Power Network Analysis

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Week-04

Lecture-18

Lecture 18: Transmission line parameters-inductance due to internal flux

Hello everyone, welcome to Lecture 3 of Week 4 of the course Power Network Analysis. This lecture is going to be the second lecture on the main fourth module, which is the evaluation of transmission line parameters, in which we will discuss the evaluation of line inductance due to internal magnetic flux lines or field lines. In the previous lecture, we had concluded our discussion on what line resistance is, to be precise. We also discussed what corona loss is, how conductance is difficult to model and accounts for corona loss, and how DC resistance and AC resistances are very different for a typical conductor. What is the skin effect, which is attributable to an increase in inductance at the center of the conductor, leading to more current density at the periphery, thus resulting in a lower cross-section for the current and hence higher AC resistance for a typical conductor? So in this discussion today, we will focus only on inductance evaluation, and specifically, we will take up our discussion of the evaluation of inductance due to internal magnetic flux lines. Now, to those of you who are new to this particular lecture, I request that you please revisit the previous lecture, where you will get an idea of how current-carrying conductors tend to have magnetic flux lines around them, thanks to the right-hand rule.

And because the conductors we are talking about are AC current-carrying conductors, in every cycle the current direction or orientation might change. So, the corresponding magnetic concentric flux lines might also change their orientation. And these flux lines can exist inside the conductor as well as outside the conductor. So in this discussion today, we will focus only on internal flux lines and how inductance results from these internal flux lines.

The consideration of magnetic flux lines outside the current-carrying conductor will be taken up in the next lecture. So, in a very basic nutshell, if I have to define the inductance of a conductor that is carrying current, there are certain magnetic flux linkages or lines existing around the conductor. And if I have a measure of that particular magnetic flux line, which I denote by lambda, the common symbol in magnetic literature, where lambda is the instantaneous magnetic flux and current I is the current flowing through the conductor, then inductance, whose unit is Henry (H), named after Henry himself, It is nothing but the ratio of lambda and I, and if the permeability or the magnetic permeability of the conductor or the medium is constant, then lambda and I, which are AC or time-varying, can be replaced by the corresponding phasor magnitudes; that ratio would itself give us a quantum of lambda. So this simple discussion or definition of L is actually very useful in evaluating inductance $\left(L = \frac{\lambda}{i}\right)$, and we would see that this relationship would be very useful in figuring out expressions for inductance for different arrangements of three-phase AC conductors. There's also an alternative way of figuring out or defining inductance, which is associated with the stored magnetic energy in a given medium

$$E = \frac{1}{2}Li^2 = \oiiint \frac{BH}{2} dV$$

So, energy, as we all know, if E is a symbol, then its unit is joules, named after Joule himself; capital J represents joules. And for a magnetic material or a magnetic medium carrying current I and having inductance L, E is nothing but half Li squared, so that is the magnetic energy. It is also equal to the volume integral of half of B and H. Now, B is called magnetic flux density, whose unit is Weber per meter square. So, WB here is essentially Weber, again named after him.

H is the magnetic flux intensity, whose unit is ampere-turn per meter. So, AT here is actually equal to, or it decredits or refers to, ampere in turns, where turns refer to the number of conductors that have been spiraled around a given magnetic material or a solenoid. So, the number of turns, N, refers to that particular unit. With "per meter" being the symbol for 80 per meter. And dv is the corresponding unit volume; the volume unit is meter cubed, so the unit is meter cubed.

There is also a note to be taken here that I have taken three integrals, and they are encircled in between. So, this basically refers to the volume integral. Then we also have special integrals, which we would see specifically in the case of capacitance evaluation. We call this the area integral. And then you also have a single integral, which is also called the line integral.

