Power Network Analysis

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Week - 05

Lecture-23

Everyone is welcome to lecture three of week five of the course Power Network Analysis, in which we will continue our discussion on the evaluation of transmission line parameters. This is going to be the second-to-last lecture on the main module four, which is transmission line parameter evaluation. In today's discussion, we will focus on the evaluation of the capacitance of actual three-phase transposed lines and how earth or other nearby potentially free or zero potential surfaces can increase or decrease the overall capacitance of the transmission lines that we will see. And this discussion will be based on the discussion that we had in the previous lecture wherein we concluded that if there are n charged conductors of different radii R 1 , R 2 to Rn , capital N being the number of conductors, and the charges are also different on these conductors, which would essentially mean that the current in the present conductors is also different, then if I take any two points, these two points we are, Coinciding them with the centers of conductor number i and conductor number j, the potential difference between these two points, which is vij, would be the superimposed effect or addition of potentials because of individual charges q1, q2, qy, qj , till qn , which results in this particular summation term. And the self distances dii and djj are the self radii ri and rj. And these distances are with respect to the corresponding nodes between which these distances are evaluated.

For example, distance k , m would represent the distance between the center of conductor k and the center of conductor m . That would be dkm. So with this, we directly come to the case of the capacitance of three-phase transposed lines. We are avoiding all those intermediate discussions on the capacitance of actual three-phase untransposed lines.

We are directly coming to our main topic, which is that we have a three-phase line that is transposed, and transposition is essential, as we discussed in the previous lecture, to sort of bring in an overall balanced effect, because if transposition is not done due to dissimilar spacing between these three phase lines, the overall inductance or capacitance would turn out to be unequal for different phases. So no matter whether the system is positively balanced or operating with balanced sources or loads, the network would behave or result in unbalances. So what we have for the sake of discussion is three phase lines, which are three long parallel strands or conductors. We still have not talked about composite conductors or bundle conductors yet, and these conductors are of similar nature; their radii are the same r , and the spacing between these conductors varies depending on the transposition cycle in which these particular things occur. So basically, transposition ensures that every conductor or every phase occupies similar distances compared to the other two phases.

So often in a full transpose system, you would find at least three equal sections or three sections where the distances are more or less similar, so that every phase occupies the same position as other phase conductors. So essentially, if I am marking position 1 , position 2, and position 3, then this triangle here essentially indicates the distance between positions 1 and 2,2 and 3 , and 1 and 3 , which is this distance when conductors a, b, and c take up positions 1,2 , and 3 respectively for different transpose cycles. For our discussion, we will for the time being consider the effect of the earth or ground to be neglected. Field distributions are perfectly smooth. The charges are all residing on the surface of the conductor.

Their charge distribution is uniform on the surface. No distortions or non-idealities exist. And since we are considering our system to be operating in a balanced condition, a balanced three-phase system, the sum of instantaneous currents has to be zero. So, on similar lines, the sum of instantaneous charges should also be zero. So $q_a + q_b + q_c = 0$.

So now, if we have to find what the value of capacitance is. So, as I indicated, we will be heavily relying on this particular potential difference expression. So let's see how this potential difference expression can help in the evaluation of the capacitance of three-phase lines. So we'll go one by one. What we have here is this expression that is trying to find the potential difference between the center of conductor a and the center of conductor b when a is occupying position 1.
b is occupying position 2. So this is 1 here. It should not be confused with the one present over here. By 1, 2, and 3, I would mean. If I were to desegregate these transposition halves.

Into three different sections. So, if this is section number 1. Section number two, section number three, then one here essentially means section number one, not position number one. Please keep that in mind. In section number one, a is coincidentally occupying position one, b is at position two, c is at position three, and since there are three charges q_a, q_b, and q_c which are spaced apart at distances D_{12}, D_{23}, and D_{31} respectively from each other when the focus is on section 1 transposition.

So we can find potentials based on individual charge locations. Essentially, we can make use of the summation expression that we saw in slide number 3 in this particular discussion. So, if we use that, let's go step by step. If I have to find the potential between a and b when we are talking about section number 1 because of charge q_a, then since q_a is the only charge of focus now, we would have q_a divided by $2\pi\varepsilon_0$. Here is the insulating medium between these conductors.

