

Course name- Analog VLSI Design (108104193)
Professor – Dr. Imon Mondal
Department – Electrical Engineering
Institute – Indian Institute of Technology Kanpur
Week- 11
Lecture- 32, Module-1

Now, welcome back and this is lecture 32. So, in the previous lecture we were discussing a differential amplifier with a current mirror load right. So, we were on the what we were we started discussion on this structure and we found out that the short circuit current of this structure was double that of a structure which does not have a current mirror load right with the one that has a resistive load. Let us say we kept it at BCM right. So, we saw that the short circuit current that the incremental current that was flowing was the total current that was flowing through this branch was $I_{naught} \text{ over } 2$ plus $g_m \text{ times } \Delta V \text{ over } 2$ this is I_{naught} . And this current was flowing through this transistor was getting mirrored through this transistor and this becomes $I_{naught} \text{ over } 2$ plus $g_m \Delta V \text{ over } 2$.

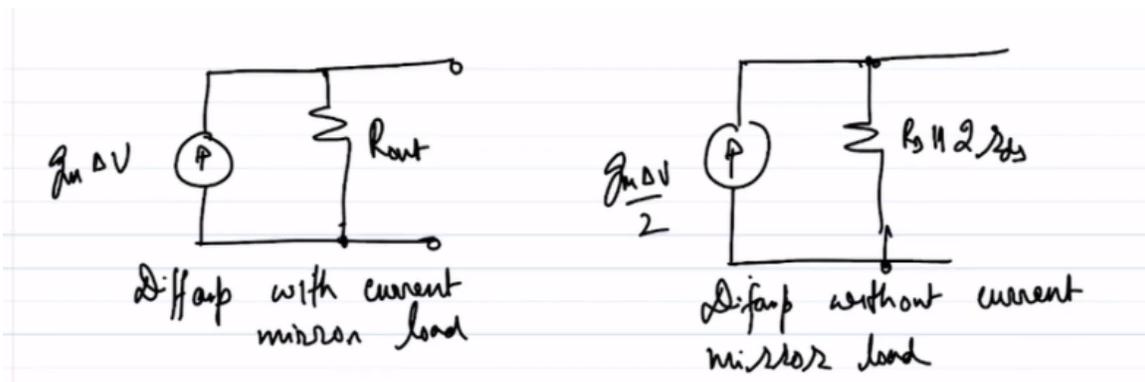
However, this current here was $I_{naught} \text{ over } 2$ minus $g_m \Delta V \text{ over } 2$ which means the difference current that is $g_m \text{ times } \Delta V$ was flowing into the incremental short circuit right. In this case I have put a battery just to be mindful of the fact that we are dealing with Poisson conditions also right. So, this is different from the case this is this was different from the case where we had we did not have the current mirror right. So, if this was $V_{cm} \text{ plus } \Delta V \text{ by } 2$ and this where $V_{cm} \text{ minus } \Delta V \text{ over } 2$ and we did something similar.

So, this current still would have been $I_{naught} \text{ over } 2$ plus $g_m \Delta V \text{ over } 2$ and this current would still have been $I_{naught} \text{ over } 2$ minus $g_m \Delta V \text{ over } 2$. However, note that there is no mirroring that is happening from the top right since there is no mirroring that is happening from the top the short circuit current right. So, assuming that ok so I should not have put I should not have put V_{cm} there because the value of battery that should that I should put in this case should have been what if this is R_d this is R_d this battery value should have been $V_{dd} \text{ minus } I_{naught} \text{ by } 2 \text{ times } R_d$ because that is what the Poisson value is right right. So, we hold it at Poisson value if we hold it at Poisson value then this current is always $I_{naught} \text{ by } 2$ right. So, the difference current that would have flown right.

So, that the difference current that would have flown out of the difference current that would have essentially flown out of your out of the V_{cm} right out of the battery would

have been $g_m \Delta V$ over 2 that is that is that is half the amount right. Oh sorry I mean it should it should not be out it should be in right the other direction because the current from top is higher right. So, the excess current should flow in this direction the excess current would have been $g_m \Delta V$ over 2 right. So, as you can see that by simply putting a current mirror in the circuit in the left right we are able to essentially double the double the short circuit current ok. So, if I have to find out the Norton equivalent of a differential amplifier with a current mirror load then what does the Norton equivalent look like now the Norton equivalent essentially looks like this.