So whether we are trying to integrate around a length, the multiplication of two lengths, or the multiplication of three lengths, the many lengths indicate the corresponding integrals around x, y, z, or r theta cubed, depending on polar or Cartesian coordinates for volume integrals, area integrals, and real integrals, respectively. So we will see this area integral specifically in the perspective of capacitance. We will be looking at the volume integral and line integral in inductance evaluation in this particular discussion, as well as in the next lecture. So the issue with inductance evaluation, like any parameter evaluation, is that we also saw this for resistance; it is also true for inductance and capacitance that exact and accurate line parameter evaluation is extremely difficult because conductors are not just single conductors. Conductors are made up of multiple strands; for example, if I talk about strands, then ACSR has more than one strand of conductors in itself. The number of strands here is more than one. There could be five steel strands in between, wound around with 12 or 13 aluminum connectors. And the type of ACSR connectors is different. Different manufacturers have different sheets that mention, in terms of tables, what ACSR connectors are and how many aluminum steel strands there are. Aluminum and steel are different types.

They have different permeabilities, so strands have different lengths; strands have different permeabilities, their permeabilities might be varying, so an individual conductor is not there; we have multiple parallel strands, so considering all those effects becomes very difficult for inductance evaluation. The same is true for capacitance; the spacing between conductors is a major factor in dictating these parameters. That also tends to vary because of increases or decreases in line length during hot or cold weather conditions, sagging, and distortions. So, a lot of these non-linearities do exist. For the sake of our discussion, to keep it simple, we will assume our conductors are straight and parallel.

We will assume the current distribution in the conductors to be uniform, which is actually a very rare practice. So you would wonder what the case would be if the current distributions were non-uniform, which is true in actual practice. The inductance or capacitance that we are evaluating will be the worst-case numbers. In reality, actual conductors might have less inductance and less capacitance, so the overall complexity tends to go down. So we are trying to get a picture of the worst-case evaluation.

And, as I have already mentioned, unlike electric field lines, which originate from positive charge and terminate at negative charge, these don't exist inside a conductor. For details, please refer to the class 12th physics discussion of charged bodies where it has been very well explained that electric fields don't exist inside a conductor. They can exist inside an insulator, but for a conductor that is well-conducting, the electric field lines will always exist at the surface. So, unlike electric field lines, magnetic flux lines can exist inside as well as outside a current-carrying conductor, and that's the reason why special consideration is needed for inductance. Basically, we would have inductance because of the inside flux lines, magnetic flux lines; we also have inductance because of outside magnetic flux lines.

So, for today's discussion, we will focus only on internal magnetic flux lines due to a current-carrying conductor, with the assumption that the current density is uniform in the conductor and that there is no skin effect. The return conductor is far away with no distortions in the considered magnetic field lines or flux lines. So, if you remember or recollect from the previous discussion, we had taken the case of a single-phase two-wire circuit, and if at a given instant of time the circuit, being an AC circuit, is carrying AC current, with the current in one conductor out of the plane of the slide and the other current inside the plane of the slide, then the magnetic flux lines drawn earlier. They were more of a bulging on the extreme left and right of the conductor, whereas in between, the density of magnetic flux lines was constructed or construed. So, the reason for this construction of inductance is that the overall inductance in this space is larger, and hence fewer magnetic field lines or flux lines would flow.

For the sake of discussion, we are not considering this non-linearity, or we are considering it to be an idle condition. So we are not bothered about the return conductor. We are assuming this return conductor is far, far off from the conductor of concern, and hence all our magnetic flux lines are always perfect concentric circles around the current-carrying conductor with different radii to the extent that we want to find the corresponding inductance. So our connectors, again, are straight and parallel. We are not considering the effect of strands at the moment.

We're talking about an individual single strand of connector for inductance evaluation. So what I have here is a conductor that is carrying current I, and this current I is out of the plane of the conductor. By the right-hand rule, if I put my thumb outside the plane of the slide, the fingers curl in the anticlockwise direction. So the blue color line is the magnetic flux line that is outside the conductor. The magnetic flux lines marked in red are the inside flux lines, and for today's discussion, we will focus only on these red flux lines and how inductance tends to appear because of these inside magnetic flux lines.

