Power Network Analysis

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Week - 03

Lecture-14

Hello everyone, welcome to lecture number 4 of week 3 in the course Power Network Analysis, in which we continue our discussion on the module synchronous generators. In today's discussion, we will understand in detail how the real and reactive power output of a synchronous generator, to be specific, can be controlled, regulated, and monitored so that the corresponding control power can be regulated accordingly through manual intervention, or there are also automatic control loops that do that, but these control loops also have to manually trigger or perturb some entity in the setup of synchronous generators so that the corresponding real and reactive powers are regulated as and when required. So, we will understand those points, items, or quantities that are regulated and how the consequences appear on real and reactive power exchange; that is what we will discuss today. In the previous lecture, we discussed at length what capability curves are, how we get the points on the PQ domain, and the complex plane domain, marking all those safe operating points for a synchronous machine to be general. Regarding the maximum armature current limit, maximum field current limit, and maximum steady-state real power limit. So, we understood all that, and that understanding also involved two critical equations: the real and reactive power expressions for a cylindrical pole synchronous generator with negligible armature winding resistance.

So we will also use those expressions in this discussion to understand how control can happen from the power perspective. So that is where these expressions of this slide come into effect. We have a cylindrical pole rotor or cylindrical pole synchronous generator with negligible armature winding resistance; therefore, we have this as our real power expression and this as our reactive power expression.

$$P_{\Phi} = \frac{V_t E_f \sin\,\delta}{X_s}$$

$$Q_{\Phi} = \frac{V_t\left(E_f \cos\,\delta - V_t\right)}{X_s}$$

And we had also seen these expressions earlier where we understood that if the synchronous machine has to behave as a generator, wherein a generator converts input mechanical power to output electrical power for a generating mode, the real power expression P, if it has to be positive, the only possibility for it being positive is that in the P expression $V_t E_f$, the terminal voltage, induced EMF, and synchronous reactance are phasors; they are scalars, basically magnitudes of phasors, and they have to be positive.

There is no way that they can become negative; the only possibility for P to become positive is when sine delta is positive. So, for sine delta to be positive, delta has to be positive, and that is how, for real power transfer in generating mode, delta inherently being positive means the induced EMF always leads the terminal voltage in a synchronous generator. In a motoring mode, the motor converts the input electrical power to output mechanical power. So, in a way, real power is being absorbed; the P value has to be negative, and for P to be negative in motoring mode, sine delta has to be negative, which is only possible when delta is negative. That is why, in motoring mode, induced EMF always lags the corresponding terminal voltage.

Similarly, for reactive power transfer, irrespective of motoring or generating mode, if the machine has to deliver reactive power, that means the term which is sitting here, which is the part $E_f \cos\ \delta - V_t$, when it becomes positive, which is what is shown over here, then only the reactive power will be delivered, and for the machine to absorb reactive power, $E_f \cos\ \delta$ has to be less than the terminal voltage. Now, one thing I will specify or pinpoint before we go deep into our discussion is that when the machine, or synchronous machine to be specific, is delivering reactive power, that means there has to be some device that is absorbing this reactive power. So, basically, what I am trying to say is that for a machine delivering reactive power, the associated load connected to this synchronous generator is inherently an inductive load because inductive loads tend to absorb reactive power. So, in a way, reactive power is being absorbed here. Reactive power is being absorbed, and this absorption of reactive power is happening because the generator is able to deliver this amount of reactive power.

Now, when a load is absorbing reactive power as an inductive load, we tend to define our power factors specifically from the perspective of loads and not generators because loads drive the entire system, and hence the notion of power factor also comes from the concept of how loads are operating. Which means that if there is a synchronous generator that is delivering reactive power to an inductive load, we would say or infer that the synchronous generator is feeding an inductive load that is inherently of lagging power factor. So, under overexcited mode, which we will see in a moment, the intuitive load connected to this machine has a lagging power factor. Now, when the power factor is lagging from the perspective of the load, that means the corresponding armature current, which is being fed from the generator, is also lagging its own terminal voltage because power factors, as I said, are defined from the perspective of loads, not from the perspective of how the sources are operating. If there had been no loads, there would have been no need for an electrical power network, and there would have been no need for electrical sources.

