Power Network Analysis

Dr. Abheejeet Mohapatra

Department of Electrical Engineering

IIT Kanpur

Week - 03

Lecture-12

Hello everyone, welcome to lecture 2 of week 3 , in which we will continue our discussion on synchronous generators. In the previous lecture, which was the first lecture of week 3, we discussed the corresponding phasor domain equations that govern the operation of cylindrical pole synchronous generators and salient pole synchronous generators. We also saw or concluded that the cylindrical pole synchronous generator circuit can be represented in an equivalent form, but unfortunately, because there are two different reactances in a salient pole machine, Xd and Xq , we cannot have an equivalent circuit representation for a salient pole synchronous generator. So, building upon that, in this discussion today, we will look at how we evaluate the expressions for real power and reactive power delivered from a synchronous generator. These reactions or these expressions, by the way, could also be derived for a synchronous motor. So it can be applicable for any synchronous machine, but we'll keep our synchronous generator in focus, and then we'll also interestingly have a brief discussion on what these capability curves are, and we'll continue on these capability curves probably in the lectures to come.

So what we do is start with the most complex case first; that is, we consider a salient pole synchronous generator, and for this salient pole synchronous generator, we derive our power expressions. From here, if we have to find the corresponding real and reactive power expressions for a cylindrical pole machine, Then it is pretty obvious that in a salient pole machine we have two reactances, Xd and Xq , which are not equal to each other, whereas for a cylindrical pole machine, the air gap being more or less uniform along the daxis or quadrature axis, if we neglect the variations in the air gap because of distributed rotor and stator slot windings, then in a cylindrical pole machine we'll have the same reactance along any axis. For the power expression that we derive for a salient pole synchronous generator, if we equate Xd and Xq to be equal to each other, essentially being equal to synchronous reactance, we automatically have our expressions for a cylindrical pole synchronous generator. So, what are those equations for a salient pole synchronous generator? We have discussed at length this phasor diagram, explaining why the Iq current, which is a component of armature current, lies along the induced EMF axis and why the direct access current Id, which is a component of Ia, is perpendicular to the induced EMF axis.

Building on that, if we numerically represent the phasor diagram with two equations, one equation being along the Iq current, which is the first equation here, and the second equation being along the direct axis, then we have this expression depending on the corresponding torque angle, which is always positive for generating mode, and depending on the phase angle, impedance phase angle. Which is usually an inductive load, so theta is usually positive in this particular sense. Depending on these angles, the corresponding Id and Iq components of the armature current can be evaluated. So one thing before we go further into our discussion is whether we use these equations, which are the Id and Iq equations, and try to see if we have any Id and Iq currents in our first two equations; then what appears is the voltage drop along IaRa with the cosine component. Ia and the cosine components of theta and delta.

$$E_f = V_t \cos\ \delta + X_d I_d + I_a R_a \cos\ (\theta + \delta)$$
$$V_t \sin\ \delta + I_a R_a \sin\ (\theta + \delta) = X_q I_q$$

It is nothing but Iq. Whereas IaRa drops with the sine component. So Ia and sin theta. This current is basically equal to Id. So essentially, if I rewrite the second equation.

$$I_d = I_a \sin\ (\theta + \delta)$$
$$I_q = I_a \cos\ (\theta + \delta)$$

It is Vt sine delta plus Id Ra. Equal to Iq Xq, and similarly, I can rewrite the first equation as Ef Vt cos delta plus Xd Id ; instead of this term, I can have Iq Ra as my last term. So with these as our equations, which are also discussed further. So if we focus on the first two expressions, keeping in mind the last two expressions, we can expand the sine of theta and delta and the cosine of theta and delta into elongated forms, which would be the first two equations shown over here.

$$I_a(\sin\ \theta \cos\ \delta + \cos\ \theta \sin\ \delta) = I_d$$
$$I_a(\cos\ \theta \cos\ \delta - \sin\ \theta \sin\ \delta) = I_q$$

And now, if we multiply the first equation by sine delta and multiply the second equation by cos delta on both sides, and then add them up, I would essentially get my third equation; similarly, if I multiply the first equation by cos delta on both sides and multiply the second equation by sine delta on both sides, and then take the difference, I would essentially get the fourth equation.

