**Power Network Analysis**

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**Week - 05**

**Lecture-22**

Hello everyone, welcome to Lecture 2 of Week 5 on the course Power Network Analysis. In today's discussion, we will continue with our discussion on the module of transmission line parameter evaluation, but we will take up a topic that is new to this particular module, which is capacitance evaluation. In the previous few lectures, particular to module 4 , the main module 4 topic, which is transmission line parameter evaluation, we have discussed what resistance is, what inductance is, how we physically evaluate the values of resistance, be it DC resistance or AC resistance, what the gain effect is, what corona loss is, and how we evaluate inductance using magnetic flux linkages. So, we will continue with a similar discussion and understand how line capacitance can be evaluated for the given three-phase transmission lines, which is the eventual purpose. So, as I mentioned, we had finished up with our discussion of the inductance of three-phase lines in the previous lecture, and similar to inductance, which is mostly attributed to a current-carrying conductor reciprocating out magnetic flux lines. In similar lines, the same current-carrying conductor also emanates electric flux lines or field lines.

So if magnetic flux lines or field lines are associated with inductance, the corresponding electric field lines or flux lines and their interactions would probably lead to capacitance. So from a layman's perspective, capacitance is a property or attribute defined or shown by two conducting mediums that are electrically charged, definitely having some nonzero potential difference between them, which indicates that there is some electric field between these two conducting mediums. And these two conducting mediums are not electrically in contact with each other. In fact, there is a non-conducting medium present between these two conducting bodies, which are charged and have unequal charges on them, resulting in some potential difference between them.

So this non-conducting medium could be an insulator or any other device that is not a good conductor. So if we have an arrangement with two electric plates that are spaced closely together, and if we charge these two plates, the air in between them can also act as an insulator, and eventually these two electric plates will exhibit the attribute of capacitance. So that's the basic layman's definition. Air between two transmission lines or air between a transmission line and the earth also leads to a similar arrangement where we have two charged bodies or charged plates separated by air as an insulator between them. So that is essentially the reason why transmission lines also show the attribute of capacitance.

Capacitance, in its very basic essence, has the unit of farads, which is denoted by capital F. It is nothing but the per unit charge with respect to the voltage. So if the charge is in coulombs, C , and the voltage is V , the symbol is V ; I'll set the volts. So $C$ is nothing but $Q$ divided by $V$. And $Q$ and $V$ here can be instantaneous quantities, time-varying quantities, phasors, etc.

So, from an electrode or capacitor electrode perspective, $Q$ is nothing but the charge in coulombs, and $V$ is the potential that this capacitor can handle between these two electrodes. And as I mentioned, $Q$ and $V$ can be instantaneous values; they can be DC or AC phasors, depending on what type of material element is attributing this capacitance property. So our entire discussion on line capacitance evaluation would focus on this basic evaluation, and in fact, for mediums that have absolute permittivity, unlike permeability for magnetic fields, we define permittivity for electric fields or components that attribute capacitance. So if a medium has constant permittivity and $Q$ and $V$ are phasors. So, we can directly use, or I mean, $Q$ and $V$ can simply represent the RMS values in that particular notion when the medium has absolute constant permittivity.

What is permittivity? We will come to that in the upcoming few slides. So coming back to simple straight parallel conductors, the conductor shown here would be one strand in a composite conductor. We will take up our discussion gradually; I mean with simple assumptions, and then we will gradually address the case of multiple composite conductors, bundle conductors, etc. The way we did inductance evaluation for a single strand, a group of strands, a group of conductors, and so on. So what we have here is a single solid conductor, not a composite conductor, which analogously represents an overhead transmission line.

It is straight, it is perfectly cylindrical, and its radius is small $r$, which is this radius marked over here, and this conductor is carrying current. Now, since this conductor is carrying current, the current flow, if I were to associate or understand current flow in its basic definition, is nothing but the rate of flow of charges with respect to a particular time. So basically $I$ is proportional to or equal to $dQ/dt$. If current is flowing, which is usually the case for overhead transmission lines, and the transmission lines are usually carrying AC currents because we have the AC network in place. So, if current is varying in time, it is definite for sure that if $I$ is varying with time, which is proportional to or equal to $dQ$ by $dt$, then the corresponding $dQ$ by $dt$ would also vary with time.

