Course name- Analog VLSI Design (108104193)
Professor – Dr. Imon Mondal
Department – Electrical Engineering
Institute – Indian Institute of Technology Kanpur
Week- 3
Lecture- 7

Welcome back, this is lecture 7. So, in the previous lecture we were discussing about some of the characteristics of a two port network right, some of the characteristics of the two port network that are necessary in order to realize our amplifier. We are half way through and we essentially stopped where we took a generalized non-linear two port network and we were trying to linearize it right. So, what did we do? A quick recap was the fact that we took a two port network, a generalized non-linear two port and if we call these ports, this port voltages V1 and we call this V2 and the current we call this as I1, we call this as I2, this is a non-linear two port RL, this is RS and let us say you have a Vi here and we said that we understand that we require an additional power source and we assume that the power source is inside the two port network right. And the power source that was inside the two port network it can be one power source, it can be multiple power sources was instrumental in setting up the quiescent voltages right. So, they were instrumental in setting up the quiescent voltages across the two ports and then we said that what happens if we perturb the port voltages right.

So, where we stopped in the last class or in the last lecture was the fact that we said that let us say we have this generalized non-linear two port right and they have across its two ports their voltages of V1 and V2 and the currents going into the ports are I1 and I2. What happens if I perturb the voltages? So, we initially said that I1 is a function some non-linear function of the port voltages V1 and V2. Similarly, I2 are non-linear functions of these port voltages V1 and V2. Note that the functions f1 and f2 did not be identical right and then we said that what if we perturb the voltages V1 and V2.

So, if I perturb the voltages V1 and V2 this becomes V1 + delta V1 and this becomes V2 + delta V2 and my current at the input will become I1 + delta I1. Similarly, the output voltages, output currents at the port 2 on perturbation of the voltages of V1 and V2 will also be can also be represented like this and then we also said that this as long as the perturbation port voltages delta V1 and delta V2 right as long as the perturbation voltages delta V1 and delta V2 are small enough right or small enough such that the higher order terms of the non-linearity can be neglected then we can approximate this as then we can approximate this as the quiescent term or that is the term which from which everything started right the normal current the quiescent current that is f1 of V1 + V2 +

delta V1 delta f1 delta V1 around the quiescent point V1Q V2Q right + delta V2 delta f1 delta V2 around V1Q. Similarly, the second the current in the total current in the second port can also be represented as f2 of V1 comma V2 + delta V1 delta f2 + delta V2 around quiescent port voltages right and then naturally what followed was the fact that this were identical these were identical and what we simply could write was del I1 was del f1/ del V1 I am not rewriting the fact that not repeating the fact that these are around some quiescent points that is implied. Similarly, del I2 become del f2 /del V1 del V1 + del f2/ del V2 del V2 right and naturally we saw that and going from the we could realize that this is nothing but a set of a set of two linear equations right and the incremental currents right the incremental currents were related to related to the incremental port voltages in a linearized fashion right. So, now we are in the domain of linear equations linear circuit theory which means that we can leverage all the all the good things all the mathematical niceties of set of linear equations to enhance our understanding on the top right and then we also continued on another piece of jargon and the jargon was the fact that these del f del y's right.

So, this del f del y's were nothing but incremental y parameters right. Why y parameters or admittance parameters? It is admittance parameter simply because I am trying to develop a relationship of how current is changing when the voltage is getting perturbed. So, essentially the coefficient of the voltage is admittance right and that is hence these parameters are called incremental y parameters and why incremental it is because it relates the incremental port voltage change to the incremental port current changes right. Note that these are not absolute y parameters, these are not total y parameters, these are incremental y parameters ok. a set of two linear equations right and the incremental currents right the incremental currents were related to related to the incremental port voltages in a linearized fashion right.

So, now we are in the domain of linear equations linear circuit theory which means that we can leverage all the all the good things all the mathematical niceties of set of linear equations to enhance our understanding on the top right and then we also continued on another piece of jargon and the jargon was the fact that these del f del y's right. So, this del f del y's were nothing but incremental y parameters right. Why y parameters or admittance parameters? It is admittance parameter simply because I am trying to develop a relationship of how current is changing when the voltage is getting perturbed. So, essentially the coefficient of the voltage is admittance right and that is hence these parameters are called incremental y parameters and why incremental it is because it relates the incremental port voltage change to the incremental port current changes right. Note that these are not absolute y parameters, these are not total y parameters, these are incremental y parameters ok.

