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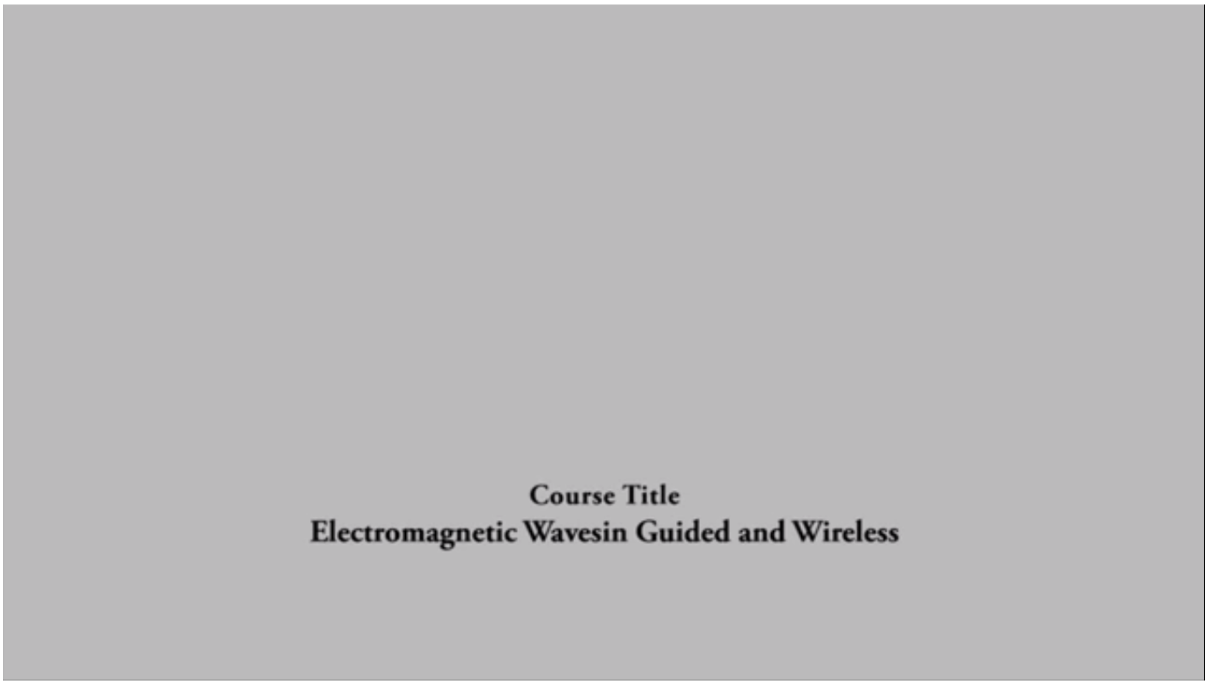