So, it is imperative that $Q$ is also a function of time, if not a function of time, but at least it would remain constant. So, if current is flowing, then it can analogously be represented as the flow of positive charges, which is the usual theoretical definition of the flow of current.

The direction of flow of current is the same as the direction of flow of positive charges inside the conductor. That is a different aspect: positive charges actually do not flow. It is the electrons that tend to dissipate around their mean position, and that energy, or electrical energy, is transferred; but we will not go into that.

We will assume, for the sake of discussion, that there is some conductor carrying current $i$, and if this current $i$ exists, that means some charge will also exist in that particular conductor for this current to exist. So we would assume that this charge, which is present in the conductor because of this current $i$, is denoted as an instant or a positive charge with small $q$ as a notation, and C being the unit of charge, which is Coulomb. And our assumptions then again come in the way we took our assumptions for inductance, that there are no nearby conductors and no nearby charges that are perturbing or disturbing the charge distribution here. So, basically, this charge is uniformly present. In fact, charges do not actually reside inside the conductor.

If it is a perfect conductor, there would not be any charge inside the conductor. The entire charge plus $Q$ would reside at the surface of the conductor, not outside, but just at the periphery of this conductor, which is, let us say, plus $Q$. And the quantum of this plus $Q$, which is present on the surface, is effectively equal to the entire small $q$. There are no nearby charges that are sort of perturbing this distribution of charge on the surface of the conductor. The conductor is a perfect conductor, with all charges residing perfectly at the periphery.

As a result of this charge, there would also be some electric field lines. These electric field lines are indicated by the red-colored arrows that you see. And it's the attribute of these electric field lines that if we have a perfect cylindrical conductor, and I mean it's a perfect surface, a cylindrical surface, the electric field lines would exist outside the conductor, which I have talked about quite a bit in the previous discussions. Unlike magnetic flux lines, electric flux lines or field lines will not exist inside a perfect conductor; they will always exist outside the conductor because the charges reside on the surface; charges don't reside inside the conductor, unlike currents. Electric field lines would be outside the conductor, as shown over here, and these electric field lines would be all radial and orthogonal since the charge density on the surface of the conductor is uniform.

Any point that I take or tend to observe the electric field lines, since the charge distribution is uniform, shows no distortion happening because of nearby charges. So electric field intensity, which is represented by this capital $E$, is also going to be uniform around any point on the periphery, on the outside of the conductor. An electric field intensity unit is volt per meter. So, with this electric field definition, let's now gradually build upon how capacitance can be evaluated. So if electric field lines exist outside the conductor, then we can also mark certain surfaces that we would call equipotential surfaces, marked by the blue dotted circles.

These equipotential surfaces mean that if I have to measure the potential at any point on this smaller circle, then no matter where this point is taken on this smaller circle, I can measure the potential difference with respect to some reference, because potential difference, as the word difference implies, can exist between two points; it cannot exist only for one absolute point. If one absolute point potential has to be measured, then it has to be measured with respect to some reference, ground, neutral point, etc., etc. So when I say that I am trying to measure the potential at any point on this smaller blue dotted circle, then at any point on this blue dotted circle, the potential value would be perfectly the same. It would be exactly the same, and the same would also be true for the larger circle where the potential difference value might be different, but the quantum of $V$ present over this surface would exactly be the same.

So, if I were to say, let us say this is $V_2$ and this is $V_1$, then $V_1$ and $V_2$ need not be the same; they have to be equal, but they are unequal, and pardon me, they are unequal. The surfaces are called equipotential surfaces because these equipotential surfaces represent sections or surfaces that are orthogonal to the electric field lines. And as I mentioned, if the electric field lines' intensity or quantum is the same, it is because the charge distribution on the conductor's surface is the same. So no matter which point I take, the electric field lines have to be the same. So if I take any surface that is orthogonal to this electric field line over here, or if I take the orthogonal surface at the same distance from the center of the conductor, let us say this is $R_1$, this is also $R_1$; $R_1$ is different from $R$.

