in parallel; there are precisely four such conductors, four bundled conductors, and the impact of the role or the necessity of having bundled conductors has been discussed at length in the previous module, as well as how this bundling leads to the impact on transmission line inductance or capacitance; that aspect we have also seen. The transmission lines do not have physical contact with respect to the towers; there are strings of insulators, the small encircled ones which you see, and typically for a 500 kV transmission line, the number of insulators would be somewhere between 21 and 23. Assuming each such insulator has a voltage handling capacity of around 20 to 22 kilovolts, the reason why these transmission lines are not directly in contact with the transmission tower is that, from the perspective of safety, number one, the personnel who are working on these transmission towers might get an electrical shock if that line touches the transmission tower. Furthermore, the purpose of having these towers is to set up a transmission line that can carry power from generating stations to the load-end stations.

If a short circuit happens in between, then the normal flow of current in that path is interrupted, and if there is physical contact, essentially the current or power would all go to the ground through the transmission tower if the tower and line come into contact, and that is the reason why insulators are present. There are also extra steel conductors that are probably visible over here. These conductors are placed on top of the tower, and they act as a good guard against lightning, providing shielding or protection to the transmission lines as well as the tower, and the impact of this ground wire is already incorporated while considering or evaluating line capacitance. So, given these typical transmission line towers and models, some of these towers could also act as transposition towers. Now let's see how we can develop a steady-state model specifically from this model perspective for these transmission lines that are overhead and erected with the help of transmission towers.

So what we understand or what we can conclude from the previous module discussion is that actual three-phase transmission lines all have distributed parameters, and that is the reason why our line resistance, line inductance, or line capacitance were evaluated on a per unit length basis. Which means that if the line length is one meter, then the effective R, L, and C are evaluated on a per meter basis; the corresponding line with one meter length would have a net resistance of small r, net inductance of small l, and net capacitance of small $c. L$ and C can also contribute to the corresponding reactance. So, basically, actual transmission lines all have these distributed parameters in the sense that every section of the line length can have these individual or uniform r, l, and c values. So if the line length is 1 meter, then the total resistance or reactance is R and X.

If the line length is 1 kilometer, then the total resistance or reactance is $1000R$ and $1000X$, assuming R and X are evaluated on a per-meter basis. Now, given the transmission line, if the question arises, "Okay, let me know where that precise or specific location is where this $1000R$ resistance or $1000X$ reactance is present in the transmission line." It would be very difficult to exactly pinpoint that specific location where $1000R$ resistance is present and $1000X$ reactance is present. Reason: the transmission line has the same material properties and the same electrical properties throughout any section that we take. So, essentially, every smallest section of the transmission line will have these distributed properties, and hence it is not possible to pinpoint one specific location where the above resistance or reactances are present in a lumped fashion.

So the question arises: is it always essential? Because if that's how our transmission lines are designed, they are supposed to have distributed properties, and that's the reason why we have evaluated our reactances and resistances in the previous module on a per unit length basis. Is it always essential to have three-phase lines modeled as circuits with distributed parameters? The answer is at times yes; the answer could also be at times no. Where the answer is no is because transmission lines, when they are modeled as lines with distributed parameters, the relevant equations that would come up are not in today's discussion but hopefully in the upcoming fourth or fifth lecture in this particular module. We would understand and see that line models with distributed parameters will involve some non-algebraic equations; to be specific, these equations would be differential equations. If we talk about these equations in the phasor domain, then yes, this would be differential phasor equations.

If you have time domain signals or a time domain representation, then the differential part would also come in from the perspective of time as well. Now, handling or solving these differential equations obviously is not so straightforward in the context of power networks as compared to handling or solving algebraic equations. That's where the attribute of having a trade-off comes in. The trade-off is whether we should complicate our model to the extent possible so that we can get the most accurate result. The time involved in getting that accurate result might be higher, or the alternative could be, okay, let's not complicate our model so much; let's try to bring in certain assumptions, and even with those assumptions, the analysis that we would get after solving those relevant equations would still be close enough to the most accurate result.

