Power Network Analysis

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Week-04

Lecture-19

Lecture 19: Transmission line parameters- inductance between points outside the Conductor

Hello everyone, welcome to lecture four of week four of the course Power Network Analysis, in which we will continue our discussion on the fourth main module, which is Transmission Line Parameter Evaluation, in which we will discuss or understand how to evaluate inductance, or basically, flux linkage and magnetic flux linkage. Due to a current-carrying conductor for points that are outside the current-carrying conductor. So, whether we evaluate the inductance or the corresponding magnetic flux linkage, both would go hand in hand, as we will see in the slides to come, and this discussion will be built on the discussion we had about the evaluation of line inductance due to internal magnetic flux. Unlike electric field lines or flux lines, which do not exist inside a conductor, magnetic flux lines can exist inside a conductor as well as outside a conductor; hence, the impact of these internal and external magnetic flux lines is different for internal inductance and external inductance. So, if you recollect, we considered a current-carrying conductor, as shown over here, which was carrying current I, and this current which the conductor is carrying we are assuming could be an AC current or a DC current. If it is an AC current, then at some point in time the current might be flowing outside the plane of the slide, and maybe for the next half cycle, the current might be flowing into the plane of the slide.

For the consideration of the figure that has been drawn here, we are assuming that if it is an AC current, then we are focusing on one half cycle; the remaining half cycle would also have a similar implication, and the current is flowing out of the plane of the slide if it is the first half of the AC, or if it is a DC current, then the current direction does not change. And we understood that if we focus only on the magnetic flux lines that are internal flux lines residing inside the conductor because of current I, then the corresponding internal inductance would have an expression of this form, which coincidentally turns out to be independent of the conductor size; it is also independent of the current I. Generally, electrical conductors are good electrical bodies and poor magnetic materials. So, we can also safely assume that the corresponding relative magnetic permeability is equal to 1; mu naught is the absolute permeability, whose value is 4 pi times 10 to the power of minus 7 Henry per meter.

$$L_{int} = \frac{\mu_0 \mu_r}{8\pi} = 0.5\mu_r * 10^{-7} H/m$$

So that is how we get this expression. And the direction of these magnetic flux lines is governed by the right-hand rule. So if we point our thumb in the direction of the flow of current, which is out of the plane of the slide, then the fingers are curling in the anticlockwise direction, and those are the red magnetic flux lines inside the conductor or the blue magnetic flux lines outside the conductor; they are concentric anticlockwise flux lines, and they are closed-loop lines as per the nature of magnetic flux lines. So, for our discussion today, we will take up the aspect of evaluating the inductance due to magnetic flux lines, which are in blue and are present outside the conductor. So our assumptions remain the same: the idealistic assumptions that, I mean, in fact, all assumptions tend to be idealistic.

So we are considering those assumptions for the sake of an easy analysis. What we are assuming is that the current density inside the conductor is always uniform. So if current is I and the cross-sectional area for a circular conductor of radius R, the corresponding current density would be nothing but I divided by the cross-sectional area of the conductor; for a conductor of radius r, that area is πr^2. So, ampere per meter, which is the current density,

$$Current\ density = \frac{I}{\pi r^2}$$

this number or current density is uniform at every point inside the conductor; that is what the meaning of uniform current density is. The return conductor for this current is far off, and that is the reason why the magnetic flux lines are all perfect concentric flux lines.

That means they share a common center; their radii may vary as the distance from the point of consideration increases or decreases from the center of the conductor. The conductor is perfectly straight; it is parallel, it doesn't have any bends, and hence these concentric flux lines don't cross or overlap with each other. That's the essence of assuming straight connectors with a parallel nature. The conductor radius is R, and the current is I out of the plane, as we discussed in the previous slide. We focus on two points, P1 and P2.

The distance of P1 is d1 from the center of the conductor, and point P2 is at a distance d2 from the center of the conductor. D1 and D2 here, both of them, are going to be greater than r; they can at best be equal to r on the surface, but they should not be less than r because we are focusing on inductance evaluation due to external magnetic flux lines. We are not focusing on internal magnetic flux lines; the impact of internal magnetic flux lines has already been considered in the previous lecture. So, by the right-hand rule, as I have already explained, if the thumb points in the direction of the flow of current, which is out of the plane of the slide, the fingers would curl in the anticlockwise direction, and that is how the directions of flux lines are dictated or decided. We are considering a similar cross-section the way we discussed in the internal inductance evaluation.

