Power Network Analysis

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Week - 03

Lecture-15

Hello everyone, welcome to Lecture 5 of Week 3 of the course Power Network Analysis. This lecture is also going to be the last lecture of week 3, in which we will continue our discussion on the module on synchronous generators. In this discussion today, we will understand a very interesting topic known as economic dispatch, which helps ensure minimum cost of generation and maintain the economics, one of the good attributes of a power network that we learned about in the first lecture, which was the introduction to the course. What is the effort involved, and what is the mathematics behind maintaining the economics of this system while dispatching generated electricity to the loads is what today's discussion will be about. This discussion that we will have will build upon the previous lecture discussion, which was extensive enough regarding the real power expressions, reactive power expressions, and how we control the flow of real power and reactive power from synchronous generators to the respective power network or individual loads, and in this entire discussion of economic dispatch, we will mostly focus on the real power aspect. So, controlling aspects we have seen in the previous lecture basically the real power control comes from the turbine action or prime mover action, and the economics behind this real power transfer is what is going to be the prime focus of discussion in this particular lecture today.

So, in a practical network, we usually don't have single generators. We have multiple generators. Some of them happen to be synchronous generators. Some of them could also be renewable-based, non-synchronous machines, such as solar PV power plants or wind generators.

They are not synchronous-based machines. These generators are placed at different locations in the power network. And often these locations are not the same as the locations where the loads are present. So, basically, there's a huge geographical disparity between the locations of the generators and the loads. And that's where the role of the transmission power network or transmission network comes in to sort of balance out this geographical disparity.

These different generators can have different operating frequencies. Their operating frequency is going to be different because the philosophy might be different, the technology may be different, and the type of fuel used to drive the synchronous unit might be different. So, depending on what type of synchronous unit we have, it could be a hydro-based plant, a nuclearbased plant, thermal, or renewable-based resources based on solar, wind, and biomass. And each of these different technologies, as well as the fuel mix type of the synchronous generator, brings in its own economic aspects. So, setting up a power plant has its own capital costs involved, and the operational aspect involves salaries to be paid to the operators or the people who are maintaining the plant.

Every power plant tends to have its own investment cost, capital cost, and operational or running cost. Depending on the type of plant, specifically for hydro and nuclear power plants, where a lot of effort is needed to set up or bring up those plants. For example, in a hydro power plant, a lot of geographical area is needed for creating the water reservoir, and building the reservoir and the dam itself is an engineering marvel, so it involves a lot of manpower and a lot of capital investment. Similar applies to nuclear power plants; a lot of safety considerations and safety designs go into setting up a nuclear power plant. So, often the capital cost of these hydro and nuclear power plants is very high.

And since the fuel that is being burned in a hydro plant is just the flow of water or a nuclear-based reaction, it is comparatively cheaper than a thermal power plant. So, the operational or running costs are low as well. There is another reason why nuclear power plants tend to have low running or operational costs: most nuclear power plants, even in India, operate as base load power plants, which means their power generation output does not change. Once they are commissioned, nuclear power plants usually tend to generate the same quantum of power under predisposed conditions throughout their lifetime. The reason why nuclear power plants generally act as base power plants is that perturbing or controlling nuclear reactions is obviously not as easy compared to controlling the flow of steam or the flow of water through the turbine to drive the prime mover or the generator.

So, that is the reason why the control perspective, or the safety perspective, I would say from the safety operation perspective, nuclear power plants tend to behave as base power plants, and if an element is operating at the same quantum of power, most likely its wear and tear would be less. So, maintenance costs would be lower, and hence operational costs would be lower. On the other hand, thermal power plants are not as capital-intensive compared to hydro and nuclear, the reason being the small geographical area that is often needed. So, the capital cost involved is comparatively lower, but the operational cost is very high because of a lot of rotating or moving devices being placed. Typically, in a thermal power plant, we would use either fossil fuels, coal, or gas to be burned in the boiler, which in turn heats up water, converting it to steam, and then the steam drives the prime mover to drive the synchronous generator.

So, a lot of wear and tear is involved, a lot of transportation is involved, and that is what often leads to higher operational costs in thermal power plants. On the other hand, with renewable-based generation or sources like solar and wind, the capital cost is also low, and the operational cost is also low because of less wear and tear involved. And more importantly, in renewable resources, the availability of fuel itself is either wind or solar radiation, which are naturally available; hence, no additional setup is needed, unlike in a hydropower plant where a dam needs to be placed. In solar and wind turbine generation, the corresponding effort required to set up is comparatively lower, and the operational cost is also lower. So, given a network that has these different types of synchronous generator technologies in place, eventually all these synchronous generators are essentially driving the network load.

