

## **Power Network Analysis**

**Dr. Abheejeet Mohapatra**

**Department of Electrical Engineering**

**IIT Kanpur**

**Week-02**

**Lecture-10**

### **Lecture 10: Synchronous Generators-Induced emf, armature reaction.**

Hello, welcome to lecture 5, the last lecture of week 2 on the course Power Network Analysis, in which we will continue our discussion on synchronous generators, wherein we will specifically talk about the expression or the behavior involved behind voltage being induced in synchronous generators in stator windings, because of rotating rotor magnetic field in the air gap and what is this armature reaction, how does the machine being loaded or unloaded, what implication does it have in the air gap MMS in between the stator and the rotor core in synchronous generator. In the previous lecture we have started with this basics of three phase machines where we arrived at just the fact that be it in cylindrical pole rotor or salient pole rotor the air gap in between stator and rotor experiences rotor magnetic field, rotor magnetic flux because of the rotor DC current and the rotor tends to operate in synchronous speed be it for the generating mode or motoring modes as a result of which The stator windings which are placed and distributed multiple slots in the stator, they experience this rotating MMF and it's well known from Faraday's law that any conductor which experiences change in magnetic flux with respect to it, voltage would be induced as a result of it. So that's how we'll take up the discussion of induced EMF in today's discussion. So we take the example or case of a two pole cylindrical rotor synchronous generator. Cylindrical rotor is pretty obvious, the air gap is uniform, this is the rotor and over here I have the stator in which I have marked few concentrated slots for phase A, phase B and phase C windings. Unlike my previous lecture discussion where I had marked 120 degree phase shift slots and each of those slots were associated with one particular phase, here I am making a little disturbance or a change here is that there are certain set of windings which refer to phase A and they may be into they may be the path 1 conductor for phase A and then we have A dash where is the return conductor for this particular filter slot so that continuity in current in sort of stator armature currents is maintained.

And similarly we have BB dash and CC dash referring to phase B and phase C filter windings respectively. If I have to particularly see what is the orientation of phase A,

phase B, phase C, incoming conductors, then if I have to mark the corresponding arrows somewhere like this and I have to find the angle between each of these arrows, these angles almost tend to be equal to 180 degrees apart, which essentially means that stator windings are again 180 degree electrically spaced apart in the stator core. So that serves the purpose of electrical construction of stator core. We will focus more on the rotor part here because that's how the winding sound stators will experience some induced EMF.

So what we have here is the rotor current, rotor core which is carrying rotor current which is DC in nature and it is being driven by a turbine for synchronous generator. The turbine is driving this rotor at a mechanical speed of  $\omega_s$  radian per second. which can also be converted into synchronous speed  $N_s$  as rotations or revolutions per minute.

$$N_s = \frac{60\omega_s}{2\pi} rpm$$

RPM here is revolution or rotation per minute, that is the RPM is the short form of revolution per minute. So because of this rotor rotating at synchronous speed which is carrying DC current, if I have to place a magnetic needle on the stator and on this needle I were to see how the magnetic field is varying or magnetic flux is varying with respect to this needle.

Now why do you have a magnetic flux? It is obvious that the rotor is carrying DC current and because of the nature of DC current this two pole cylindrical rotor is behaving as a electromagnet with north and south pole as marked here and as this magnet rotates in the anti-clockwise direction at a mechanical speed of  $\omega_s$  radian per second it is likely that this magnetic needle which is placed over here would also experience different fluxes at different point of time. So, basically if we have this magnetic needle then if this magnetic needle aligns with the position of rotor where the north pole is just close to this needle and the south pole is opposite of this north pole probably in that situation this magnetic needle will have the maximum deviation whereas on the position where the north pole and the south pole are in a perpendicular direction to the current needle position let's say north is here and south is here and this angle here is 90 degrees it's likely that this magnetic needle will have the minimum or the smallest possible deviation because the magnetic field axis is not aligned with the magnetic needle axis that's how maximum minimum deviations were to be observed. If we plot these deviations over a period of time, let's say  $\omega_s t$  or  $\omega_s t$  to be general and over here we have the deviations in the magnetic needle. These deviations if they are plotted, they would typically follow a sinusoidal curve. The sinusoidal curve may be a sine wave, may be a cos wave because sine and cosine functions are the two common functions for defining sinusoidal signals in networks.

