Power Network Analysis

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Week – 01

Lecture-02

Hello welcome to the second lecture of week one on power network analysis. In this lecture we will start with the first module of this course which is basic principles of circuits. Specifically we will be talking about or discussing about phasor diagrams. which lays the premise for understanding the different models specifically steady state models of generators, transmission lines that we would be discussing in the upcoming modules and that is what would be the focus of today's discussion. In the previous lecture which was the first lecture of week one, we had a general introduction to power networks, how the power networks have evolved over the time and what are the few typical trending issues that exist in modern current day power networks, what are smart grids and lastly we touched upon few aspects of network protection, few good attributes of a power network which was sort of part of the last lecture. So in today's discussion, as we have seen in the previous lecture also, power network is nothing but a very complex man-made network which consists of several elements.

We had also talked about these elements a bit in the last lecture. The different components or elements that exist in this power network, they range from generators which typically generate electrical power at few tens or twenties of kVs and then this power is stepped up to high voltage through step up transformers so that the associated I square R loss are minimized while transmitting this power from generating stations to the load ends. This power essentially flows through the network backbone which is formed by the transmission lines and the devices here specifically mean the different reactive power compensation devices, FACTS devices, etc., which we will briefly touch upon as the time comes.

And since this power is at very high voltage levels, the loads directly cannot consume this power at hundreds or thousands of kV. So we need step-down transformers. And essentially from these step-down transformers, the voltage is scaled down to 11 kV, 33 kV or 415 volt, 3-phase or 230 volt single phase. And this step down power flows through the distribution lines through which our electrical loads which could be industrial loads or residential loads get their input. So, in this complex network that we have whatever essential circuit principles, circuit laws or power calculations that you might have learned about in the previous relevant courses The same laws, rules, principles are still applicable in defining the behavior of this power network.

If I were to give you a few examples of what these circuit laws are, they could refer to the Kirchhoff's, well-known Kirchhoff's law, Kirchhoff's current law, Kirchhoff's voltage law. And then you also have superposition theorem, Thevenin's theorem, Norton theorem. all those theorems, principles, rules they are still applicable in understanding the behavior of this complex power network. So if we talk about the typical signals that are available in the conventional AC power network to be very precise. The signals they are mostly sinusoidal and they are periodic in nature reason being that thanks to the natural form of power generation, the way our synchronous generators, the way they operate, the way they are designed, they inherently produce these sinusoidal periodic signals.

So in this particular slide, if you see, i have tried explaining the nature of these signals through two examples these are current waveforms time domain form of currents i(t) which could be in form of a sine signal or a cosine signal because these two are the typical functions which are used to define the sinusoidal behavior of the ac voltages or currents so in this signal i(t) if we talk about the amplitude or peak that is defined by the number $\sqrt{2} \cdot I$ and then in this time domain signal we also have a factor or a variable or a constant known as omega which refers to the angular frequency defined by 2 pi f. This frequency f is inverse of the time period t and for Indian power network this frequency is supposed to be or nominally supposed to be equal to 50 hertz. then this signal I(t) could also have a phase angle alpha which is indicating how much the signal has time shifted or phase angle shifted from the reference of omega t being equal to 0. So alpha is essentially known as the phase angle which is Typically defined in terms of radians, this could also have a unit of degrees, but then omega t should also be of appropriate unit, so that sine or cosine of omega t plus alpha, which is this entire term can be appropriately evaluated. Now this sine and cosine so similar analogy is also applicable for the cosine waveform and through simple mathematical representations we can define this sine wave as a function of a cos wave.

Now how is that defined or how is that possible? Suppose we have a angle theta whose sine is to be defined then by the first principles of trigonometry sine theta is also equal to cosine of 90 degrees minus theta and I am assuming that theta is in degrees. And this can also be further rewritten as cosine of theta minus 90 degrees because The argument inside the cosine function does not have an impact on the cosine value. So based on this analogy we can redefine our sine waves in terms of cos waves and similarly the cosine signal here which is the second signal can be rewritten in terms of the sine signal. If we have to define the certain statistical or certain parameters for this time varying signals, in the previous slide we had the example of current signals, the same could also be true for voltage signals. And if we have to sort of understand the importance of the signals through certain matrices or certain indices, then there are two important numbers or factors which come into picture.

They are known as the root mean square error which is also in short known as RMS and then we have the average value. So, what is this root mean square or average value? Root mean square RMS is nothing but a alternate form of representing AC signal in terms of a DC equivalent. whereas average is nothing but the value of the AC signal over a typical time period or a full wave which is typically for t seconds capital T seconds. So, if we were to evaluate these values the expressions of these RMS values and averages are shown over here as the term says root means square. So, essentially the signal is first squared and then it is integrated or averaged upon the squared value is averaged upon a entire cycle.

