Power Network Analysis

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Lecture-13

Hello everyone, welcome to lecture 3 of week 3 , which is a continuation of the module on synchronous generators in this course, Power Network Analysis. In this discussion, we will continue with capability curves, and in the previous lecture, we briefly touched upon one aspect of capability curves, which was built on the discussion that we had regarding real and reactive power expressions for synchronous generators, to be specific. So, if we recollect, like any other electrical equipment, a synchronous generator also has certain ratings for safe operating points, which are constrained by typically three limits. The first one is the maximum armature current limit or stator current limit, which indicates the maximum current that can flow in the three-phase stator windings. If the current exceeds this maximum armature current limit, the stator windings may get damaged, and the associated insulation may also get damaged. Similarly, there also comes a limitation in terms of the maximum current that the rotor winding can carry in the synchronous generator.

Rotor winding is also known as field winding; hence, the field current is analogous to the rotor current, which is DC current. And there is also a maximum limit to it. If rotor windings carry current beyond this maximum field current limit, then rotor windings can get damaged, and the associated insulations may get damaged. And indirectly since the induced EMF, if we recollect the lecture discussion on induced EMF, where we also talked about armature reaction in a generator being loaded and unloaded.

In that discussion, we also saw that the field current, or rotor current, in a way influences the EMF that is induced in the three-phase stator windings, which can also result in or be replicated as an equivalent maximum limit on this induced EMF that can be generated without compromising the safety of the rotor winding or the rotor insulation. So, in a way, the field current limit maximum can be linked with the maximum of the induced EMF or associated voltage. And the third constraint, which comes from the synchronous generator perspective, is the maximum power that can be extracted from this synchronous generator without compromising the stability limit. And that's where the maximum steady-state limit comes in, which is going to be a discussion of length in the last module on stability analysis. In this discussion, we will only understand where this limit comes from and what the associated numeric value is.

More details on the stability limit will come in the last module of stability analysis. The synchronous generator or machine that we have considered was also discussed in the previous discussion. To make it clearer, the synchronous generator or machine that we have is a cylindrical pole machine with negligible armature winding resistance. So that is what the machine is considering. The discussion that we have can also be applied to salient pole machines or similar types.

We are not discussing the salient pole machine capability curves for the sake of simplicity. And this synchronous generator or machine, which has a cylindrical pole, often operates at a rated terminal voltage, Vt, with a fixed synchronous reactance, Xs , and this synchronous reactance is considered to be fixed. The non-idealities or non-linearities involved in the rotor air gap or the stator air gap between the rotor and stator are being neglected for the time being. So, with this as our premise, we looked at this locus with respect to the armature current limit, which essentially comes from the first limit that we discussed in the previous discussion. This limit pertains to a fixed terminal voltage; there is no need for synchronous reactance while defining this locus because the maximum armature current limit, in a way, indicates what the maximum amount of armature current per phase is that can flow in this rotor winding, which can be replicated or transferred as that if Ia max is the corresponding maximum armature current limit and Vt is the specified terminal voltage at which the machine is supposed to operate.

Then the corresponding apparent power that can be handled at maximum by the synchronous generator will at best be Vt times Ia max. And we also know that apparent power squared is nothing but the square sum of the squares of real power and reactive power. So, P is the real power and Q is the reactive power. So, in a way, if I have to understand this locus, this locus is essentially telling me that P squared plus Q squared can at best be equal to Vt times Ia max squared. The expression, the equality that I have written, can have a less than or equal to form, but definitely not a more than or equal to form.

If the current goes beyond Ia max, the stator winding may be damaged. So, if I have to understand or plot this curve on a PQ plane, which is the complex plane with P as the x -axis and Q as the y -axis. This equation is very similar to the equation of a square on an xy plane, which indicates that if x squared plus y squared is equal to r squared, then on an xy plane, this equation represents a circle. With the center as the origin and R being the radius. So, if you apply the same notion, we have the armature current limit as all points inside this circle whose center is the origin and whose radius is Vt into Ia max.