If the load had not been there, then there would not have been any need for a power network, there would not have been any need for generation, and no need for a transmission network. Now that is point number one. Point number two: this load often tends to have variation. So what is shown here is a beautiful duck curve. It's the source of this particular duck curve.

It's also provided here in this link. So basically, it is the predicted load for 2024-25 for a typical day, and it also has certain curves for the previous years for some particular resource or location, which can be seen in this particular link. What is important to observe here is this load that we are experiencing across 24 hours, starting from 6 am in the morning until 6 or 5: 55 am the next day. The load does not have a constant profile; if the load had remained constant, then the effort needed to maintain system economics would have probably been reduced and not as substantial. Now, why does the load have variation? It's pretty normal; it's pretty obvious.

It depends on how we live our lifestyle. So typical of human beings, they tend to get up at 6 a.m., get ready for their day, prepare for their work or school or office or college, go out, and follow their daily routine. And then around, typically when people are at their offices, the corresponding domestic loads tend to consume less energy.

Washing machine lesser heating lightning load is needed because it's 12 noon, and again as evening sets in, lightning load tends to go up; people tend to return to their homes or cottages wherever they stay, and in the evening we need light, so the power or the load again tends to go up around evening. Once post dinner, usually normal human beings tend to sleep, so again after 12 noon or so, or around 8 or 9 pm , the curve tends to go down, and again from 6 a.m. it tends to go up. So that is how a typical load for a typical day, or for a typical load for a community, or a society, or a city, tends to have two peaks and one duck, or one flat lower profile, and if we try to fit it in, it looks like a duck curve, with the head of the duck representing one peak and the tail representing the other peak.

The belly of the duck represents the lower bottom half, and this is not just for one day; the overall load, or aggregated load, in a network also tends to have annual variation. Typically, in extreme summer and winter weather, the power consumption goes up; in summer, the consumption is for cooling, and in winter, the consumption is for heating, while during the spring and autumn seasons, the weather is pleasant, and the corresponding aggregate load requirement tends to go down. So, the daily load curve will also typically follow a duck shape if you have to plot it for a particular city, community, society, or country. So, the question is, if the load itself is not constant and individual aggregated loads tend to have this variation, how do we ensure our generators are operating at minimum cost, that is, economics is being maintained, and how do we ensure that profits are being made by the generators, because every entity would tend to make a profit to the extent possible? So, that's where the idea of dispatch comes in, or economic dispatch comes in; total generation must always meet the demand or the load. In addition to meeting this demand to ensure system security, we also expect, or it is also expected of a network operator, to ensure some generation margin that is over and above the committed generation which would meet the demand; we call this a spinning reserve or a rotating reserve.

More details will come as we venture through the upcoming discussions, specifically in stability analysis and power flow analysis. So, over and above the scheduled generation to meet the demand, some generation margin is always committed in a power network to address any contingent conditions, such as load changes, that might not have been anticipated. The typical generation dispatch philosophy one would think of is that we have different types of units with different efficiencies, so let's try to prioritize or operate the highly efficient units first, so that during light loads these highly efficient units deliver power to these loads, and as the demand goes up, lesser efficient or more costly units are brought online. The problem with this conventional thought process is that it might not always lead to the lowest cost of generation. By least cost of generation, I do not mean individual cost of generation.

The power network consists of several generators. When I talk about system economics, it refers to the total net cost involved in generating power from all these individual units. Minimizing the cost of one such generation unit will not suffice. If economics has to be maintained from the system perspective and fairness is to be ensured, one should always be concerned about the total cost involved in generating power in a given power network and not the individual units. So this conventional philosophy will have this problem and that's where economic dispatch comes in, which is a very... integrated, beautiful, optimized way of ensuring least cost of power balance between generation and loads. Specifically, our discussion of economic dispatch will focus only on thermal units because their operational costs are comparatively higher, capital costs are moderately lower, while ensuring system security and reliability; one of the ways of ensuring security is through maintaining spinning reserve or sufficient generation margin. Again, for the simplicity of our discussion, we will not consider this notion of spinning reserve. I will briefly touch upon the notion of spinning reserve when the right time comes in upcoming lectures. And depending on the time frame in which this power balance is to be maintained, if we try to maintain the power balance between generation and load at 9 PM at night, since I am trying to maintain optimization only for one time snapshot, I would call that a static economic dispatch.