So, that is how the notion should be that for reactive power delivered, the machine behaves as a capacitor; its own power factor becomes leading, but then to compensate for this leading power factor or reactive power delivery, the load is of inductive nature and its power factor is lagging. Similarly, for under-threaded mode, the load has to be a capacitive load because the machine is absorbing reactive power. So the load is behaving as a capacitor, which usually operates under leading power factor mode. So when we say a machine is connected to a particular load having some power factor, the power factor is inherently the load power factor, not the machine power factor. So, having discussed these expressions, let's now see how the control aspect can come in.

We have talked about isolated synchronous generators at length, but if we talk about the entire power network, we have seen that generators need not be present at the same locations where the loads are, and that's where there is a need for a transmission network. Since there is a network that acts as a passage between power generated and power delivered, we often don't have isolated synchronous generators that operate in isolation. So that's the reason why this statement says that an alternator rarely delivers power to individual loads. It transfers its powers through a power network that consists of several such buses. And when this alternator is connected to the corresponding power network, which has a few far-off distributed loads, there is a process that is followed, which we will see in the next immediate slide.

And often, for the sake of analysis, we consider the bus to which the alternator is connected in the power network. We call that bus an infinite bus. The term infinite here refers to the fact that it can absorb or deliver any amount of real or reactive power. Ideally, yes, it is good to have this notion. Practically, we don't have infinite buses in the power network because no such bus, device, element, or load will have the capacity to absorb or generate any amount of regular reactive power.

It is only from the mathematical analysis perspective, for the sake of simplicity, that I would say we can mathematically analyze our systems or generators. We consider the bus to which the generator is connected to be the infinite bus. To some extent, it might make sense because when we say a generator is being connected to a power network, the power network is a huge, geographically spanning network operating at different voltages, having several transmission lines, distribution lines, transformers, and loads of different types, etc. So, if we compare the capacity of power that is being handled in this power network, which could be in the order of a few gigawatts, remember India's installed generation capacity is close to around 450-460 gigawatts. If we compare one generator whose capacity is, let's say, a few megawatts to a network that is handling power of a few gigawatts, there is actually no comparison.

This standalone generator will probably not significantly perturb the voltage and frequency of a network that is handling gigawatts of power capacity, and that's where the notion of an infinite bus makes sense in some practical sense. So that's the reason why practically a bus to which a large load or a large generator is connected behaves or tends to behave as an infinite bus that doesn't have much perturbation in its terminal voltage and frequency, and hence we can call that an infinite bus. It also has to do with the inertia present in the bus from the effective inertia perspective, which we will also discuss in detail in the last module of stability analysis, regarding how system inertia helps in strengthening or weakening a bus from the infinite bus perspective. So the process of how this generator is connected to the power network is known as synchronization. There are four simple steps to ensure that the machine is synchronized.

The first one is ensuring that the terminal voltage of the generator and the terminal voltage of the bus to which it is connected match. The frequency of the generator and the infinite bus should match. The corresponding phase sequence ABC or RYB should have the same sequences connected, and the phase connection of the phase difference should be zero. Now the reason why these steps are important to ensure before loading the generator to generate some power is that if we have a generator whose terminal voltage magnitude is different from the terminal voltage of the infinite bus, that means we are trying to connect an active device or a generator whose magnitude is greater compared to a bus where the voltage magnitude is lower. So, essentially, these two points will have a potential difference.

And in between this potential difference, if we have a switch that is supposed to be closed to ensure synchronization, there are safety hazards involved in closing switches when the potential difference across the switch is not zero. So ensuring the terminal voltage magnitude is the same, phase angle differences are the same; even if the magnitudes are the same, phase angle differences don't exist. If we start connecting A, B, and C of the generator to capital ABC on the grid side, let's say this is the generator terminal and this is the infinite bus; if we have ensured that the terminal voltage magnitude and phase angles are the same, and now we incorrectly connect phase A to phase B, phase B to phase C, and phase C to phase A, We would definitely have line-to-line voltages between these switches, and again, if we close the switch with some non-zero potential difference, there might be safety hazards. So those are the issues or reasons behind ensuring that these quantities match. The last term that is to be matched is frequency again.