And then we said that we could simply represent those currents and voltages as I1 is equal to y11 V1 + y12 V2 and I2 become y21 V1 + y22 V2 right and what are the context of this y11 and y22? So, the context was now if I have the same two port network right, if I have the same two port network and I want to relate only the incremental port voltages with the incremental currents port currents, how can I relate them with? We can relate these incremental voltages and incremental currents through this set of equations which are governed by, they are linear equations, but they are governed by the y parameters right and hence I mean we can write rewrite these equations in a matrix form and if I rewrite these equations in the matrix form this is what we get right. So, it is customary often to express this two port network with its y parameters in the matrix form ok. So, now let us take a step back, let us take a step back and see where we are right now and where from did we start. We started with the understanding that we require a non-linear two port right, I require at least a non-linear two port network in order to get an amplifier which can amplify power right. So, once we understood that we figured that we need a framework to realize or we need a framework to understand analyze a two port network which is non-linear because a generalized non-linear solution does not exist then we played a trick and what trick did we play we played a trick that saying that I mean understand we understand and we appreciate that a generalized non-linear solution for any sort of non-linearity is not possible.

However, if we if we do not seek a generalized non-linear solution instead if we see what happens if I change the voltages and the currents of a non-linear system by a small amount right, what is small can be can be related later on, but if we if we change the currents and the voltages in a non-linear network by a small amount then we saw that we can relate the changes between voltages and the currents in the non-linear network with a linear relationship right. So, we got away with basically we cheated the system and we said that I understand we have a non-linear network I am not good at doing non-linear mathematics, but I know how to do linear algebra. So, I will I will twist the given system in such a way that I will bring it to my circle of competence and what is my circle of competence I will make everything linear and how did we make everything linear we said that we will perturb the system by a small amount we perturb the system by a small amount and as long as this perturbation is small then the relationship between the currents and the voltages remain linear and in case of a two-port network I have two voltage variables and two current variables right. So, which means that I should be able to relate them with a set of linear equations and that is what we have done right. Note that this is not yet job is only half done because we have only set up a framework to to analyze a non-linear two-port network, but we have not yet come to that place where we have figured out what is the what will be the input output characteristics when we apply a voltage source right.

So, that is something that we will be doing next. So, what am I hinting at , am hinting at the fact that ultimately what are you looking for we are looking for the fact that I have a voltage source Vi it has it has a resistance associated with it within that is Rs I have stuck it between this non-linear two-port network right. So, this is f1 (V1 V2), so f2 (V1 V2) right. So, I have stuck this between a non-linear two-port network and with the understanding is that there are currents and voltage sources maybe inside maybe outside maybe somewhere right, but we are not interested in finding out at present we are not interested in finding out what is the relationship between the total V1 and total V2 at present we are interested in finding out what happens if this input Vi right changes by a small amount right. What happens if this input Vi changes by a small amount what happens to the voltage across the Rl.

In other words what we are saying is that we understand that if I apply a small perturbation Vi the code voltage V1 will change it will change to + some small V1 the port voltage on the other port, port 2 will change it will change to some small voltage delta some small voltage which is small V2. Similarly the currents will also change from I1 to I1 + small I1, I2 to I2 + small I2 right. What are we interested in now we are interested in in finding the relation small V2 or small I2 right and Vi right. Ultimately this is what we are interested in because that is what the perturbations are and now the fact that we have developed a framework to relate perturbations of the ports we can switch from this full-blown non-linear network to our incremental non-linear network and solve our set of linear equations right and that is what we will do next ok. So let us do it so what should we do we replace this non-linear 2 port with their incremental equivalent ok.