Course Title
Electromagnetic Waves in Guided and Wireless

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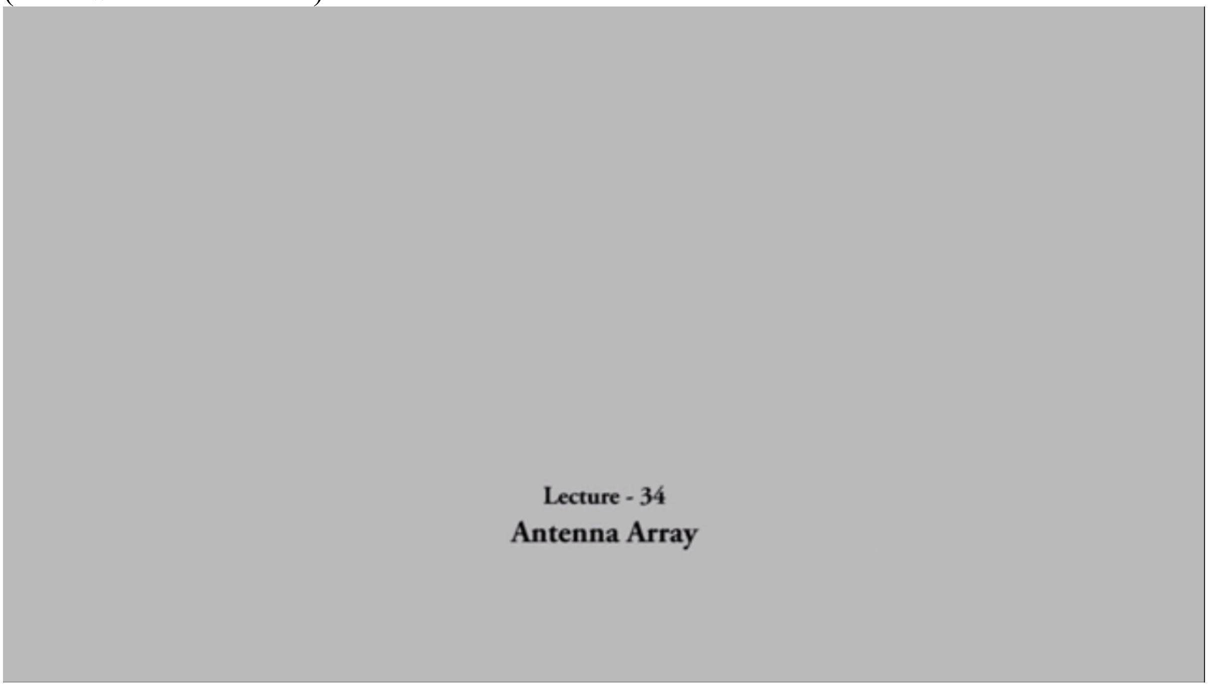

Lecture - 34
Antenna Array

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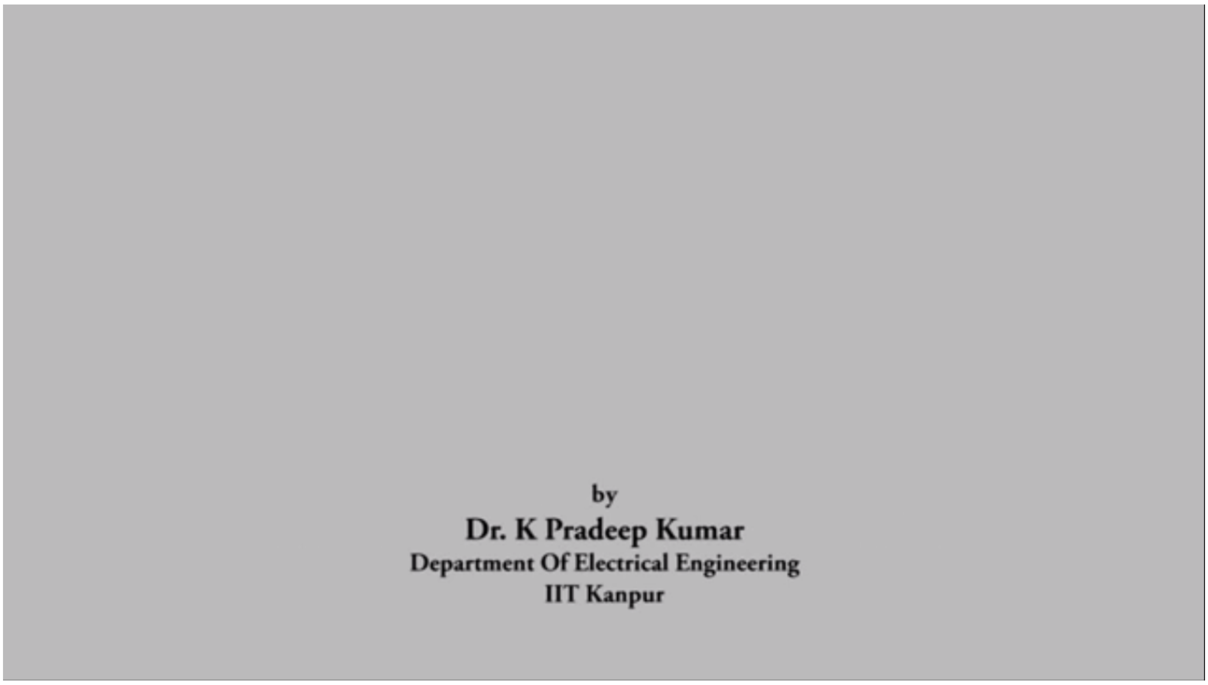

by
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Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media. In this module, we will finish one sub-topic of antennas called antenna arrays. Although we have said antenna arrays, we will confine ourselves to the simplest case of two element array. An extension of two element to n element array is possible. I will tell you the basic idea and then I will leave the final results to you as an exercise. Okay.