Which may make sense for that given period of time, but in practice, the load keeps changing, and that's where the beauty of economic dispatch comes in. So if we try to ensure power balance while maintaining security and reliability for a time period, let's say 24 hours, then that would be called dynamic optimization-based economic dispatch. This is just for the sake of discussion. For the sake of discussion, we'll focus on a static framework for economic dispatch, for the simplicity of discussion. So, the question is, why do we need economic dispatch only for thermal units? As I have already mentioned, nuclear units mostly operate as base plant units from the perspective of safety reasons and from the control complexity involved in controlling these plants.

Hydro units have low operating costs; in fact, they have costs close to negligible, with high ramp-up and high ramp-down capability. Now, what is meant by "ramp-up" and "ramp-down" capability? Suppose there is a generator which is generating, let us say, this is generator I which is generating P_{gi} at a given instant of time, and as the load is varying with time. So, let us say at T plus delta T time the load earlier was P_{DT}, and at T plus delta T the load has changed to $P_{T+\Delta T}$; the load could have increased or could have decreased. Since generations, total generation has to follow the entire system's demand; it is expected that the generation of the ith unit at time t plus delta t might be different from the generation at time instant t in order to maintain the balance of load at different time instances. Now, when the generation tends to change, that means some control action has to come in.

So, basically, real power control has to come in, which means that the turbine has to respond so that the corresponding generation output, specifically real power output, can change. Now, this change, which happens, has to occur within a time instant or duration of delta t ; generation might go up or down depending on whether the load goes up or down accordingly. Often our power plants, specifically thermal power plants, have a lot of rotating devices involved; there is a boiler involved that has its own mechanical inertia. Temperatures in a boiler cannot change instantaneously; a boiler takes time to heat up and also to cool down. So, if the boiler is taking time to heat up and cool down, the corresponding steam exchange would also take time, and as a consequence, the turbine action will also take time.

On the other hand, for hydro units, the hydro units' power control will just be conducted through the flow of water. So, if you open up the valve in a hydro turbine, more water gushes in, the turbine rotates at more speed, and hence more power is developed. Since the water is involved, its controllability is faster, and that is what leads to ramping up and ramping down. Ramping up essentially means generation increases from its previous instant of generation; ramping down means generation decreases from its previous instant of generation. Hydro units have faster controllability and response times; hence, changes in generation can occur at a quicker pace, whereas in thermal units, the boiler and turbine, along with the generator itself, are involved, which have their own mechanical inertia to changes in temperature and movement or rotation.

Thermal units tend to have very slow ramp-up and ramp-down capability. Their operating costs are also higher because of transportation and maintenance involved. The important part that is critical here is the slow ramp-up and ramp-down capability, since the load is not stagnant, and things become much more difficult when we have these renewable resources, which are non-conventional sources; their generation pattern depends on weather, solar availability, or wind availability. So, if the generation is also changing from a renewable perspective. There has to be some generator that should act as a balance or buffer for this mismatch between generation and load, as well as changes in load.

So, the difficulty lies in the slow ramp-up or ramp-down capability; thermal units have a difficult time adjusting themselves while catering to system power balance and security. And that is where the dynamic economic dispatch makes even more sense, which we will not consider for our discussion. The idea in dynamic economic dispatch is that we plan well ahead of time rather than focusing on a specific time of interest, and we would be prepared economically with our thermal generation schedule, which would ensure economics, security, and reliability while meeting the system demand. When we are talking about economics, it makes sense that there has to be some quantification for the economics behind power generation. So, often from a thermal unit perspective, each of these units generates their power, which is a net output of P megawatts; it is associated or linked with some fuel cost or other costs, which are usually obtained from the heat rate curves of the boilers and the corresponding combined boiler-turbine action.

We tend to assume this cost to be an hourly cost for the sake of our discussion, and it need not; this cost curve, which is being shown here, has the cost on the Y axis and the generation on the X axis. This cost need not pertain to instantaneous cost for the sake of simplicity, again for the sake of discussion. We will assume this cost to be an hourly cost, meaning the generation tends not to change for the simplicity of discussion; again, practically, it would definitely change. The generation is not changing for a given time slot, which is typically one hour, and hence the cost associated with generating that particular power is also constant, and that is where hourly costs are involved. Typically, out of this total power P , which is generated, around 6 to 2 percent of the power is consumed as auxiliary power for pump action, condensers, rotor excitation, etc. , etc. 94 to 98 percent of net power P is observed as the gross output. And usual cost characteristics are not smooth, they are not continuous, they are not differentiable, which we will not be bothered about as of now. We will assume ideal conditions where the cost characteristics are smooth and non-ideality or non-practicality comes in because generators, typically thermal units, don't have a continuous mode of operation. As the temperature changes from a few hundred degrees to thousands of degrees, the corresponding power output changes with respect to boiler input or fuel input, which is coal or gas. The corresponding power output has a nonlinear relationship, and that is what we call valve point effects.

