Power Network Analysis

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Week-04

Lecture-20

Lec 20: Transmission line parameters- magnetic flux linkage of a conductor in a group of conductors

Hello everyone, welcome to the last lecture of week 4 of the course Power Network Analysis, in which we will continue our discussion on the fourth main module, which is Transmission Line Parameter Evaluation, and in today's discussion, we will try to find expressions for. Magnetic flux linkages of a conductor with respect to a point because of a group of conductors. So, essentially, what we have discussed until the previous lecture or lectures is the evaluation of inductance due to internal flux lines and external flux lines, basically for points outside the conductor and inside the current-carrying conductor. So, we have evaluated or figured out the expression for inductance for a single conductor with respect to internal magnetic flux lines as well as external magnetic flux lines. What we would take up or expand further is how you would find the inductance of actual transmission lines, which need not be just one conductor. We have taken or understood one example or initial lecture of this module; we understood about ACSR conductors, aluminium conductors with steel reinforcement, and aluminium or ACSR conductors that have strands of steel and strands of aluminium wound around the steel strands to increase mechanical strength.

What we have found so far or evaluated so far is probably the inductance of a single strand or a single conductor, that too under certain assumptions. So, let us see how, in a group of conductors or a group of strands, the inductance expressions or flux linkage expressions could be well understood, because once we know the value of lambda, we can also jump to or evaluate the corresponding value of inductance. So it could need not only to be flux linkage; we can also try to find expressions for stored magnetic energy E, and from there we can also find L. So we will probably keep our focus on trying to find this lambda, from which we can find the corresponding Ls.

So let us go over what we have. So we will try to sort of complicate the case one by one. So earlier, our focus was only on one conductor or one strand. Now I am bringing in another conductor that could act as a return conductor for the actual conductor. So the return conductor is carrying current I2; the actual conductor might carry current I1, and I1 and I2 have to be in different directions because one conductor is acting as the return for the other conductor.

So essentially, at any point in time, I2 is always the negative of I1, and that is how balanced operation between a single-phase two-wire circuit can be ensured. The directions of these currents might change if they are AC, or they may remain the same, as shown here: I1 is entering the plane of the slide, whereas I2 is coming out of the plane of the slide if these are DC currents. And the individual conductors, wires, or strands have radii R1 and R2, and the center-to-center distance is capital D. And the connectors are so spaced or so placed that this distance d is exceptionally large or higher compared to individual resistances, pardon me for the word resistance, their individual radii. So, our idealistic assumptions still hold true: the magnetic flux lines remain concentric, their distribution remains uniform, and the current density in the respective wires also remains uniform.

Although the figure shows some distortion, for the sake of simplicity in our discussion, we will not focus on these distortions for now. We will still assume these flux lines to be concentric and that the two conductors are at a large distance. So, these magnetic flux lines near the conductor in the space outside the conductor do not get distorted; they still remain concentric. And as I mentioned, the current densities are also assumed to be uniform because of this idealistic condition and the flux lines due to currents I1 and I2, as per the symbols or the flows that we have, since I1 is into the plane of the slide. So, if by the right-hand rule the thumb points into the plane of the slide, the fingers would curl in a clockwise direction; the corresponding magnetic flux lines are actually clockwise, whereas for I2, the same flux lines appear to be in an anticlockwise direction.

So, if we look at the space in between these two conductors, the magnetic flux lines because of the individual conductors carrying I1 and I2 are additive in this space between the radii of the conductors at distance d. So, we will make use of this fact towards the end of today's discussion that in the space between these individual conductors, the magnetic flux lines are additive, whereas in the space apart from the conductors, the corresponding flux lines are not additive. So, if you focus on this part here from the space outside the region where we are looking outside the two radii or two connector spaces placed at distance d, the corresponding flux lines need not be in the same position. So, by Ampere's law, if we draw a surface that is at a distance of d plus r2 more than the center of conductor 1, whose radius is r1. So, basically what we are trying to say is if I draw a surface, let me mark the distance of that surface; that distance is given as d center to center, and this additional distance is r2.