So, this is g_m times ΔV and this is some R out we have not yet figured out what this R out is. So, this is something we will do next. So, this is the defamped with current mirror load ok and what was the condition for the defamped without current mirror load or we can simply say defamped with resistive load. So, this was short circuit current was $g_m \Delta V$ over 2 and the output resistance was R_d parallel twice R_d s correct. So, if you recall we did this why the output resistance looking down is twice R_d s right ok.



So, this is for defamped with resistive load or without current mirror load ok fine. So, that is as far as the short circuit current is concerned. So, let us now let us now investigate what will be the what will be the effect of output resistance right ok. So, in order to figure out output resistance what should I do incrementally what should we do we are talking about incremental output resistances. Incrementally we should short the inputs correct.

So, this current source also goes off this guy also gets shorted right. So, this is M_1 , M_2 , M_3 , M_4 again we are assuming M_1 is exactly equal to M_2 and M_3 is exactly equal to M_4 ok fine. And this is an incremental picture again let me be mind let us be mindful of the fact that this is an incremental picture and not the not the picture where we are having where we are having all the biases all the quiescent points in place right ok. So, what we are essentially trying to figure out we are trying to figure out that if we apply a

test voltage V_{test} right. So, what will be the incremental I_{test} flowing in that is all essentially what we are trying to figure out right.

And naturally if I neglect channel length modulation what is incremental i_{test} it is 0 right because if because we clearly see that it is looking into the drain of two transistors if we neglect channel length modulation looking into the drain of the transistor we get infinite impedance which means R_{out} is infinite right. But that is a trivial solution and as we have emphasized multiple times in this course you can neglect a high value with respect you can neglect a low value with respect to a high value right, but you cannot neglect anything with respect to 0 or with respect to infinity right. So, essentially or rather let me rephrase you cannot neglect anything with respect to 0 and if you if something is finite and your model is telling you some it is going to be infinite then which means the model is making an infinite percent error right. So, you should we should be mindful of these things while making approximations right. So, neglecting channel length modulation is an approximation, but that approximation holds under certain circumstances where there is a separate smaller amount of load connected to the transistor right.

But if there is no small load connected to the transistor we cannot neglect we cannot neglect channel length modulation ok. Ok which means we will have to put back the resistances let us assume M_1 and M_2 are the symbols M_1 and M_2 represents the transconductance of the transistor right. So, that is what we have been doing throughout when I put the symbol let us assume it means the transconductance and if we have to incorporate channel length modulation we will we will incorporate them as separate resistors incremental resistors attached to attached to them ok. So, this is R_1 this is R_2 right similarly we will have R_4 and R_3 ok. So now, we are trying to figure out what is i_{test} in terms of V_{test} ok.