So essentially that's the reason behind that. So basically the stator windings which are similar to these magnetic needles, the stator windings would also experience a sinusoidal variation in the magnetic flux which is being created because of the rotor carrying DC current and rotating at the synchronous speed. And if we have to write a mathematical function for the system or the slide under consideration, Since the north and south pole they are correspondingly placed at time  $t$  is equal to 0. So if this magnetic needle is placed along phase A and A dash winding then at  $t$  is equal to 0 A and A dash winding they are experiencing the maximum magnetic flux where the flux is denoted by the symbol  $\lambda$  usually and A corresponds to the phase A flux.

$$\lambda_a = N\phi_f \cos \omega t$$

So since the needle or the electromagnet is aligned along AA dash at  $t$  is equal to zero it is likely to observe a maximum flux linkage to it and as the rotor rotates so at  $t$  is equal to zero the flux linkage is maximum and at  $\Omega t$  is equal to  $\pi$  by 2 radian specifically when the rotor rotates and the north pole comes over here, south pole comes over here.

The magnetic field because of this analogous electromagnet is not aligned with the phase A and A dash finding. At  $\omega t$  is equal to  $\pi$  by 2, it is likely to observe the minimum flux linkage. Ideally, it could be zero also. If we were to associate these variations with the sinusoidal behavior of the magnetic flux, we can then represent the flux linkage as a cosine function of  $\omega$  and  $t$ . For this two-pole machine here,  $\omega$  is same as the  $\omega_s$ , which is the synchronous speed determined by the turbine. Similar expression can also exist for flux linkage of phase B or for phase C also. And now if we apply Faraday's law and Lenz law, the induced EMF in time domain is nothing but time derivative of the associated flux linkage. The negative sign comes because of Lenz law.

$$E_a = -\frac{d\lambda_a}{dt} = N\phi_f \omega \sin \omega t$$

and having defined  $\lambda_a$  as  $n \phi_f \cos \omega t$  where  $n$  is the number of turns in coil a a dash phase winding we have  $E_a$  as  $n \phi_f \omega \sin \omega t$  and since remember the phase windings on the stator they are intently placed 120 degrees electrical apart. So, if we write or find the expression for  $\lambda_B$  or  $\lambda_C$ , they would inherently have a  $\cos \omega t$  plus minus 120 degree as the argument over here. So, from there if we try to find the corresponding induced EMFs in phase B winding and phase C winding.

$$E_b = \sqrt{2}E_{rms} \sin(\omega t - 120^\circ)$$

$$E_c = \sqrt{2}E_{rms} \sin(\omega t + 120^\circ)$$

Because the windings are 120 degree electrically spaced apart, the induced EMFs also turn out to be balanced voltages with the same RMS value as defined by the corresponding  $n \phi f \omega$  expression.  $\phi$ , by the way, I forgot to mention, it is the maximum rotor flux along the d-axis, which is where I was mentioning about the permanent magnet aligning along coil AA dash direction and that depends on the, so  $\phi$  here essentially depends on number of turns on the rotor winding and it also depends on what is the winding current, the reducing current flowing in the rotor winding which we also often called as field current and hence the term IF comes there. So depending on the number of turns on rotor and the field current, the induced EMF expression is correspondingly obtained and coincidentally it turns out to be a balanced voltage. That's how synchronous generators or alternators inherently produced positive sequence balanced voltages.

$$E_{rms} = \frac{N\phi_f\omega}{\sqrt{2}} = \sqrt{2}\pi N\phi_f f = 4.44N\phi_f f$$

For a machine where the number of poles are different from two let us say P number of poles remember when you talk about poles these poles refer to magnetic poles and magnetic poles usually appear in pairs we have often called the term known as dipoles they are not single poles. So, P here is going to be a even number for a P pole machine. The relationship between the omega at which these fluxes are being induced and the actual speed, it depends on p by 2 omega s relationship as a result of which we can also find. So, if omega is 2 pi f, we can also find the relationship between the electrical frequency and the synchronous speed  $N_s$  in rpm.

$$\omega = \frac{P}{2} \omega_s \Rightarrow f = \frac{N_s P}{120}$$

Under no load condition, when these stator windings are just experiencing induced EMF along them and the stator windings are not carrying any current, whatever is the induced EMF, same would be observed at the terminal voltage of the generator because the generator windings are not carrying any current. There is no voltage drop in the threader windings or armature windings. Things however, so in order to continue this discussion on induced EMF, if we were to plot the open circuit characteristic through some OCC experiment for synchronous generator, this OCC characteristic is very similar to the characteristic which is also evident or present for DC generators which essentially says that the induced EMF is proportional to the field current It's also proportional to the synchronous speed and for constant speed this OCC is also known as magnetization curve as is shown over here which is very similar to the OCC for separately excited DC machines.

$$E_f = E_{rms} \propto \phi_f f \propto I_f N_s$$

Using this characteristic, one can also find few relevant important parameters of the corresponding stator windings, which we are avoiding for case of simplicity in our discussion. So if we talk about the case when stator windings carry load currents, the situation changes dramatically under no load conditions.