The moment we open up the turbine valve for more steam to flow in or close the valve for the steam to flow in less, the corresponding output change is not linear with respect to this valve change effect, which in a way results in burning more fuel in the boiler or less fuel in the boiler. So practical characteristics tend to have discontinuities because of non-linear responses, and from a safety perspective, every generator has a minimum generation limit and a maximum generation limit. Now the minimum generation limit typically comes from the perspective of the boiler, whereas the maximum limit typically comes from the perspective of the turbine. Now, what are these boiler and turbine considerations? Typical boiler, if it has to generate or heat up or sort of warm itself up, generates real power so that steam can be produced. The temperature below which a boiler should not cool down is usually specified from an operational perspective.

So, if the boiler cools down below a particular temperature, it is as good as the thermal power plant becoming non-operational, and it would take a few hours to a few days to warm up the boiler again to generate the corresponding steam for power generation. So, from an operational perspective, the boiler temperature specifically, or boiler operational temperature, determines what the minimum limit is for what the generator can generate or should generate, and the turbine, being a mechanical device, operates beyond a particular limit. So, a turbine beyond a particular speed or a particular power cannot deliver more mechanical power to the generator. So, the turbine limitation, in a way, also dictates the corresponding maximum real power that can be extracted from this synchronous generator. So for the sake of our discussion, we will assume this cost to be a smooth continuous curve, and often in literature, this cost has been considered to be a quadratic function of the real power output P ; in some contexts, this cost can also be considered a linear cost depending on the type of application.

Before we go deep into the economic dispatch perspective, let us understand a bit about how we go about maintaining system economics. So let us say we consider a function f of x, where f is the function, x is the corresponding argument of the function, f is continuous, and we assume it to be at least twice differentiable. That means if $f(x)$ exists, then f prime x, which is also df by dx. This will also exist and its double derivative also exists. That means d squared f by dx squared; they are continuous functions.

Their values are well defined for all values of x , and we want to find the value of x where f takes its minimum possible value. So, how do we go about doing that? Let's take a typical function, a typical function f of x . I've just plotted it here. Typically, if this is my f of x, then if f of x is known, I also need to see what f dash x or f double dash x is.

So let me rewrite it again. f prime x . It is nothing but the first derivative of the function f of x with respect to x itself, so if we think about what df by dx is from the first principles of calculus. df by dx is nothing but the slope of the function f at different values of x. So, let us say we focus on this blue part of f of x. If we try to choose one particular point, which is, let us say, point number a, which can also be associated and marked over here, let us say point number a.

And from A, we tend to move along the curve. So let us say we tend to move down the curve. If I now have to draw a line or a tangent at this point A along the function f , then this tangent is nothing but df by dx . Since the function value is decreasing from point A along the tangent df/dx, the slope of the function is negative. And that's the reason why, for this blue curve, the corresponding f prime value, or f dash value, is also negative. This negative keeps happening until the function value at this particular point, let's say point number B, becomes or reaches its least possible value, and if I now try to draw a slope or a tangent at this point B for the function f of x , this tangent will have a value or a slope equal to 0 .

Basically, it means that df by dx would be 0 . So that's the reason why point B is over here. And similarly from B to D , let's say this is point C ; at points C and D , the curve's first derivative tends to take a maximum value. And again at point number D , the slope is 0 . So, if we continue this logic, we have three points. among a to f where the slope or df/dx is becoming 0 . So, from the first principles of calculus, in order to find the value of x where f takes its minimum value, what we do is since the minimum of f of x is either at point b or at point f , also df by dx tends to be 0 at point d as well. So the first principle of calculus is to take the first derivative of the function f , equate it to 0 , solve this function, find its root, and we will likely be able to find our x star at which f becomes a minimum. Now, as I have mentioned, points B and F are the points where F is taking its minimum value; point D appears to be a point where F is tending to take a maximum value among the local neighborhood of the points around D , or B and F are minimum in the local neighborhood of points around B and F. If we now take the double derivative or try to understand how the double derivative would look, like f of x or f prime of x is the slope of f of x at different points. The double derivative function is nothing but the slope of different points of f prime x at different instances.