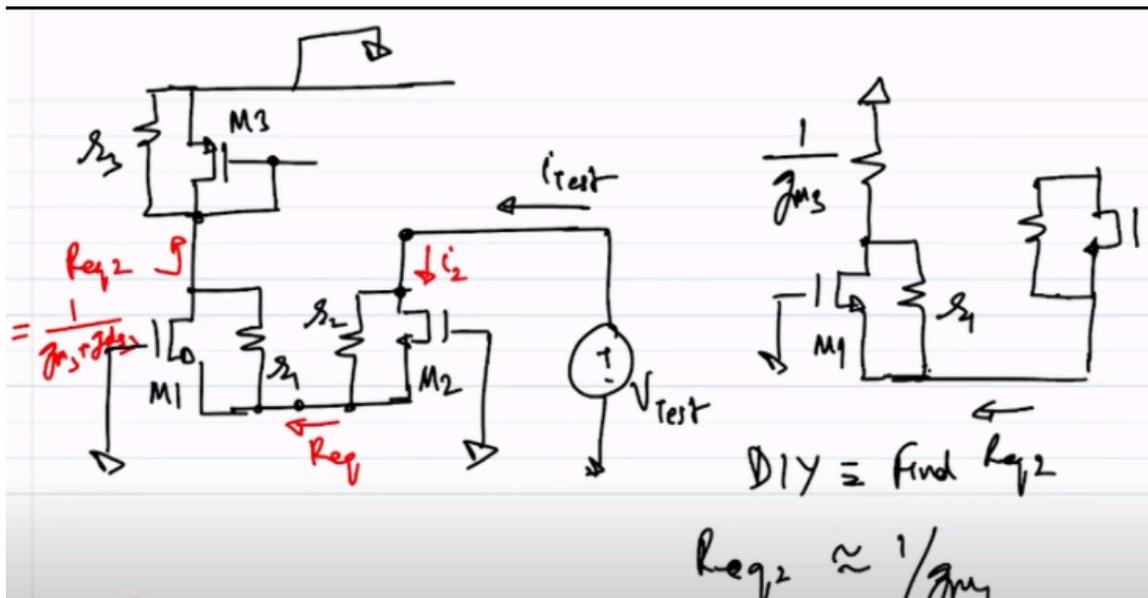
So, let us again let us break the problem down into multiple parts because this seems like a messy way of going about analyzing all 4 transistors together right. So, just like in previous cases we will like to break this part and break this problem into multiple parts. What we would like to do is we recognize the fact that there will be an incremental current flowing into the drain of M_2 let us call this I_2 . And we also recognize that there will be some current flowing into the drain of M_4 let us call this I_4 right. So, we also which means that we know that i_{test} is equal to I_2 plus I_4 and we would like to figure out what is I_2 and I_4 in terms of i_{test} and V_{test} right.

So, if we can if we can do that then essentially the our job is done right. So, let us do the I_2 first because that seems familiar we have done this in a separate context right. So, what should we do in order to figure out I_2 ? So, we break the circuit again into sub parts

right. So, let us break them break it down into sub parts right. So, when you are doing I2 let us let us not bother about I4 fine.

So, let us only concentrate on I2. So, when we in the previous examples when we were trying to find out the output impedance of a differential amplifier how did we approach this? We approach this from the point of view of we understand this M2 is the drain of that source of the M2 is not grounded, but it is loaded with something right. It is loaded with it is loaded with some equivalent resistance right, it is loaded with some equivalent resistance. What is the equivalent resistance? That equivalent resistance is the resistance looking into the source of M1 right. So, this is R equivalent that we are trying to figure.

If we know R equivalent then we are all good right. So, in the earlier examples that we did while trying to find out R equivalent we saw that this node was rounded right, but in this case we see that the that the drain of M1 is not rounded right. So, we will have to then invoke we have to figure out what is this resistance looking up again yes right. So, let us call this R equivalent 2 right. So, what will be R equivalent 2? So, clearly this is M3 the diode connected transistor.

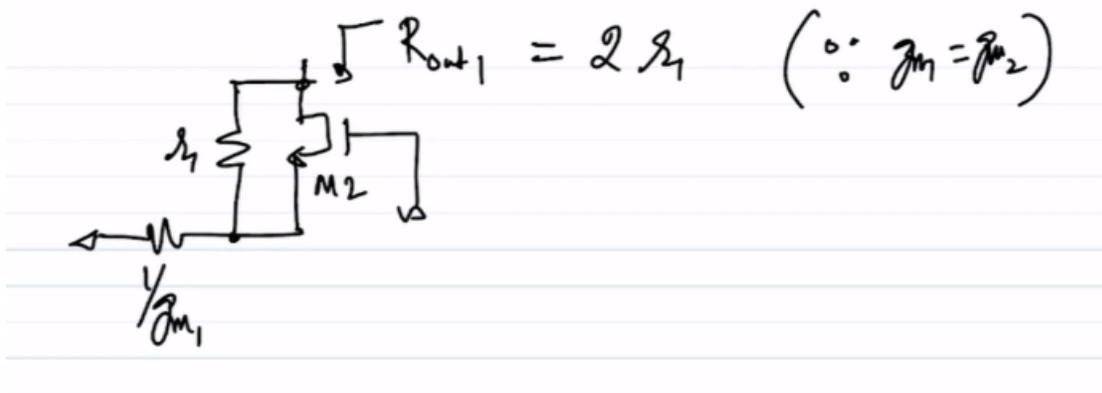


So, R equivalent 2 is 1 over gm3 plus gds3 right and you can and we as we have talked about multiple times we can neglect gds with respect to gm. So, this is approximately equal to 1 over gm3 right. So, let us put our blocks back. So, this becomes 1 over gm3 right. So, now what is R equivalent 2? So, what is the impedance looking into the source of this M1? So, again how will you how will you figure this out? We can actually write out the we can actually what I would request you to do is to do this take this structure,

take this structure sorry, take this structure and find out the impedance looking into the liquid looking into the source of the of the MOSFET right in the presence of $1/gm3$ ok.