The first three terms, to be precise, the terminal voltage magnitude has to do with how much reactive power is being generated or absorbed dominantly. When we close the switch to ensure synchronization between the generator and the infinite bus, we are ideally ensuring that there is no reactive power exchange happening between the generator and the infinite bus. Similarly, if we ensure the frequency to match for both the generator and the infinite bus, we are ensuring that at the point of synchronization, ideally no real power is being exchanged. And those are again from the perspective of safety for the operational purpose, as well as the synchronous generator that is being synchronized. Practically, there are few differences.

In practice, the frequency of the incoming alternator or generator is kept a little higher, probably somewhere around 0.01

A machine behaving like a motor will start driving the turbine, which can have its own implications if it is a steam turbine or a hydro turbine as well. So from the safety perspective, the synchronous generator should always deliver real power to ensure the delivery of real power. The frequency of the incoming generator is kept a little higher, maybe somewhere around 0.01

That is the reason. So essentially, if we recollect the capability curve discussion, there we also emphasized that all our capability curves are from the perspective of fixed terminal voltage and fixed synchronous reactance. In the process of synchronization, we are also ensuring that the terminal voltage of the synchronous generator matches the terminal voltage of the infinite bus or the grid supply. So that's the reason why when it is connected to an infinite bus or a grid, it will typically always have a fixed terminal voltage and also a fixed synchronous speed or frequency. Frequency and synchronous speed go hand in hand. In fact, synchronous speed in RPM is $120\mathrm{f}/\mathrm{p}$, where p is the number of poles induced in the rotor or the stator windings or from the MMF perspective, and f is the frequency.

To understand the control action behind real power and reactive power, we have two entities that can be controlled from the outset. The first is regulating the rotor or the field current, which is DC current from the rotor excitation perspective, and the next control that can come in is from the perspective of the turbine, which is feeding in mechanical torque or mechanical power to the synchronous generator. The variation of rotor field winding primarily governs the reactive power delivered or absorbed by the machine, whereas the mechanical torque control primarily governs the real power generated by the corresponding synchronous generator. So in this particular discussion we will talk about these controls at length and then we will try to understand how their implications can come in. So we will first start with the excitation control, power factor control, or reactive power control, in which the rotor field winding is regulated or varied as per the requirement, and by doing so, what happens is the induced EMF, E_f or E_{rms} value changes in a way that the corresponding torque angle might also change depending on whether the real power is kept fixed or not, and the change in the induced EMF significantly regulates whether the stator current is leading or lagging the terminal voltage.

What we would do for the sake of discussion is to make it simple: we will consider the reactive power control for a specific machine; that machine is a cylindrical pole synchronous generator with negligible armature winding resistance, R_a, almost being 0 . We will also sort of recollect that when the machine armature current is lagging behind the terminal voltage, the machine is, in a way, feeding an inductive load that is hungry for reactive power. So, the machine is actually delivering reactive power. With that notion in mind, we will now try to see how the reactive power control comes into effect. So we are talking about this particular machine, a cylindrical loader synchronous generator with negligible armature resistance, whose expression for real power is P equal to $\frac{E_f V_t \sin\,\delta}{X_s}$, and here the terminal voltage is a fixed quantity because the generator is synchronized.

$$P_\Phi = V_t I_a \cos\,(\theta) = \frac{V_t E_f}{X_s} \sin\,(\delta)$$

The synchronous reactance is also fixed because it's a machine operating in a steady state, so these cannot be changed at all. The things that can be regulated individually are E_f with sine delta and also I_a with cos theta. These four quantities- armature current, impedance angle theta, induced EMF, and torque angle - can change. Now, if we put a constraint that we are doing or trying to understand reactive power control for fixed real power, that means if P phase has to remain constant, then this constancy can only happen when $I_a \cos\,\theta$, where I_a and θ individually are varying, $I_a \cos\,\theta$ and $E_f \sin\,\delta$ remain constant because V_t anyway is constant, because the machine is synchronized and excess is a fixed quantity. So in order to understand reactive power control, we'll have this constraint where these two are the enforcements or constraints.