Why should one consider arrays of an antenna? Why not just a single antenna be sufficient? It turns out that single antenna with whatever the antenna pattern that you have, there is very little manoeuvring with that antenna pattern, right?

For some reason, suppose say you are stuck with a dipole antenna whose radiation pattern given the length of the antenna that is a linear dipole antenna that I am talking about or linear wire antenna, given the length of the antenna with respect to λ, you know that you can get different antenna patterns.

But if you wish to adjust the directivity of an antenna to maximise the directivity in a particular direction or if there are some side lobes that are occurring and you want to remove or at least trim the side lobes, then these things are not possible when you just have a single antenna because its pattern is fixed.

However, by the principle of superposition and here it is very important, by adjusting the phase of not only the antenna, you know, antenna currents that you can supply like, you know, one antenna can have a current of I, the other antenna can have a current of I but at different angle, by also positioning them at different points in space, at a given observation point, you can actually have the total field adding up or total field being completely destroyed into zero.

So because you can do these two things, then it actually means that you can also tailor the antenna patterns, okay, in such a way that you get maximum energy in one side and kind of trim the side lobes or unwanted energies in the other side. Okay. This antenna arrays is not something new. It is been there for many decades now.

However, it is still a very interesting topic because you can do so many things with this antenna arrays by adjusting the currents and by adjusting the positions. Okay. Moreover, you can also adaptively change these currents and the phases or the positions also in some cases so as to get what is called as adaptive antennas, so whose beam patterns can actually be tuned over time.

That's a very interesting thing, but we unfortunately don't have a lot of, you know, I mean, we don't have any scope in this course to get into those interesting details, but I will show you the basic idea of an antenna by considering the case of two identical antennas, identical meaning that if you supply the same amount of current, their pattern would essentially be the same. Okay. Or if you normalise them, the powers with that one, they both have the same power pattern or the field pattern. Okay. And we place these antennas at a certain distance apart.

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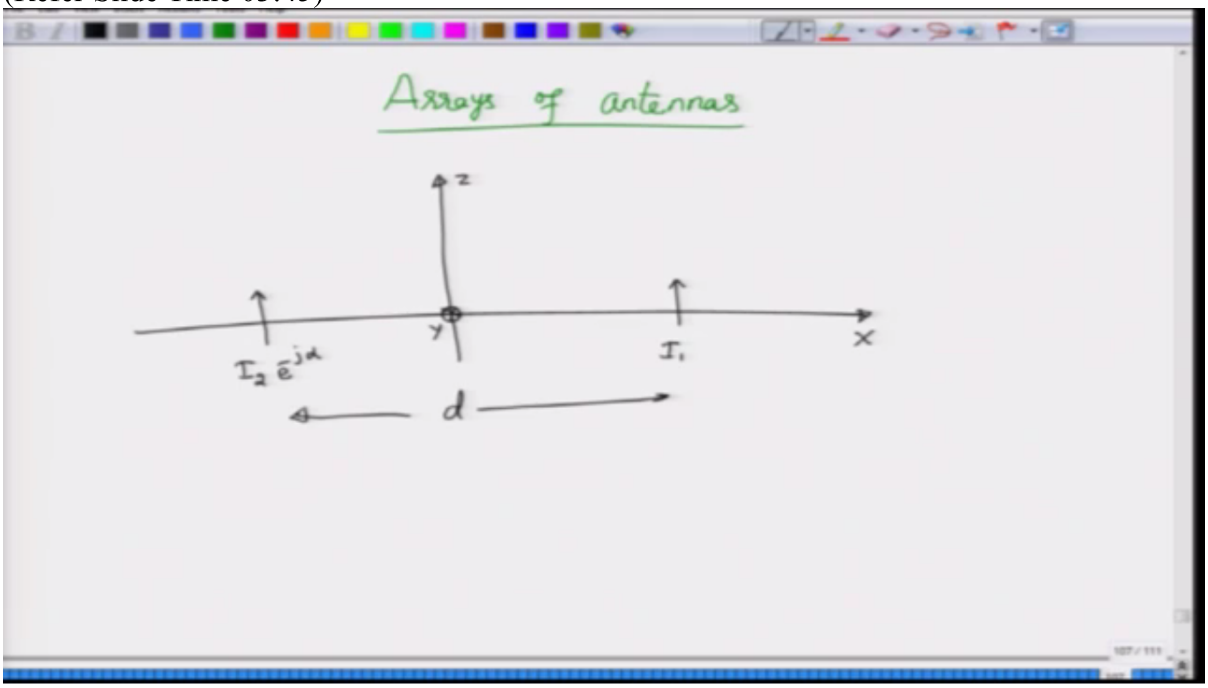