Before I proceed further to the next slide, let us also try to understand that if our current density is uniform for this given conductor, whose cross section is shown and has a radius R, it is a perfect single strand cylindrical conductor. Since I have to find the current density in this conductor, the current density is basically the surface area shown or the frontal sectional area seen on the slide. If I know that particular cross-sectional area because current is essentially flowing through this cross section, then the current density, which also tends to have the symbol J, different from joules, for our discussion, let us say we do not use the symbol; we only say if I have to find the current density in this particular conductor, then current density is nothing but current divided by the area, the area through which this current is actually flowing. That area happens to be a circular area with radius r for a circle with radius r. The surface area or the area of the circle is nothing but pi r squared, so the current density for this particular conductor is current I divided by pi r square, which is ampere per meter squared; this is the corresponding current density.

$$current\ dendity = \frac{I}{\pi r^2}$$

So with this, what we're trying to say is that this number i by pi r squared is uniform at any point in this particular connector. It is the same at the center, the same at point A, the same at point B, point C, point D, anywhere. There is no difference in this value of the current density. There is no skin effect. So what we have is a current-carrying connector I of radius R.

We are focusing only on the internal magnetic flux lines, which are shown by these dotted lines. There are 200 dotted lines. These are all internal magnetic flux lines; we are not bothered about flux lines that are originating outside the conductor. Now one might wonder why magnetic flux lines exist inside the conductor; I have not given a reason for that. Now, the reason they can exist inside a connector is that if I focus on one part of the cross-section for this entire cross-section, assuming that it is carrying a uniform current, the density is uniform, so even this cross-section would also have some current.

Which is for this cross-section, let's say I have taken a cross-section of width dx, length dl, and its distance is x from the center of the conductor, where this x is less than r. If I take out or plug out this particular cross-sectional area of x with dx and length dl, Then this particular cross section, because the current density is uniform, would also carry a current Ix, and any current-carrying conductor, be it inside or outside, would have the right-hand rule applicable to it. The current is still the same because the time variation has not occurred. So, the current is still outside the plane, and because of this current inside the conductor core, the current outside the plane would be called in the right hand's anticlockwise direction. So, magnetic flux can also exist inside the conductor.

And for this particular cross-section, we have marked the length vector \(dl \). So, dl is basically the length along this tubular section; this is dl. We have also marked the magnetic flux, or field intensity, and the magnetic flux density, BX. Now, why are these arrows or directions marked like this? Let me explain the length vector first. If DL is being measured with respect to some reference, then basically I'm trying to move along this periphery of the cross-sectional area that I have taken.

As the direction keeps moving up, the DL vector is nothing but a tangent at the point from where DL is being measured. So DL is actually the tangent line here, tangent to the surface of a circle with radius X. Coincidentally, magnetic flux density, or flux intensity, basically H—sorry, magnetic flux intensity H—is also a line that is tangent to the surface carrying the current. That's the nature of the magnetic flux intensity or field intensity H. B, which is the flux density, is nothing but mu times H, where mu is called permeability.

$$B = \mu H \ \ (\mu \rightarrow permiability)$$

We will see more about this permeability in the upcoming slides. If the medium permeability is constant, whatever the direction of this H vector, the same would be the direction of this B vector, provided mu remains constant. We are assuming, as per our assumption, that this is a single strand whose conductor property is not changing and whose permeability is not changing. So if H is along the tangent, B would also be along the tangent of the surface with a circle of radius X. And my focus here is on this shaded area along which I have taken the section dL and width dx.

Basically, this is a cylindrical tube of radius x and width dx, and along this cylindrical tube, I'm trying to focus on or find the corresponding inductance. Since the current density is uniform, remember that the current density is i divided by pi r squared, which is uniform along this conductor. If I have to find the current in this cross-section of radius x, I would multiply this current density by the corresponding current area. The radius is x, so the area is πx squared. So that gives me the value of I of x. I of x is in the same direction as we have chosen for direction i. Because of I of x, these internal magnetic flux lines can still exist.

$$I_x = \frac{\pi x^2}{\pi r^2} \Rightarrow I_x = \frac{x^2}{r^2} I$$

If I of x is defined in this manner, then what we make use of is Ampere's Law, thanks to Ampere himself, who stated that the net magnetomotive flux (MMF) or force around the closed path is equal to the current enclosed by the path, which is also equal to the line integral of magnetic flux intensity that is tangent to the circumference of the closed path. So this property or definition given by Ampère is essentially also responsible for dictating the direction of H along the tangent.