If I am trying to measure or find an orthogonal surface at the same distance from the center of the conductor along two electric field lines where the electric field lines are equal in quantity and the directions are different, then since the field lines have the same magnitude and the same intensity, that would mean that these orthogonal surfaces and their relative parameters would also be the same. And in terms of the capacitance of electric field lines, that orthogonal surface is nothing but the potential or the voltage itself. So if these two surfaces were to have the same attributes, that would mean that their voltages with respect to a common reference would also be the same. And that is the reason why if I draw any concentric circle around the charged conductor, these surfaces would all represent the blue-colored circles, which are equipotential surfaces. So we consider one such equipotential surface at a distance $x$ from the center of the conductor.

So basically, this green circle has a radius of $x$, where $x$ is definitely more than $r$, small $r$ being the radius of the charged conductor, and we want to find what the potential electric flux intensity or field intensity at this surface could be. If we know the quantum of $E$ that is present at a distance $x$ from the center of the conductor, then probably we can also find the corresponding value of the potential again with respect to some reference. So that is the basic idea; basically, we want to find what the value of $E$ at a distance $x$ from the center of the conductor is. So, in order to find this electric field intensity, we choose an elemental or small surface marked by this element $dS$; $dS$ is the surface area or elemental surface area along this cylindrical surface that we are considering, the cylindrical surface being the cylinder with a cross section of radius $x$ and the length of the cylinder being the same as the length of the conductor. So that is why this green dotted line also represents a cylindrical surface; if it were to be drawn on a 3D plot, then it would represent a cylindrical surface that is parallel to the cross-sectional length of the charged conductor.

That's the idea. So along the cylindrical surface, we want to find what the electric field intensity is at the surface of this equipotential surface. So $dS$ is the corresponding elemental normal area vector for the surface area. As per the dots that have been observed and the directions that you can easily see, it appears that the $dS$ vector is parallel to the electric field intensity vectors, basically $E$, and $dS$ are parallel vectors; if we recall that for two parallel vectors, if one takes their corresponding dot product, it is nothing but the magnitude of $E$ times the magnitude of $dS$ times the cosine of the angle between these two vectors. If there are two parallel vectors, the corresponding angle is perfectly zero degrees, which means the dot product of the two vectors would simply be the corresponding magnitudes of the respective vectors. So with that, because essentially we'll be using this dot product, I'm coming to that part in a moment.

So, coincidentally, $E$ and $dS$ are both orthogonal to the surface of radius $x$ that we have considered. So the corresponding dot product becomes a simple scalar quantity. Please don't confuse this cross with the cross product. It is simply a multiplication factor or represents the product of the magnitudes of $E$ and $dS$. So with $E$ and $dS$ being parallel, $E \cdot dS$ resulting in a scalar, there comes the beauty of Gauss's law.

Where the law says that along a surface or across a surface, if one has to figure out the net electric flux that is emerging from or protruding out of the surface, assuming that the surface contains a charged conductor inside it, the surface itself need not be charged, but there is some charged body inside that particular surface along which this electric field line or flux line is to be considered. Then, by Gauss's law, this electric flux line, which is nothing but the surface area or the surface integral of $E \cdot dS$, is equal to or proportional to the charge enclosed inside this surface. That's the beauty of Gauss's law. It doesn't say that this charge has to be at a particular location. Inside this surface, this charge could be present at the center.

This charge could be present at any other random point inside the surface, but not on the surface because we're trying to find Gauss's law. In fact, it can tend to be at the boundary of the surface, but again, the surface itself should not be charged because we are not considering those specific cases. So inside this surface, this charge $Q$, instead of being present over here, could have been present over here; it could be present over here at any point. It need not be present at the center. So with this, since $E_x \cdot dS$ is nothing but $E_x$ times

$dS$ simply, and $E$ being uniform or $E_x$ to be precise, $E_x$... The $E_x$ is uniform at any point I take along this cylindrical surface of radius $x$. So this surface integral can be simplified as the surface integral of $E_x$ into $dS$. $E_x$ is the same along any point I take on the cylindrical surface of radius $x$. So this can also be simplified as $E_x$ with the integral of $dS$. $E_s$ is a constant. It does not depend on the integral. So, $E_s$ can come out of it, and $Q$ is the net charge that is present over here. $\varepsilon_0$ is the absolute permittivity, a universal constant of free space, whose value is $8.854 \times 10^{-12}$ farads per meter. $\varepsilon_r$ is the relative permittivity of the non-conducting medium that is segregating the charged surface, the charged body, and the surface.