Benefits could be in terms of time. One can save time in terms of obtaining these accurate and inaccurate results. So that's where the trade-off aspect comes in. So the takeaway is that not all lines need to be modeled as circuits with distributed parameters. The equations that come up need not always be differential.

We can simplify our model, assuming these line resistances and reactances to be lumped at a specific line location rather than being distributed across any section of the line. Given that trade-off of how we want to model our transmission line, the line could also have lumped parameters. Again, the lumped parameter is not an actual representation. It comes from the perspective of the trade-off required between the complexity of the model and the accuracy of the solution in defining the transmission line performance. Also, we have understood in the previous module that transmission line operation is often balanced thanks to line transposition at the transposition tower, and since it is mostly balanced due to transposition, we could also have our per unit analysis or per phase analysis instead of analyzing the entire three-phase circuit.

Our focus would be on understanding where we should use distributed parameters, where we should use lumped parameters, and using these distributed and lumped representations, how we can get the equivalent perphase model of the three-phase transmission line, because our transmission lines are often balanced in nature in terms of operations. So in the context of transmission line models, there are different types of models that can arise. These mostly depend on the physical line length, whether the line length is a few miles, a few thousand miles, and also the operating voltage, whether it is a distributed line, a sub-transmission model line, or a transmission level line. And by operating voltage, I specifically mean the line-to-line RMS. So typically in the literature on power networks, there are three typical categories or models of transmission lines.

The first one is called the short line model, which is essentially applicable and has enough accuracy with simpler complexity to understand the performance of lines operating at less than 69 kilovolts line to line and with a line length not exceeding 80 kilometers. The second line category model is a medium line model, where the line length is typically between a few hundred kilometers to about 300 or 250 kilometers, and the operating voltage is between 69 kV and 400 kV . And the last line model is the long line model where line length is very high and operating voltage is beyond 400 kV , operating at extra high voltage. Now given these three typical line models, the terms that you often see are the term short, the term medium, and the term long. It might appear that these terminologies of short, medium, and long are coming in because we are associating these models with respect to the corresponding line lengths.

There is a word of caution that one should heed before understanding or having this understanding. The word of caution is, please don't associate short, medium, and long with only the line length. The operating voltages are also important. So, for example, if there is a line that is, let's say, 200 kilometers long and the operating voltage is at 765 kV . Then, if you look at the numbers or definitions of the medium line model, the distance very well qualifies it to be called a medium line model, but the operating voltage doesn't qualify it to be a medium line model.

Since the operating voltage is more than 400 kV , this attribute indicates that it should be modeled as a long line model. And that's how accurate performance analysis can be done. So overall, if the line length is 200 kilometers and the operating voltage is above 400 kV , it is preferred to model this line as a long line, irrespective of what the line length is. Again, as power engineers, we do understand and consider the aspect of trade-off. If the line length is substantially small, let's say a few kilometers or hundreds of kilometers, then unnecessarily modeling this particular line as a long line might result in taking more time.

So, depending on the need for the performance that needs to be done, whether it is for steady state or dynamic performance, the corresponding line could be modeled as a medium or short line. So given this, it is understood that short, medium, and long need not always refer to the line length. Each of these transmission line models differs in terms of whether a distributed parameter or a lumped parameter is considered. What type of line parameters is considered? Usually, you would think that all these line models would have resistance, inductance, and capacitance.

No, it's not always true. And depending on these representations of distributed or lumped and which quantity or attribute is being modeled or considered, the governing equations could be pure algebraic equations or they could also be differential equations, and all these considered together result in what trade-off is necessary between the line complexity, the line model complexity, and the accuracy of the solution that is desired or defined to assess the precise performance. The bottom line is that no matter what the line length or operating voltages are, each of these transmission lines can have the same ACSR conductor. The type of material used for short lines, medium lines, and long lines need not be different. It is the same transmission line that has the same R, L, and C on a per unit basis. Depending on the line length and the operating voltage, the corresponding model could be different.