We also understood what the blue, green, purple, and orange regions are. All points on the right-hand side of the Q axis pertain to generating mode, whereas all points on the left-hand side of the Q axis refer to motoring mode because, on the right-hand side, P is positive. P positive means the machine is delivering real power, so it is behaving as a generator. For the left-hand side of Q, P is negative, so the machine is behaving as a motor, consuming real power. A similar notion can also be applied to points that are above and below the P axis; at points above the P axis, reactive power is being delivered or generated by the machine.

So, in that sense, the blue and the orange colors refer to overexcited mode, whereas points below the P -axis where Q is negative mean the machine is absorbing reactive power; we would call them under-excited mode. We will talk more about over-excitation and under-excitation in the next lecture. And for a specific point or a specific angle, theta is equal to 0 degrees, where theta is the load impedance angle, the corresponding power factor would be equal to 1 . The notion of why theta is marked like this over here is that if S is the radius of this circle, then S cos theta would be the real power, which is this measure, and Ssin theta would be the reactive power, which is on the y-axis, and that's how the angle theta can be marked. So theta is equal to zero, meaning theta can be zero both for generating mode, which is this yellow color line.

It can also be zero in the motoring mode. And that's how the yellow color line represents unity power factor operation, where no reactive power exchange is happening. Building on this, now let's talk about what the locus for maximum field current limit looks like. As already mentioned, the maximum field current limit, if the current exceeds this limit in the rotor winding, may damage the rotor winding as well as the corresponding insulation. And since the field current is responsible for the creation or induction of this induced EMF in the generator winding.

So, if the max value can be correlated as the maximum excitation voltage, not the terminal voltage, for a given synchronous generator. The reason why any operation beyond the maximum induced EMF is prohibited comes from the open circuit characteristics of the synchronous generator, beyond which any point operated on the open circuit characteristic is beyond the saturation point. It may damage the stator and rotor, and it may affect their magnetic properties. And as already discussed, for every such field current limit, armature current limit, or steady state limit, the corresponding loci would pertain to a fixed terminal voltage and fixed synchronous reactance for a negligible armature winding resistance cylindrical pole synchronous generator. So if we have to understand the locus or corresponding mathematical expression that comes from the maximum field current limit, it is essential that this locus or equation we are trying to find should be in terms of known quantities, not the unknowns.

Given that the machine is operating at a fixed terminal voltage, fixed synchronous reactance, and the maximum induced emf value is known from the corresponding maximum field current. It is imperative that the locus or equation we derive to understand the field current limit locus be in terms of these three encircled items. So, if we start from the basics we already understand or know that for a cylindrical pole synchronous generator with negligible armature winding resistance, this is the expression of per-phase armature current.

$$\mathbf{I}_{\mathrm{a}} = \frac{\mathbf{E}_{\mathrm{f}} - \mathbf{V}_{\mathrm{t}}}{jX_{\mathrm{s}}} = \frac{E_f \angle \boldsymbol{\delta} - V_{\mathrm{f}} \angle \mathbf{0}}{jX_{\mathrm{s}}} = I_a \angle \boldsymbol{\theta}$$

Again, please note that subscript " a" here does not mean phase " a"; subscript "a" here is indicating armature winding. So, if this is the current expression for a cylindrical pole generator with negligible armature resistance, we can also expand this current into the corresponding real and imaginary parts.

So if we do that and focus on the term sitting over here, we will most likely have - not most likely, we'll definitely have one real part as well as an imaginary part. The real part would be of the form Ef cos delta minus Vt. Pardon me for the mistake in defining the real and imaginary parts; let's look at how this expansion looks. So if we expand Ef at an angle delta as Ef cos delta minus Vt, which is sitting over here, then we'll also have j , which is the complex root or imaginary number root of minus 1 . It will be Ef sine delta divided by jXs, which, if expanded, would look like Ef sine delta by Xs minus jEf cos delta minus Vt by Xs.