So since we are talking about a transmission line, specifically over a transmission line, you would find an ample amount of air. So $\varepsilon_r$ here indicates the relative permittivity of air, which is equal to 1 because air is not a good conductor. And since the value of $E_x$ is uniform, this integral simplifies into this form, where if we now focus on what this integral is, then this integral is nothing but the surface area or the curved surface area of the cylindrical surface we have chosen. Again, remember we are trying to find all our parameters with respect to the per unit length of the conductor. So if the conductor length is equal to 1 , it could be 1 meter, 1 kilometer, or 1 mile depending on what units we have considered for distance.

We are using it to be 1 meter by default for the sake of our discussion. The conductor length is 1 meter, and if we remember, the curved surface area of a cylinder is the circumference of the circle, which is represented in the curved surface area multiplied by the length of the cylinder. Circumference is essentially $2\pi x$ because this circle is a perfect circle with radius $x$. So $2\pi x$ is the circumference of the circle, and 1 is the length of the connector along which the entire surface area is to be found. So this surface area gets simplified into this, and now once we have the $dS$ elemental vector.

$$\oint_S \oint \mathbf{E}_x dS = E_x \oint_S \oint dS = E_x \times (2\pi x \cdot 1) = \frac{q}{\varepsilon_0}$$

So our $E_x$ is a function of $xq$ divided by $2\pi\varepsilon_0$ times $x$v volts per meter squared. Once we know what this electric field intensity is at the surface of radius $X$, then we can probably understand what this potential is that we are trying to find. Because once we know the potential, $C$ is equal to $Q$ divided by $V$. So once we know the potential, then we can probably also find the value of $C$, which exists because of this charge $Q$. So the potential difference, in its basic definition or layman's perspective, is nothing but the work done to move a unit charge; the unit charge could be one coulomb, one microcoulomb, or one nanocoulomb depending on what unit is to be considered.

It is the work done to move a unit charge in an electric field, in the presence of an electric field. So basically, if I were to explain, if I have two electric plates that are charged and air is in between these two plates, then if I intentionally capture a positive charge of, let us say, one coulomb and move it from one plate to the other plate, in between which there is an electric field present, then the amount of energy dissipated or energy used to move this charge from one point to another between these two charged plates in the presence of an electric field would be nothing but the potential difference between these plates. So here, $v_{ij}$ means that $i$ and $j$ are two distinct points between which the potential is to be measured, and between these points, $i$ and $j$, there definitely exists a non-zero electric field, and we are trying to understand how much work is being done to move a unit charge from point $i$ to point $j$ in the presence of an electric field. If we were to overlap this discussion on the conductor that we have, then essentially we could choose two surfaces, two different equipotential surfaces, and that would give us essentially the potential difference between these two equipotential surfaces. And in this context, the electric field that we are talking about for potential difference to exist is because of the electric field intensity $E$, which is the force per unit charge, which we have evaluated here.

So, if we know $E_x$, then we can also probably find the potential difference. So, in order to find the potential difference, what we do is take a hollow cylindrical tube whose width is $dx$, and this $dx$ hollow tube is at a distance $x$ from the center of this charged conductor. As I mentioned, or as is told over here, or discussed over here, the potential difference is the work done to move the unit charge, so essentially. The incremental potential $dV$ is nothing but the product of $E_x$, which is the electric field intensity, multiplied by the width of the tube across which the potential is to be found. So, since we know $E_x$, we substitute $E_x$, which is $Q$ by $2\pi x, \varepsilon_0$. $dX$ is the width along which the potential is to be found. Essentially, the charge is moving along this $dX$ tube, and the work that would be done is equal to the field intensity multiplied by the shortest displacement, or the width along which the charge is moving. It might happen that the charge is residing at the surface of $x$, let's say over here, and this charge, in order to move from this point or this surface to a surface whose width is $x$ plus $dx$, could move along this small hollow tube through a lot of distances covered, but essentially it reaches this point over here. No matter what the distance traveled by the corresponding charge is, the electric field intensity multiplied by the width, which is the shortest displacement or the shortest distance, to be precise, displacement is the actual shortest distance; it would depend on the shortest displacement that would exist between these two surfaces. So that is the reason why $dV$ is equal to $E_x$ times $dx$, and now once we know what $dV$ is, we can take two such points or two such potential surfaces; we mark two points $D_1$ and $D_2$.