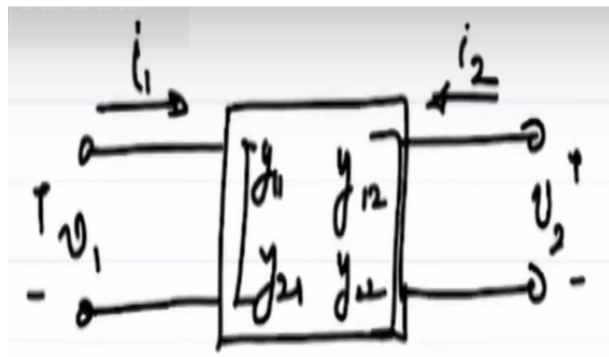
And we and we rewrite our equations whatever what are our equations our equations are I1 is equal to Y11 V1 + Y12 V2 ok. Now note that I am interested between them I am not interested to get V2 or I2 in terms of V1 right. I am interested to get V2 and I2 in terms of Vi so I have to replace I have to somehow juggle this equation to figure out what to do right. So I can I recognize the fact that I1 is equal to Vi - V1 over Rs right or I can instead of going in the division root I can I recognize the fact that this is equal to Vi times Gs - V1 times Gs where Gs is equal to 1/Rs similarly GL=1/RL juggle this equation to figure out what to do right. So I can I recognize the fact that I1 is equal to Vi - V1 over Rs right or I can instead of going in the division root I can I recognize the fact that this is equal to Vi times Gs - V1 times Gs where Gs is equal to 1 over it down up front.

So what I will do I will replace I1 I will replace I1 with this. So this becomes Vi times Gs is equal to Y11 + Gs times V1 + Y12 V2 ok. Similarly what is I2? I2 is Y21 V1 +

Y22 V2 ok. Now note that I2 is equal to also I can write I2 is equal to - V2 over RL right. So this is V2 by the way the notations of the two port networks are I am pulling current into the port right.

So this becomes - V2 over RL or - V2 times GL. So I will replace I will replace this I2 with - V2 times GL. So this becomes - V2 times GL or in other words this becomes 0 is equal to Y21 times V1 + Y22 + GL times V2 ok. So now have a look at these two equations this one and this one. So what do you notice? You see these are two sets of equations which can be written in a matrix form right.

$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



So why am I interested in writing in a matrix form? Because it is a 2 by 2 matrix is easy to solve right. So let me just write it in that format. So this becomes Vi times Gs 0 equal to V1 right. So now once you have it in this format it is easy to it is easy to find out what V2 will be. Now we know that V2 is equal to V0 right.

So in other words V2 is nothing but If I use Cramer's rule this becomes right. So if I expand these determinants what do I get in the denominator? I get Y11 Gs times Y22 Gs - Y12 21 and what do I get in the numerator? Get - Y21 Vi times Gs right. So in other words in other words this is equal to V0 right V0. In other words what is V0 over Vi? Then denominator we get Y11 + Gs times Y22 + Gl - Y12 Y21 and in the numerator we get - Y21 times Gs ok right. So now what is our goal? Our goal is to maximize maximize V0 over Vi right.

I want amplification right. If I can for a given Vi right if I can maximize V0 and note that V0 is also across a register RL. So if I can maximize V0 I can maximize the power delivered to the RL. So I would like to I can achieve that right. So how can I achieve that? Mathematically it seems like mathematically it seems like if I simply set the denominator to 0 assuming that all the all the incremental Y parameters are all positive right.

By the way a note of caution is not necessary that the incremental Y parameters should always be positive because at the end of the day they are derivatives of a non-linear function. It is not necessary that the derivative of a non-linear function will always be positive, but as it turns out in most of the regular devices that we use they are positive and so we will limit ourselves in this discussion to those conditions where all these Y parameters are positive right. So what do you think how can I maximize this? I mean the curious among you will obviously point to the fact that I can choose Y12 and Y21 and Y11 and Y22 in such a way that the denominator goes to 0 then I get V0 over Vi to be -infinity right. Great I mean I am looking for infinite I am looking for a high amplification

$$\frac{v_o}{v_i} = \frac{-y_{21} G_s}{(y_{11}+G_s)(y_{22}+G_L) - y_{12}y_{21}}$$

what is better than infinite amplification right. Well you might be right, but there is a problem and let me let me try to let me try to convince you why the problem is a problem why is it actually a problem.

So in order to appreciate that let us express these Y parameters in a in a circuit form right. So one one one overwhelming theme that you will see in this course is we will try to express everything in the form of a in the form of a circuit because ultimately it is a circuit theory course not because only it is a circuit theory course, but in order to develop an intuition in order to develop an intuition of figuring out what these equations actually mean because ultimately all these equations are are pointing to some sorts of currents and some sorts of voltages. that I can choose Y12 and Y21 and Y11 and Y22 in such a way that the denominator goes to 0 then I get V0 over Vi to be - infinity right. Great I mean I am looking for infinite I am looking for a high amplification what is better than infinite amplification right. Well you might be right, but there is a problem and let me let me try to let me try to convince you why the problem is a problem why is it actually a problem.

