Power Network Analysis

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Week - 03

Lecture-11

Hello everyone, welcome to the first lecture of week 3 on power network analysis, in which we will continue our discussion on synchronous generators. In the previous lecture, we discussed the basic phenomena of how voltage is induced in the stator windings of a three-phase synchronous generator due to the rotor windings carrying DC current and the rotor rotating at synchronous speed, which is driven by the turbine or the prime mover. As a result, the stator windings experience sinusoidal time-varying flux linkage, and by Faraday's law and Lenz's law, a voltage is induced in these threephase stator windings. Coincidentally, these induced voltages are inherently positive sequence voltages, and as long as the synchronous generator is not feeding any electrical load, there is no notion of the stator winding reacting to this rotor flux, which is present through the air gap. So when the rotor windings carry any current, that is, they are feeding in some electrical load, the stator windings also carry three-phase balanced currents, which leads to the creation of stator MMF that opposes the rotor MMF, and as a result, the air gap net MMF, magnetomotive force is a combination of rotor flux MMF and the stator flux or MMF. So in today's discussion, we will build upon these discussions that we had in the previous lecture and try to understand how the electrical equivalent circuit of a typical cylindrical pole synchronous generator looks, and what the governing equations in the phasor domain are that help in understanding the steady-state behavior of a synchronous generator.

Just to recapitulate, the rotor current, which is DC current, induces a rotor flux phi in the air gap. And this rotor flux is rotating at the same speed as the synchronous speed of the rotor, which in a way induces Ef in the phasor domain; the corresponding RMS value is Erms in the stator windings, and when the generator is loaded, the armature currents also exist, which means Ia is not zero. I am choosing Ia not to be the current for phase A, but in fact Ia represents the armature current or stator current, and it is the per-phase equivalent representation; it is not the phase A current. So, as a result, when the reactor is loaded, there exists a non-zero armature current, which also creates its own flux in the air gap.

And we call that flux the armature flux phi a, which is nothing but the effect of armature reaction. And it has two components. One is the minor component, known as the leakage flux. This leakage flux is linked only to the stator in the form of core loss components and the magnetization component. The other major component, which is the reaction flux, links the stator winding, the air gap, and the rotor windings.

So the armature reaction effect that we discussed in the previous lecture was essentially the effect of this reaction flux. We had not considered the effect of the minor component of this armature flux, which was the leakage flux. And as a result, since the air gap consists of the rotor flux as well as the reaction flux from the stator, the net air gap flux is a vector combination of the rotor flux phi f and the reaction flux from the stator winding. So if we have to plot these fluxes, which we also saw preliminarily in the previous lecture, assuming the rotor flux to be my reference by Faraday's law, the induced EMF is the time derivative of the corresponding rotor flux. So basically, the induced EMF leads the corresponding rotor flux phi f by 90 degrees.

And consequently, because of the armature current, which happens to be non-zero when the generator is loaded, there also exists this major reaction flux from the armature of the stator, which is known as phi ar. And in the previous lecture, we had seen the phi gap to be the vectorial combination of phi ar and phi f, which for the time being I have not shown in this phasor diagram for the sake of simplicity. The important part to understand here is the induced EMF and the armature reaction voltage; they don't go one by one. They are not aligned with each other. It is thanks to the inherent behavior or law given by Lenz, which is called the Lenz law.

Every action has opposite reaction. Since the rotor flux is inducing voltage in the stator, the stator is trying to oppose this induction through reaction voltage. And hence, the effective stator voltage that might appear on the stator voltage windings is the vector sum of the induced EMF Ef and the opposing action of the armature reaction flux because of the negative opposing effect of Lenz's law. So if we have to plot the reaction voltage Ear on the same phasor diagram, since the armature current is non-zero and the reaction flux is the major component of the armature current. So phi ar is in a collinear direction with IA, and as a result, the induced EMF leads the corresponding flux by 90 degrees.

The armature reaction voltage would also lead the reaction flux by 90 degrees, and the induced armature voltage or stator voltage Er is the vector sum of Ef and this negative minus Ear, so this is the dotted line indicating minus Ear. When we superimpose this on the tip of Erms, we get Er as Ef minus Ear. Please remember these are all vectors or phasors; they are not simple scalar quantities. As I have already mentioned, phi ar will be collinear or in phase with the stator current, which is the majority component. So essentially, the induced reaction voltage leads the corresponding armature reaction voltage by 90 degrees.

