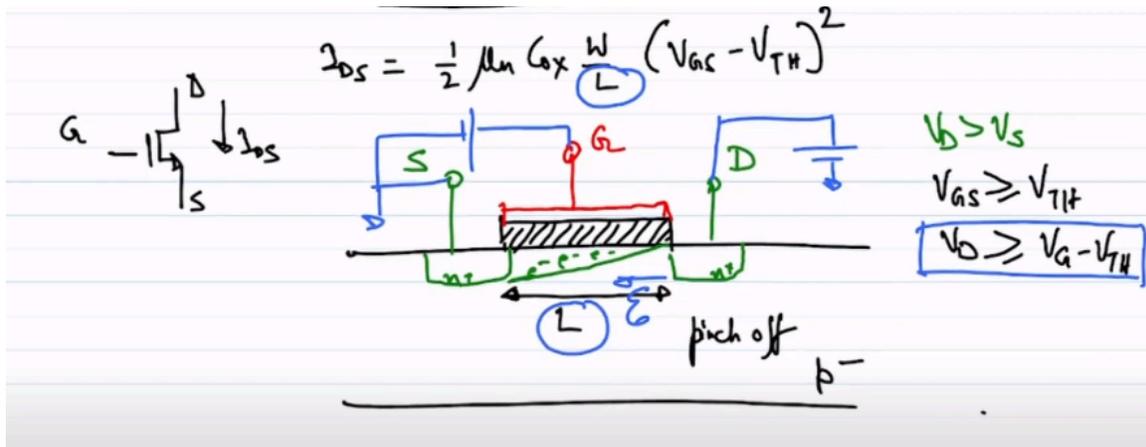


Course name- Analog VLSI Design (108104193)
Professor – Dr. Imon Mondal
Department – Electrical Engineering
Institute – Indian Institute of Technology Kanpur
Week- 8
Lecture- 22, Module-1

Welcome back, this is lecture 22. So, up until now we have been discussing various topologies of amplifiers and we were also discussing how to make different voltage control with different control sources namely the voltage control current source, the voltage control voltage source, current control current source and current control voltage sources using a single transistor circuit right. So, that is what we have been doing till now. So, what I would like to do in this lecture is to bring you back to one of the transistor properties that we have been taking for granted and see its impact and the possible solutions in the circuits that we traditionally deal with right and we will take examples of the common source amplifier to see its impact right ok. So, let us get started. So, if this is our MOS transistor, this is drain, this is gate, this is source.



So, what is the current voltage equation in saturation that we are using till now? We assume that the drain to source current this is I_{DS} is half $\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$ right. What was the genesis of this equation? The genesis of this equation was if I look at the cross section of a MOSFET, if I take you back to the one of the earlier lectures where we were introducing the cross section of a MOSFET. What did the cross section of a MOSFET look like? It took like this. So, let us say you have a we have a p type wafer right, we have a p type doping wafer p type wafer which on top of which we have let us say introduced or deposited some oxide right.

This is oxide which acts as a dielectric. On top of that we have introduced a layer of metal, this is our gate and how did we introduce a source and drains? We had to dope something on either side with n plus with n plus dopants and we call one of them drain and one of them source and the one that is at the higher potential we call it as a drain and one that that is at the lower potential we call it call it the source right. So, this is drain this is source let us assume I mean under the constraint that V_d is greater than V_s ok. So, in saturation what was the condition of saturation? Firstly, the channel had to be on. So, V_{gs} had to be greater than threshold voltage and V_{ds} or rather I can as well forget about the source requirement was V_d should be greater than equal to V_g minus threshold voltage.

And what was the genesis of this? The genesis of this was the fact that the charge distribution in the channel that is just under the oxide the charge distribution was something like this right. So, this were electrons all the way in the channel and at the drain channel junction right at around here at around here we were having pinch on here we were having pinch on ok. And this was the electrical length or physical length of the channel right. So, this physical length was getting translated to the length in the in the current equation of our MOSFET ok. So, now as it turns out what happens is if you increase the drain voltage.