So, if we have a well defined time domain signal for example, i(t) which is let us say equal to root 2 i sin omega t plus alpha. then we take the square of the signal take the entire average over a full cycle which is for capital T seconds. And then essentially the root of this entire term is taken into account which eventually turns out to be the capital I which was defined earlier and hence this capital I refers to the same amplitude by root 2 factor and if we were to take the average value so the average value as is given over here it would always be 0 irrespective of the phase angle the reason why this would be 0 because if I were to plot this time domain signal on the x-axis which refers to omega t and the y-axis which refers to i(t), then depending on the phase angle which is let us say alpha, it could be zero, it could not be zero. The value of i(t), it starts from some non-zero value, let us say which is, let us say if alpha is non-zero, then this particular value refers to root 2i sine of alpha. And then depending on omega t being equal to 0, so let's say this is the 0 instant, alpha is non-zero.

So the wave originates from omega t is equal to 0 and initially takes a value of root 2i sine alpha. It starts from omega t being equal to 0 and then reaches a peak and eventually has a shifting of this particular waveform so this entire time period is let's say cap omega capital t so if i take where to take the average i'm sorry can i retake this oh yeah So, having discussed about periodic voltage and current signals AC signals, there are certain matrices which often become useful in defining the behavior of these signals and those two popular matrices are one of them is the root mean square value which is nothing but in short known as RMS and the other one is known as the average value. RMS as the term says if I of t is the same signal which we have taken earlier which is root 2 I sine omega t plus alpha. To find the RMS value, we take the square of this I of t, which is the term given over here. We take the average of this square over an entire time period, which is for capital T seconds.

And then essentially, we take the square root of it, which essentially defines the RMS value. And for the AC signals that we have considered in the previous slide, this coincidentally turns out to be the same value as capital I, which is the capital I marked over here. And if we were to find the average value of these periodic signals, then irrespective of whether they are sine signals or cosine signals, their average value would always be zero irrespective of the phase angle. The reason being that if we were to plot this signal I of t, which is the signal over here, over y-axis, and omega t being the x-axis, then from omega t being equal to zero, the initial value of this I of t would be equal to root 2 i sin alpha, so depending on whether alpha is 0 or non-zero, let us assume that alpha is non-zero, the signal would start with some non-zero value and then would reach its peak at let us say omega t is equal to omega t plus alpha being equal to pi by 2 radians and then when omega t plus alpha reaches equal to a value of pi. let us say omega t plus alpha is equal to pi radians.

So, this signal crosses a zero value and a similar period is also obtained for omega t plus alpha being 3 pi by 2 and omega t plus alpha being equal to 2 pi. So, if we were to take the average of this signal over a entire time span of omega t being equal to capital T, what we see here is that for one first half we have a positive area because this average nothing but the area under the curve. So, for one first half we have a positive area and then for similar other half we have a negative area and since this signal is periodic over the entire omega t is equal to capital T cycle this positive number and negative number they are equal to each other in terms of magnitude and hence the average value is 0. So having defined the RMS and phasor values, let's talk about what do we mean by phasors first of all. So AC signals, they are characterized by, so let's say if I take the AC signal, omega t is equal to root 2i sine omega t plus alpha, which is the time domain representation of this periodic signal.

Then capital I here refers to the RMS value and alpha here is the phase angle, which is defined with respect to a particular frequency. In terms of phasor domain representation, this being the time domain representation, phasor representation is nothing but the frequency domain representation of AC signals. In phasor domain representation, the frequency reference is very important. So, typically since sine and cosine form as examples or basis for defining periodic signals. So, depending on whether sine omega t or cos omega t is being defined as a reference for phasor representation, This phasor would essentially have a factor of or a value of capital I which is nothing but the rms value and depending on what the value of alpha is which is the phase angle that would be part of also the phasor angle or phasor representation.

Essentially the point that is being made is if capital I of t is a time domain representation and this needs to be converted into a phasor representation assuming sin omega t as the reference for phasor essentially capital I which is the phasor notation will say with a arrow factor would nothing be but equal to capital I at an angle alpha. So, essentially in phasor representation the definition of reference is very important whether we take sin omega t as reference or cos omega t as reference this phasor form is going to change. This is essentially indicated in this particular representation or slide. If we have I of t defined as root 2 I sine omega t plus alpha and we choose sine omega t as reference then alpha being the phase angle with respect to omega t and sine and sine they are the same in both the time domain representations. So our phasor notation here with the arrow not being marked here is nothing but capital I at an angle alpha which is basically the polar representation of a phasor which can also be expanded as I cos alpha plus j sin alpha where j is nothing but the complex operator defined as square root of minus 1 and we can represent or draw the phasor which is I at an angle alpha in this particular form when omega t acts as a reference.

Now if we were to redraw or represent this phasor choosing cos omega t as a reference. So, we can what we have to do the first step would be we have to reconvert or re-represent this I of t which is a sine wave in terms of a cos wave and going by the trigonometric first principles this can be easily written as root 2 I cos omega t plus alpha minus 90 degrees. and when we choose now omega cos omega t as a reference then additional to angle phase alpha we also have a minus 90 degree phase shift which is essentially what is written here pi by 2 being the radian equivalent of 90 degrees. So these angles they match and if we now redraw this Phasor with cos omega t as reference, it would look something like this. Cos omega t leads the corresponding phasor which is marked over here.