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Let us say this port is I call this V1 this is this port I call this V2 what is this telling us, this is telling us these currents at least this equation is telling us this current I1 is related to is related to port V1 right current I1 is directly proportional to the port voltage V1 right. So what do you, what do you call an element whose current is proportional to its voltage it is a resistance right in this case it is a conductance. So I have a I can denote this the conductance of value Y 1 1 or a resistance of value 1 by Y 1 1 right. So this is as far as this term is concerned. What about the next term what about Y 1 to V 2 what is this telling us this is telling us that the current I 1 is related to the port V 2 the port voltage V 2 with a proportionally constant Y 1 2.

Now note that the relationship is between current in one port and voltage in some other port right. So this is not a this is not a resistance per se right it is that change of current is not happening because the voltage across its terminals have changed the change of current is happening because voltage in some other part of the circuit has changed some other terminal has changed right. So what type of controls what type of sources do you think or what type of element do you think will represent such a behavior it is a voltage controlled current source right it is a trans conductance. So what is the voltage control current source telling us here so this essentially will be a voltage control current source whose control terminals will be somewhere else that is V2 and the current through it will be Y12 times V2 what will be the direction of the current what will be the direction of the voltage current going up or going down this will be going down because ultimately I1 I1 is the current through this + the current through this right okay. Similarly what is I2? I2 is Y21 V1 + Y22 V2 so exactly like mathematically this is exactly like the port 1 just that whatever we have done in port 1 will have to do in port 2.

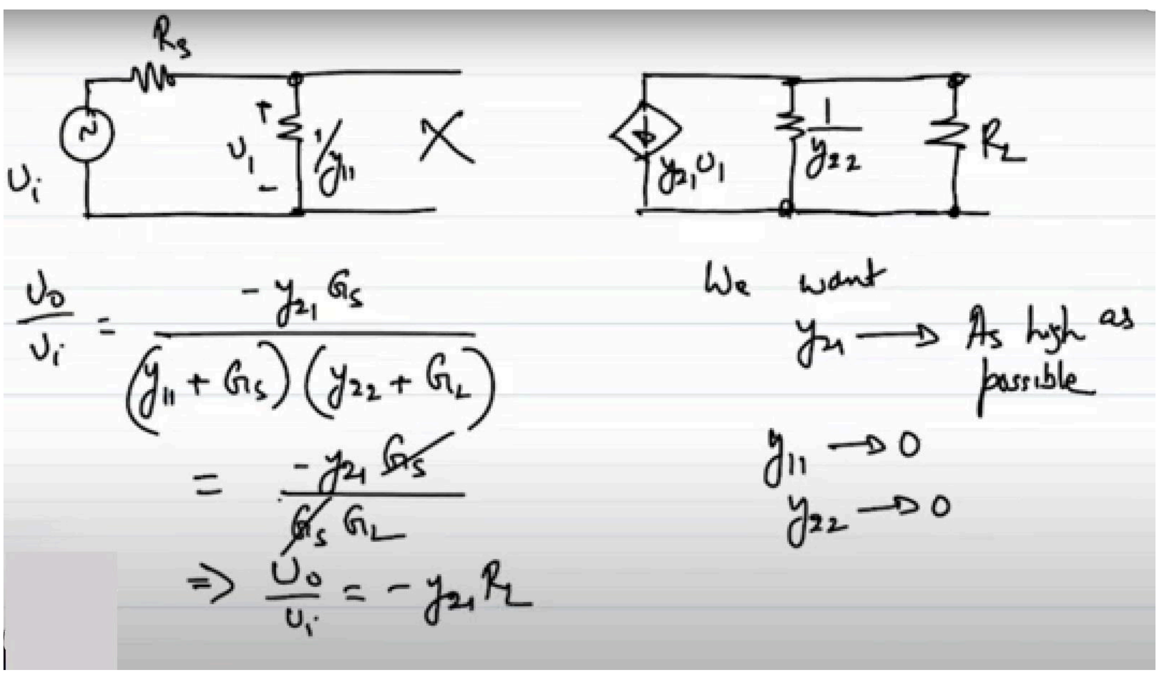
So let me make space for it so this is V2 so you will have 1/ Y22 here and you will have a port or rather you will have a current source control current source which is dependent on the on the voltage of port 1 so this becomes Y21 this becomes your incremental Y parameter okay or rather this is I2 okay and now when you applied the incremental voltage source but what did we do? So we have simply applied Vi between ports through resistance Rs right and we applied we stuck this resistance RL across, okay. So now what is this equation here telling us? What happens if Y12 becomes equal to Y, what happens if the denominator becomes 0? So when denominator becomes 0 you get infinite gain right what does infinite gain mean in real sense? In infinite gain in real sense means the fact that even without any input you can get some finite output right when you have finite gain a very large gain in order to get an output you still require an input a small input can lead to a large output right so that is what a large gain means. Now if the gain is infinite what does it mean you do not even have to give an input you will get an output okay so what does such as I mean do you require such a system? We obviously do not require such a system at least we do not need such a system to be an amplifier because one of the requirements of an amplifier is to amplify if something is presented to it at its input it should not start singing on its own right. So this is not a good characteristics that even without an input you get an output right so we will have to steer clear of we will have to steer clear of any such cases where we will get infinite gain right so infinite gain infinite gain implies borderline instability right. So often you will hear that infinite gain leads to oscillation which is at times true but essentially what one one would like to convey is the fact that when you have infinite gain you can get a finite output even without given an input and that is not a characteristics of an amplifier whatever it is it's not an amplifier since our goal is to get an amplifier we will have to steer clear of infinite gain.