$$\oint H_x . dl = I_x$$

So, Ampere's law relates the line integral of H with the current that is enclosed inside this magnetic material or current-carrying conductor I of X. Now, if we focus on this line integral H of X and DL, this is basically called the dot product; there are two vectors, and the dot product of two vectors, let us say vector a dot vector b, if I have to take the dot product, then the dot product is always a scalar quantity and results in the modulus of a times the modulus of b times the cosine of the angle between vector a and vector b. So, theta is nothing but, if I have a common reference for A and B, if this is vector A and this is vector B, then theta is the angle between these two vectors. So, the dot product results in a scalar quantity; the dot product of vectors A and B is |A| |B| cos θ. So if A is H of X and B is DL, and if we recollect our previous discussion, or if we focus on these alignments of the DL vector and the HS vector, if two vectors are parallel to each other or they are along the same direction, then the angle between these two vectors would always be 0; essentially, in this expression, theta is 0 when a vector is the same as the H of X vector and B is the same as the DL vector. So, essentially, h(x) · dl is nothing but h(x) · dl · cos(0), which is 1, and h(x), if we focus or look at it, since this is a circular cross-section.

If I were to mark h(x) instead of this point, if I were to mark it over here, then along this point, also h(x) would be a tangent line. Sorry for marking b; let us say this point is point P1. And over here I have point P2; then along point P2, h of x will still be a tangent to point P2 because P2 lies along the periphery of a circle, and the quantum of h of x will also remain the same because the current density is uniform. So, the value of x has not changed between P1 and P2. So, h of x would remain uniform throughout the periphery of the corresponding area. So, in a way, I can rewrite this as h of x line integral of d l equal to i x, where h x and d l are simply magnitudes;

$$H_x \oint dl = I_x$$

they are not vectors. The reason H of X is constant is that the current density is constant and uniform, so H of X will be constant at all points on the hollow tube of radius X. HX and DL are collinear vectors,

$$H_x . dl = H_x dl cos 0 = H_x dl$$

so their dot product is also a scalar quantity, which is simply H of X multiplied by DL. Since HX is constant, this can be removed from the line integral, and hence this integral simply becomes H of X line integral of DL.

$$\oint H_x . dl = I_x \oint dl = I_x$$

Now, what do you think the line integral of DL would be? dl is the length that is being taken along this cross-section. So, if the entire dl is integrated, the integral of dl would be nothing but the circumference of this circle whose radius is x, which is going to be 2 pi x, and we also know I as x squared by r squared into r. So, if we plug in these numbers and substitute those numbers, we would have h of x as x into I by 2 pi r squared ampere-turns per meter.

$$H_x \oint dl = I_x \Rightarrow H_x = \frac{I_x}{2\pi r} = \frac{xl}{2\pi r^2} At/m$$

Here, the number of turns is only one because we have only one conductor. So, if we know the magnetic flux intensity, now if we want to find the associated magnetic flux density, which is B of X, as I mentioned, for a conductor or a material of uniform permeability, magnetic flux density is nothing but permeability times magnetic flux intensity.

$$B_x = \mu H_x = \mu_r \mu_0 H_x = \frac{\mu_x l}{2\pi r^2} wb/m^2$$

Since we know H of X, we can also find the corresponding B of X, where mu is nothing but mu naught times mu r. Mu naught is called the permeability of free space. It's a universal constant: 4 pi times 10 to the power of minus 7 Henry per meter($\mu_0 = 4\pi * 10^{-7} H/m$). Whereas, μr is a dimensionless quantity, and materials with poor magnetic properties have μr equal to or close to 1. So, most of our electrical conductors are not made of good permeable materials.