So this lead effect of voltage with respect to current can be represented through an analogous inductive circuit, where if Xar is the armature reaction reactance or magnetizing reactance, then the induced armature reaction voltage, which is Ear, can be mathematically represented in terms of Ia with the help of the inductive reactance, which is the armature reactance or armature reaction reactance. So jXar, Iar is nothing but Ear. And now, if we put the value of the armature reaction voltage into the effective stator voltage, we get the equation that is underlined over here,

$$E_f = E_r + jX_{ar}I_a$$

and when we reorient it, the induced EMF, Ef, remember these are all phasor quantities being measured with respect to a particular reference. So induced EMF is nothing but the effective stator voltage plus the voltage drop across the armature reaction reactance or magnetizing reactance. The stator windings will definitely have some resistance.

$$E_r = E_f - jX_{ar}I_a = V_t + I_a(R_a + jX_{al})$$
$$\Rightarrow E_f = V_t + I_a(R_a + jX_{al} + jX_{ar})$$
$$\Rightarrow E_f = V_t + I_a(R_a + jX_s)$$

We have also not considered the effect of leakage flux, which is linked only to the stator and does not go beyond the air gap. So we also have to consider those voltage drops, and effectively, we will also have to evaluate what the net terminal voltage is, per phase terminal voltage across the synchronous generator winding. So if the filter winding has a non-zero resistance, which is usually the case, the leakage reactance Xal accounts for the leakage flux. And there is also a terminal voltage per phase Vt across the synchronous generator. So, in addition to the induced EMF and the armature reaction reactance, which in a way leads to the effective or filter voltage, this Er induced voltage is responsible for the voltage drop across the leakage reactance; it's also responsible for the voltage drop across the winding resistance, and then effectively we would have Vt as our per-phase terminal voltage.

So in a way, this represents an equivalent circuit, which is the one shown over here: the equivalent per-phase circuit of a typical cylindrical pole synchronous generator. The reason I am emphasizing the cylindrical pole generator will become obvious in the next few slides. So effectively, if we have this as our equivalent circuit representation for a cylindrical pole synchronous generator, we can also have typical mathematical expressions that can explain how this equation or circuit looks in terms of an equation. So if we put all of that together, then remember we had defined Ef to be the sum of Er and this voltage drop, where if we reorient Er, it is nothing but Vt plus Ia, the voltage drop across winding resistance and leakage reactance as per the circuit shown over here. So if we put all those variable numbers there, effectively we have induced EMF, which is nothing but the terminal voltage across the per-phase terminal voltage of the synchronous generator plus the voltage drop across the winding resistance and the leakage reactance and magnetizing reactance; together, they can be combined as the reactance Xs, which we call per-phase synchronous reactance, which is basically a combination of magnetizing reactance, the major component, and the minor component, which is the leakage reactance Xal.

So if we put it in effectively, this is our governing equation for a typical cylindrical pole synchronous generator: all the above equations, the ones which are shown here, the currents and voltages, are all phasor domain quantities with some reference quantity as their corresponding phasor reference. So if we put in or choose the terminal voltage per phase as the reference for the measurement of all other phasors, then the induced EMF will have an angle delta with respect to the terminal voltage, which on this phasor diagram is the angle being marked as delta, and depending on the load connected to the synchronous generator, assuming that load to be an inductive load for the phasor diagram shown over here. which means the current is lagging behind the terminal voltage. So the angle between the terminal voltage and the current is theta, which is what is shown over here, i . e., at an angle of minus theta. We can represent or draw the phasor diagram, which in a way correlates to this entire equation. An induced EMF is nothing but the terminal voltage plus the voltage drop across the winding resistance plus the voltage drop across the corresponding synchronous reactance.

$$E_f = E_{rms}\angle\delta = V_t\angle 0 + (R_a + jX_s)I_a\angle(-\theta)$$

Where Erns is typically 4.44 N , being the number of turns on the stator, phi f is the maximum flux linkage from the rotor to the stator, and f is the corresponding source frequency, which depends on the synchronous speed.

Delta is the torque or power angle, as I have marked here, and theta is the phase angle or the impedance angle between the terminal voltage and the current. So this is just a slide that recapitulates what we discussed in the previous few slides. Remember, we are choosing the per-phase terminal voltage as the reference for all other phasor quantities. One thing to note here is that the phasor diagram we have seen here is specifically for a cylindrical rotor synchronous generator. Things will become a little different when this synchronous machine behaves as a synchronous motor.