If I draw a surface that is just touching the extreme rightmost surface of a conductor with radius r2 and this surface is again concentric. Thanks to our concentric magnetic flux lines, the surface is concentric and could be considered a circle—the dotted circle that I have drawn, whose radius is D plus R2, from the center of the conductor carrying current I1. And on this surface, if I were to apply Ampere's law, it simply states that the integral of the dot product of magnetic flux intensity and the length vector DL should be equal to the net current that is encompassed within this surface. Now, let us look at this surface, which I have considered to be a circle. Inside this circle, there are going to be two conductors: conductor number 1 is carrying current I1, conductor number 2 is carrying current I2, and since it is a single-phase two-wire circuit, I1 plus I2 is always 0 because I2 is the negative of I1.

So, inside this surface, even though there are two conductors, the net current at any point in time inside the surface is always 0. So, if the MMF itself is 0, that means no matter what distance I take at a distance D plus R2 or more than that, the corresponding Ampere's law simply says that no meaningful magnetic flux interactions are going to happen for the single-phase two-wire circuit. So, that is the reason why it is mentioned that the flux line set up by current I1 at a distance D plus R2 from the center of the conductor does not link conductor 2 because the net current inside this surface is always 0 due to I1 plus I2 being equal to 0. The same could also be said for flux lines with respect to current I2 at a distance D plus R1 from the center of conductor I2; the corresponding net current will again be 0, so no MMF would actually result. So, only at a distance less than d, since we are assuming the current densities to be uniform on both surfaces, and also d is much greater than the individual radii.

So essentially, at a distance of around D, or to be precise, D plus R, to less than that particular distance, the flux lines of conductor 1 are linking the current I2 in some fashion, in some manner, as per Ampere's law. And since d plus r2 or d minus r2 is approximately equal to d because d is much, much larger than r2 and r1. So, that is where this distance d comes into consideration. So, if we apply the logic of net inductance at a point D from the center of a conductor carrying current I1 with radius R1, then the inductance between the point, which is the center of the conductor, let us say point A at a distance B, which is just outside the surface of the conductor carrying current I2, would be nothing but d l1 is equal to 2 times 10 to the power of minus 7 as per the expression that we saw in the previous lecture, considering that mu r is equal to 1 because between these two spaces we essentially have air or vacuum. The relative permeability of air is equal to 1. So, we have this simple expression, which is the inductance of the circuit due to current I1.

$$L_1 = 2 \times 10^{-7} \ln\left(\frac{D}{r_1'}\right) H/m$$

$$r_1' = r_1 e^{-\mu_r/4}$$

Similar inductance will also exist because of current I2; these inductance expressions are independent of current.

$$L_2 = 2 \times 10^{-7} \ln\left(\frac{D}{r_2'}\right) H/m$$

$$r_2' = r_2 e^{-\mu_r/4}$$

So, their values do not affect, but individual radii yes they might be the same, they might be different, and μr here is the relative permeability of the conductor, not the air; for air, the relative permeability is exactly equal to 1. And since the space between these two conductors, the MMFs or the flux lines are perfectly additive, as we have discussed earlier. So, we can find the overall inductance, which is the addition of L1 and L2 for the entire circuit, which is the single-phase two-wire circuit, and we get this expression as our overall expression.

$$L = L_1 + L_2 = 4 \times 10^{-7} \ln\left(\frac{D}{\sqrt{r_1' r_2'}}\right) H/m$$

How do I get this expression? It is pretty simple. So, if I have to sort of evaluate or cross-check this, then l1 plus l2 is ln of 2 times 10 to the power of minus 7 plus the natural log of d over r1 dash plus the natural log of d over r2 dash, with the entire factor of 2 times 10 to the power of minus 7 outside. This can be rewritten as 2 multiplied by 10 to the power of minus 7; the sum of two logs with a common base is also equal to the log of the product of these two arguments. So, basically, I have d squared divided by r1 dash into r2 dash, which can also be rewritten as 4 into 10 to the power of minus 7 divided by one-half natural log of d squared by r1 dash r2 dash. This term is nothing but the term present over here, and 4 into 10 to the power of minus 7 is the term present over here.