So, let us for this for this purpose let us assume that the source is grounded right and gate is connected to some higher potential. So, that inversion has been established ok. And let us assume that the drain is now connected to a higher potential. So, that this condition has been satisfied. Now all you are saying is let us assume that we are increasing the drain voltage.

So, if we increase the drain voltage the assumption that we took was the fact that nothing is going to happen to the channel anymore because the channel has been pinched off right. Now as it turns out as it turns out that is not entirely accurate. So, one can appreciate the fact that if I increase the if I increase the drain voltage the lateral electric field that is the electric field going from the drain to the source inside the channel right. So, let us say let us mark it with the. So, the lateral electric field is going to increase right.

So, even though there is a pinch off even though there is a pinch off around the around the drain channel junction the increased electric field will have an effect. And what is that effect? The effect will be the effect will be to shorten the length of the channel right. So, what ends up happening is that the slope of the slope of this channel increases and maybe it the channel the starting point of the channel on the drain side gets pushed from

the physical junction of the gate and of the gate and the drain, but it gets pushed towards inside the channel right. So, since this happens. So, we can say that let us assume that this is because of this increase of electric field the channel got shortened by ΔL .

If the channel got shortened by ΔL what will be my new I_{DS} ? The new I_{DS} I mean you can write out the full device physics models, but the easier hack is to say that what I will do? I will now instead of plugging in the physical length of the channel in the current voltage equation I will introduce the actual length of the channel that is the shorted version of the channel right, shortened version of the channel. So, the I_{DS} now becomes $\frac{1}{2} \mu_n C_{ox} W (V_{GS} - V_{th})^2$ ok. So, I am expressing $V_{GS} - V_{th}$ as V_{DS} . So, this becomes a new equation new current voltage equation for the transistor in saturation. So, we can massage this equation slightly.

So, what we will do we will say that if ΔL is much much less than L right. So, $L - \Delta L$ can be expressed as $L (1 - \frac{\Delta L}{L})$ correct or rather I can say $1 - \frac{\Delta L}{L}$ since it can be expressed like this. So, I can use Taylor series and say this is equivalent to $1 + \frac{\Delta L}{L}$ by L . So, this is approximately right. So, now if I plug in this modified expressions into I_{DS} what should we get? We will get I_{DS} equal to $\frac{1}{2} \mu_n C_{ox} W (V_{DS})^2 (1 + \frac{\Delta L}{L})$ right.

So, this becomes the modified this becomes a modified equation of our of our MOSFET in saturation ok. So, we can do one step better right as it turns out we can do one step better. So, what is the cause of ΔL ? The cause of ΔL was the fact that just at the if we increase the drain voltage if we increase the drain to source voltage right beyond the pinch off point then the effect was that of shortening the length of the channel. So, ΔL is the amount of shortening of the length and the cause of ΔL is increase of the drain to source voltage beyond the pinch off point right. So, what we can further approximate is we can approximate ΔL as we can say that approximately ΔL is proportional to ΔV_{DS} where ΔV_{DS} is the increase in V_{DS} over the pinched off V_{DS} right.

$$\text{If } \Delta L \ll L \quad \frac{1}{L - \Delta L} = \frac{1}{L(1 - \frac{\Delta L}{L})} \approx \frac{1 + \frac{\Delta L}{L}}{L}$$

$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \left(1 + \frac{\Delta L}{L}\right)$$

Approximately $\Delta L \propto \Delta V_{DS}$ (where ΔV_{DS} is the increase in V_{DS} over the pinched off V_{DS})