We are considering a cross section of width dx that is at a distance x, which is more than r, indicating that the distance x refers to a point outside the conductor, and at a distance x from the center of the conductor, we have taken a cross section dx, and the length of this cross section is dl, which is again being measured along the periphery of this circle whose radius is x from the center of the conductor. So the way we understood why the length vector, magnetic flux intensity, magnetic field density, and flux density should align along the tangent at the point along which this distance dl or dx is being measured is simple. As I mentioned, since we are measuring the distance dl along the circumference of the circle whose radius is x, the corresponding DL vector will definitely lie along the tangent. So, basically, this is the circle; this is the circle, and then this line is nothing but the tangent along this circle, which has a slope of 90 degrees with respect to the radius being drawn from the center. So, this angle is 90 degrees, and along the D L vector, magnetic flux intensity or field intensity would also lie because we are looking at the impact of the flux lines from the current consideration perspective, and it is by the nature of this magnetic flux or field intensity that the H x vector is also parallel to the corresponding D L vector.

The magnetic field density for a medium where the medium has constant permeability, which is not changing with time, is the magnetic flux density, calculated as permeability multiplied by the magnetic field intensity, where this permeability is mu naught multiplied by mu r, mu naught being the absolute permeability and mu r the relative permeability of the medium in which these magnetic flux lines exist. So if Hx is along the DL vector, then it is imperative that mu, being a scalar quantity and constant, is not changing with time; Bx would also be parallel to the DL vector, and that is the reason how the corresponding vectors are marked. So if we now focus on the cross section that we have considered, and if we were to continue measuring or continue shifting these dl along the circumference of the circle of radius x, then eventually we would get a hollow cylindrical tube whose internal radius would be x, the width would be dx, and the length of this cylindrical hollow tube would be equal to the length of the conductor that we are considering. So, that is how we focus on this. If I were to complete this entire cross section, pardon me for the inconsistency in not being able to draw a perfect circle, then this entire tube, which is going to be our point of concern for finding out the impact of external magnetic flux lines, would be a cylindrical hollow tube whose attributes or parameters are given over here: the radius is x and the distance or the width is dx.

Also, I would like to briefly point out one thing: we will also be discussing this aspect in detail when we go to the upcoming slides regarding our magnetic flux lines as they are now. These magnetic flux lines are not along the length of the conductor that is carrying current. These magnetic flux lines are tangent to the surface of the point or the circumference where we are considering this point. So, basically, if I were to focus on the magnetic flux lines at the periphery of this conductor, then the corresponding magnetic flux lines and their corresponding strength are along the tangent; the reason being that magnetic flux lines are always closed loop lines, they are concentric lines, and they are parallel to the surface of the conductor that is carrying the current I. So that is the reason, and also another reason why h of x is along the tangent; and if h of x is along the tangent, with permeability remaining constant, b of x would also be along the tangent.

So if we have to understand that this magnetic field density or flux density is not along the length of the conductor. So, let us say that if I were to draw the corresponding cross-sectional or frontal section of this conductor, which, if I were to mark it over here, then the current is flowing along this length I, and the magnetic flux lines or field lines are not along this current; they are actually aligned like this if this is the front section that I were to draw. For this corresponding, sorry not the cross-section, this is actually the front section, which is what is shown over here. This would actually serve as the side section of the cylindrical portion that I am looking at. So if I look at it from the side, then this is the entire length along which current is flowing, and the field lines or the flux lines will appear to be perpendicular to the direction of the current.