So epsilon R is either perfectly equal to 1 or close to 1 . The natural log of some distance ratio; the numerator distance, if you remember, is the distance between the position of the charge that is resulting in this potential with respect to the second point where the potential is being measured. So, if you look at q_a, the distance with respect to the b conductor positioning, which is q_b, the distance of concern is D_{12}. Therefore, we have D_{12} as the numerator; the denominator would be the distance between the charge that is resulting in this potential with respect to the first point between which the potential is to be measured. Since q_a is coinciding with the conductor itself, and again, electric field lines don't exist inside the conductor, they can exist only outside the conductor.

Thus, the surface distance with respect to the center of the conductor itself is nothing but the radius itself, which is r , and that is how we get our first term, which is shown over here. On similar lines, we have a potential difference because of charge q_b and charge q_c only for section number 1.

When conductor a is in position 1,

$$V_{ab}^{1} = \frac{1}{2\pi\varepsilon_0}\left\{q_a \ln\left(\frac{D_{12}}{r}\right) + q_b \ln\left(\frac{r}{D_{12}}\right) + q_c \ln\left(\frac{D_{32}}{D_{31}}\right)\right\} \text{V/m}$$

Similarly for other two positions

$$V_{ab}^{2} = \frac{1}{2\pi\varepsilon_0}\left\{q_a \ln\left(\frac{D_{23}}{r}\right) + q_b \ln\left(\frac{r}{D_{23}}\right) + q_c \ln\left(\frac{D_{31}}{D_{21}}\right)\right\} \text{V/m}$$

$$V_{ab}^{3} = \frac{1}{2\pi\varepsilon_0}\left\{q_a \ln\left(\frac{D_{31}}{r}\right) + q_b \ln\left(\frac{r}{D_{31}}\right) + q_c \ln\left(\frac{D_{21}}{D_{23}}\right)\right\} \text{V/m}$$

If we rewrite or extend this for section number 2, wherein 2 refers to section 2 here and 3 refers to section 3 here, then we have similar expressions. I'm sure these expressions would take time to digest or adjust, but the way I explain the potential difference expression with respect to q_a in section number 1, if the same logic is applied, I'm sure you can easily verify or cross-check the other expressions or terms that have been explained or written here in this particular slide. So, since I have three potential differences for the same two points, a and b, where a and b themselves are changing their positions with respect to different transposition sections.

So, we can always approximate these values using the well-known concept of the arithmetic mean or the average voltage. So, V_{ab} is the average voltage of the individual credentials that are coming in because of different sections, so if we do that, then the V_{ab} expression turns out to be this simple term. The question here is what happened to the point or term associated with respect to q_c; why do we have only terms representing q_a and q_b, wherein we have brought the notion of GMD here, geometric mean distance, similar to the inductance discussion? Why are only q_a and q_b terms appearing here, whereas in the previous slide there are terms with respect to q_c as well? Now, if you focus on the terms with respect to q_c, the first terms are common in all three expressions. If we focus on the term q_c and try to understand what is actually happening here, q_c, natural log of $\frac{D_{32}}{D_{31}}$ plus natural log of $\frac{D_{31}}{D_{21}}$ plus natural log of $\frac{D_{21}}{D_{23}}$, is also equal to q_c, which is equal to natural log of $\frac{D_{32} D_{31} D_{21}}{D_{31} D_{21} D_{23}}$. D_{23} is the same as D_{32}, so it gets nullified. D_{31} is the same, D_{21} is the same, so essentially we are left with the q_c of the natural log of 1 . The natural log or any quantum log of 1 is always equal to 0 . That's how this term is nullified, and we have only q_a and q_b terms. Similarly, if one wants to find V_{ac}, one would have similar terms with respect to q_a and $q_c . q_b$ terms would all be nullified or approximated.