So, the first term here is the real component of the armature winding current, and the second term here is the imaginary component. Now, if we take or try to find the magnitude of this armature current what we can do is we can probably evaluate let us say if we do it here let us say we take the square of mod of I a so it should look like I a mod square should be equal to Square of this real term square of this imaginary term and then addition of both of them So if we do that it should look like

$$\hat{I}_a^2 = \frac{E_f^2}{X_s^2} + \frac{V_i^2}{X_s^2} - 2\frac{E_f V_i}{X_s^2} \cos\ \delta$$

wherein if we see the first two terms they can be added together and we would simply have Ef square by Xs square followed by Vt square plus Xs square and then this cosine term. So, from there, if we let us say have this sort of expression where Ia squared is defined in terms of induced EMF, terminal voltage, cosine of torque angle, and synchronous reactance, we can then further elaborate on it. We also know from the armature current limit locus that P squared plus Q squared for a cylindrical pole or any generator is equal to

Vt squared times Ia squared. Why? Because P is nothing but Vt times Ia, which is the apparent power, cos theta; theta is the power factor angle, and Q is Vt Ia sin theta.

Depending on the power factor angle, Q can be positive or negative. So from here we get this expression; essentially, we can have P squared plus Q squared equal to Vt squared plus Ia squared, and now if we substitute Ia squared into this term, it looks a little complicated, but when you plug in those numbers appropriately, we will have this as our equation.

$$P^2 + Q^2 = V_t^2 \left(\frac{E_f^2}{X_s^2} + \frac{V_t^2}{X_s^2} - 2\frac{E_f V_t}{X_s^2} \cos\ \delta \right)$$

$$P^2 + Q^2 - \frac{V_t^4}{X_s^2} + 2V_t^2 \frac{E_f V_t}{X_s^2} \cos\ \delta = \frac{E_f^2 V_t^2}{X_s^2}$$

Now what we see here is that for defining the locus, P squared plus Q squared are the obvious terms because we are trying to plot this locus or limit on the PQ plane. The third term is also okay because Vt is fixed; synchronous reactance is also known as the last or the fourth term on the right-hand side, or the left-hand side, pardon me, is a troublesome term because with a change in the excitation current, it is not just the induced EMF that will change; the corresponding torque angle can also change. So, what we need to do is get rid of or have an equation that defines the maximum field current limit only in terms of known quantities; we have to figure out a way to avoid or substitute this term, which carries a variable quantity, specifically the cosine of delta.

So, let us try to see how it can be done. Before we do that, let us try to recollect the real and reactive power per-phase expressions for a cylindrical pole machine with negligible armature resistance. If we recollect or revisit our previous lecture discussion, we remember that the real power for a cylindrical pole machine with negligible armature resistance is nothing but Ef Vt sin delta divided by Xs , and Q is nothing but Vt into Ef cos delta minus Vt , the entire term divided by Xs. Now, if we try to look at the terms that are residing in this, we can probably figure out that this particular term, which is EfVt cos delta, is also partially present in the expression of Q. So can we make use of this Ef cos delta term sitting in the reactive power expression and try to substitute this here? If we do that, then this is the equation I was trying to refer to: EfVt cos delta by Xs can be simply written as Q plus Vt squared by Xs.

And now, if we substitute this term into the expression over here, it would simply be all the terms that you would see after substitution; they would all be in terms of either P or Q, or only in terms of the known quantities. And the little simplification would reveal that the locus for the maximum field current limit, wherein Efmax is the maximum induced EMF due to maximum field current limitation from the operational perspective, can again be defined in the form of an equation as a circle on the PQ plane whose center is 0 with minus Vt squared by Xs as coordinates, and the radius of the circle is Ef max Vt by Xs. So if we plot this circle on the PQ plane again, this is how the green color locus would look. All points inside this locus are the safe operating points for the synchronous machine. Any point beyond this circle would indicate that we are trying to operate the machine at an induced EMF that is greater than Efmax for fixed terminal voltage and synchronous reactance, which may compromise the safety of the rotor winding and the insulation.