So, if I have to find the impact of the magnetic flux lines in terms of flux linkages, I should be very careful in figuring out the cross-section through which these flux lines are flowing. So, please keep that aspect in mind. We will come back to the same aspect again in a short while. So, our Ampere's law would be very useful in figuring out the corresponding values in evaluating the corresponding values of flux intensity or flux density. Ampere's law simply states that the line integral of the dot product of the magnetic field intensity and the length vector \(dl \) is equal to the current enclosed is nothing but equal to the net current present inside the line integral that is carrying this surface.

$$\oint H_x.dl = I$$

So, basically, the net magnetomotive force (MMF) around a closed path, as per Ampere's law, is equal to the net current enclosed within this line integral path, which is also equal to the dot product of the magnetic field intensity and the corresponding DL vector. And similar to the discussion that we had in the previous slide or the previous lecture, we are considering a straight connector parallel conductor whose current density is uniform. So, if we try to find the value of h of x at a distance x from the center, where this distance x could refer to n number of points along this circumference, then at any of these points, the value of h of x is going to remain the same. The current density is the same; I by pi r squared is the same throughout these points along the conductor, which is straight and parallel.

So h of x, no matter the value of x distances—I mean not the value of x, but no matter what point I take along this circumference or circle, which is at a distance x from the center—the value of h of x is going to remain uniform as well. H x and d l are parallel vectors, so if you recollect our previous discussion, the dot product of two vectors is nothing but equal to the corresponding magnitudes. with the cosine of the angle between these vectors. For parallel vectors, the angle is going to be 0 degrees. So this dot product, which eventually results in a scalar quantity, would be nothing but h of x times dl. So, in order to simplify that, if I have to find this line integral, I can make use of two properties.

$$H_x . dl = |H_x||dl| \cos 0 = H_x dl$$

One, the dot product is a simple scalar term, and h of x is going to be the same along any point I take on the circle of radius x. So essentially, this integral can be simply stated as h of x integral of dL, which should be equal to I. Since our point of concern is outside the conductor, the net current present is I itself, which is the total current present inside the conductor. For the consideration of internal magnetic flux lines, this I was replaced by I of x, where I of x was a portion of the current depending on the distance from the center x along which the circle consideration was made. So x was also a function of I of x, whereas here in the expression, I is independent of x. So, given that the current density is uniform, h of x is going to be uniform at all points in the circumference of circle x, so we can make use of that property along with the property of the dot product of two collinear vectors, and we get this beautiful expression for h of x where the integral of dL is nothing but the circumference of the circle whose radius is x; the circumference is going to be 2 pi x, so we have h of x as i over 2 pi x. Now, once we know the magnetic field intensity or flux intensity, we can also find the corresponding magnetic flux density b of x, where we multiply mu with h of x. The medium outside the conductor is likely to be air.

$$\oint H_x . dl = I \Rightarrow H_x \oint dl = I$$

$$H_x = \frac{1}{2\pi x} At/m$$

And that is the reason why mu is replaced by mu naught, which is absolute permeability equal to 4 pi times 10 to the power of minus 7 Henry per meter.And the relative permeability of air is also considered, where air is very bad. Conductor not only from an electrical perspective but also from a magnetic perspective. So the mu r of air is equal to 1; it is as good as the conductor being considered in a vacuum space outside the conductor. So the b of x value can also be found from the expression of h of x. Once we know the field intensity and the field density, we can evaluate the corresponding stored magnetic energy density in exactly the same way we evaluated for internal magnetic flux line considerations.

$$B_x = \mu H_x = \mu_0 \mu_{r,air} H_x = \frac{\mu_0 I}{2\pi x} wb/m^2,$$

$$\mu_{r,air} = 1$$

So, the field energy density, magnetic energy density is half of b(x) into h(x). When we substitute those values of b(x) and h(x), we get this expression.

$$u_{Bx} = \frac{B_x H_x}{2} = \frac{\mu_0 I^2}{8\pi^2 x^2} J/m^3$$

And now we will go back to the cross section or side section that we were talking about again. This is the front section of the corresponding one which we are looking at. If we look at the side section along the length of the conductor, it appears that the current is flowing like this, whereas the field lines or flux lines are concentric and parallel to each other.

The field lines or flux lines B or H are perpendicular to the side section that we are looking at. And now, if these field lines are flowing along the periphery or circumference of the point of concern that we are looking at, then it is not along the length of the conductor. So, if we have to find the energy density, it is U bx multiplied by the volume in which this magnetic energy density is present. Then it would be nothing equal to the circumference along which the DL vector is being measured, which is essentially the volume of the cylindrical hollow tube that we looked at. This tube is the same length as the conductor since we are evaluating our parameters on a per unit length basis of the conductor.