That is how we get our overall inductance for the single-phase 2-wire circuit in henries per meter because we are considering the length of these 2 conductors on a per unit length basis. Essentially, it is equal to 1 meter if the per-unit length is 1 kilometer or 1 mile; then the corresponding distance units could be kilometers or miles, depending on the line per-unit length that we are considering. Moving on to a set of conductors, we are basically complicating the situation more and more; earlier, we focused on only two strands or conductors. Now we are focusing on n number of strands or n number of conductors because actual ACSR conductors are made up of different strands, and the number need not be only one; it can be more than one. And since these strands in this set of strands of conductors, we have the assumption that at least one of those conductors is acting as a return conductor for the entire circuit.

So, that is the reason that in the previous discussion of the single-phase tube wire circuit, I1 plus I2 was equal to 0 at every point in time in a group of conductors, which we are not specifically pinpointing. Which conductor is acting as a return conductor? The overall sum of all these instantaneous currents, whether AC or DC, is always zero. So there may be one or more conductors that could be acting as return conductors for this entire group of conductors. So, given this condition, let us try to find the flux linkage of one conductor with respect to a point P because of all these individual current-carrying conductors carrying currents I1, I2, IN; their radii might also be different, which could be R1, R2, up to Rn. For the sake of our discussion, we will have a simple notation, and we will again reuse this aspect that the overall flux linkage we have seen; this expression is nothing but mu naught i divided by 2 pi times the natural log of d2 divided by d1, where d2 and d1 are two points outside a current-carrying conductor of current strength i.

$$\lambda = \frac{\mu_0 I}{2\pi} \ln\left(\frac{D_2}{D_1}\right) wbt/m$$

So, we will reuse this and try to find the corresponding flux linkages. So, let us do that one by one. So, if I have a point P that is at a distance d1P from the center of a conductor carrying current I1, it is also at a distance d2P from the center of a conductor carrying current I2, similarly d3P, d4P, and so on until dnP. Where I1 plus I2 plus I3 up to IN are all 0 at any point in time, I focus only on one conductor at a time. So, let us focus only on current I1; for current I1, we have seen that the net inductance or net flux linkage, basically looking at flux linkages here, net flux linkage at a point P from the center of the conductor, which is the distance d1p due to current I1, is nothing but this expression here, where λ1p essentially indicates the current.

$$\lambda_{P,1} = \frac{\mu_0 I_1}{2\pi} \ln\left(\frac{D_{1,P}}{r_1'}\right) wbt/m$$

$$r_1' = r_1 e^{-\mu_r/4}$$

Indicates the points, one of which represents the center of the conductor carrying current I1; P is the point of concern with respect to which the flux linkage is being evaluated, and comma 1 could become comma 2, comma 3, comma n, indicating that the flux linkage we are evaluating is due to this current. So, comma 1 here indicates that this flux linkage is due to current I1. And the remaining terms, I believe, are pretty obvious or understood by now since we are talking about individual conductors. So, the current I1 and the current center of conductor I1 are rated by r1 dash, where r1 dash is r1 nu mu 2 to the power of minus mu r divided by 4, mu r being the relative permeability of the conductor carrying current I1. So, similarly, if we now try to find lambda 1 p 2.

So, going by the same expression given over here, if we have to find lambda 1 P, 2, it would be nothing but mu naught I 2 by 2 pi natural log of the denominator distance, which is the distance between the first number sitting over here and, pardon me, the first distance in the natural log denominator; 1, P, 2 is the distance between the second point P. and the current which is responsible. So, basically, this would be d^2p divided by the distance between the first current, the first number, and the second current, I_2. So, basically, it would be d21. Now, d21 is the center-to-center distance between these connectors, which is mentioned here. So if we go on and try to find flux linkages due to other current-carrying conductors, these will be the individual expressions that we will get.