But the material or type is going to be common in all these different line models. So before we dig deep into what these line models are, we would significantly make use of the concept known as two-port networks. Now, two-port networks: what do they do? The two port networks help in defining and establishing these analytical expressions, which could again be differential equations or could be algebraic equations in the phasor domain, and they also help in figuring out the per phase equivalent of this three-phase transmission line because essentially when we talk about the line model's performance, we need to understand how the line losses are, how the line is behaving at different operating voltages, different voltage levels, and different load voltages. So, as the name suggests, a two-port network has two pairs of ports or terminals, both of which could be used. And one pair, or one port, or one pair of ports is usually assigned the notion of input end or sending end.

And the other pair of ports could be called the output end or the receiving end. Usually, sending and receiving come from the perspective that if a line parameter or line port is called the sending end, that end usually has some sort of source voltage or generating source, and the receiving end is generally associated with the fact that there is some physical load drawing power from the source through this transmission line. That is the notion of the sending and receiving ends. So, it is mostly from the perspective of how power is flowing, essentially effective power. Different types of parameters have been defined to explain or represent these two-port networks.

In our discussion, we will take up four typical parameters useful for defining two-port networks. Before we go deep into what these parameters are, I just want to make it clear what type of two-port network I am talking about. So I have this transmission line with a sending end, again on a per-phase basis. I have this transmission line whose length is a few kilometers, and its operating voltage is also a few kilometers. The second port that I have drawn here could refer to the equivalent neutral port, or it could refer to the equivalent ground from which the sending end voltage V_S and receiving end voltage V_R are being measured on a per-phase basis, and this entire transmission line can be considered to be modeled through this black box, which we call the two-port network.

This black box could have different parameters, which we will see at length. And since this black box has a dedicated sending end that has the subscript S for voltage V_S and I_S for the sending end current. Similarly, the other part is being assigned as a notion of the receiving end, essentially meaning there is some load present here, and that is the reason for having a small R in I_R and V_R, I_R being the receiving end current and V_R being the receiving end per phase voltage. Different types of representations could be present in different textbooks and in different literature. In our discussions to come, we would simply assume I_S to always be an input current to the two-port network, whereas I_R would always be an output current for the two-port network.

Certain references could also have I_R represented as an input. And since this is the sending end, the corresponding return current could be in the reverse direction. We are not going to use this terminology. So if you understand these aspects well, you can also apply them to the reverse notation of receiving-end current. First, the common parameter that we would see would be called the Z parameters, and usually, if you recollect the variable or symbol Z, Z is often associated with line impedance in power network discussions or the power network context.

So, Z parameters are essentially a few impedance-based parameters that define the properties or attributes of this two-port network. With sending end and receiving end quantities marked the way they are, as we saw in the previous slide. So, if these parameters are impedance-based parameters, that means the corresponding equations, which are likely to come up, relate some voltage with respect to some current multiplied by the corresponding impedance. That's how the impedance or basic definition of Ohm's law is: V is equal to IR.

So IR is also a form of impedance. If you are talking about Z parameters, they are going to be impedances. So basically, for an equation on the lefthand side, there should be a voltage expression equal to a current expression multiplied by the corresponding impedance; that is how impedance or ohms are defined. So in the context of Z parameters, these expressions or equations would be parameters; let us say we call them $Z_{11}, Z_{12}, Z_{21}, Z_{22}$. These Z s would essentially indicate an equation which is of the form V is equal to Z times I, where Z are the Z parameters, V is voltage, the parameters are in ohms, and here I is in terms of currents, and we have four such Z parameters. How do we relate these Z parameters in terms of the equations that are going to come up? So for this two-port network, Z parameters are nothing but V_S, so we have two voltages: one on the receiving end and one on the sending end.

If we are using Z parameters to represent these voltages and currents, then both these voltages should be present, so V_R is over here. In terms of Z parameters, V_S is Z_{11} times I_S minus Z_{12} times I_R, and similarly, we have Z_{21} times I_S minus Z_{22} times I_R. Essentially, we have all voltages on one side, the corresponding impedances on the other side, and the corresponding currents being multiplied. So, in matrix form, we can rewrite this equation in this order. The reason why you have a minus sign here is that typically when Z parameters are defined conventionally, the receiving and current notion has been defined in a manner different from the one that I am using here.