There is a charge that has to move from point $D_1$ to point $D_2$. As I was saying, it could directly go from here to here, with the charge moving in the presence of the electric field, or it can take any particular path in this Fagner and then also reach $D_2$. So no matter what distance is covered, it is always the shortest displacement between these two potential surfaces. These two distances, potential surfaces, and the radii are $D_1$ and $D_2$. So essentially, the points $D_1$ and $D_2$ indicate that they are at a distance $D_1$ from the center of the conductor, and the $D_2$ point is again at a distance $D_2$ from the center of the conductor.

And since these points are along two individual equipotential surfaces, the actual positioning of $D_1$ and $D_2$ doesn't matter. It is the radius of these equipotential surfaces between which the potential is found. So if $dV$ is the incremental or unit potential difference and we have to find the potential between $D_1$ and $D_2$, which is also equal to $V_{D_2}$ minus $V_{D_1}$, in certain textbooks or references you may also find this to be equal to $V_{D_2}$ minus $V_{D_1}$, but essentially no matter what the context is, we are trying to find the potential difference between two surfaces, and that is the reason why there are two voltages for the $D_2$ surface and the $D_1$ surface. It would be nothing but equal to the integral of $dV$, and when we substitute $dV$ in terms of $x$ and $dx$, we have this expression of the potential, which is $q$ by $2\pi\varepsilon_0$ natural log of $D_2$ by $D_1$. Now, once we know what the potential difference is, it is, as I mentioned, independent of the path traversed by the unit charge to move from two points on two equipotential surfaces.

$$V_{D_1D_2} = \int_{V_{D_1}}^{V_{D_2}} dV = \int_{D_1}^{D_2} E_x dx = \frac{q}{2\pi\epsilon_0}\ln\left(\frac{D_2}{D_1}\right)$$

It is equal; it actually depends on the shortest equipotential displacement between these points, not the actual distance. And as you can recollect, $C$ was $Q$ by $V$. $Q$ is the reason why all this exists. So once we know these two points, $D_1$ and $D_2$, the capacitance between these points $D_1$ and $D_2$ would be $Q$ divided by the potential difference, which is this beautiful expression in terms of farads per meter. Why do we have farads per meter? Remember, we had considered the conductor length to be one unit.

$$C_{D_1D_2} = \frac{q}{V_{D_1D_2}} = \frac{2\pi\varepsilon_0}{\ln{(D_2/D_1)}} F/m$$

So now, if point $D_1$ is exactly same or is residing on the surface of the conductor, Remember inside the conductor charges don't exist. This charge would exist on the periphery of the conductor. So if we have to move inside the conductor, then, as I mentioned, charges will not exist inside the conductor. Charges will always exist on the periphery.

They will always exist on the surface of the conductor. I think I have erased that particular discussion. So if the charges exist inside, not inside the conductor, and they have to be present on the surface, if I have to find the potential inside the conductor, then according to Gauss's law, which states that the net electric flux emanating out of a particular surface is equal to or proportional to the charge present inside that surface. So inside a conductor, since charges won't exist, there would not be any potential or electric flux lines at all along this surface. The worst-case point that $D_1$ can take is the surface of the conductor itself, which is at a distance $R$ from the center of the conductor. And $D_2$ could be any point in general that is at a distance $D$ from the center of the conductor.

So in that case, the effective capacitance at a distance $D$ from the center of the conductor would be this particular expression where $D_1$ is equal to $R$ and $D_2$ is equal to $D$. And remember, or recollect, that for inductance evaluation, we had seen the effect of the skin effect. Over here for capacitance evaluation, there is no deterioration or change in the effective radius of the conductor, and hence capacitance results in the skin effect. It's only the inductance effect that leads to the skin effect and increases AC resistance. So now, if we move on and try to complicate our discussion gradually, we have understood the capacitance evaluation for a single conductor.