$$\lambda_{1,P,2} = \frac{\mu_0 I_2}{2\pi} \ln\left(\frac{D_{2P}}{D_{21}}\right) wb/m,$$

$$\lambda_{1P,x} = \frac{\mu_0 I_n}{2\pi} \ln\left(\frac{D_{nP}}{D_{n1}}\right) wb/m,$$

And since our focus is on the center of the conductor carrying current I1 and point P, we can add the individual flux linkages because these are actually components of the same flux linkage due to different currents. So, if you want to find the total flux linkage, we can add up these flux linkages, and since we know that the instantaneous sum of all currents is equal to 0, this is a net 0 arrangement of conductors. So, if we reuse this expression, or basically if we replace I of n with this negative sum present over here, then we get two distinct terms or two distinct components. So let us look at those two distinct components.

So what I have done here is keep the first two expressions the same, and I have broken up their corresponding natural logarithm numerators and denominators separately. So essentially I have one term, which is the sum of all natural log numerators with their respective currents. The other sum is basically, since the log of 1 by r is equal to the log of 1 minus the log of r, with the log of r being 0, so this term is also minus the log of r. So the corresponding natural log denominators are represented by this expression here, which consists of the denominator terms with their respective current components, and from here I replace I of n with minus I1 plus I2, continuing until I of n minus 1. So if I put that or do that, what do I get? I am making this substitution only for this term.

So, if I do that, then if I focus only on the first addition, I will have I_1 log of d_1p, and the other term, which would come from over here, is minus I_1 log of d_np. I can combine these two and write it as the natural log of (D_1P / D_nP), which is what you are seeing over here. Similar terms could also apply to other expressions, and in this expression, what we see is that after substituting this current IN into the expressions, we are now obtaining ratios of distances of point P from individual centers. Now think of the fact that if this point P tends towards infinity, it means point P is being stretched out and is going far off from this connector arrangement. Don't you think that the individual distances D1P, D2P, D3P, and DNP would more or less tend to be equal to each other? If not equal, their differences would become marginally small because point P is very far off from this arrangement of conductors.

$$\lambda_{1P} = \frac{\mu_0}{2\pi}\left\{I_1 \ln\left(\frac{D_{1P}}{r_1'}\right) + I_2 \ln\left(\frac{D_{2P}}{D_{21}}\right) + \ldots + I_n \ln\left(\frac{D_{nP}}{D_{n1}}\right)\right\}$$

$$\lambda_{1P} = \frac{\mu_0}{2\pi}\left\{\begin{matrix} I_1 \ln D_{1P} + I_2 \ln D_{2P} \ldots + I_n \ln D_{nP} \\ -\left(I_1 \ln r_1' + I_2 \ln D_{21} \ldots + I_n \ln D_{n1}\right)\end{matrix}\right\}$$

$$\lambda_{1P} = \frac{\mu_0}{2\pi}\left\{\begin{matrix} I_1 \ln\left(\frac{D_{1P}}{D_{nP}}\right) + I_2 \ln\left(\frac{D_{2P}}{D_{nP}}\right) \ldots + I_{n-1} \ln\left(\frac{D_{n-1P}}{D_{nP}}\right) \\ -\left(I_1 \ln r_1' + I_2 \ln D_{21} \ldots + I_n \ln D_{n1}\right)\end{matrix}\right\}$$

So, essentially, with the point P tending to infinity, we would have this particular condition where individual distances become more or less similar. And if individual distances become more or less similar, then this number inside the natural log term would start tending to 1. In fact, all these distances would start tending to 1 individually, and we also know that the natural log of 1 is always 0. So, essentially, we would have the first terms, which are over here, all individually tending to 0, and we would only be left with the negative term sitting upside down inside, and that is how lambda 1 infinity is minus mu naught over 2 pi. Sum of individual self-radii, the distance to the center points, and the corresponding current components. This is essentially the flux linkage of one conductor in a group of conductors whose instantaneous sum of currents is always zero.