So, you will see that you will see that. So, this is I request you to do it yourself right. So, find out in so, by find R equivalent 2. So, what will you see what you will see that after you find R equivalent 2 under these conditions you will see that R equivalent 2 can be approximated as $1/gm1$. This is almost similar to the case where where I am neglecting that R1 is present or even $1/gm3$ is present right.

So, so once you do that I mean it this should make sense because we are looking into the source of the transistor right. If I if I excite the source of the transistor then gm times that current will flow through the transistor which means I should be expecting resistances of the order of $1/gm3$ right. So, once you do the math you will see that this approximation will hold good ok fine. So, if looking into the source of looking into this node if it is $1/gm1$ right. So, where does it lead us to? It essentially leads us to this place where this node this equivalent impedance becomes $1/gm1$ right and we are trying to figure out what is the impedance looking down right.



So, let us say R out 1 this is m m2 right. So, this is familiar territory we have solved for this multiple times before. So, what is R out 1? R out 1 is is equal to twice of R 1 since $gm1$ is equal to $gm2$ right ok fine. So, this solves for this solves for R out 1 right. So, let us let us go back and put this guy I copy this guy again ok.

So, if R out 1 is or rather I should have said R out 2 right. Let us see R out 2 because I was everything was suffix to 2 this is R out 2 right. So, if R out 2 is R out 2 is twice of R 2 right twice of R 2. If R out 2 is twice of R 2 then then what do you think I 2 is? So, I 2 becomes the test over twice of R 2 make sense right. So, if this I 2 is flowing into the

drain of m_2 this I_2 has to come out of the source of m_2 it cannot go anywhere else right.

So, this has to be I_2 . So, again if the if this I_2 is flowing into the source of m_1 it has to come out from the other side. It cannot go anywhere else. So, that is also I_2 right. So, if this I_2 flows into this diode connected configuration of m_3 it will generate some voltage at the gate of m_3 . What will that voltage be? This voltage will be I_2 by g_{m3} plus g_{ds3} which I can again say is approximately equal to I_2 by g_{m3} correct.

So, if it generates this voltage of I_2 by g_{m3} how much current do you think will flow through m_4 I mean which direction. So, let me split this part up into multiple right. So, what I am essentially saying is this voltage is how much? This voltage is let us say this voltage is V_x . If that voltage is V_x what is the incremental current through m_4 ? Incremental current through m_4 is in this case g_{m4} times V_x because source is rounded. What is the direction? Going up because that is what we have been using to characterize our P MOSFETs right.