The induced voltage in stator is due to sinusoidal rotor flux is a balanced positive sequence voltage but the things change dramatically when stator windings carry current that means if there is some electrical load connected to the stator of the synchronous generator the induced emf would try to supply the load as a result of which this supply can only happen when current tends to flow so when current tends to flow Stator windings will also carry current and remember stator windings when they carry current depending on what type of electrical load is connected assuming that the load is a balanced load so every phase of the load is having a impedance of mod  $Z$  with an angle of  $\theta$ . We can also use the relationships as we deduced for induced EMF to find corresponding relationship for the phase currents. Now remember these phase currents because of induced emf being balanced and the fact that the load is also balanced the armature currents or stator currents in the synchronous generator also turn out to be balanced.

$$I_a = \sqrt{2}I_{rms} \sin(\omega t - \theta)$$

$$I_b = \sqrt{2}I_{rms} \sin(\omega t - 120^\circ - \theta)$$

$$I_c = \sqrt{2}I_{rms} \sin(\omega t + 120^\circ - \theta)$$

$$I_{rms} = E_{rms}/|Z|$$

Now what is the criticality about this balance? What is this term reaction here? Let us talk about that. So suppose we focus only on the stator windings or armature windings in synchronous generator which are carrying these respective currents  $I_a$ ,  $I_b$  and  $I_c$  because there is some electrical load connected to the synchronous generator and the way these windings are connected A end of coil A is carrying current out of the plane whereas A dash is the corresponding return conductor and if we were to correspondingly find or apply the same logic which we applied for rotor core carrying DC current.

Now let us try to understand that at a given point of time let us say at some particular time instant. phase A or coil A is carrying current out of the plane and if we apply right hand rule to understand what would be the instantaneous magnetic flux because of coil A winding carrying current out of the plane by applying right hand rule then the corresponding magnetic flux would be in a anticlockwise direction whereas for coil A dash carrying current into the plane if we apply right hand rule then the corresponding magnetic flux would be in the clockwise direction. If we take the superposition effect of both these corresponding fluxes then the resultant magnetic MMF or magnetic motive

force which would result in because of armature current  $I_a$  would likely be in a direction shown over here which is  $NiA$ .  $NiA$  is a resultant MMF which depends on number of turns on stator and  $I_a$  being the instantaneous armature current in phase A. If  $NiA$  can exist like this then correspondingly phase B would also have its induced MMF.

Phase C currents would also have their induced MMF which are shown by the corresponding orange and green lines. And now what I'm trying to focus here is I'm not bothered what is happening because of rotor, my focus is completely on stator. So because of these stator currents which are resulting in the corresponding MMFs, I'm trying to understand If I have to place a magnetic needle in this air gap assuming there is no rotor and I have to observe how this magnetic needle behaves because of these MMFs resulting because of stator currents, then how would that expression look like? So what I have chosen here is, I have chosen an arbitrary axis along which I have placed this magnetic needle. And I'm trying to see what is the corresponding magnetic deviation that it would observe. So effectively, I'm trying to find the effective MMF, which is  $F$  at an angle  $\phi$ .  $\phi$  is the angle with respect to filter phase A MMF. So if I have to find what is  $\phi$ ,  $F \phi$ , what I can do, I can simply take the component of individual MMFs along this axis. So if I take the component of phase A MMF, then it would probably along this  $F \phi$  would look like  $NiA \cos$  of  $\phi$  ( $Ni_a \cos \phi$ ) If I have to take the component of  $Ni_b$  MMF along this axis then it will typically look like  $Ni_b \cos$  of  $\phi$  minus 120 degrees ( $Ni_b \cos(\phi - 120^\circ)$ ) and similarly for  $I_c$  MMF it would be  $\cos$  of  $Ni_c \phi$  plus 120 degrees ( $Ni_c \cos(\phi + 120^\circ)$ ). This is nothing but the resultant MMF which is being absorbed by this magnetic needle. Now let's see what this expression turns out to be.