$$\lambda_{1P} = \frac{\mu_0}{2\pi}\left\{\begin{matrix} I_1 \ln\left(\frac{D_{1P}}{D_{nP}}\right) + I_2 \ln\left(\frac{D_{2P}}{D_{nP}}\right) \ldots + I_{n-1} \ln\left(\frac{D_{n-1P}}{D_{nP}}\right) \\ -\left(I_1 \ln r_1' + I_2 \ln D_{21} \ldots + I_n \ln D_{n1}\right)\end{matrix}\right\}$$

$$\lambda_{1\infty} = -\frac{\mu_0}{2\pi}\left(I_1 \ln r_1' + I_2 \ln D_{21} \ldots + I_n \ln D_{n1}\right) Wbt/m$$

Build our next discussion on the basis of this particular flux link. So, in case you have any doubt or confusion regarding this, I would request all the viewers to please revisit or go back a couple of minutes in this particular discussion to recheck whether the numbers of the expression that we are getting are actually correct. Beyond this, after this, we will build our next discussion on the expression we are getting here. So now and again, as I mentioned, similar flux linkage expressions could also be obtained for other conductors with respect to point P, tending to infinity.

Now let's go to a relatively more complicated case where we discuss composite conductors. Composite conductors are nothing but the ACSI conductors that we saw in the first lecture of this particular module, where practical transmission lines have multiple strands, and depending on the voltage level, they can have different bundles. We will discuss the bundled part later, not today, and not in this discussion. We'll focus only on stranded conductors and composite conductors. There could be several conductors that contain several strands in parallel to each other for either a single-phase circuit or a three-phase circuit; both could be applicable.

We are focusing on a single-phase arrangement where one set of composite conductors, one conductor that is composite conductor A, has different strands. Those strands are labeled from A to N, so you can also define your own label. For my reference, I'm choosing composite conductor A to carry strands whose numbers or labels could be from A, B, C to small n. Small n is the important number here. And composite conductor B also has strands that could be labeled as A-, B-, C-, and so on.

Basically, this set here is a set of n alphabets or n numbers, and similarly, composite conductor B also has different strands with different labels totaling m. And we are talking about an arrangement where these two composite conductors refer to a single-phase circuit. And that's the reason why one composite conductor is acting as the return path for the other conductor. So essentially, if I have to take the net sum of the currents of all conductors, Ia plus Ib plus Ic, which is the sum of the currents of the composite conductors present here, and add them to the currents of A dash, B dash, C dash, et cetera, the instantaneous sums would always turn out to be zero. And for the sake of simplicity in discussion, we are assuming the individual currents in composite conductor A to be equal to the carrying current I divided by N; the current distribution is uniform.

And since the sum of currents is always 0, if I is the net current in composite conductor A, then composite conductor B would carry a current of minus I because of this relationship and the current distribution being the same or uniform. So, minus I by M is the individual current in the individual strands of this composite conductor, B. Our assumptions remain the same; they are idealistic assumptions that the strands are all similar, with similar radii and diameters, magnetic flux lines that are concentric, uniform current density, no distortion occurring, and perfect concentric flux lines existing; there are no distortions. So, if we apply our logic of the flux linkage expression that we saw here to the flux linkage evaluation in this set of composite conductors A and B, where we could have different strands, we can derive the results accordingly. So the way we focus on one strand and try to evaluate inductance with respect to its own current, as well as the currents from other strands, is important.

So, if we use that, then the flux linkage would have two distinct terms. The first term would be due to the flux linkage of conductor A in composite conductor A, which would have two terms. One term would come because of the strands present in conductor A. Each of these strands is carrying current I through N. So if we have two natural logs, let's say X log of Z plus X log of Y, it is equal to X log of ZY.