The difference is that the induced EMF always leads the terminal voltage in a synchronous generator, which means the torque angle or the power angle is always a positive quantity for generators, whereas in a motor, the expression shown here transforms into the expression that Vt , which is the terminal voltage, should be equal to the induced EMF plus the voltage drop across the corresponding winding resistance and the corresponding synchronous reactance in the case of a synchronous motor. In a motor, the objective is to convert electrical input power into mechanical power. So the current direction is reversed, as a result of which induced voltage is output, or resulting from the terminal voltage that is applied at the input of the synchronous motor. So with that equation, what happens is the induced EMF Ef tends to lag the corresponding terminal voltage, which means the power or torque angle becomes negative in the case of a synchronous motor. The same aspect is essentially shown here, and the other quantities remain the same as we discussed in the previous few slides.

The phasor diagram which I have discussed here or shown over here inherently has the rotor flux chosen as our reference. Rotor flux is DC flux or comes from the DC current. So essentially, we can have a phasor diagram where we have only AC quantities. And with Vt as our reference terminal voltage for phasors, we can again redraw the phasor diagram, which would look like this for a cylindrical pole machine, with this KVL equation being the governing equation for synchronous generators in cylindrical pole rotors. For salient pole rotors, where things become a little different and interesting, is that in a cylindrical pole rotor we have well understood that the air gap length, or the path between the direct axis and quadrature axis, remains the same.

So, as a result, one would think that, oh, if I have to measure the re-actance along any length of the air gap in a cylindrical pole rotor, where the synchronous reactance is inversely proportional to the air gap length, a shorter air gap length would result in a higher value of reactance, indicating that more linkage occurs in terms of flux between the stator and rotor. One would imagine that the synchronous reactance would remain constant along the direct axis and quadrature axis for a cylindrical pole machine. That's theoretically possible; in practice, synchronous reactance never remains a constant quantity thanks to the non-linear open circuit characteristic. One of the reasons is the non-linear open circuit characteristics of the synchronous generator and also the non-linearity involved in the stator cores and rotor cores of the synchronous generators. As a result, the synchronous reactance is not a constant quantity.

It has variations. It has a nonlinear variation with respect to the machine's magnetization behavior. We had briefly discussed the open circuit characteristic in the previous few lectures on synchronous generators, where we saw that the OCC is not a linear curve. And that non-linearity is responsible for the synchronous reactants not being constant when one of the reasons is considered. The other reason is the rotor slots; they are distributed. The same is true for the stator slots where windings are placed.

So effectively, the distribution of these stator and rotor slots for windings creates variations in the air gaps along the direct and quadrature axes, even in a cylindrical pole machine. And furthermore, in a salient pole machine, the direct axis reactance Xd is much greater than the quadrature axis reactance because, along the direct axis, the air gap length is small. And on the quadrature axis, the air gap length is greater. So it is furthermore relevant that the synchronous reactance will not be uniform at all in a salient pole machine because of the non-uniform air gaps. As a result, we have different values of synchronous reactance in a salient pole machine.

So the question then is, what should be the governing equation in a salient pole synchronous generator, because according to this discussion, different values of synchronous reactance exist along the written quadrature axis in a salient pole machine? This beautiful equation will no longer be applicable for a salient pole machine because the reactance value itself is going to be different. So, what is the discussion? What should be done? So, one thing: let's go step by step. How do we consider the effect of rotor saliency in understanding the steady-state behavior of the salient pole synchronous generator? So one thing is pretty sure: , the direct axis reactance, Xd, is much greater than the quadrature axis reactance because the air gap length here is much smaller compared to the air gap length along the quadrature axis. In order to consider this rotor saliency, we have to take into account these two different reactances, that is for sure.

So let us see how we do that. So what we do is resolve the stator MMF, specifically the armature reaction MMF, not the leakage component, along two axes. One is the direct axis; the other is the quadrature axis. The corresponding filter MMF resulting from direct axis current is not aligned with the corresponding filter MMF coming from the quadrature axis reactance. So basically what we do is we indirectly decompose the armature current or stator current Ia along the direct axis, which we call Id, the direct axis current, and Iq, which is the quadrature axis current, and these currents look something like this.

$$I_d = I_a \sin(\theta + \delta), I_q = I_a \cos(\theta + \delta)$$

The quadrature axis current Iq is parallel to the induced EMF phasor Ef, whereas the direct axis component is perpendicular to the quadrature axis component.