So let's see how this enforcement can come about. So before I dig deep into the next slide, let us also recollect that for a cylindrical pole synchronous generator with negligible armature winding resistance, the corresponding phasor equation that defines its operation with terminal voltage being a reference is E_f at an angle delta is equal to V_t at an angle 0 , where V_t is the terminal voltage per phase acting as a reference for all phasor quantities plus I_a phasor into jX_s, where j is the complex operator and X_s is the synchronous reactance. Depending on whether the current is leading or lagging, this I_a phasor can lead or lag the corresponding terminal voltage. Delta is the torque angle of the induced EMF with respect to the terminal voltage. So if we have this as our equation, we also know how its phasor diagram would look like.

All these quantities are phasors. So that phasor diagram essentially consists of the yellow line, orange line, and red line. I have intently marked these lines so that we can correlate this orange, yellow, and red line with the corresponding voltages that are marked or written in this equation. So I am initially considering a condition where the armature current is lagging the terminal voltage by an angle theta, and having defined this phase angle, I can also probably get to know what the torque angle, which is this angle delta, and also the induced emf are. Now, in this phasor diagram, if I have to ensure that no matter what quantity is being regulated for reactive power control, $I_a\cos\ \theta$ and $E_f\sin\ \delta$ have to remain constant because the machine's real power output is being kept constant. So the locus that indicates $I_a\cos\ \theta$ and $E_f\sin\ \delta$ to be constant are the respective blue and purple colored lines.

Now, how do I get that? Now that I have already marked the angle theta, remember that the perpendicular distance in front of the angle theta is associated with sine theta. The corresponding base dictates the quantity
that is cos theta. Now, if theta is known and cosine theta is the base that defines this cos theta, and it gets multiplied by the armature current I_a, that means the distance of this origin or point from this blue color line, which is a measure of $I_a\cos\ \theta$, should remain constant. That means, if I have to draw the locus of $I_a\cos\ \theta$, it is a line that is perpendicular to the yellow line. That's how $I_a\cos\ \theta$ is getting marked.

Similarly, if delta is this angle, then the perpendicular in front of angle delta is sine delta. If sine delta is the component here, which is multiplied. Basically, the origin of the point with respect to $E_f\sin\ \delta$ should always be constant. So basically, the locus of $E_f\sin\ \delta$ is a line parallel to the yellow color reference, which is the voltage terminal reference. So having understood these loci, let us see how reactive power control happens.

Reactive power control occurs by variation in the rotor current, which is important because it changes the maximum flux per pole, which in turn changes the induced EMF. Induced EMF is typically mathematically defined as $4.44\text{n}\phi_f \times \text{f}$, where ϕ_f is the particular maximum flux linkage per rotor pole defined by the armature and the field current I_f. So we consider one condition compared to the field current, which I have considered here.

I now reduce the field current to I_{f1}. If I_{f1} is reduced, it means the induced EMF will also reduce. So E_f has to reduce it. But E_f can be reduced only on the condition that $E_f\sin\ \delta$ remains constant. So, now if E_f has reduced to, let us say, E_{f1}, which is this number here, to maintain $E_f\sin\ \delta$, which is a constant number, the corresponding torque angle should increase. So, δ_1 is that particular increase with respect to delta so that $E_f\sin\ \delta$ can remain constant; likewise, if $E_f\sin\ \delta$ is marked, I can also get to know what the corresponding voltage drop across the synchronous reactance is, from where I can indirectly figure out what my armature current is.

So what I see is that my armature current magnitude has gone down. If the magnitude of the armature current has gone down, $I_a\cos\ \theta$ should remain constant. That means cos theta should increase. Cosine theta can increase only when the impedance angle is reduced. I_{a1}

and θ_1 adjust accordingly so that $I_a \cos\ \theta$ is also constant while ensuring the KVL satisfaction for the synchronous generator.

And in this case, what I see if I compare or revisit my reactive power expression $E_f \cos\ \delta$, which is the component of E_f (orange color line) along the yellow color line, is the entire $E_{f1} \cos\ \delta_1$ component. For this excitation current, what we see here is $E_f \cos\ \delta_1$ is definitely more than the terminal voltage, so reactive power is positive, which means the machine is delivering reactive power; it is overexcited and hence it is delivering reactive power. This power has to go somewhere, and that's where the load is lagging; the load is basically an inductive load. Now, if we keep reducing the field current, let us say we come to a field current I_{f2} where $E_{f2} \cos\ \delta_2$ is exactly the same as V_t, which means the reactive power output is perfectly 0 . Then, in that condition for $E_f \sin\ \delta$, so $E_f^2 \cos\ \delta$ equal to 0 means the δ_2 angle has definitely increased from δ_1.