Now, why are we trying to find V_{ab} and V_{ac} ? Remember, in our previous discussion, we talked about capacitance, which is similar to inductance; when we evaluate inductance, it is with respect to a particular phase. Capacitance, when we are finding it, starts our discussion with the potential difference between two points, and these two points can be at different potentials with respect to each other. It's often convenient to evaluate quantities on a per-phase basis where, for per-phase, the neutral point or the ground point acts as a zero reference or a zero potential reference. So since we are talking about the potential difference between two points and we don't know where the neutral point exists, we are trying to come up with an evaluation or expression where we can find phase a potential with respect to a neutral point, and once we know V_{an}, where n is a neutral point, then we can also derive the capacitance of phase a with respect to the neutral point rather than finding the capacitance between phase a and phase b or phase a and phase c because the line-to-line capacitance doesn't make much sense. So, for a balanced system, if we see, we have understood the notion of line-to-line voltages and phase voltages.

For balanced three-phase voltage phasors, V_{ab}, which is a line-to-line voltage across phase a and phase b, choosing V_{an}, where small n is the neutral point, V_{an} is the potential of phase a with respect to neutral. As per the definitions of line-to-line and phase voltages, V_{ab} is $\sqrt{3}$ times at an angle of 30° from V_{an}, and similarly, V_{ca} is $\sqrt{3}$ times V_{an} at an angle of $+150°$, which is also justified or indicated through this particular phasor diagram. If V_{an} is the reference, V_{ab}, V_{ca}, and V_{bc} are also shown. To be specific, if V_{an} exists in this fashion and if it is a positive sequence voltage, then V_{bn} would exist somewhere over here.

If it is balanced, then here we would have V_{cn}. And if we have to find where V_{ca} is, pardon me, yeah, I think it is correct. So V_{ca} is V_{cn} minus V_{an}. So to find V_{ca}, we take the negative of V_{an}, which would lie over here, and the addition of V_{cn} with minus V_{an} would essentially lead to the V_{ca} vector, which is also shown over here. So with that line-to-line and phase-to-phase voltage relationship, we know that this is for sure. And coincidentally, if we add V_{ab} and V_{ca} or V_{ac}, we have V_{ab} plus V_{ac} equal to three times V_{an}, where V_{an} is the phase a voltage with respect to the neutral point.

So essentially, if we add up these two terms, we would get $3V_{an}$ as the effective term. And now, once we add them up, we have two terms. The first term has the same form as q_a natural log of $\frac{GMD}{r}$, and the other terms are q_b natural log $\frac{r}{GMD}$ and q_c natural log $\frac{r}{GMD}$. So let's look at it for a bit: we have $q_b$log of natural log of $\frac{r}{GMD}$ to be added with q_c natural log of $\frac{r}{GMD}$, in which the second terms are common. So this is also equal to $(q_b + q_c)$ natural log of $\frac{r}{GMD}$.

And remember, this is a balanced system. So $q_a + q_b + q_c$ are always zero. So it means this is also equal to-sorry, I'm using arrows here. It should actually be equal to, it is equal to $-q_a$ times the natural log of $\frac{r}{GMD}$, which is also equal to q_a times the natural log of $\frac{GMD}{r}$. Minus 1, when taken as a log, is the inverse of the ratio.

So effectively, I would have $V_{ab} + V_{ac}$, which is equal to $3V_{an}$. I have one q_a natural log $\frac{GMD}{r}$ term. I have the second term, q_a natural log of $\frac{GMD}{r}$. I would also have the third term, which is q_a natural log of $\frac{GMD}{r}$. So I have three $\frac{q_a}{2\pi\varepsilon_0}$ natural log of $\frac{GMD}{r}$, which means $V_{an} = \frac{q_a}{2\pi\varepsilon_0}\ln\left(\frac{GMD}{r}\right)$.

Such a simple, beautiful expression that we are receiving. So essentially, that is what is mentioned here in terms of volts per meter because we are choosing our unit length of the conductor. Once we know V_{an}, we can use the capacitance expression to find the phase a to neutral capacitance, which is $C_{an} = \frac{2\pi\varepsilon_0}{\ln(GMD/r)}$. The same expression can be found for other phases with respect to neutral. And this capacitance, actually, if we think of it from the perspective of network impact, would be a very detailed discussion in the next module of transmission line performance evaluation.