Since I am using receiving and current as an output current and for me this is not an input, I have a negative sign. And as I mentioned, these Z parameters are impedances, so they are all in ohms. For a given twoport network, if I have to evaluate or figure out a way to evaluate these Z parameters, then it's very simple in the sense that these Z parameters are essentially, let's say, if we have a case where the receiving end is open, in the sense that the receiving end is not feeding in any load. So in that sense, I_R would be zero, and Z_{11}, as per this equation, would simply be V_S divided by I_S for Z_{11}. I_R equals 0 , and similarly, Z_{21} would be V_R divided by I_S for I_R equal to 0 .

So, if I have an open circuit at the receiving end and I measure V_S, I_S, and V_R using appropriate meters, I can physically evaluate these numbers Z_{11}, Z_{22}, and Z_{21}. Similarly, Z_{12} and Z_{22} could be defined when the receiving end has some sort of electrical quantity, such as a source or load, and the sending end is open. Similar lines, if Z parameters can exist, which are impedances on similar lines, Y parameters, which are admittances, can also exist. So in terms of admittance parameters or Y parameters, the relevant equation indicates the relationship between the currents at the sending and receiving ends, in terms of the sending and receiving end voltages. The current direction is reversed from the conventional nature, so that is why you would see a negative sign here.

In typical textbooks, you may find a positive sign instead of a negative one. And essentially, in matrix form, if the inverse of Z exists, then the Y parameters are nothing but the inverse of the Z parameters. The unit of admittance is siemens, or Ω^{-1}. So that's how the Ω^{-1} sign comes in. The logic behind evaluating these y parameters again remains the same since we are relating currents to voltages; instead of open-circuiting the terminal ends, now what we would do is short-circuit the sending end.

Let us say if we short-circuit the sending end, the corresponding voltage here would be 0 . Sending and receiving voltages are being measured with respect to some neutral or ground. So, sending and voltage, once it is 0 , the corresponding Y_{11} and Y_{21} would be ratios of I_S and V_R; sorry, this is not true for Y_{11} and Y_{21}; this is true for Y_{12} and Y_{22} where V_S is 0 . A similar notion is also applicable for defining or evaluating Y_{11} and Y_{21}. So, given the physical network, I can use this sort of mathematical relationship, carry out certain experiments, open circuit and short circuit experiments, the way open circuit and short circuit tests are done for evaluating typical parameters of a given electrical element, be it a transformer or generator; the same notion can be applied or used here for transmission lines.

The third common property of parameters is ABCD parameters, in which the notion is a little bit different. On one hand, the quantities they all refer to sending in, both sending in voltage and current, and on the other hand, the quantities they pertain to all receiving end quantities, receiving end voltage and receiving end current, ABCD as relevant parameters. So if one has to understand what the units of A, B, C, and D are, and if V_S is equal to AV_R plus BI_R, and I_S is equal to CV_R plus DI_R as per the definition of ABCD parameters, then it's pretty obvious that V_S and V_R are voltages, and I_S and I_R are currents. So, A and D are dimensionless quantities. V_S is voltage; I_R is current. So, B has to be impedance, whose unit is going to be ohm. I_S is current; V_R is voltage. So, C is going to be admittance, whose unit is going to be Ω^{-1} or siemens. So, that is how this notion or logic comes in.

These are ABCD parameters. They are common parameters for defining transmission line models. And again, by carrying out different open circuit and short circuit tests at the sending and receiving ends, there's a logical way of evaluating these ABCD parameters. The last set is hybrid parameters or H-parameters, where again it's basically a combination of ABCD and admittance parameters. So, on similar lines, H_{12} and H_{21} are dimensionless, whereas H_{11} is impedance and H_{22} is admittance. It's a very common model representation or parameter representation for modeling power electronicsbased circuits or power electronics-based switches.

By again carrying out different open circuit and short circuit tests, one can also evaluate the corresponding H parameters. So if one has to understand how these different parameters can be correlated or co-represented, we have seen that this is the relevant equation for Z parameters. So if one has to find the corresponding Y-parameters. So basically, if the inverse of Z exists and we carry out this mathematical operation, we would get I_S in terms of $V_S V_R$; we would get a similar equation for I_R. So from where, if we now plug in that $I_S I_R$ is equal to Y_{11} ? Y_{12} minus Y_{21} minus Y_{22}, V_S, and V_R are all in vector or matrix form.