So let us call that distance apart that we have placed as d and we locate the coordinate system at the centre and we have oriented two antennas. One antenna is fed with the current of I_1. The other antenna is fed with the current of $I_2E^{-j\alpha}$. Okay. There are circuits which allow you to do this. So you can, in fact, take the transmission line, bifurcate the transmission line and then feed it, and if you choose different lengths of the transmission line, you are obviously going to change the phase shift of the current that could reach the antenna. So you are controlling α there and also by adjusting some other conditions, you can actually change the amount of current that goes into one antenna to the other antenna.

Anyway, the antennas are oriented along the z-axis. Please note that the page I am showing you is the x and z plane. So y would, of course, be going into the page as this is a three-dimensional picture. Okay.

Now let’s say my observation point is here P, okay, and because this is antenna problem, we are forced to deal with spherical coordinates. So I am positioning this antenna at point P. I do know, of course, that the angle θ is measured with respect to z and φ is, of course, measured with respect to the angle, sorry, with respect to the x-axis, okay, in the spherical coordinate system.

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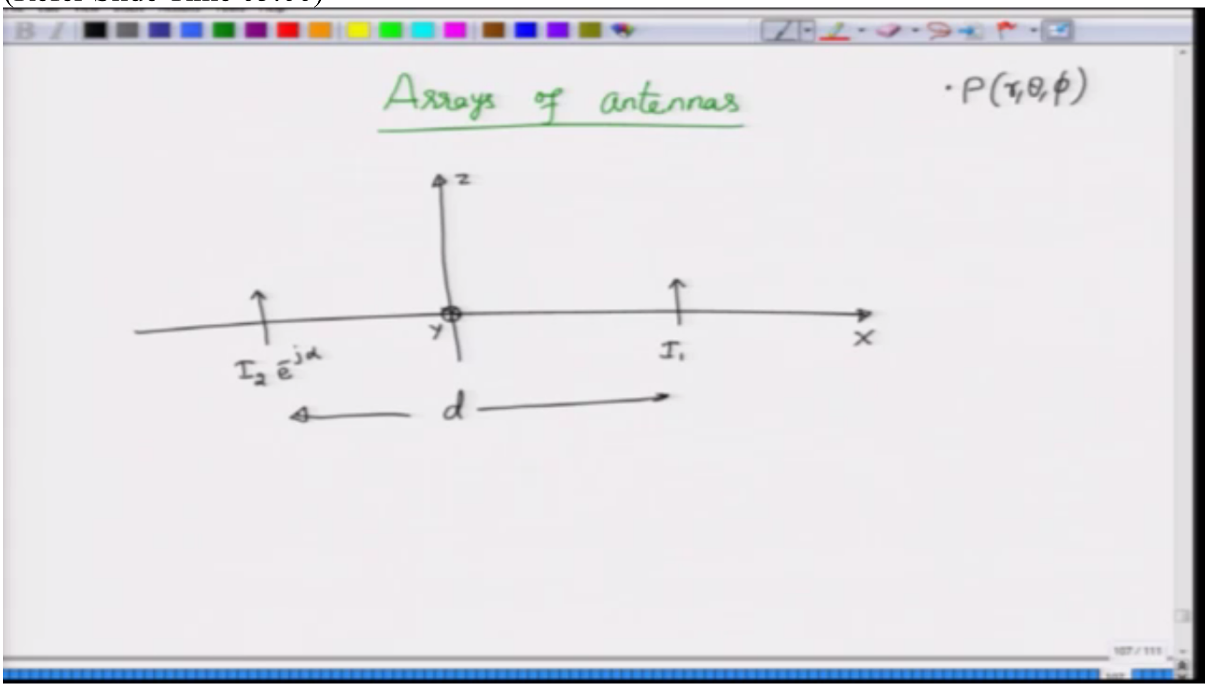