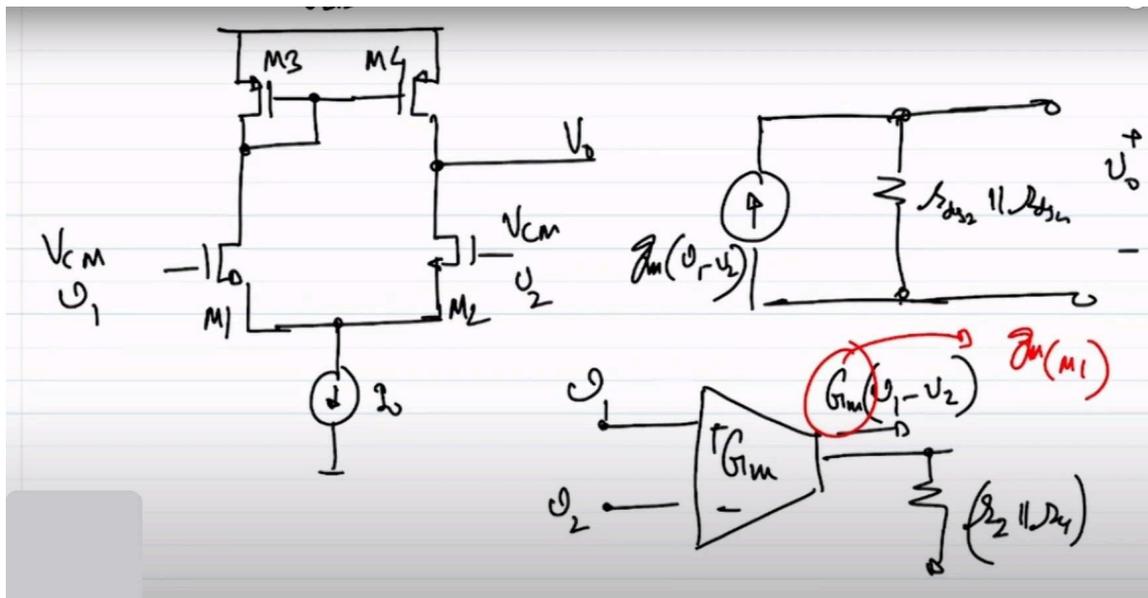
So, the current direction will be up this will be g_{m4} times V_x . So, what is g_{m4} times V_x ? What is V_x ? V_x is I_2 over g_{m3} and what is g_{m3} and g_{m4} relation between g_{m3} and g_{m4} ? g_{m3} is exactly equal to g_{m4} because quiescent current through m_3 and m_4 are identical and so on and so forth right. So, this becomes equal to I_2 right. So, note that the I_2 circulates back and it gets mirrored into the right branch and it flows on top also right ok. What is the current through this resistance that was R_4 right.

So, this current is I_2 over R_4 right. So, that in what is I_4 ? I_4 is I_2 plus I_2 over R_4 correct. So, what is then I_{test} ? I_{test} is I_2 plus I_4 . What is I_2 ? I_2 is V_{test} by twice R_2 plus what is I_4 ? I_4 is again I_2 which is V_{test} over twice R_2 plus V_{test} over R_4 which is V_{test} times $\frac{1}{R_2} + \frac{1}{R_4}$ correct. So which implies what is V_{test} over I_{test} ? It implies this is equivalent to R_2 parallel R_4 right.

So essentially you see that we end up with a pretty similar expression that we had earlier but the reason it is similar is a slightly bit convoluted right. So, eventually it seems like the output impedance of a differential amplifier with a current mirror load is only dependent it seems like it is a parallel combination of the two transistors M_2 and M_4 that is all it seems like. But the reason it appears to be parallel combination of R_2 and R_4 is slightly more profound because this happens because as you see there is a there is a path extra path that creates current mirroring and so on and so forth right. But eventually the result is quite simplified eventually the result is almost like a common source amplifier with an active load right. If you have a common source amplifier with an active load then output resistance is the parallel combination of the main transistor and the load transistor and that is what it this seems like.

So, one good thing is that this is easy to remember right. So, what is a so eventually what becomes the Norton equivalent of a differential amplifier with a current mirror load the Norton equivalent effectively becomes this right. So let me I will just sketch the full thing now. So, the incremental equivalent incremental Norton equivalent of this becomes this right. So, g_m times ΔV and R_{out} becomes R_{out2} parallel R_{out4} or rather I should say R_{ds2} parallel R_{ds4} and incrementally this becomes the output voltage right.

So, the another popular way of expressing the same thing is this. We know that this is a voltage control current source what is what does this voltage control current source depend on? It depends on the voltages V_1 and V_2 and this is a transconductance of value g_m this current is g_m times V_1 minus V_2 and we have a resistance here of R_{ds2} parallel R_{ds4} . What is this g_m ? This g_m is exactly equal to g_m of M_1 . Note that in the previous case where we did not have a current mirror load this g_m was equivalent to the g_m of M_1 divided by 2, but in this case it is equivalent to the g_m of a single transistor right. So, again this is plus minus and I should sketch it in a more relevant manner.



So, this is V_1 this is V_2 and this is g_m times ΔV only if V_1 is plus ΔV by 2 and V_2 is minus ΔV by 2 right. So, this should ideally be g_m times V_1 minus V_2 right ok. Thank you.