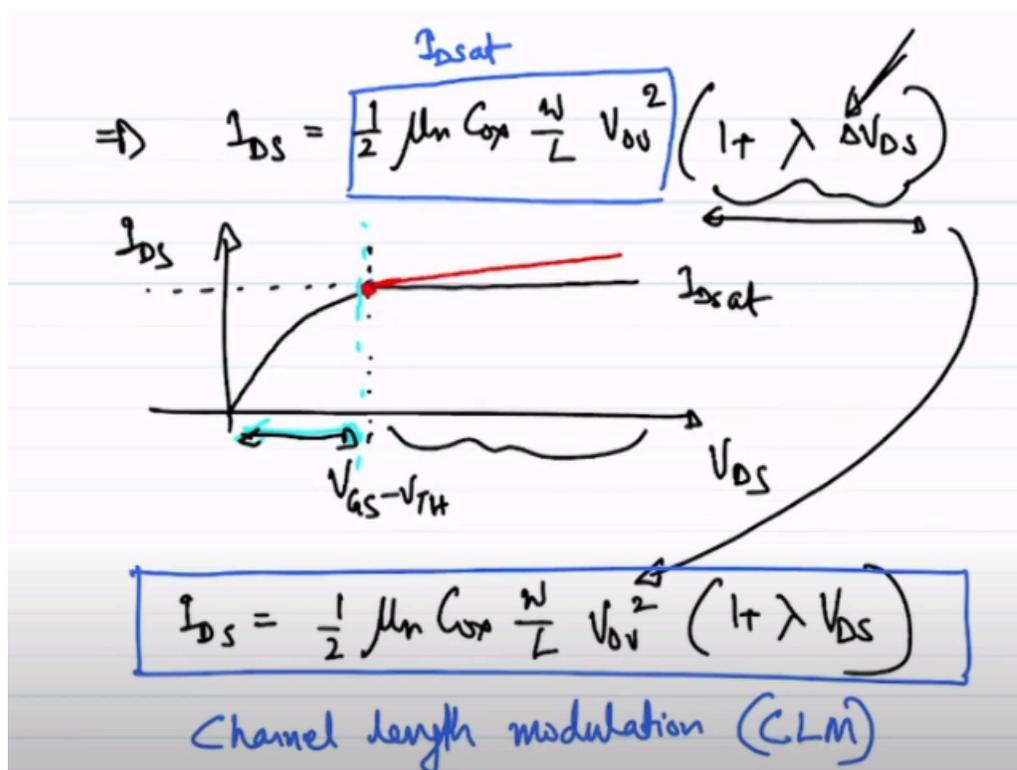
So, till pinch off we do not have any the transistor is not in saturation right till pinch off the transistor is in linear region just when you hit the pinch off voltage that is V_D becomes gate voltage minus threshold voltage you you hit saturation beyond that beyond that if you increase V_{DS} what is the effect? The effect is the shortening of the length of the channel right. So, that is why we can approximate ΔL as proportional to ΔV_{DS} right and this is this is again an approximation because we purposefully are saying that it is a directly proportional it did not be directly proportional as it turns out it is not directly proportional, but as long as we are dealing with small changes in ΔL for small changes in V_{DS} right. So, then we can we can assume that again this good old Taylor series approximation we can neglect the higher order terms if ΔL is a function of V_{DS} right. So, what I am essentially saying is ΔL is actually a function of ΔV_{DS} and we are neglecting the higher order terms if we neglect the higher order terms then we can say that ΔL is proportional to ΔV_{DS} this is approximately this is an approximate ok. So, when I say that ΔL is proportional to ΔV_{DS} then we have to introduce a proportionality constant and the proportionality constant that we normally use is called lambda call this lambda times ΔV_{DS} ok.

What is the unit of lambda? I am sorry I am sorry. So, let me not do this right now. So, let us write it out fully right. So, let us call it let us ΔL is equal to η times ΔV_{DS} where η is some proportionally constant and let us plug in this value in the I_{DS} expression. So, what is my new I_{DS} expression? So, I_{DS} becomes half $\mu_n C_{ox} W$ over $L V_{ov}^2$ right times $1 + \frac{\Delta L}{L}$, but ΔL is η over L times ΔV_{DS} correct.