So, we use this property, and that is how we get this expression. Similarly, the other composite conductor B has strands that are carrying currents of minus i over m, and the corresponding distances that you see are the pairwise distances between individual strands. So, we have the second term because of composite conductor B. If we now rearrange this a bit, this is what would be the net flux linkage of conductor A with respect to a point P, which is at infinity because of all the individual strands. And DAA is the effective self-radius of conductor A itself; thus, all connectors have the same radius, so R remains the same for all of them.Their material properties are also the same, and the relative permeability is the same.

$$\lambda_a = \frac{-\mu_0}{2\pi}\left\{\frac{I}{n}\ln(r_a' D_{ab} D_{ac} .. D_{an}) - \frac{I}{m}\ln(D_{aa'} D_{ab'} .. D_{am})\right\} Wbt/m$$

$$\Rightarrow \lambda_a = \frac{\mu_0 I}{2\pi}\left\{\ln\left(\frac{(D_{aa'} D_{ab'} .. D_{am})^{1/m}}{(D_{aa} D_{ab} .. D_{an})^{1/n}}\right)\right\}, D_{aa} = r_a' = re^{-\mu_r/4}$$

Similarly, if we now try to find the inductance expression of conductor A, which is carrying current I by N, L A would be nothing but the flux linkage divided by the actual current. So we obtain the expression for inductance here. This expression looks very complex and difficult to evaluate. Please have patience; we will first understand, and I mean this understanding is important, of how the expressions come in, and then we will find simpler ways of how actual inductance expressions can be evaluated while trying to minimize the complexity of evaluating these distances.

Taking their 1 by mth root or the mth root, 1 by m is the mth root of the expression here, whereas 1 by n is the nth root of this particular denominator product or distance product. We can also find a similar inductance expression for strand B in composite conductor A.

$$L_a = \frac{\lambda_a}{I/n} = \frac{\mu_0 n}{2\pi}\left\{\ln\left(\frac{(D_{aa'}D_{ab'}..D_{am})^{1/m}}{(D_{aa}D_{ab}..D_{an})^{1/n}}\right)\right\} H/m$$

We can also find a similar inductance expression for any other strand in this entire composite conductor, A or B. And if we start doing that, we can find, as I mentioned, the inductance of any other strand in the composite conductors A or B.

$$L_b = \frac{\mu_0 n}{2\pi}\left\{\ln\left(\frac{(D_{ba'}D_{bb'}..D_{bm})^{1/m}}{(D_{ba}D_{bb}..D_{bn})^{1/n}}\right)\right\} H/m$$

$$D_{aa} = D_{bb} = \cdots = D_{nn} = re^{-\frac{\mu_r}{4}}$$

The average inductance, if we were to sort of average out or normalize these individual inductances, would be different because the distances are going to vary.

So, for LB and for LA, the individual distances are going to be different. Because that depends on the spacing between the strands, how the ACSR connector is arranged, and how these two ACSR connectors—one return and one live—are also arranged, the distances might be different. So, individual LA, LB, and LC could also be different. In order to normalize this individual strand effect from the perspective of the entire composite conductor. What we can do is average out these individual inductances, and probably that average would be associated with the average inductance of any strand in conductor A.

Which would be the sum of the individual inductances in the strands of composite conductor A divided by the number of strands that are present in composite conductor A. So, if we substitute those expressions, the overall expression becomes even more complex and becomes much more boggling.

$$L_{av,A} = \frac{L_a + L_b + \cdots + L_n}{n} = \frac{\mu_0}{2\pi}\left\{\ln\left(\frac{\{(D_{aa'}D_{ab'}..D_{am})(D_{ba'}D_{bb'}..D_{bm})\ldots(D_{na'}D_{nb'}..D_{nm})\}^{1/m}}{\{(D_{aa}D_{ab}..D_{an})(D_{ba}D_{bb}..D_{bn})\ldots(D_{na}D_{nb}..D_{nn})\}^{1/n}}\right)\right\}$$

Please have patience; we will rebuild our discussion on how this could be simplified or if there are simpler ways of finding these complex distances, which we will definitely take up in our next discussion, in which we will start with the evaluation of inductance for actual three-phase lines, where three-phase lines may also have composite conductors and bundled conductors. And our discussion will build upon the expression that we obtained here. So please revise, recheck, or browse back to this particular video to go through the concepts clearly once again.

Thank you.