This charged capacitance is also leading to a current known as line charging current, which is the current in the line due to phase-to-neutral capacitance because the potential V_{an} exists in the transmission line. So no matter whether the line is loaded or unloaded, meaning whether the line is delivering any load or not, irrespective of the load, this current would always exist in the transmission line because the line is charged, it is activated, and it is just carrying some potential. So because of this capacitance, whose expression is given over here, jx_{can}, x_{can} is $\frac{1}{j\omega C_{an}}$, where ω is $2\pi f$. So if I have to find the charging current

I_a, it is x , which is just $\frac{1}{\omega C_{an}}$. So if I have to find the charging current, then $\frac{V_{an}}{-jX_{can}}$, which when I substitute it here, results in this particular term.

So this is a line charging current that is flowing even when the load current is zero. That means the line is not delivering any actual loads. And it results in a very interesting effect known as the Ferranti effect, wherein, due to light load conditions, the receiving end voltage tends to be higher than the sending end voltage. It would sound pretty awkward how the load voltage tends to be more than the source voltage because it is the source voltage or the source energy that is transferring to the load. So how can the load voltage go up? The basic reason why that goes up is because of this charging current, as this serves as a source of additional reactive power for the system, which leads to an increase in voltage at the receiving end.

We will discuss these aspects in detail when we discuss the next module. The effect of bundling and the effect of composite conductors are going to be exactly the same. Instead of actual r, you can get the expressions in terms of GMR, which is the geometric mean radius. Remember, the geometric mean radius in capacitance is the actual radius or the actual overall radius, not the skin effect-based radius which we saw in inductance.

Please don't be perturbed. We'll have one more discussion, which will be the next discussion, wherein we will see numeric examples of how these evaluations can happen. And as I mentioned, the log term in the capacitance would have GMR instead of the actual r when GMR is calculated based on the actual radius, not the R dash which is commonly used in inductance evaluation for bundled conductors or composite conductors. With that, we conclude our discussion of the capacitance of three-phase lines. We have one more interesting aspect, which is the role of earth or any other potentially charged free surface on line capacitance since our transmission lines are overhead lines and the earth is always at the bottom of these transmission lines and towers. There is a concept known as right of way, so earth has a role in capacitance, and it influences transmission tower design.

So why is that? It is because, as I mentioned, Earth influences the value of capacitance. So how that influence comes in is what we would understand, and then we will see what corresponding actions are in place to minimize the effect on the Earth. So there can be two effects to begin with. Either the effect of the earth increases the overall capacitance or decreases the role of capacitance. If the Earth's role were to decrease the value of capacitance, then it would probably have had a beneficial effect because, in that sense, the charging current would also have been lower, and other consequences could have resulted as well.

But the reason we are discussing this would have become obvious by now: the effect of Earth leads to an increase in line capacitance. So how is that happening? Let's see that.

Earth acts as an infinite source or sink of charges. It can be treated as an equipotential surface with zero potential. And if we have an overhead line that is carrying current or is charged, assuming it to be a positive charge, then the electric field lines, as I mentioned, always originate from positive surfaces and terminate at negative surfaces.

So since Earth acts as a sink or source of charges, it's always at neutral potential or at zero potential. So for a positively charged body, plus Q , the arrows or the lines that you see here are the directions of the electric field lines. These electric field lines would originate from the surface of the connector in an orthogonal fashion, and they would terminate at the earth's surface again in an orthogonal fashion because electric field lines with respect to corresponding charged surfaces always have to be orthogonal. So, electric field lines are originating from the conductor, which are orthogonal, but they are not radial anymore because distortion has occurred; the earth's effect is now coming in, and field lines are also orthogonal to the surface of the earth. So how do we encounter this effect of field lines coming in from the charged body to the Earth? Because if the field lines can exist, then we can also mark corresponding orthogonal surfaces, which would turn out to be equipotential surfaces, and the equipotential surface zero is appearing over here, so at a distance a little above over here.

The potential difference might be higher and higher until the line whose operating voltage could be a few hundred kVs, or let's say 220 kV or 400 kV . Such operating voltages are the usual transmission line voltage, 765 kV . These are line-to-line voltages. Phase voltages could also be obtained automatically. If the electric field lines originate from the charged body and terminate at the Earth, and the Earth has zero potential, then as I move higher and higher, the potential would increase and eventually become equal to the line potential.
So if potential exists, that means capacitance will definitely exist because C is inversely proportional to V or it is equal to Q divided by V . Q exists, V exists, so definitely in between this surface which is occupied by air, the value of C should definitely exist. So that's the reason why Earth influences the capacitance value. So, how do we do that? Kelvin came up with a very beautiful idea. He suggested that if Earth acts as a zero potential plane or a ground, that means we can denote a negative charge of the same quantum as the positive charge Q to be present at an equal distance below the surface of Earth, H being the distance between the charge plus Q and the surface of Earth.