So, from there we can correlate the corresponding Y parameters in terms of relevant Z s, and remember, as I mentioned, the inverse of Z should exist; that means the inverse of Z can only exist when this matrix here, which is the Z matrix, is considered in terms of the matrix shown over here. Its determinant is not zero. So if I find the corresponding determinant, the determinant here is $Z_{11}Z_{22} - Z_{21}Z_{12}$. That's the determinant of the corresponding Z matrix. If this matrix inverse is to exist, the determinant should not be zero.

And that's the reason why you see the corresponding similar term present in all the y parameters in terms of z parameters. For y to exist, these terms or divisions must be defined. Additionally, if we now find the Y_{11} and Y_{22} products, we can use these expressions to correlate or find these relationships along the terms of the Z parameters. Similarly, ABCD parameters can be represented in terms of Z parameters through similar logic. I'm not going to spend much time on this because the discussion we had about Y parameters represented in terms of Z parameters will apply here for ABCD parameters as well.

And on similar lines, the H parameters can also be defined in terms of Z parameters through the corresponding mathematical analysis, provided the inverses or divisions do hold true. So what we have seen here so far in the last three slides are Y parameters, ABCD parameters, and H parameters represented in terms of Z parameters. Other parameters can also be represented in different forms. So similar interchanges in terms of exchanges between these parameters are also possible depending on what the need or requirement is. A few generic properties of these two-port networks are that they are linear and lumped, which means the superposition theorem holds true if we apply different open-circuit tests and short-circuit tests, or if we attach different types of voltage sources with similar frequencies; then the superposition theorem will still hold true in the sense that we analyze the circuit from one voltage source to get the results.

We analyze the circuit for a different voltage source, analyze the results, and then, by adding these two voltage sources, we can still obtain the overall effect by combining the overall results. That's the beauty or property of linearity. And we can precisely point out specific locations in this two-port network against a fictitious representation where parameters are lumped and not distributed. Two port networks often tend to be symmetric in the sense that the effective impedance observed by this two port network, either at the sending end or receiving end, has the other port remaining open. It's going to be the same because we consider it to be a passive network.

The two-port networks are often also reciprocal; that means the cross impedances or cross admittances of currents and voltages, in terms of cross impedance, are the ratio of sending voltage with respect to receiving current, under certain conditions, or in terms of cross admittance, are the ratio of receiving current with respect to sending voltage; these numbers remain the same irrespective of what the port positions are. That means the ratio of V_S to I_R and the ratio of V_R to I_S would remain the same; they would be equal to each other irrespective of the port position. That is the property of reciprocality, and these network parameters are time invariant, so basically, the performance remains the same no matter if we analyze the circuit now or at time t equals infinity. The overall steady-state behavior is not going to change unless the inputs themselves don't vary in time. In the sense of the phasor domain, for fixed inputs, you would get a fixed set of outputs.

The time parameter or behavior won't change because the throughput networks are basically passive elements. So for conditions of symmetry, if we want to understand in terms of Z parameters, this is our equation: the model with the sending end excited by a voltage V and the receiving end open, which means there is no load on the receiving end, so I_R is 0 . The associated sending end impedance is Z_{11}, which is V by I_S, and again, if we do the same thing on the receiving end, that means we excite the receiving end by V and open the sending end port, then we have Z_{22}. For symmetry, V by I_S should be equal to V by $-I_R$; that is the property of symmetry. So essentially, in terms of Z parameters, Z_{11} and Z_{22}, if they are equal for a two-port network, the two-port network is called symmetric.

Other relationships in terms of Y parameters ABCD and H_{11}, H_{22} can also be defined using the relation that we talked about. For the reciprocity property, the ratio of the cross impedances is matched. So basically, if the network is reciprocal, the values of Y_{12} and Y_{21} would be equal. And on similar lines, the corresponding relationship in terms of other parameters, Z , ABCD , and H , can be defined or evaluated based on the discussion that we had earlier.

So that's all for today's discussion. In the next lecture, we will take up the basics of the two-port network and apply it to understanding the short-line model. Thank you.