- If $D_1 = r$ and $D_2 = D$, then effective capacitance at any point $D$ outside the conductor is

$$C_{rD} = \frac{2\pi\varepsilon_0}{\ln{(D/r)}} F/m$$

Now let's see how we evaluate capacitance for a single-phase two-wire system. Remember the discussion that we had for inductance evaluation? A similar logic would be applicable here. We have two charged conductors; we have two current-carrying conductors. Essentially, one conductor is acting as a return path for the other. If we associate this in terms of the charges, if $Q_1$ is existing on one conductor, at the same point in time, the other conductor should have the negative charge of it, and that's how the current balancing would happen because it's a single-phase two-wire system where one wire is acting as a return path for the other conductor.

So the distance $D$ between these two charged conductors is very high compared to the individual radii $R_1$ and $R_2$. The electric field distortions are not occurring. $Q_1$ and $Q_2$ are perfectly distributed along their respective surfaces. There is no distortion in this charge density or surface here.

The effect of earth or ground is neglected for the time being. So, if we apply the similar logic of finding potential, which is $V_{D_1D_2}$, the one which we discussed over here, and try to understand the potential difference between two such charges. Since our focus is to find the potential between two points, point number one is the center of conductor, and the other point is point number two, which is the center of the conductor of charge $Q_2$. We have to find the potential difference between points one and two, which are the centers of the conductors because of charges $Q_1$ and $Q_2$. So if we find the potential difference due to charge $Q_1$, then there is a little bit of a suggestion here. If we choose 1 as our first point of reference for measuring displacement for point number 2, and the charge that we are looking at is charge $Q_1$, then it's obvious that since we are talking about charge $Q_1, Q_1$ would appear in the potential difference expression because there is no other charge apart from $Q_1$ in this particular expression to be discussed.

The distance that appears in the natural log term, the numerator, is always the distance between the actual position of charge $Q_1$ and the second point with which the potential is being measured. So if we talk about $Q_1$ at a distance from conductor $Q_2$, the actual distance is $D$. And so the numerator would always refer to the distance between charge $Q_1$ and the second surface, whereas the denominator and the natural log term would refer to the distance between the actual charge and the first point at which the potential is being measured. So if we just keep that aspect in mind, because that was how we evaluated our potential difference expressions, then the overall discussion would become simpler.

So we have two such charges. $Q_1$ and $Q_2$. So, we would have two such potential differences. One is $V_{12}$. Because of charge $Q_1$, Similarly, we would have $V_{21}$.

Because of charge $Q_2$. The expressions look similar. The only difference is in terms of the charges themselves. And since $Q_1$ is the negative of $Q_2$. So, $V_{12Q_2}$. Which would be the negative of $V_{21Q_2}$.

$$V_{12,q_1} = \frac{q_1}{2\pi\epsilon_0}\ln\left(\frac{D}{r_1}\right)V/m \text{ and } V_{21,q_2} = \frac{q_2}{2\pi\epsilon_0}\ln\left(\frac{D}{r_2}\right)V/m$$

However, $$V_{12,q_2} = -V_{21,q_2} = -\frac{q_2}{2\pi\epsilon_0}\ln\left(\frac{D}{r_2}\right) = \frac{q_1}{2\pi\epsilon_0}\ln\left(\frac{D}{r_2}\right)V/m$$

Why? Because $V_{12Q_1}$ is nothing but $V_{2Q_1}$ minus $V_{1Q_1}$. Similarly, $V_{21Q_2}$ is $V_{1Q_2}$ minus $V_{2Q_2}$, which is also equal to minus $V_{2Q_2}$ minus $V_{1Q_2}$, which is also equal to minus $V_{12Q_2}$. And that's how this negative term is coming in, and since $q_2$ is equal to minus $q_1, v_{1toq_2}$ is this expression itself. Since the potentials exist because of two different charges, the effective potential difference between these wires would be the additive sum of these two potentials. So if we add those two, then it would actually come as it would actually be equal to.