If zero potential exists, that means this particular surface, which is a zero potential surface, is orthogonal to the electric field lines that are originating from plus Q and just terminating at the surface. I am increasing the arrows so that they can appear to be originating from plus Q , the values of E , and terminating at negative Q again in an orthogonal surface perspective. And if zero potential exists, that means if this is at plus V , I can also associate a corresponding minus V at a depth h from the surface of the Earth. And that's how the consideration of effects can be taken care of.

So, Kelvin suggested the method of charge or images. Basically, they are image charges. This negative Q does not exist. It is a fictitious or an image charge that is being considered to account for the factor of the presence of earth potential or zero potential. Field lines are orthogonal to the Earth's zero potential; thus, the zero potential surface can be removed when evaluating the capacitance due to the Earth. So what we have here is the most complicated case: we have our three-phase transposed lines, which are transposed at different sections.

Remember, 1,2,3 can be different from the section markings, so this can be section 1, this can be section 2, this can be section 3, and Conductors a, b, and c are taking different positions, 1,2,3, over here. These distances 1,2 , and 3 are also present, and the same distance arrangement is shown through this particular triangle D_{12}, D_{31}, D_{23}, with these charged surfaces or conductors at distances h_1, h_2, and h_3 from the surface of the earth. Their in-between distances are d_{12}, d_{23}, and d_{31}. Since we have to consider the effect of ground or earth, we are assuming image charges or negative charges of a negative quantum of charge and at a distance of minus, let's say, okay, so this has not been particularly shown over here. So if this is h_1 over 2 , then this distance is also h_1 over 2.

This distance is h_2 by 2 . The image of charges says this is h_2 by 2 in half. And similarly, this is h_3 divided by 2 . Then the image charge here is also at h_3 over 2 . Similar inferences could also be made for other distances correspondingly.

I'm not going into the details of that. But given this tower arrangement, and the heights of these conductors with respect to the earth, the other distances, h_3, h_1, h_2, can all be obtained and evaluated. So we have three phase lines of the same radius. Their charges are q_a, q_b, and q_c. It's a positive balanced system.

So at any point of time, the sum of charges is zero. Image charges have the same charge but with a negative quantum. Field lines are all orthogonal. Surface charge density is uniform. There are no distortion effects. So we now go into finding capacitance because of the effect of earth for a three-phase transposed line, and we use the same notion of finding the potential with respect to different points.

So what we have here is us trying to find the potential between conductor a's center and conductor b's center when a and b are at section 1. So let's spend some time here. So if we talk about section 1's position.

And we want to find V_{ab}. When ab is in section 1 . With respect to charge q_a. Then this would be equal to $\frac{q_a}{2\pi\varepsilon_0}$. The natural log of the first distance in the numerator would be the distance between q_a and b . In this case, q_a is over here, q_b is over here, so the distance is D_{12}, which is this distance, divided by the radius of the charge, because q_a with respect to the other conductor a is the self-radius, so we have plus r .

Similarly, we would also have V_{ab} with respect to $-q_a$ when a and b are in section 1, so our expression becomes $-\frac{q_a}{2\pi\varepsilon_0}$ with the natural log. The first numerator should be the distance between $-q_a$ and b . If we look at $-q_a$, it is sitting over here. b is in this position. The distance is h_{12}. So the natural log of $\frac{h_{12}}{h_1}$. If we add these two effects, let's say V_{ab} overall 1 , then this is nothing but $\frac{q_a}{2\pi\varepsilon_0}\left[\ln\left(\frac{D_{12}}{r}\right) - \ln\left(\frac{h_{12}}{h_1}\right)\right]$, which can again be simplified as $\frac{q_a}{2\pi\varepsilon_0}\ln\left(\frac{D_{12}h_1}{rh_{12}}\right)$. So this is the first term only because of the charges q_a and $-q_a$, which is also appearing over here. And that is how the first term is obtained. Similar terms can be obtained for q_b and q_c, and then you can also find the potentials with respect to other sections 2 and 3.