So if we, let's say, mark points a, b, c, d, the six points that we have, say around f, the way we try to find slope values of f of x at different points, let us also try to find the slope values of f prime x at different points. If we look at point number c and point number e , the double derivative, which is the slope along this line, is considered. For f prime x at c and e, the corresponding f prime or double prime f double dash value, let's say if this is f double prime x on the x -axis, we have x ; all these are x values. The f double prime value is taking a value of 0 at points c and e, and in between c and e, at point number d , the function value is decreasing.

So basically, if I draw the slope, which is the f prime value. C to E, the double prime value would be a negative value because the slope is negative here. So the slope typically has a negative value. It would have a lower peak at point D where it has a zero crossing. And from E to F , the curve increases.

So, the slope is also increasing. So basically, it would look something like this along here. C to F and between C to A , which I have sort of missed here, between C to A , the slope is again increasing. So basically, around point F , it would have a dip, and from B to A, it would look something like this: so double prime x , which is the slope along this point. Now, if we see if f double prime x is the slope of f prime x at point C and f, where f prime x was 0 at point B , and the double prime value is positive.

So here, this value is positive; here it is positive, where at D . where the value was taking its maximum around the neighborhood of x , the value here of double prime is negative. So, if we apply this first principle logic, this is a standard way of finding the minimum and maximum points. At x star value where the double derivative is positive, it would refer to a minimum point. Again, this all refers to local minima.

There is a difference between local and global minima. All these first principles will always give us the local minima or maxima. For global minima and maxima, there are certain additional conditions that are beyond the scope of the current discussion. If the double prime value is negative, X star refers to a maximum point, and if F prime is again zero like the first derivative, we call it a saddle point. Our point of interest from an economics perspective is to ensure this particular condition when we have a cost function that is denoted by f of x . So what we do is we have an economic dispatch problem where there are n units that are all connected across the same fictitious bus, boiler, turbine generators; they are marked, and each of these generators has its individual cost curves.

Usually, these cost curves tend to be quadratic functions of f of g, and the idea is that all these generations together should satisfy the common load p of d, which is connected over here, and each of these generators should respect their respective minimum and maximum limits. So what do we do? How do we solve this problem? In the unconstrained form, there were no constraints involved, whereas in economic dispatch, there are also additional constraints. We call this an equality constraint, whereas this constraint becomes an inequality constraint. So how do we go about solving or handling these inequality constraints in optimization problems? Basically, we're talking about constrained optimization. So what we do is try to find an unconstrained function that is equivalent to this constrained problem.

So, what we do is focus only on the equality constraint for timing, which ensures that total generation is equal to total load, and we ignore the inequality constraints for timing. So we will handle them separately. And when we try to append the equality constraint to the overall cost function, which is f of t , there has to be some scalar or some variable because, remember, the unit of f of t is typically in dollars or rupees per hour, whereas the unit of this equality constraint is typically in megawatts or per unit. So, if we have to combine rupees with megawatts or per unit, there has to be some variable or multiplier that should handle this change in unit, and that's where the variable or term lambda comes in. A similar notion could also be applied to inequality, but we will not be considering them as of now.

So this function here represents an unconstrained form of the actual constrained form, and L comes from the term known as the Lagrange or the Lagrange function. In this Lagrange function, there are two sets of variables. One is the variable lambda, and the other is the generation of each i-th unit. And if this is equal to the Lagrange function, it can also be considered equivalent to f of x in the previous slide, where x is lambda or pgi for each ith unit. So, the way we applied the first order condition there, the similar logic could also be applied here.

So, if we do that we come up with our first order optimality conditions. So, we have two variables or two sets of variables; the first derivative with respect to pgi gives us this equation here, and the second, being the lambda variable, gives us our usual power balance equation. What we see here is that we have n plus 1 variables, n pgs as unknowns, and 1 lambda as an unknown. So, we have n plus one unknowns. We have n plus 1 equations; all are cost curves that are smooth, continuous, and differentiable. So, with n plus 1 equations and n plus 1 variables, we can easily solve them and get our solution.

And these equations also have a good name known as coordination equations. Essentially, lambda is defining the incremental cost of supplying any additional change in load over and above the current demand, Pd. Now all is good as long as the solutions of these n plus 1 coordination equations automatically satisfy the generation limits. There is no guarantee that these solutions of coordination equations would ensure satisfaction of generation limits. So, what do we do about them? If a generator happens to be generating within its minimum and maximum limits, that's where the simple curves come in. As per the coordination equation, which is the one over here, this should always be true; however, if there is a generator whose generation happens to violate the lower limit, actual generators should not violate their limits.