So, we can easily consider the length to be equal to 1, where 1 could be 1 meter, 1 kilometer, or 1 mile; and accordingly, this per meter unit could also change, and 2 pi x into dx is nothing but the front cross-section of this. cylindrical hollow tube that we are multiplying by a unit length of 1, which gives us the volume of this entire cylindrical hollow tube. So, we can find the unit of energy that is being stored in this hollow tube. So, the stored magnetic energy density multiplied by the corresponding volume of the cylindrical hollow tube gives us this expression for the unit energy that is stored. Now, if we want to find the entire energy, then this entire energy E would be nothing but the integral of dE, which, when substituted with this expression over here, gives us our value of total E equal to mu naught I square by 4 pi natural log of d2 by d1.

We are considering or trying to find the stored magnetic energy between points P1 and P2, which are at distances d1 and d2 from the center of the conductor.

$$dE = u_{Bx} * 2\pi x.1.dx = \frac{\mu_0 I^2}{4\pi x} dx\, J/m$$

$$E = \int dE = \int_{D_1}^{D_2} \frac{\mu_0 I^2}{4\pi x} dx = \frac{\mu_0 I^2}{4\pi} \ln \frac{D_2}{D_1} \frac{J}{m} = \frac{1}{2} L I^2\, J/m$$

So, the corresponding energy stored in this section for the current-carrying conductor, which is the value of E that we have evaluated, is nothing but μ_0 I^2 / (4π) natural log of (d2 / d1). This energy is also equal to half L I squared, where I is the net current present in the conductor, and from here, if we correlate or try to find the expression of L, which is going to be the inductance between points P1 and P2 outside the conductor at distances d1 and d2, which are more than R, then we have this natural law of expression as the entire external inductance between points P1 and P2. Outside the conductor. We can, so in a way, we have got to know that this is going to be our external inductance due to external magnetic flux lines.

$$L_{ext\, P_1 P_2} = \frac{\mu_0}{2\pi} \ln \left(\frac{D_2}{D_1} \right)\ H/m$$

We also know that the internal inductance, which is L_int, if I were to represent it, is nothing but $\mu_0\mu_r/8\pi$; that is the only expression that we get, and it is independent of the radius and independent of the current I. So, basically, L_internal is the entire inductance of this. The conductor, which is carrying current I of radius R, and L internal is the internal inductance between the center of the conductor and the radius at the surface; that is what L internal means. So, if we have to find the net inductance of a point P that is at a distance d from the center of the conductor, we have two inductances in series. The one inductance that is inside the conductor is L internal; the other inductance is the inductance outside the conductor, which is L external, and if I were to mark a point C here, then it is L external C, P.

Where c is at a distance r and p is at a distance d. So, if I now combine these two inductances to find the overall inductance at point p from a distance d from the center of the conductor, then I can easily get net L to be equal to L int plus L external c, p. Please keep this in mind because why are we adding these inductances? Because in the center it appears that they are series combinations, that is how we can easily add these two inductances. We will come back to this expression again in the next few slides. There is also an alternate way of finding this inductance between the outside points of a conductor due to external magnetic flux lines.

$$L = L_{int} + L_{ext,c,p}$$

So, we have seen in our course of discussion that the magnetic flux density is nothing but $\mu_0 I / (2\pi x)$, where x is the distance from the center of the conductor and it is outside the conductor. So, x is essentially greater than R here. So, if we know the corresponding magnetic flux density and we again recall our side section where the current is along I, the flux lines are perpendicular to the side section. Since the flux lines whose strength is a density B are along the periphery or outside the conductor. So, essentially B of x is not flowing inside the conductor or outside of the conductor in terms of the slide of the plane; the magnetic flux lines are actually along this section, which has a width of dx.