So all points inside this circle are safe operating points. All points outside the circle are prohibited operating points from the perspective of the maximum field current limit, and if we also look at the expressions for P and Q that we saw for a cylindrical pole machine, let's try to look at that. So we remember P is Ef Vt sine delta by Xs and Q is Vt Ef cos delta minus Vt over Xs. Using these expressions, is there a way to mark the angle delta on this PQ complex plane that we have? Now, if you look at the first term of real power, it is part of EfVt excess, which is also similar to the radius we have over here. So if we have to find how delta should be marked, then suppose we know that delta, the corresponding sign component, if it is multiplied by the radius of the circle, should be nothing but the real power.
So, the way we marked the load impedance angle in the armature current limit discussion, if we draw a similar analogy here, it would be pretty obvious that the angle delta is nothing but the angle of this radius with respect to the reactive power axis. Because if we take the sign of this delta marked over here, it would essentially be a component shown as the dotted line, which would indicate the corresponding real power being generated or absorbed by the corresponding synchronous machine. So that's how we can know how the delta angle can be marked. This also goes well with the expression of reactive power because if sine delta is the component over here, then the corresponding perpendicular line would be cos delta, which is nothing but Vt into Ef cos delta by Xs, and in order to find the total reactive power, the Ef cos delta by Xs term has to be subtracted from Vt square by Xs, which is inherently sitting over here. So, if we all combine or merge these ideas, then this is how the delta angle looks, which is the torque angle or the power angle.

And now, if we look at the third constraint, which comes from the steady state limit, it was also talked about in the previous discussion that any operation of a synchronous machine for an angle of more than 90 degrees, with the torque angle being more than 90 degrees, is a prohibited operation because the generator might lose its synchronism, which is a stability concern that we will discuss at length in the last module. Just to sort of compromise or just to collate how the steady state limit locus would look, it is defined by the fact that for a cylindrical pole synchronous machine with negligible armature resistance, this is the expression for real power; any angle of delta greater than 90 degrees is prohibited. So, in a way, this limit here is indicative of the steady-state limit. If this is the limit of the steady-state limit Pmax equal to Ef Vt Xs for fixed terminal voltage, fixed synchronous reactance, and fixed field winding current, then is there a way to mark this locus or limit on the PQ plane? Yes, the locus can be obtained from the locus or from the field current limit itself.

So let's see how we do that. And that was the reason why we had a discussion on how the torque angle or load angle could be marked. That was a necessity. So now, if the load angle is the angle of the radius with respect to the reactive power axis, delta equal to 90 degrees would essentially mean that in the generating mode, the corresponding radius, instead of being in this notion for delta equal to 90 degrees, would mean that this radius has rotated, and the angle which is now being formed is equal to 90 degrees with respect to the Q axis, and probably this is the line which is the maximum steadystate real power that can be delivered by the machine in generating mode. A similar line can also exist for delta being equal to minus 90 degrees, where delta is negative for motoring mode. This is the corresponding limit that can be absorbed by the cylindrical synchronous generator in the motoring mode.

So, if we know or understand that having defined the angle delta, and delta is equal to plus 90 for generating mode, and delta is equal to minus 90 for motoring mode, which defines the steady state limit, then essentially this blue line is the locus line that is the steady state limit. Now, is there a way to have a numeric equation that defines this blue color steady state limit? So essentially, all points that are above this blue line indicate the safe operating points. All points below the blue line are prohibited points because the torque angle would be more than 90 numerically, which is not allowed. So all points above this limit, line, or blue line are the safe points. Now, is there a way to find this equation that identifies this blue line similar to the locus line for the field current limit or the armature current limit? If you look closely, this blue line is a line that is parallel to the p-axis and crosses the q -axis at the point zero with minus Vt squared by Xs.