And similar to vectors, the way they can be added, subtracted, multiplied, divided. On similar lines, these phasors, they can also have similar operation. Essentially speaking, addition of vectors would be very useful in understanding the Kirchhoff's voltage law application to synchronous generator modeling and transmission network modeling. Both addition and subtraction would be useful in that perspective, whereas multiplication would be very useful in understanding how power is evaluated in these steady state models. So going forward, specifically taking few network elements specifically, so let's say if we have a pure resistor whose resistances are ohms and through this resistor a current is flowing which is this time domain signal.

If I choose cos omega t as a reference, for this phasor then cos omega t with being the reference this current signal in time can be also rewritten as a phasor. And by Ohm's law which defines the principle or operation of a resistor voltage is nothing but current into the resistance. So, if I represent or bring in this phasor notation of current then the voltage is capital IR at an angle 0. So, essentially I have capital V defined as capital or bold I into R. And if I have to draw a phasor diagram which explains the current through a resistor and voltage drop across the resistor, since these two phasors they have the same phase angles, so essentially there is no phase difference between the current phasor and voltage phasor through a resistor and hence they are in phase.

So we have this phasor diagram for a resistor. If we have to understand the similar thing for a capacitor, so we assume the same current representation which was there in the previous slide and we choose cos omega t as a reference for my phasors. So time domain signal I of t is written as in this phasor form with phase angle being zero and the magnitude being the RMS value. And if we define or understand the behavior of a pure capacitor then the corresponding capacitance property indicates that current through a capacitor is nothing but equal to capacitance C which whose unit is farad and dv by dt where v of t is the time varying or time dependent voltage across the capacitor. Now this equation it can always be written into the integral form and if we use this capital small i of t which is defined over here and put it over here and take the indefinite integral assuming integration of constant to be 0.

So, we have v of t which is a time domain voltage as a function of this particular root 2 i omega by omega c into sin omega t. And accordingly or appropriately we can always redefine or rewrite this time domain signal with cos omega t as a reference for phasor into the corresponding phasor form. Now in this particular capacitance property what we see is that Current is having a phase angle of zero, whereas voltage is having a phase angle of minus 90. So if we plot this or represent this into a phasor diagram, then this is how our phasors would look like. In the first phasor diagram, current is being chosen as a reference for phasor, whereas in the second phasor diagram, voltage is being chosen as phasor.

Similarly for a inductor if we choose the same current variation and try to find the voltage across this inductor then this is the equation which defines the property of inductance which is V of t equal to L di by dt and if you substitute this I of t function in this expression then accordingly we get a cosine waveform for the voltage which can again be re-represented or rewritten with omega cos omega t as reference into the corresponding phasors and in this now we see that the current lacks the voltage or alternatively voltage needs the current. So if we redraw this phasor diagram for an inductor then we have two alternate options one being the current as reference whereas in the next one the voltage is the reference. Typically in AC circuits these currents and voltages they are represented in terms of their rms values and why those rms values are important because the way our analog voltmeter ammeter or for that matter multimeters they inherently measure the rms values of these voltage and current waveforms. So if we have a single phase circuit or a source which is of 110 volt with frequency of 50 hertz it essentially means that it's a AC voltage source with 110 volt as the rms voltage and the angular frequency being 100 pi because omega is 2 pi f and if you put f as 50 hertz we get omega 100 pi as a relationship alternatively if we have a current source of 10 ampere with frequency as 10 hertz it would mean that the maximum current which can be observed through this current source is root 2 I, where I is nothing but 10 amperes, which results as 14.14 amperes, and source frequency or angular frequency is 20 pi.

We will conclude our discussion today with example on phasor diagrams. In this question it has been given that I is which is the source current is nothing but 10 at an angle 0 degrees and you have to draw a phasor diagram in which all our phasors which is the source voltage V current I1 and I2 are to be evaluated or represent those we have to first find what these phasors are. So, if we apply KVL across this particular branch by KVL we would get

$I_1 = V / (8 + j6) = V / 10\angle 36.87°$

$I_2 = V / -j6 = V / 6\angle -90°$

$I_1 + I_2 = I$

$V / (8 + j6) + V / -j6 = I$

$(8V) / (10\angle 36.87° \times 6\angle -90°) = I$

$V = 75\angle -53.13°$ V

since we now know what the voltage is we can always substitute this into the current expressions which is I1 and I2 and from here we would get I1 as 7.5∠-90 A and I2 as 12.5 ∠36.87 A. So if you now plot or choose capital I as reference which is 10∠0 which is the source current then voltage would nothing be but 75∠-53.13 volt and I1 current is lagging the source current by 90 degrees which is 7.5 ∠-90 A and subsequently I2 is leading the source current by 36.87 degrees amperes. That is all for today's discussion. In the next lecture, we would extend this to evaluation of power for single phase circuits, AC circuits. Thank you.