So, suppose there is a generator whose generation happens to be less than the corresponding lower limit; then what should be the case? What we should observe or understand here is why this generator is generating power less than its minimum limits. It is probably generating power less than its minimum limit because it is most likely a costlier generator. If it had been a cheaper generator for a given load, it would have tried to generate as much as possible to ensure the least cost of generation for the entire network. It is typically a costlier generator, so its incremental cost of generation, which is indicated by this first derivative, is more than the current value of incremental supply of load. If it had been a cheaper generator, it would not have generated beyond its lower limit.

The same goes for the case when a generator tends to generate more than the maximum limit. It is most likely the case that this generator is a much cheaper unit and hence its marginal cost of generation, which is the first derivative, is less than the actual cost of supplying the incremental cost of load. If it had been costlier, then it would not have gone beyond its maximum limit. There are other ways to explain or understand how these relationships should come about. But I believe I have given my best to simply explain to you all why these units or why these relationships should hold true.

So this gives us a clue of how inequality constraints can be checked. So how do we go about that? This slide presents a simple flow process. Solve n plus 1 coordination equations for n plus 1 unknowns without considering limits. If the limits happen to be within boundaries or limits, the solution has been obtained and nothing needs to be done. But in most cases, it won't happen. So what you do is figure out the generators that are violating their minimum limits, call that set of generators small p , figure out the generators that are trying to violate their minimum limit, and call that set of generators q.

So basically, p refers to generators that are tending to violate their maximum limit, and q refers to the set of generators that are trying to violate their corresponding minimum limit. Evaluate these violations, which are shown over here or encircled over here, and then evaluate delta H. Basically, delta H is a measure of how much the deviation is in the respective generators violating their respective maximum or minimum limits. If delta H is positive, it means there are more generators that are violating their maximum limit. So let's first reset these generators to their maximum limit because violation should not happen.

If delta H is negative at a given point in time, it would mean more generators are trying to violate their minimum limit, so we reset them to the minimum limit. If delta H is perfectly zero, we can set the respective generators to the respective limit. Once the generators are set at their respective limits, these thermal units are out of the economic dispatch problem. So the coordination equations tend to reduce because generations are fixed at their limits; economics is no longer applicable to them. We again resolve a new economic dispatch with the new load as the original load minus the generators that are frozen at their limits.

So they basically tend to minimize the overall load, resolve the ED problem, come back again to step two, and repeat this until all violations are gone. I'll just conclude this discussion with a simple example. There are three units that are present in a given power plant. The corresponding quadratic cost curves are given, the respective generation limits are provided, and you have to solve the economic dispatch for a total load of 800 megawatts. So what do we do? We first have three Pg's, which are unknowns Pg1, Pg2, and Pg3 representing three generations, and we also have an unknown lambda, which is the incremental cost of supplying the load.

So, for four unknowns, we figure out our four coordination equations, which are shown over here; we solve them, and what happens is that when we solve them, we can see generation number one and generation number three. They are not within their limits; Generation 1 is violating its maximum limit, Generation 3 is violating its minimum limit, and Generation 2 is in between the limits. Basically, we have a case where a solution is still not obtained, so what we do is find delta h. Pg1 is violating its maximum limit, and Pg3 is violating its minimum limit, so we find delta h . Delta h turns out to be positive, so we first set Pg 1 at its maximum value; Generation Pg 1 is at its maximum value.

The total load was actually 800 megawatts. PG1 is frozen at 300 megawatts. So essentially the new load is 500 megawatts, which is 800 minus 300 . For 400 megawatts, we again resolve our economic dispatch while not considering unit 1 to be part of the economic dispatch.

We are left with only two units. PG2 and PG3 are variables. Lambda is a new variable. And new demand is 500 . We resolve these for three unknowns. And for this, it happens that PG1 is 300 megawatts, while PG2 and PG3 are within their limits.

The total cost of generation is 7559.91 rupees per hour; an optimal solution has been obtained. And we can also check this condition that when PG1 was violating its limit, the Generation 1 being costlier, its incremental unit or supply is actually less than the current lambda, which is 9.364 megawatts, while for the other two generators that are within their limit, their incremental cost of generation is the same as the incremental cost of supplying the load. In the next lecture, we will take up the last aspect of the parallel operation of synchronous generators, which will be the last part of module number 3. Thank you.