So essentially this is what has been mentioned here and in addition to  $I_A$ ,  $I_B$ ,  $I_C$  currents as a result of Erms and balanced load, I've also substituted the corresponding phase current expressions and now if I simplify this, this is the beautiful equation which comes in,

$$F(\phi) = Ni_a \cos \phi + Ni_b \cos(\phi - 120^\circ) + Ni_c \cos(\phi + 120^\circ).$$

$$F(\phi) = N\sqrt{2}I_{rms}\{\sin(\omega t - \theta) \cos \phi \\ + \sin(\omega t - 120^\circ - \theta) \cos(\phi - 120^\circ) \\ + \sin(\omega t + 120^\circ - \theta) \cos(\phi + 120^\circ)\}$$

$$F(\phi) = \frac{3}{\sqrt{2}}Ni_{rms} \sin(\omega t - \theta - \phi)$$

which tells me that The MMF which is being observed by this magnetic needle, it is also inherently sinusoidal in nature and it is rotating inside the air gap at a corresponding synchronous speed or relevant speed, angular speed known as omega radian per second.

For different positions of angle  $\phi$  between 0 to  $2\pi$  radian 0 to 360 degrees, the MMF which is absorbed by the corresponding magnetic needle, it is also sinusoidal in nature because of stator currents. And as a result, what we can conclude is that the stator MMF which is observed by any particular point on the air gap in between rotor and rotor that is also sinusoidal in nature. It is also rotating at speed  $\omega$  radian per second and depending on the number of poles present this could also relate to the synchronous speed. So to summarize the effective MMF and that is where the armature is trying to react.

It is trying to sort of show its own reaction by creating its own individual MMF in the air gap. So the effective MMF due to stator currents sensed by rotor winding is equivalent to rotation of a magnet which is rotating at  $\omega$  radian per second. In actual generators, there are multiple stator poles.  $P$  need not necessarily be equal to 2 for practical machines. There can be multiple number of poles, And no matter the number of poles, these stator MMFs and rotor MMFs, they both rotate at the same synchronous speeds. Essentially, rotor MMF and rotor MMF, they are magnetically locked. So as a result, the effective poles on stator and rotor, they are essentially the same. Both MMF rotate at the same synchronous speed because in both the expressions we saw  $\sin \omega t$  expression appearing in both for rotor MMF as well as for stator MMF. The stator MMF, it tends to oppose the rotor MMF which is causing the currents to flow and that's where the term reaction comes in.

As a result of this reaction, the terminal voltage of generator and induced EMFs, they are unequal because of voltage drop across the stator windings and the net air gap MMF is vector sum of both the rotor and stator MMFs. If we have to plot it on a phasor diagram, then  $I_f$  is the corresponding rotor current which is DC current and  $\phi_f$  is the corresponding flux which is resulting in throtler currents being induced if it is loaded. Because of  $I_f$  or  $\phi_f$ ,  $E_{RMS}$  is the induced EMF on the stator winding. And depending on the type of load, which I am assuming to be an inductive load connected to the synchronous generator only for phase A, the corresponding air gap MMF is vector sum of  $\phi_A$ , which is flux due to stator winding A current and the corresponding field current. We also have additional windings in synchronous generators known as damper windings.

They have the same windings which are also present in induction machines. Essentially damper windings don't carry any DC current under certain state condition in synchronous generators. Basically, they are windings which are shorted at both the ends like the squirrel cage induction motor in the rotor case. And the importance of these windings are that under transient conditions, specifically from the perspective of fault analysis and stability analysis, these damper windings help the rotor speed or stator MMF speed to retain back to the synchronous speed in case of disturbances the MMF do not rotate at synchronous speed. They rotate at synchronous speed only under steady state conditions.

So, in order to ensure that the slip of the synchronous generator motor becomes zero, the damper windings play a very important role. During transient conditions, these damper windings carry AC currents which is resultant of slip frequency current similar to rotor winding currents in induction machine and these damper windings they help to strengthen up or diminish the corresponding air gap MMF so that the stator and rotor MMF, they balance out under steady state conditions. That's the importance of damper windings, and that's where their roles would come into picture for fault analysis or stability analysis conditions. That's all for today's lecture. In the next lecture, which would be first lecture of week three, we'll start our discussion on equivalent circuits for synchronous generators.

Thank you.