$$\Rightarrow I_a \cos\ \theta = I_d \sin\ \delta + I_q \cos\ \delta$$
$$\Rightarrow I_a \sin\ \theta = I_d \cos\ \delta - I_q \sin\ \delta$$

So, essentially, I am trying to represent Ia cos theta, which is generally a known quantity for the armature current, choosing terminal voltage as a reference. I am trying to represent the phasor currents or the armature current quantities into the corresponding Id and Iq currents and the torque angle delta. Now, if we recollect our initial discussions of basic circuit principles, we have seen that the complex power is usually, or not usually, always the product of the terminal voltage phasor multiplied by the conjugate of the corresponding current. If we equate or correlate these quantities for a synchronous generator, then we have chosen the terminal voltage as a reference for all phases, so its phase angle is zero. And assuming that the corresponding armature current is essential, there is a mistake here; I'll probably correct it here itself.

I'm assuming the current to be an inductive current or a lagging current, so essentially if Ia is equal to Ia at an angle minus theta, then Ia conjugate should actually be Ia at an angle theta. So, essentially, this negative sign here is a mistake; I am erasing it with this red color. So, if I have the current lagging the corresponding terminal voltage, which is usually true for inductive loads, then the complex power is nothing but Vt Ia cos theta plus j sine theta, where j is the complex operator, essentially the square root of minus 1 .

$$S = V_t I_a^* = V_t I_a (\cos\ \theta + j\sin\ \theta)$$

Now in this equation, I can correlate or decompose it into the corresponding real component and reactive component. The real component is nothing but the per-phase real power delivered.

$$P_\Phi = V_t I_a \cos\ \theta$$

Which is nothing but Vt Ia cos theta. And the imaginary component is per-phase reactive power delivered, assuming that the generator is feeding an inductive load, which is why I have the current lagging the corresponding terminal voltage. Then Q_Φ, which is the per-phase reactive power delivered, is nothing but $V_t I_a \sin\ \theta$.

$$Q_\Phi = V_t I_a \sin\ \theta$$

And remember, I have already derived the equation that I am encircling here. where Ia cos theta and Ia sin theta have been expressed in terms of Id, Iq, and the torque angle.

$$P_\Phi = V_t I_a \cos\ \theta = V_t (I_d \sin\ \delta + I_q \cos\ \delta)$$
$$Q_\Phi = V_t I_a \sin\ \theta = V_t (I_d \cos\ \delta - I_q \sin\ \delta)$$

So from there, I can also rewrite or have expressions of the P phase and Q phase in terms of terminal voltage, Id current, Iq current, and torque angle. Now, if these expressions are so commonly available or useful, is that it, or do we need to do anything further? There is a catch here. The catch is, generally, what we know for a given machine is the terminal voltage information. We can also know the corresponding armature current information. And from there, it is pretty difficult to directly identify what the corresponding direct axis current or quadrature axis current is.

Remember, these currents are fictitious. They are not actual currents flowing anywhere, not in the stator windings. The stator winding currents are still carrying armature current. For the sake of simplicity in our discussion, we are decomposing the armature current into two components, Id and Iq, which are fictitious currents. So if we have expressions wherein we have fictitious quantities that are difficult to identify, then something else needs to be done so that we can get a better form of these equations P and Q per phase real power and per phase reactive power, which should not be in terms of Id and Iq, and that is where the next discussion comes in.

So if you recollect the discussion that we had here about the equations that were related in terms of this and this, it's now time to reuse those equations. So let's see how we can reuse them. So, the first two equations on slide number three can be rewritten into these two equations, which are shown over here; it's just a minor rearrangement. And now, if we try to solve these two simultaneous equations for two unknowns, the unknowns are Id and Iq, because these are fictitious quantities. As I said earlier, one can usually get to know what Vt is and what the induced EMF is.