So, if I have to find the corresponding flux linkage or the unit flux linkage, which I denote by d of l, remember that flux linkage we have seen is represented by the variable lambda. So, if I find the unit flux linkage, that flux linkage will be nothing but the flux density multiplied by the area through which this flux density or these flux lines are passing. Now the area I have to focus on is not the cross-sectional area of the conductor, nor is it the surface area of the conductor; it is essentially the area along which this width of section dx and dl is considered. So, if the cylindrical hollow tube is of width dx and the area is not with respect to the periphery or circumference of the circle of radius x, but it is actually width dx multiplied by the length using unit length 1, so this side section is of unit length 1, then the area of this cross-section along which magnetic flux density exists is 1 dot dx. So, from here we can find d lambda, which is mu naught i dx by 2 pi x Weber per turn per meter.

$$d\lambda = B_x.1.dx = \frac{\mu_0 I dx}{2\pi x}\ wbt/m$$

Now we can also integrate this lambda, so the net lambda net flux linkage between points P1 and P2 would be the integral of D of L, which turns out to be this expression again.

$$\lambda = \int d\lambda = \int_{D_1}^{D_2} \frac{\mu_0 I}{2\pi x} dx = \frac{\mu_0 I}{2\pi} \ln\left(\frac{D_2}{D_1}\right) wbt/m$$

And we have also seen or recollected that the inductance value L for a medium where permeability remains constant is also equal to lambda by I, where lambda could represent the phasor quantity and I could also represent the phasor quantity. We are considering I and lambda to be the phasors here; they could be AC current or DC current. So, L by lambda or lambda by I is also a measure of L.

From here, we can also find lambda by I. Lambda by I would essentially be mu naught 2 pi ln of d 2 by d 1 Weber turn per meter, which is also the same expression we obtained from the magnetic stored energy expression perspective.

$$L = \frac{\lambda}{I}$$

$$L_{ext\ P_1,P_2} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)\ H/m$$

And now, with this, if we try to find the net inductance at a point that is at a distance d from the center of the conductor, and this d is outside the conductor, then we have two inductances: one is the internal inductance between the center and the surface of the conductor, which has a radius r. Coincidentally, independent of R, the external inductance is between a point on the surface of the conductor and the point that is at a distance d from the center of the conductor. So, according to this expression, this is the corresponding external inductance. Now, if we combine these two, we can easily get the overall expression L d to be of this form.

Now, how do I get this? It involves a little bit of mathematical reorientation. So, mu naught by mu naught into mu r by 8 pi can also be rewritten as mu naught by 2 pi into mu r by 4, which is also equal to mu naught by 2 pi times the natural log of e to the power mu r by 4 divided by 1. This could also be rewritten as mu naught divided by 2 pi times the natural log of 1 over e to the power of minus mu r divided by 4. Now, if I try to add this term to the term sitting over here. Then, essentially, I would get d as the same d which is present over here; d into 1 is d, and r into e to the power of minus mu r by 4 is this particular term. So, that is how the corresponding net inductance expression can be evaluated.

$$L_D = L_{int} + L_{ext,r,D} = \frac{\mu_0 \mu_r}{8\pi} + \frac{\mu_r}{2\pi} \ln\left(\frac{D}{r}\right) = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{re^{-\mu_r/4}}\right)\ H/m$$

Now, there is a very specific aspect about the expression that we have obtained here. What we see here is that generally, for conductors, the value of μr is close to 1. So, essentially, mu r by 4 would always be a positive quantity, which means 1 by mu r, which means, let us say, e to the power minus mu r by 4 would also be a positive quantity, and this positive quantity is getting multiplied by the corresponding, sorry, it would be more than 0, but it would definitely be less than 1 because mu r by 4 is positive.

So, e to the minus mu r by 4 is a value less than 1. So we are trying to multiply a quantum number, which is positive and less than 1, with the effective r. So this term is actually less than r for practical conductors. And this gives a clue or an indication that, because of external flux lines or magnetic flux lines, the overall effective radius of the current-carrying conductor reduces, which is also responsible for the skin effect. The overall cross section of the conductor is reducing, the current density is getting affected, and more current is rising to flow on the periphery; the reason being that the overall resistance and the overall cross-sectional area are getting compressed because of inductance in the conductor, and that is how we can also understand why AC resistance is more than the actual DC resistance of conductors.

That would be all for today's discussion. We will take up the discussion of evaluating the magnetic flux linkage or flux line values of a conductor in a group of conductors in the next discussion.

Thank you.