Now let's spend some time discussing why this phasor diagram is applicable to a salient pole machine. In order to understand this, we'll probably have to go back a few slides. So let's do that. What we saw here in this particular phasor diagram, if we see, in a cylindrical pole machine, where the air gap is more or less uniform and assuming the synchronous reactance is not drastically different compared to a salient pole machine, is one single armature current that is leading to the major armature reaction flux, and as a result, the corresponding armature reaction voltage is perpendicular, which is this orange line, to the corresponding armature reaction voltage. Now, in the case of a salient pole machine, the armature current Ia is being decomposed into the direct axis current Id and the quadrature axis current, which would also have their corresponding armature reaction voltage.

So let's say we mark the Ead as the armature reaction voltage because of the direct axis current and Eaq as the quadrature axis reaction voltage because of the quadrature axis current; how would these reaction voltages look for a given salient pole machine? Now it is for sure that the induced EMF and the corresponding reaction voltage don't align with each other. They are a phase apart from each other. And if we remember, in a salient pole machine where the rotor is tapered with salient poles, with the north pole, let's say, being marked here and the south pole being marked here, the magnetic axis along which the air gap is minimum in a given salient pole rotor is called the direct axis component, whereas the quadrature axis component is perpendicular to the corresponding direct axis component. So now, if we look at the salient pole rotor, one can easily understand that the induced EMF, which is a result of the rotor current, is aligned along this direct axis. Let's say that if we mark Ef, which is also Erms at an angle delta, then the corresponding direct axis component reaction voltage would typically be in the opposite direction to the direct axis current.

So let's say if Id is the corresponding armature component of the direct axis, then minus Ead should be opposite to the induced EMF. And consequently, the quadrature axis reaction voltage, which is perpendicular to the direct axis component, would more or less be along this particular axis. Let's say this is minus Eaq. Then, if we correlate these induced EMF reaction voltages because of the Id and Iq components of Ia, the corresponding direct axis current, it should lead to the corresponding minus Ead, so essentially the Id current should be aligned along this particular axis, whereas the Iq current should be aligned along this particular axis in correlation to the corresponding currents that have been shown here for the corresponding cylindrical pole machine. So if we correlate or superimpose this, what we see here is that the induced EMF and quadrature axis current are parallel, whereas the direct axis component is perpendicular to the direction of the induced EMF.

So that is the reason behind having this as our phasor diagram for a salient pole machine, and mathematically we can define Id to be the sine component of armature current, which is dependent on the impedance angle and torque angle, and the corresponding quadrature axis current is the cosine component of armature current, which again depends on the phase angle between voltage and current, terminal voltage, and the power angle. So, if we have to write an equation that mathematically represents this phasor diagram, then this is what the corresponding equation looks like.

$$E_f = E_{rms}\angle\delta = V_t\angle 0 + R_a I_a \angle(-\theta) + jX_d I_d + jX_q I_q$$

The first two terms remain similar to the terms in a cylindrical pole machine. The last two terms are replacements for jXsIa in a cylindrical pole machine, and the induced EMF is leading the terminal voltage for the generator. So if we break this equation along two components, along two axes, one along the Q axis current Iq and the other along the D axis current, we can get two analogous equations.

$$E_f = V_t \cos(\delta) + X_d I_d + I_a R_a \cos(\theta + \delta) \quad \{\text{along } q \text{ axis}\}$$
$$V_t \sin(\delta) + I_a R_a \sin(\theta + \delta) = X_q I_q \quad \{\text{ along } d \text{ axis }\}$$

The first equation is along the Q-axis. How do we get it? It's very clear that if we look at the vectors that are aligned along Iq, then we have Ef, and we will also have Vt cos delta, the component of Vt along the cube axis, which is Vt cos delta. And then we have jdXd, which is a term sitting over here. And then IaRa will also have a component along this Iq axis, which will depend on the cosine of theta and delta. And along the direct axis, if we see which is along this component, we have IqXq as one component. And the other component would be the component of Vt along Iq, which is Vt sine delta, and the component of IaRa, which is this component.

So that's how we obtain two analogous equations. Unfortunately, we cannot have a single equivalent circuit where we have two different reactances in series with two different currents in series. And that's the reason why you would not see an exact equivalent circuit for salient pole machines in typical references or textbooks. That's all for today. In the next lecture, we will take up the discussion on evaluating power expressions and capability curves for synchronous generators based on the equations we have seen here. Thank you.