Which is nothing but the magnitude being 4.44 n phi f and f , where n is the number of turns on the stator, phi f is the flux linkage, f is the source frequency, which depends on synchronous speed, so it's easy to know what the induced emf magnitude is going to be; also, it is useful to know what the corresponding torque angle is. So usually, with these quantities known and Xd, Xq, and Ra being the physical parameters of synchronous generators,
which can be obtained from equivalent open-circuit and short-circuit tests. So it is now time to find Id and Iq, which are fictitious quantities in terms of usually known quantities. So if we solve these two simultaneous equations, we get these two equations where everything on the right-hand side is usually known.

$$I_d = \frac{(E_f - V_t\cos\,\delta)X_q - V_t R_a \sin\,\delta}{X_d X_q + R_a^2}$$

$$I_q = \frac{(E_f - V_t\cos\,\delta)R_a + V_t X_d \sin\,\delta}{X_d X_q + R_a^2}$$

The unknowns, however, or fictitious quantities, are Id and Iq. So when we know everything on the RHS side, we can substitute numbers here and also get these LHS quantities. So if we put these Id and Iq expressions into the expressions we saw in slide number four, then plugging this Id current and Iq current into the positions over here, we will have the per-phase real power delivered by the salient pole synchronous generator in terms of this particular equation here. One can again cross-check the validity of these expressions, and similarly, we can also have the per-phase reactive power delivered by a salient pole machine in terms of this form.

$$P_\Phi = \frac{V_t\{E_f(X_q \sin\ \delta + R_a \cos\ \delta) + V_t(X_d - X_q)\sin\ \delta \cos\ \delta - V_t R_a\}}{X_d X_q + R_a^2}$$

$$Q_\Phi = \frac{V_t\{E_f(X_q \cos\ \delta - R_a \sin\ \delta) - V_t(X_d \sin^2\ \delta + X_q \cos^2\ \delta)\}}{X_d X_q + R_a^2}$$

So this essentially is the perfect accurate expression of a salient pole synchronous generator having winding resistance Ra and direct axis reactance Xd, and quadrature axis reactance Xq.

With terminal voltage being Vt, the phasor reference for all of the phasors, Ef is the induced EMF, delta is the torque angle, and that's it. So from there, if we now try to see, can there be any approximations? So yes, usually in a well-defined or properly designed machine, the armature reactance is essentially the armature resistance, and the winding resistance, Ra, is very small. So if we try to sort of neglect the resistance value in the windings in a properly well-designed machine, if we put Ra as zero in these expressions here, these two equations are respectively reduced to these two forms.

$$P_\Phi = \frac{V_t E_f \sin\ \delta}{X_d} + \frac{V_t^2 (X_d - X_q)\sin\ 2\delta}{2 X_d X_q}$$

$$Q_\Phi = \frac{V_t E_f \cos\ \delta}{X_d} - \frac{V_t(X_d \sin^2\ \delta + X_q \cos^2\ \delta)}{X_d X_q}$$

And from here, the expression that is shown over here is the power delivered by the salient pole generator with zero winding resistance. Now, if we have to find the expressions for a cylindrical pole machine, what we do is recollect that we will put Xd equal to Xq equal to Xs .

So if we do that in the expressions p phi and q phi shown over here, if we put Xd equal to Xq, this term will become zero and coincidentally we will be left only with this expression and this expression here.

$$P_{\Phi}=\frac{V_t\{E_f(X_s\sin\ \delta+R_a\cos\ \delta)-V_tR_a\}}{X_s^2+R_a^2}$$
$$Q_{\Phi}=\frac{V_t\{E_f(X_s\cos\ \delta-R_a\sin\ \delta)-V_tX_s\}}{X_s^2+R_a^2}$$

This is the expression for the per-phase real and reactive power delivered by a cylindrical pole synchronous generator. If we have a well-designed synchronous generator, the stator resistance Ra is zero. So, if we neglect that, then we have these two simple equations.

$$P_{\Phi}=\frac{V_tE_f\sin\ \delta}{X_s}$$
$$Q_{\Phi}=\frac{V_t(E_f\cos\ \delta-V_t)}{X_s}$$

Now, for these two equations, let's try to spend some time on them for a cylindrical pole machine.

What can we infer here? The inference we can draw here is that if we have to find the maximum possible value of the per-phase real power that can be delivered by a cylindrical pole synchronous generator, usually what is constant in a cylindrical pole machine is the terminal voltage, which is typically fixed because it is synchronized to a grid; synchronous reactance is a physical parameter and cannot change, and as long as the machine is synchronized, the internal EMF is also most likely fixed. What can change, however, is the value of the torque angle, so if you need to find the maximum possible value of real power delivered by the corresponding cylindrical pole synchronous generator. This maximum will happen only when sine delta takes its maximum possible value because other quantities are more or less fixed, and sine delta will be maximum at a value equal to 1 ; for this, delta would be equal to 90 degrees. So, this is the corresponding maximum possible real power that can be delivered per phase from a cylindrical pole machine. Similarly, if it spends some time in the Q phase, for an inductive load, the machine would be delivering reactive power.