So, ΔV_{DS} I should characterize as $V_{DS} - V_{DS, pinch}$ equal to $V_{DS} - V_{DS, pinch}$

right on top of this. So, in other words in other words this becomes I_{DS} is equal to half $\mu_n C_{ox} \frac{W}{L} V_{ov}^2$ plus some constant because L is a physical length of the device λ is some proportionally constant. So, we call this λ some constant times ΔV_{DS} ok. So, what is the unit of λ ? The unit of λ is clearly volt inverse right ok. So, what impact does this have on the current voltage characteristics of a device? So, note that the current voltage characteristics of the device in the absence of this extra term was so it was linear it was in a linear region for till the pinch off point that is V_{GS} minus threshold voltage and after that this was flat.

What was it flat with? It was flat with the value of I_{Dsat} where I_{Dsat} was this voltage correct. So, this was the I_{Dsat} correct, but now what is it telling us? This is telling us that there is a extra term on top of I_{Dsat} . So, when at when V_{DS} is exactly at V_{DS} minus threshold voltage then we are here we are we are just on the dot because the linear region current equation has not changed. If we increase if we go beyond this point the current is no longer flat at I_{Dsat} the current will increase linearly right. So, that is all it is just saying ok.



So, now what impact will this have on our incremental characteristics? Clearly you see that the $\frac{\partial I_{DS}}{\partial V_{DS}}$ for this characteristics will not be 0 right. So, that will be a significant that will have a significant impact on our current voltage characteristics of the device, but

before before going there let me talk about one more important thing and the one the thing that I wanted to talk about was the following that this equation can further be approximated. So, as it turns out this this slope the slope of the red line that I talked about is is is not significant at least it cannot be significant because if if if that slope is very steep what will happen to Y^2 ? Y^2 will be will become higher and higher and which essentially means that will not be able to make an amplifier. So, that what our fellow device physics colleagues or the device engineers have done is to ensure that we get devices in which the the even though there there will be a slope in the I_d sat in in the in the saturation region characteristics, but they will not be significant then. The other thing that we should keep in mind is the fact that this point is even though I have shown it in such a way that such a way that this is quite far from the this is quite far from the 0 0 point from the origin this point is not significantly away from the origin that will be I mean in most practical devices it the it will be q I mean less than 100 milli volt right.

So, so hence since that since that is the I mean one might argue that if V_{gs} is higher V_{gs} is larger and larger then obviously, this voltage will be this this knee point will go to the to the right further to the right, but even though that is the case, but in most practical devices you would not be biasing your V_{gs} with very large voltages simply because you see that if you supply if you if you bias it with very large voltages then you there is a tendency of the transistor going towards towards linear region. So, in most practical cases you limit the overdrive to 100 milli volt or few 100s of milli volts right. So, so the point that I am trying to make here is that this knee point is much closer to the origin and you have a lot of room on the right of the knee. So, that is why this equation is further approximated as instead of using this term ΔV_{ds} we get rid of Δ and we say that I mean ΔV_{ds} is approximately equal to V_{ds} because this extra V_{ds} minus threshold voltage we can we can neglect without incurring too much of error ok. So, essentially what we are saying is a new I_{ds} equation is essentially this is $\frac{1}{2} \mu_n n C_{ox} \frac{W}{L} \frac{V_{drive}^2}{1 + \lambda V_{ds}}$.

So, this is the famous current voltage such equation of a transistor in saturation in the presence of the effect that causes reduction of the channel or the modulation of the channel based on the applied gain to source voltage and this effect is often called channel length modulation right. This effect is called channel length modulation and often abbreviated as CLM ok ok. So, now so let us spend some time understanding the effect of channel length modulation and as you have already guessed the primary effect of channel length modulation will be on the y^2 of the device right. So, I_{ds} since I_{ds} is $\frac{1}{2} \mu_n n C_{ox} \frac{W}{L} \frac{V_{drive}^2}{1 + \lambda V_{ds}}$ what is y^2 ? Y^2 is $\Delta I_d \Delta V_{ds}$ right and at some quiescent operating point of I_d of now the operating points are $V_{ds} Q$ $V_{ds} Q$ we cannot now neglect $V_{ds} Q$ because it is the current it actually depends on $V_{ds} Q$. So, this becomes $\frac{1}{2} \mu_n n C_{ox} \frac{W}{L} \frac{V_{drive}^2}{1 + \lambda V_{ds}}$.

squared times lambda right.