Just give me a moment. So let us do that. So, we have $V_{2Q_1}$ as $Q_1$ divided by $2\pi\varepsilon_0$ natural log of $D$ divided by $R_1$. We have $V_{1toQ_2}$ as $Q_1$ by $2\pi\varepsilon_0$ natural log of $D$ by $R_2$, and remember when we add these two terms where the first terms are common, the natural log terms, the arguments of natural logs, they get multiplied, and that's how you get $D^2$ by $R_1R_2$. Once we know $V_{12}$, we can use the same capacitance evaluation because the effective charge is $Q_1$ or $Q_2$, which is the negative of $Q_1$. So $C_{12}$ is $Q_1$ divided by $V_{12}$, which gives us this expression in terms of farads per meter. Often it is convenient to find capacitance not between line to line but between line and phase or phase and the neutral point. ¿ Hence, effective potential difference between wi to their charges is

$$V_{12} = V_{12,q_1} + V_{12,q_2} = \frac{q_1}{2\pi\varepsilon_0} \ln \left(\frac{D^2}{r_1 r_2}\right) V/m$$

$$C_{12} = \frac{q_1}{V_{12}}$$

So if we look at this particular arrangement of $Q_1$ and $Q_2$. And the distance is since $Q_1$ is equal to minus $Q_2$. So for $Q_1$, if I were to draw the electric field intensity line, then it would be orthogonal to this surface, whose quantum, let us say, would be $E_1$, and similarly, I would have the $E_2$ electric field coming out, which would be $E_2$, and these are all orthogonal. Since $Q_1$ is the negative of $Q_2$, it would mean that $E_1$ would be the negative of $E_2$. So if I talk about a point that is at the midpoint between these two surfaces, the effective electric field intensities cancel out. So there comes a point between these two current-carrying charges or current-carrying conductors where the potential difference would be perfectly zero, and we can denote that point as a neutral point or a zero potential point.

And remember, unlike resistance and inductance, when they are in series, their corresponding sums or individual inductances are added up. If they are in parallel, then the effective inductance is the inverse of the sum of the inverses of the inductances. The similar vice versa logic is applicable to capacitances. If I have two capacitances in series, then the effective capacitance in terms of $C_{12}$ would be 2 times $C_{12}$. Since $C_{12}$ is $\pi$ times $\varepsilon_0$, our capacitance or phase capacitance is this particular expression, which is $2\pi\varepsilon_0$ divided by the natural log of $d$ divided by the square root of $R_1 R_2$ farads per meter.

Coming to a much more complicated system, which is a multi-connector system, where we have N conductors carrying currents or carrying different charges. These charges could differ from $Q_1$ to $Q_n$. Their radii are also different from $R_1$ to $R_n$. And these conductors are in large spaces. The space between them is particularly large, so there is no distortion of charge on the surfaces of individual conductors.

There is no distortion in terms of electric field. And the distances are marked as $D_{12}, D_{21}, D_{2j}$, etc. We want to find the potential difference between the center of charge conductor I and the center of charge conductor J. We want to find the potential difference between these two points I and J with respect to all possible charges that are present inside this particular arrangement. So if you remember, if I were to find $V_{ij}$ with respect to $Q_1, Q_1$ being the single sole charge that would come over here, and around these connectors, there is all air; there is no other medium, so $\varepsilon_r$ is perfectly one. In the natural log term, the numerator should be the distance between the position of charge $Q_1$ with respect to the center of charge, the center of the conductor, which is carrying charge $Q_j$.

So the first distance that should come is $d_{1j}$, which is the term present over here, and the denominator should be the distance between $q_1$ and the conductor, which is numbered i , and that's how $d_{1i}$ comes in, which is a denominator over here. If we apply the same logic, we can also find the potentials with respect to other charges. For charge i or j, $d_{ii}$ and $d_{jj}$ could refer to $r_i$ or $r_j$, respectively; they are the self-radii. And with all these individual potentials, we can find the applied superposition theorem and determine the overall effective potential, which is the summation of all individual potentials with respect to the individual charges.

$$V_{ij,q_1} = \frac{q_1}{2\pi\epsilon_0} \ln\left(D_{1j}/D_{1i}\right) V/m$$
$$V_{ij,q_2} = \frac{q_2}{2\pi\epsilon_0} \ln\left(D_{2j}/D_{2i}\right) V/m$$

$$V_{ij,q_i} = \frac{q_i}{2\pi\epsilon_0} \ln\left(D_{ij}/D_{ii}\right) V/m$$

That's all for today's discussion. We will take up our next discussion on the capacitance of three-phase lines and the effect of earth based on the multi-conductor potential difference discussion. Thank you.