So if there is a line that is parallel to the p -axis or x -axis, its equation is by default in terms of the Q or Y axis. So, if we have to write the numeric equation for this blue line, it is nothing but Q equals minus Vt squared over Xs. This is a generic line that can extend beyond these blue lines. The blue line is limited by the fact that the synchronous generator has to abide by all limits at a particular point. So, this blue line is a fraction of this dotted line limited by the steady state limit because remember all points inside the steady state limits are also safe points from a field current perspective.

So, in a way, this entire section of the line is coming from the superimposition of the field current limit as well as the steady state limit, which is for the angle delta equal to 90 degrees, and Q equal to minus Vt squared by Xs is the numeric mathematical form for defining this blue line. So now, if we sort of superimpose or combine all of this, the locus of the field current limit is a circle with its center at 0 minus Vt squared by Xs and a radius of Ef max Vt by Xs. The steady state limit is a line parallel to the p-axis with the equation defined as q equal to minus Vt squared by Xs , and it is also limited on both ends by the field current limit locus. All operating points below the limited locus are inoperable as the machine might lose synchronism. So if we draw or plot the feasible operating points or safe operating points for this synchronous generator, which is cylindrical, then on similar lines, the way we had the armature current limit locus, we have our generator limit locus for both over-excited and under-excited operation.

We also have our safe operating limit for the motor for both over-excited and under-excited operation, the yellow line still pertaining to the unity power factor point. And now just to sort of pick your brains, if we have to find these specific points on the PQ plane, that's where the equations of the blue line and the green line would come in handy. If you were to find the point which I have, pardon me, marked over here, then this point would typically be Ef Vt, let's say Ef max, to be specific Ef max Vt by Xs, with minus Vt square by Xs at the corresponding Q point, and similarly, this point over here is nothing but the intersection of the blue line and the green line. So, here we would have minus Ef max Vt by Xs, minus Vt squared by Xs as corresponding P and Q axes. We can also verify these points by revisiting the equations for the field current limit locus and the steady state limit locus.

Now, remember that when we are drawing all these points, we still have one more limit, which we discussed in the last lecture, and any machine, at any given point in time, has to abide by all operating points. So, it is time to superimpose the armature current limit locus on these two loci. So, if we do that without marking the under-saturation or over-saturation points, then the armature current limit locus equation is P squared plus Q squared equal to Vt multiplied by Ia max squared. The green color equation is p squared plus q plus Vt squared divided by Xs squared is equal to E f max Vt divided by Xs squared. And the blue color line equation is q equals minus Vt squared by Xs; if we have to identify the common operating points, the common operating points should be all points that are overall safe from all limits' perspective.

So, for the armature current limit, if you recollect, all points inside the black circle are safe points; for the green color locus, which is the field current limit locus, all points inside the green circle are common or feasible points, and from a steady state limit perspective, all points above the blue line are safe points. So, if we now try to find a set of points that are common to these three sets-the black circle set, the green circle set, and the points above the blue line - the only safe points that would appear are the areas that I have been marking over here on the positive p side. The armature current limit locus is limiting the safe operating points on the top, we have the field current limit locus limiting the points below, we have the steadystate limit, and similarly, on the LHS, we have the safe operating point from the armature current limit locus. So if we now mark our or figure out the common points, this is the locus that looks like from the operating limits restrictions, and now if we mark our generating or motoring modes for under-excitation and over-excitation, those points look like this. So this, in a way, is the overall capability curve, the one which is shown over here for a cylindrical pole synchronous generator operating in synchronous mode with negligible armature winding resistance.

In the next lecture, we will take up our discussion on finding out how the real power and reactive power of a synchronous generator can be controlled physically through certain action points. Thank you.