And if reactive power is 0 , that means $V_t I_a \sin\ \theta$ is also $0. V_t$ cannot be zero. Armature current is a quantity. So I_a is the magnitude of the armature current. So that cannot be 0 because real power is still being maintained constant.

So, theta has to be 0 for Q to be 0 , and that is where the green line and the yellow line are parallel. So, if $E_f \sin\ \delta$ is constant, δ_2 is more than δ_1, θ_2, theta is perfectly 0 , no reactive power exchange is happening, and basically, the machine is operating under unity power factor mode. If we furthermore reduce the excitation current to I_{f3}, the E_f magnitude would decrease, the torque angle would increase, so that E_f and delta become positive. Now the current would start leading the corresponding terminal voltage, and it would behave as an understated operation mode where reactive power is being absorbed, meaning the load is a capacitive load, and hence it's operating at a leading power factor. That is all about reactive power control under excitation and overexcitation.

If we talk about real power control, which is essentially driven by the output of the prime mover of the turbine, it, in a way, regulates or has a significant impact on the torque angle delta, which occurs by opening or closing the turbine valves, feeding in more power, more water, or more steam; that means the mechanical output of the turbine is increasing, so the machine tends to get more mechanical input. This happens electrically through the physical observation of changes in the torque angle, and this torque angle essentially creates advancement or retardation between rotor magnetomotive force and stator magnetomotive force temporarily. So, for that moment of perturbation with constant excitation and an increase in turbine power, the synchronous rotor speed momentarily increases; the speed tends to go beyond synchronous speed, and hence the torque angle also increases, but eventually, in steady state, The increase in delta tends to increase $I_a \cos\ \theta$, so the machine tends to deliver more real power, but in steady state, because of armature reaction and thanks to damper windings, the rotor MMF and stator MMF again balance each other, the speed restores to synchronous speed, and that's the reason why synchronous machines operate at synchronous speed only in steady state conditions during disturbances or perturbations. There are deviations of the speed different from synchronous speed in synchronous machines, and the restoring action comes in because of armature reaction and the damper windings, which we discussed at length and will also talk about in detail when we cover the stability analysis module as well as the fault analysis module. So in a way, through real power control, the prime mover and the rotor, along with the rotor MMF, re-establish the matching of the system frequency and generator frequency.

So just to summarize power factor control, if we have a generator with rotor current I_f that is just sufficient for the machine to operate at UPF, that means the machine is not delivering any reactive power. If the current is less than I_f, the machine will start drawing reactive power because the corresponding load would be a capacitive load. If the rotor current is more than the field current I_f, the power factor of the load will be lagging, which means the generator would be delivering reactive power. A synchronous machine can operate at no load; ideally, no real power transfer should happen, and it can still behave as an inductor or a capacitor depending on what the reactive power control is, which sets the corresponding field current. There are special synchronous motors installed in power networks specifically for reactive power control purposes.

We will also talk about this reactive power control perspective when we discuss transmission line models, where we will discuss at length reactive power compensation, and synchronous motors behaving as synchronous condensers are a beautiful way of maintaining or compensating reactive power in three-phase power networks. So, synchronous motors do not transfer or absorb any real power; ideally, no machine can have no real power transfer. It will definitely be consuming some real power, but that real power can be minimized with proper design. Synchronous motors can behave as inductors or capacitors, and we call those special motors synchronous condensers, which are very useful for reactive power compensation.

We'll conclude today's discussion with a simple example. We have a threephase, $4\text{kV}, 400\text{kVA}$ synchronous motor, a small motor that is installed with other induction motor loads in a given plant. Those induction motor loads are 500 kVA operating at a 0.8 power factor lagging. The mechanical load on the synchronous motor is 300 kVA at unity power factor and at 4 kV rated voltage.