So, this can further be approximated as lambda times I_d ok ok. So, now you will see that in many cases in many cases we I_d as it turns out this the values of the lambda are not significant by not significant what I mean is if I take a typical device lambda is approximately I mean lambda is often less than less than 0.1 volt inverse right in devices with L greater than 180 nanometer ok. So, if this is the case right so what do you think is the significance of this term? Significance of this term on the absolute I_d s as it turns out the significance as you can see the significance of this term on absolute I_d s is not very high under the assumption that lambda V_d s is much less than 1 right. So, let us say V_d s is 1 volt lambda is 0.

1 volt. So, lambda V_d s is 1 point is 0.1 essentially right. So, for the calculation of I_d s right we can often neglect the effect of channel length modulation right. So, what I am essentially saying is that we can neglect the effect of CLM for calculating I_d s in the first order. This is obviously unless otherwise mentioned or required.

Sometimes you will see that we are interested in very accurate values of current then obviously we cannot neglect, but we have first order if we are ok with some values of current which are not exactly accurate to the second or third decimal places we can essentially say that fine let us get let us not use CLM right. So, because it helps our maths and helps our calculations ok. So, if that is the case what is Y_{22} ? Y_{22} can be lambda times I_d s ok. And as it turns out we have a better expression for we have a better symbol for Y_{22} . Y_{22} is a generic 2 port a network parameter we call Y_{22} as G_{ds} that is the conductance between the drain and the and the source ok.

So, what is what is the other parameter that can get impacted? The g_m of the transistor can also get impacted right. So, what is the g_m ? g_m is $\mu_n C_{ox} W / L (V_{gs} - V_{th})$ which is equal to $\mu_n C_{ox} W / L (V_{gs} - V_{th})$ times $1 + \lambda V_{ds}$ right. So, so this is often I mean so if we can approximate the total current by neglecting lambda V_{ds} right. So, what is generally done is to say that while approximating g_m we will not use the will not use the effect of lambda V_{ds} in most devices right. So, approximation is approximation since lambda V_{ds} is much much less than 1 we approximate g_m as $\mu_n C_{ox} W / L (V_{gs} - V_{th})$.

So, essentially the expression for the g_m remains unchanged you can use whichever expression you want it does not really ok. But we cannot make these approximations while calculating g_{ds} because when you if we assume g_{ds} to be equal to 0 or y_t to be equal to 0 and we introduce g_{ds} the difference can be start right. I mean you can neglect let us say 10 with 1 with respect to 10, but you cannot neglect 1 with respect to 0 you

cannot neglect anything with respect to 0 right. So, sometimes it you will see that it is not very prudent to neglect g_{ds} , but in some other cases you will see that it might be prudent to neglect g_{ds} right. So, we will we will look into that as we as we as we go forward ok.

$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

$\lambda < 0.1 V^{-1}$
(in $L > 180nm$)

$$y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS}, V_{DS}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \lambda = \lambda I_{DSAT}$$

We can neglect the effect of CLM for calculating I_{DS} in the first order. (unless otherwise mentioned or required)

$$y_{22} = \boxed{\lambda I_{DS} = g_{ds}}$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) (1 + \lambda V_{DS})$$

Approximation: $\because \lambda V_{DS} \ll 1$ we approximate

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_{DS}}$$

So, now, what we will do? So, let us in the next part of this lecture we will switch our attention to the impact of channel length modulation on our small signal small signal incremental amplifier right. We will we will go back to a common source amplifier and then we will see what impact does the channel length modulation have on on its operation ok. Thank you.