So when you say we have to steer clear of infinite gain then one might say that I will choose these values of Y1 and Y2 to in such a way that the denominator is not actually 0 but very close to 0 right but there is a problem with that and what is the problem? The problem is you would expect your amplifier to work under all conditions right you make an amplifier you are selling it to your customer and you don't know what type of Gs and what type of Gl your customer is going to apply right. You don't know also whether this incremental Y parameters will stay constant or it will change with temperature right because ultimately everything is made out of real devices and the characteristics of the devices might change if you take it to a very cold glacier or you can take it to a very hot desert right. So you don't know whether these Y parameters will remain constant across different humidity conditions different temperature conditions and so on and also you don't know what type of Gs and Gl user might be interested in plugging in so given that you don't know all these things you wouldn't want to take such a risk ultimately you are selling a product in the market you would want it to behave in a proper way regardless of whatever, you I mean regardless of whatever conditions it is being used at right. So what do you do, what is are selling a product in the market you would want it to behave in a proper way regardless of whatever you I mean regardless of whatever conditions it is being used at right. So what do you do what is the obvious thing to do so to prevent instability so to prevent infinite gain ensure Y12 times Y21 tends to 0.

Okay so okay one might say that why should we end Y12 times Y21 to be 0 why not the why not the other terms right. So as you might have already guessed the other terms have these GS and GL associated with them which might not totally be in your control right. So that is one that is one aspect and the other aspect is the fact that if you stare at this circuit what do you think is the what do you think is the desired signal flow direction right. What am I getting at is your signal is applied at port essentially you are applying a signal at port 1 right you are applying a signal at port 1 and you are getting the output at port 2 right. So essentially you are trying to transfer your signal from port 1 to port 2 correct and through which through which part of the system do you think this transference of signal is happening not that this transference is happening through this critical element Y21 right.

Your signal when you are applying a VI when you are applying a VI it is generating some voltage across the port port P1 and your port 2 is getting excited because the control source the control current source at port 2 is sensing that voltage V1 and creating a current which is which is which it is using to excite the output resistance right. So Y21 is a core of the amplifier right Y21 is causing the transference of signal from port 1 to port 2. What is Y12 doing? Y12 is sensing some of the output voltage and transferring it back to port 1 right. So this is Y21's job and this is Y12's job right. And in an amplifier given that you want signal transference from input to output which one do you think is more important? Obviously Y21 will be more important you would want Y21 to be as high as possible because that's what is causing the transference of the signal and what is Y12 doing? It is stealing some of the signal from output and bringing it to the input and in that process also might might lead which which might also lead to instability right.

So essentially what you want you don't want this you don't want this back transmission this reverse propagation of signal from output to input through the through the network which can cause cause potential instability. So essentially you would want Y12 to be 0 right. So we want Y12 to be 0 to prevent instability right ok. What about the rest? So let's go to the next page and redraw this circuit again. So if Y12 is 0 I don't have



anything there so let me not sketch it.