So for the Q phase to be positive, Vt and Xs , as I mentioned, are positive specifications. They are not negative quantities. So if the Q phase has to be positive, then it's pretty clear that Ef cos delta is positive. should be more than VT. That means the machine will be delivering reactive power.

The machine can also absorb reactive power, which is evident here from this expression. So if Qphi becomes negative, which means the machine is absorbing reactive power, in that case, Ef cos delta would be less than Vt. We will discuss more about this expression of Qphi being positive or negative when we talk about the reactive power control of synchronous generators in the next few lectures. So essentially, with negligible winding resistance and stator winding resistance, the real power delivered at the filter is also fed through the air gap. So one can equivalently find the torque developed across the rotor or the corresponding equivalent torque delivered by the prime mover in generating mode. which would be nothing but the three-phase total power divided by the synchronous speed. Synchronous speed is dependent on the number of poles and the frequency of the machine. For a round rotor machine, I have already explained that the maximum real power would occur at an angle of delta equal to 90 . Any operation beyond a delta equal to 90 will lead to an unstable operation. That's the steady-state real power transfer limit.

We'll discuss more about this, that is why delta equal to 90 is the maximum allowable limit for the machine to safely operate. We'll discuss more about this in the last module of this course, which is going to be on stability analysis. So until that time, please have patience. We'll come back to this particular aspect of why a delta equal to 90 is the safest possible operation, and any operation beyond this would be unstable for the synchronous generator as well as the grid. We'll talk about it in the final module.

The other aspects I have already discussed for non-zero real power transfer indicate that the generator delta should be positive, and that is the reason why, in generating mode, delta remains positive because the generator is supposed to deliver real power. So sine delta has to be positive. If phi delta has to be positive, delta must also be positive. In a synchronous motor, real power is absorbed from the grid. So it basically converts electrical power to mechanical power: the synchronous motor.
So in that case, real power is absorbed. So sine delta has to be negative. So in motoring mode, the delta will be negative or sine delta will be negative only when delta is negative. That is the reason why the torque angle is positive and negative in generators and motors. And regarding the last two aspects, we'll talk more when we discuss reactive bar control in detail.

So, until that time, please be patient. What we would take up in between is the notion of the capability curve. In fact, all equipment, all electrical equipment, has its own safe operating limits, and any machine that is designed to operate will always have certain suggested conditions or operating conditions beyond which, if it is operated, it would be detrimental to the corresponding element. The same is also true for a synchronous machine, and for the capability curve discussion, we will consider the cylindrical pole machine specifically. Similar discussions are also applicable to salient pole machines, but we will avoid that for the sake of simplicity in this particular discussion. So for a synchronous machine, there can be several such limits, or for any electrical equipment, there can be several such limits.

Those limits come from three major components, the first being the current. Any equipment can withstand a typical amount of current that it can run through, which it can deliver, or a conductor has a maximum currentcarrying capacity. If the conductor carries current beyond a certain limit, it gets heated up and may melt down, which could cause a rupture. The same is also applicable for voltage. Insulation strength determines the operating voltage at which an element or equipment can operate.

The higher the voltage level, the greater the insulation requirement. And over age or over time, these insulation properties sort of degrade. So basically, the voltage buildup is an issue in electrical equipment. And lastly, it is the amount of power that can be transferred, delivered, or absorbed by a particular piece of equipment without losing safety and stability. So similar restrictions also exist for synchronous machine.

And if we have a mathematical plot or a two-axis plot, wherein we can jot down these constraints or limits, then that is nothing but the capability curve. Essentially, locus or safe operating points depend on these constraints if they are plotted on the complex power plane. By complex power plane, I essentially mean the x -axis referring to real power p and the reactive y axis corresponding to reactive power. This is called the complex plane in the electrical power network domain. If we plot these safe operating points based on these constraints, then we will have the capability curve.

How do these constraints or limits come in? Specifically for the synchronous generator, the same are shown here. Every synchronous machine would specifically have a maximum armature current limit. Armature windings, if they carry current beyond this limit, always have the possibility of the corresponding windings melting or rupturing. Then we also have the maximum field current that can be induced; indirectly, the maximum field current limit comes from the maximum rotor current limit, which we will see in the next lecture, not in this particular lecture. So, the voltage limitation in a synchronous machine specifically comes from the maximum rotor current limit, and third, the maximum power that can be extracted from this particular machine.