So, their mu r is going to be close to 1. Now, since we know h of x and b of x for this tubular section d of x, can we not find the energy density? Yes. The stored magnetic energy density at any point in this hollow tube of width dx is nothing but half of b of x times j of x.

$$u_{Bx} = \frac{B_x H_x}{2} J/m^3$$

The unit is joules per meter cubed, and once we know this current density—or, sorry, stored magnetic energy density—since the cross-section is uniform, the conductor is circular, and d of x is uniform along any length across this particular hollow tube. So, the current density and the stored magnetic energy density are uniform at all the points along this hollow tube, and if I were to somehow know the volume of this hollow tube, then I could also find the incremental or delta energy that is stored in this hollow tube. So, let us see how we find the volume of this hollow tube.

Remember, this hollow tube is actually a cylindrical hollow tube. So if I were to expand this cross section, then this is how the conductor cross section looks. And inside the cross-section of induct radius x, this is how the tube is expanding along the width of the conductor. So this is a cylindrical hollow tube whose radius is x and width is dx. So if I have to find the volume, let me try to find the cross-sectional surface area of this hollow tube.

The cross-sectional surface area of this hollow tube is nothing but the circumference, which is 2π times dx, because dx is very small. So, this is the cross-sectional area of this hollow tube, and this hollow tube might have a length of a few meters or kilometers. So, if I assume that the length is 1 meter, which is the new notion of unit length here, then 2 pi times 1, which is the unit length, times dx is the volume of this hollow tube. The volume has uniform energy or stored magnetic energy in it because, irrespective of x or irrespective of dx length or dl length, the stored magnetic energy is going to remain uniform. So for this width of tube dx, I know what my elemental stored magnetic energy density is.

$$dE = u_{Bx} * 2\pi x . 1 . dx = \mu H_x^2 \pi x dx$$

$$dE = \mu \left(\frac{xl}{2\pi r^2}\right)^2 \pi x dx = \frac{\mu x^3 l^2}{4\pi r^4} dx\, J/m$$

So, if I plug in those numbers, this is what my d value comes to, and now if I have to find the net energy, then the overall stored magnetic energy density because of internal flux lines would be nothing but the integral of d of e, which, if I plug in those substitutions from this particular area here or part here. In the end, I am left with total E being equal to mu times I squared divided by 16 pi.

$$E = \int dE = \int_0^r \frac{\mu x^3 I^2}{4\pi r^4} dx = \frac{\mu I^2}{16\pi}\, J/m = \frac{L_{int} I^2}{2}\, J/m$$

$$E = \frac{1}{2} L I^2$$

The unit of energy is still joule per meter and not joules because I have chosen a conductor of length that per unit could be 1 meter, 1 kilometer, or 1 mile depending on the line length that has been considered. And if we recollect our initial slides to the precise third slide in this particular discussion, the energy that is stored, magnetic energy that is stored, is nothing but L times I squared divided by 2, and for this discussion, we are only focusing on internal magnetic flux lines. So, the corresponding inductance that we might get from this energy expression would be called L_int, referring to the term that it is because of internal magnetic flux lines. So, from here, if we find L internal, it is nothing but μ by 16 μ by 8π, which, when expanded, is μ_0 by μ_r into $8\pi\mu_0$, which, when simplified, results in L internal as 0.5 μ_r into the power of -7 henries per meter.

$$L_{int} = \frac{\mu_0 \mu_r}{8\pi} = 0.5\mu_r * 10^{-7} H/m$$

The beauty of this evaluation is that this internal inductance is independent of the current I because L internal does not have any current value in it. It is also independent of the material width; it is also independent of the material conductor diameter, etcetera, etcetera. It only depends on the magnetic relative permeability of the conductor, which, coincidentally, for good conductors, is usually non-magnetic. So, their value of μr is close to 1. And hence, the essence that we get here is that irrespective of what electrical conductor we use, if it is carrying current, the corresponding internal inductance because of internal magnetic flux lines is independent of current, it is independent of conductor radius, and it only depends on the magnetic permeability. That's all for today's discussion. We will resume our discussion on evaluating inductance due to external magnetic flux lines, during which we will talk about points outside the conductor. So far, we have been concerned about points inside the conductor because we're talking about internal magnetic flux lines. In the next discussion, we will talk about evaluating magnetic flux lines outside the conductor points, and we'll see how the same discussion of ampere flow can help in that.

Thank you.