Now the condition that I want to ask is that this observation distance P from the origin be very, very large compared to the, okay, let’s move the observation point because I could not draw the line correctly. So this is the observation point. Okay.

So the only condition that I want to ask from our analysis is that this distance r that we have, okay, between the observation point P and the origin be much, much larger than either the linear dimensions of the antenna. These are linear because I have kept two, you know, linear antennas.

In general, it would be the two apertures of the antenna or two physical dimensions of the antenna as well as the distance at which I have kept them. Okay. So r is really in the far field of this antenna. So no near field business we have. It’s all in the far field and this is the usual application of an antenna array. Okay.

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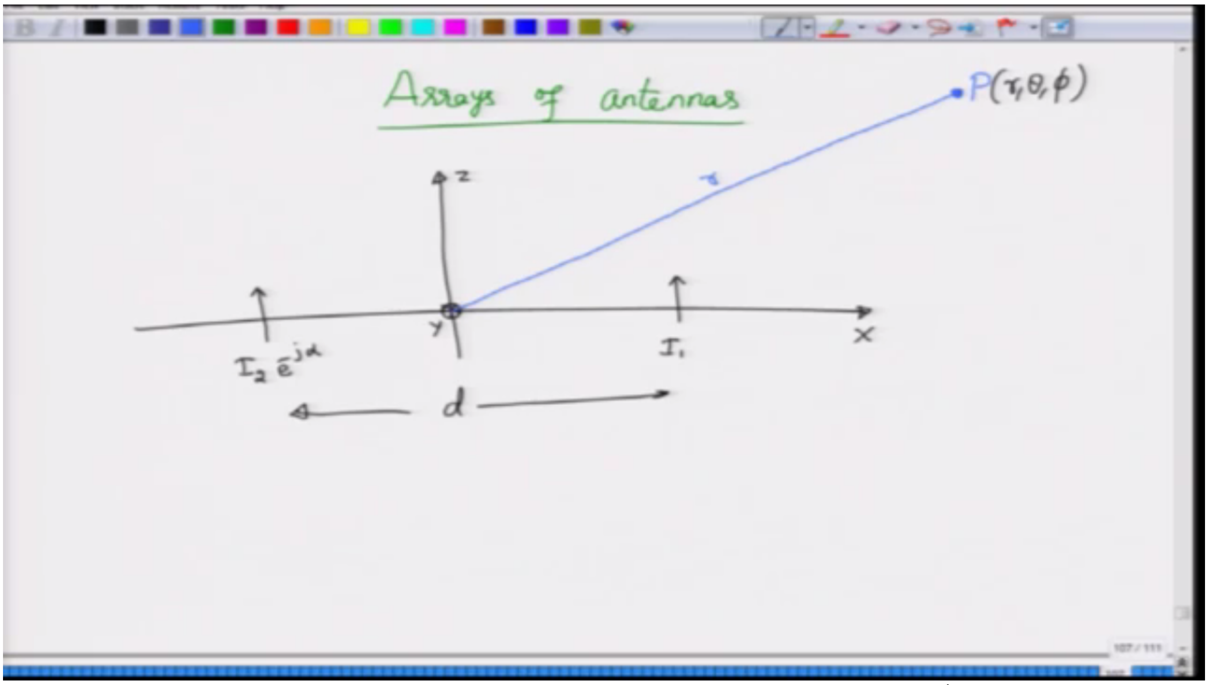

So I have kept this in the far field. I have fed this with current I_1 and $I_2E^{-j\alpha}$. Let us denote this angle as zeta (ζ). Okay. This is an angle ζ as measured from the x-axis. Okay. Please you have to imagine that you have a sphere here, okay, and then I have this z-axis and then I have this x-axis. My position point can lie anywhere on the sphere. Okay. On the sphere, I am writing this as (r, θ, φ). The only angle that I can properly measure or rather I can give in this diagram is the angle θ.