So if this is Y21 V1 this is Y22 and you have a load resistance right. Here I S over Y22 and this is V1 and again what is our V0 over Vi? V0 over Vi is -Y21 Gs divided by Y11 plus Gs times Y22 plus GL and given that Y12 is 0 I can, I can neglect this term. So what is the wish list now? What do I need Y21 to be? You would want Y21 to be , Y21 you would want Y21 to be as high as possible right.

Correct. So that's fine. So what about Y11 and Y22? What do you want them to be? If we stare at the equation what is the goal? The goal is to maximize V0 over Vi so which means I have to minimize denominator which means we would want Y11 should tend to 0 and also Y22 should tend to 0 correct and if that happens what is V0 over Vi? Then V0 over Vi becomes -Y21 Gs by Gs times GL this goes this means V0 over Vi becomes -Y21 times RL right. So this turns out mathematically but we have to also understand whether this makes any physical sense right. So let's stare at the output for some time being so this is my V0 right. So how is the voltage V0 getting generated? The voltage V0 is getting generated when this current Y21 times V1 right when this current is flowing into RL. Now note that this Y21 times V1 is flowing into a parallel combination of 1 over Y22 and RL.

In other words this current is getting divided between two resistors since the current is getting divided between two resistors obviously the voltage V0 is not getting maximized right. The voltage V0 is not getting maximized because RL is not getting the entire current right. Ideally I would want Y21 times V1 to flow completely into RL in order to get the maximum possible output voltage but in the presence of 1 over Y22 right part of this current Y21 times V1 is getting into 1 over Y22 which is preventing all the current to flow into RL. So what is the wish list? What is the obvious thing to do? Obvious thing to say is that hey can we make 1 over Y22 to be infinity or rather 1 over Y22 to be 0 right. I am sorry if we can make this 1 over Y22 to be infinity then obviously all the current will flow into RL which means that we would want Y22 to tend to 0.

So what is the physical significance of Y22 tending to 0? The physical significance of this is no current division at the output port right. Okay so what about Y11 tending to 0? What does it signify? Now note that this main current Y21 times V1 this is the main current Y21 times V1 is dependent on obviously V1 right and if I have to maximize this current what should I do? What should I ensure? I should ensure that V1 is maximized right. Now what is V1? Note that V1 is the voltage across the input port 1 over Y11 and when can V1 be maximized and under what condition can V1 be maximized? The condition under which the V1 can be maximized is when 1 over Y11 is infinity right because ultimately if you have a finite resistance at the input port there will be a finite current and moment you have a finite current you will have a finite drop across RS right and moment you have a finite drop across RS V1 will not be equal to Vi right. Ultimately if the maximum value of V1 is Vi that can only be possible if there is no input current or in other words you have infinite input impedance right. In other words if you have this impedance to be infinity right if this impedance is infinity if there is no loading if your network if your two-port network doesn't incrementally load your input then there will be no voltage drop across no voltage drop of the input when you measure it across V1 right.

So again so this means this Y11 should tend to 0 to prevent or to maximize V1 right. So note that we understood I hope you are convinced both mathematically, algebraically and from the standpoint of circuit theory that why would we would want Y12 to be 0, Y21 to be as high as possible and Y11 to be 0 and Y22 to be 0 right. This is our wish list note that we don't yet know how to make such a network but if somebody asks us what are you looking for we can at least tell that person that hey can you give us a network which has these properties because if you can give it to us we will be able to make a good amplifier right. So the one last thing before we leave is if all the wish lists are met right what is the incremental amplifier what is the incremental model of an amplifier. So if all wish lists are met our amplifier should look like this this is port V1 this is port V2 or Y11 should be 0, Y12 should be 0 so nothing is connected to V1 in V2 I should have one voltage incremental voltage control voltage source Y21 times V1 and that's it this is my ideal incremental model of an amplifier and if I have to stick a input source Vi and this is what and if we stick a resistance RL at the output source at the output node and this becomes V0 what do you think will be V0 over Vi V1 is now equal to Vi this current Y21 times V1 becomes so this becomes Vi this current becomes Y21 times Vi so naturally this voltage becomes minus Y21 times Vi times RL which means your V0 /Vi becomes -Y21 times RL and that is the best that we can do if somebody gives us a network whose incremental equivalent looks like this right okay.