So, usually for the consideration of capability curves for cylindrical postsynchronous machines, we'll assume that the terminal voltage is fixed. It is operating at its rated voltage, and the synchronous reactance value is also fixed. So Vt and Xs are fixed quantities for this given machine. And every capability curve essentially pertains to these fixed parameters. If the terminal voltage and synchronous reactance change, the capability curves will also change.

They will also have deviations. So if we take up our first limit, which is the armature current limit for a cylindrical pole synchronous generator, any current beyond Ia max might damage the shunt winding as well as the insulation and capability curves since they pertain to a particular terminal voltage. So, this armature current limit, in a way, can be attributed to the maximum MVA that the machine can transfer without violating, disturbing, or damaging the winding or the corresponding insulation. So if Ia max is defined by the manufacturer and Vt , the terminal voltage, is known from the operational perspective or the synchronization perspective of a synchronous generator, we can have the maximum MVA as S max. Remember, this is apparent power, so there is no need for a conjugate here; S max is equal to Vt times Ia max. This is the maximum apparent power that is coming in or limited by the corresponding armature current limit.

So if we put our apparent power, apparent power squared is nothing but P squared plus Q squared. So essentially, the armature current limit locus, if it were to be observed on a PQ plane, would look like a beautiful circle whose center is the origin. Essentially, if I have P as the Q axis (X axis) and Q on the Y axis, then the center of the circle is the origin. And remember, For circles, the typical equation in the XY domain is X squared plus Y squared equals R squared. This represents a circle on the XY plane with the origin as the center and small r as the radius.
So, for this locus of the armature current limit, Vt multiplied by Ia max is nothing but the corresponding radius. That's what is mentioned here, and the circle looks like this. This is the locus. All points that are inside this particular circle are safe operating points from the perspective of armature current limit for the cylindrical pole synchronous machine. Now, why did I talk about machines? I will come to that in a moment.

And on this plane, if I have to mark the corresponding impedance angle or load angle, which is theta, because if you remember or recollect, P , which is P over here, is nothing but apparent power times cos theta, and Q is nothing but apparent power times sine theta, where theta is the power factor angle or load angle. So, is there a way to mark this load angle or impedance angle on this PQ plane? Yes. If we focus on the radius, which is Vt Ia max, then this angle, if it is theta, which is the impedance angle, the cosine component of this radius is nothing but the real power part, and the corresponding sine component, which is over here, is the corresponding reactive power part. So this representation of theta is nothing but the impedance angle, which is essentially what is shown over here, and now let's come back to why I talked about this locus representing the safe operating limit from the armature current perspective for a cylindrical pole synchronous machine. Now, in a synchronous machine, to behave as a generator, the corresponding real power should be positive.

So basically, all points where P remains positive refer to the parts marked over here, and hence we would call these our points pertaining to a generator where P remains positive; in a motor, the real power is absorbed from the grid. So, all points on the left-hand side that are marked here would refer to the points pertaining to a cylindrical pole synchronous motor. And there is also one specific point: if this is the corresponding impedance angle, remember this angle is also known as the power factor angle. And for the power factor to be equal to one, which is also known as unity power factor operation, the corresponding theta has to be zero. So if theta has to be zero, then there is also a way to mark this unity power factor operation on this locus plane, which is essentially the one shown over here.

The yellow line, which is marked here, represents the unity power factor operation. The blue and the green regions refer to generating points, while the purple and the orange ones refer to motor points. Please have patience; we will talk about why I have described the blue part as overexcited or the orange part as overexcited, and vice versa for the green and the purple one as underexcited. We'll talk more about it when we discuss the reactive power control aspect, but just to give you an idea. The over and under excited operation comes from the perspective of whether reactive power is positive or negative.

From an over-excitation perspective, the synchronous machine is likely to deliver reactive power. So Q would remain positive. And that's the reason why the blue color region and the orange color region, with Q being positive, indicate that the machine is overexcited. It is delivering reactive power to an inductive load. Whereas for the lower half, which is the purple and green one, Q is negative, so the machine is under-excited.

That's all for today's lecture. We will continue this discussion on capability curves in the next lecture. Thank you.