So, 4 kV is the rated line-to-line voltage. What we have to find is the overall power factor of the entire factory, which consists of different induction motor loads as well as the synchronous motor, and what can be the maximum power factor of this

particular factory without overloading the synchronous motor and without changing its real power absorption. So, the synchronous motor is basically observing real power, and we have to find to what extent the power factor of the entire factory can be maximized or improved without overloading the motor, without changing the real power, and under that condition, we have to find the motor current, synchronous motor current, and its corresponding power factor. So, if we have to find the power factor, we basically have to find the net real power and the net reactive power at all instances. If you look at the induction motor loads that are operating at 500kVA, 0.8 power factor lagging, then P is the apparent power multiplied by cos theta, which is 0 .
8. So basically, 400 kilowatts is the power that is absorbed by the induction motors. Since the power factor is lagging, which is true for induction machines, these induction motors are also consuming power. 300 kVAR of reactive power. How do I get 300 kVAR ? If cos theta is 0 .

8, it would essentially mean sin theta is 0.6 , which is what is multiplied by 500 kVAR to get 300 kVAR . Induction motors are consuming 400 kW of real power and 300 kVA of reactive power. The synchronous motor is operating at 300 kVA with a unity power factor, which means the reactive power handled by the synchronous motor is perfectly zero. If we now have to find the net real power observed by all motors in the factory, this will be 400 kW for induction motors and 300 kW for the synchronous motor. So, net real power is 700 kilowatts, and net reactive power at this instant is only 300 kilovolt-amperes reactive.

So, in a way, the overall power factor of this factory load, which is basically the cosine of theta, where tan theta (theta being the impedance angle) is nothing but Q net by P net. So, what we do is take the tan inverse of Q net by P net; Q net is 300, P net is 700 , and then we take the cosine of it, which would give us 0.9191 as the lagging power factor because the net reactive power is still being absorbed by all the motors in the given factory. Now, if you have to find the maximum KVA or reactive power that the synchronous motor can generate without getting overloaded, remember the synchronous motor is rated at 400 KVA, and its real power should not change; its real power is basically 300 kilowatts, which it is already consuming. So, if this is S and this is P , we can also find the corresponding reactive power that this machine can generate without getting overloaded.

So, that would essentially be the root of S squared minus P squared, which is what is mentioned here. So, 264.575 kVAR can be generated by the synchronous motor. So, in a way, earlier, if 300 kVAR was the reactive power absorbed by the induction motors, 264.575 kVAR can be locally managed by the synchronous motor, which is placed along with the induction motor.

So essentially, the grid supply for reactive power is only 35.425 kVAR , with real power remaining as it is. And since the reactive power consumption has now sort of decreased, the corresponding power factor of the factory tends to improve. So basically, the new power factor is almost 1 , at 0.99 lagging. It is still lagging because this reactive power is still being drawn from the electrical supply by the factory.

The corresponding armature current of a synchronous motor, in terms of its maximum apparent power and 4 kV as the rated voltage, is 400 kVA , which is the rated apparent power. If we take the per-phase power, it is divided by 3 , and to find the corresponding phase voltage of the synchronous motor, the line-to-line voltage is 4 kV . If we divide it by the square root of 3, we would have the phase voltage. Therefore, phase power divided by the phase voltage is 400 divided by the square root of 3 multiplied by 4 .

The kV units all get canceled, and essentially we are left with 57.74 amperes as the armature current of the synchronous motor. The power factor of the synchronous motor is basically 0.75 leading; how? The motor is still absorbing 300 kilowatts of power, operating at 400 kVA , and is still able to generate 264.575 kilovolt-amperes reactive as reactive power. So, cos theta, which is also real power divided by apparent power, where P is 300 and apparent power is 400 , is because the machine is operating at its maximum capacity without getting overloaded.

So, the power factor number becomes 0.75 ; it is now leading because the machine is able to generate this much reactive power. With this, we conclude today's discussion. In the next discussion, we'll take up another interesting topic of synchronous generators on economic dispatch, and we'll understand how these different generators are able to generate or share real power while maintaining system economics. Remember, as engineers, we all tend to design our systems or behave, usually keeping the economics in consideration.

Each one of us wants to maximize profit and minimize operational costs. The same is also true for generators or the corresponding owners or operators who operate these plants. So with that, I thank you and conclude this discussion. Thank you so much.