Hence,

$$V_{ab}^1 = \frac{1}{2\pi\epsilon_0}\left\{q_a\ln\left(\frac{D_{12}h_1}{rh_{12}}\right) + q_b\ln\left(\frac{rh_{12}}{D_{12}h_2}\right) + q_c\ln\left(\frac{D_{32}h_{31}}{D_{31}h_{23}}\right)\right\}\text{V/m}$$
$$V_{ab}^2 = \frac{1}{2\pi\epsilon_0}\left\{q_a\ln\left(\frac{D_{23}h_2}{rh_{23}}\right) + q_b\ln\left(\frac{rh_{23}}{D_{23}h_3}\right) + q_c\ln\left(\frac{D_{31}h_{12}}{D_{21}h_{31}}\right)\right\}\text{V/m}$$
$$V_{ab}^3 = \frac{1}{2\pi\epsilon_0}\left\{q_a\ln\left(\frac{D_{31}h_3}{rh_{31}}\right) + q_b\ln\left(\frac{rh_{31}}{D_{31}h_1}\right) + q_c\ln\left(\frac{D_{21}h_{23}}{D_{23}h_{12}}\right)\right\}\text{V/m}$$

Average voltage (sum of above/3) with

$$GMD = (D_{12}D_{23}D_{31})^{1/3}\text{ m}$$
$$V_{ab} = \frac{1}{6\pi\epsilon_0}\left\{q_a\ln\left(\frac{GMD^3h_1h_2h_3}{r^3h_{12}h_{23}h_{31}}\right) + q_b\ln\left(\frac{r^3h_{12}h_{23}h_{31}}{GMD^3h_1h_2h_3}\right)\right\}\text{V/m}$$

And if we take their average, we get V_{ab} as this particular expression over here. We can also find the V_{ac}. Trying to find V_{an}, actually, so $V_{an} = \frac{V_{ab}+V_{ac}}{3}$, and also $q_b + q_c = -q_a$, so we get V_{an} as this particular term. If we now find capacitance, then we have this term over here. Remember, when the effect of earth was not considered, this term was not present at all.

Now, with consideration of Earth, we have this additional term, and let's focus on this term for a moment. Let's not worry about the negative signs. Let's talk about this ratio for a moment. What do you think? The ratio of $\frac{h_{12}h_{23}h_{31}}{h_1h_2h_3}$. Is it going to be less than 1, or is it going to be more than 1? It appears that no matter how the tower arrangement is, h_{31}.

.. h_{23} and h_{12}, their product is obviously more than h_1, h_2, and h_3 if we compare these distances individually. It may not be an individual comparison, but definitely, these distances are larger than the self-distances. So the ratio inside the log term here is a term that is more than 1 . That means the natural log of this term is going to be a positive number, which means when I multiply minus 1 by 3 , this term overall is trying to bring down the denominator value. If the denominator goes up in a particular ratio, the corresponding capacitance value should accordingly go up.

And that's the reason why the ratio is positive: overall capacitance due to the earth increases. What is the alternative? How can we minimize this increase in capacitance? If this ratio tends to be equal to 1 , then the natural log of 1 will become 0 . When this term can become 1, try to design your transmission towers or place conductors that have a high right of way from the surface of the earth. So as the distance between the conductor and the earth increases, these products become more or less equal, and that's where the concept of ROW, also known as right of way, comes in. The higher the potential of a transmission line, the higher its ROW; the lower the potential of the line, the ROW can decrease so that the capacitance effect due to earth is minimized or neglected.

So, as an additional term, when this ratio becomes close to 1 , the additional term would tend to 0 , and hence the effect of the earth on capacitance becomes negligible. That's all for today's discussion. As I promised and discussed, I'll take up numeric examples and also try to understand how manufacturer's data can be used because GMR and GMDs can be very well obtained from manufacturer data. So we'll take up examples and numeric discussions of how GMR and GMD can be evaluated for line capacitance and line inductance in the next discussion. Thank you.