The angle of, you know, the other angle, the angle which the y-axis with respect to P makes the r, you know, the arc origin to the P makes with respect to the x-axis is denoted by ζ and please understand that the ζ is not φ because I can move this position point P. I am only giving you the angle with respect to x.

And in fact, ζ as you can see from the spherical coordinate systems can be written as sin θ cos φ, okay, because you imagine the sphere and you have this z, x, and y-axis up there. So the position point P is here with, which makes an angle θ with respect to the z-axis.

And then when you drop this perpendicular and then look at this angle φ, the perpendicular fellow will actually be given by. So this will be r cos θ or whatever. If it is just a unity gain, I mean, unity circle, then this would be cos θ along z and the projected part will actually be sin θ and this sin θ further has to be projected onto the x-axis to get sin θ cos φ.

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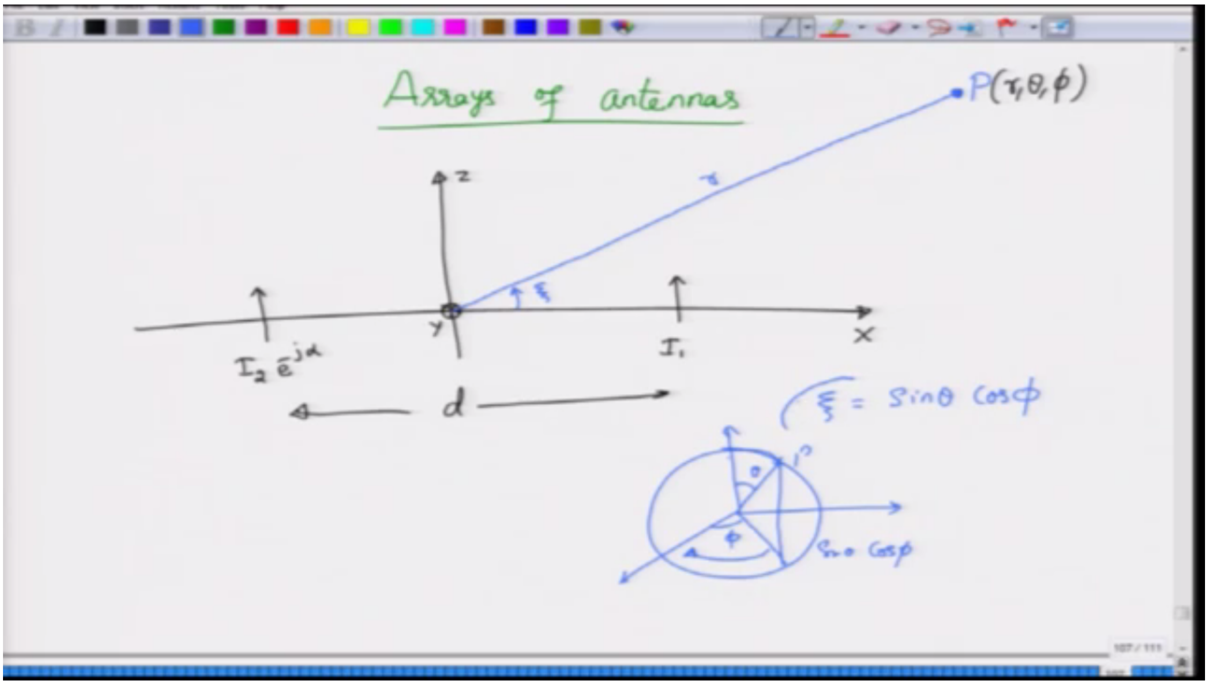

That is the overall ζ that, I am sorry, this is basically cos ζ, not ζ itself. The projection part is what I'm looking at. So cos ζ is sin θ cos φ. Okay. So please keep this in mind when we analyse the antennas later on. Okay.

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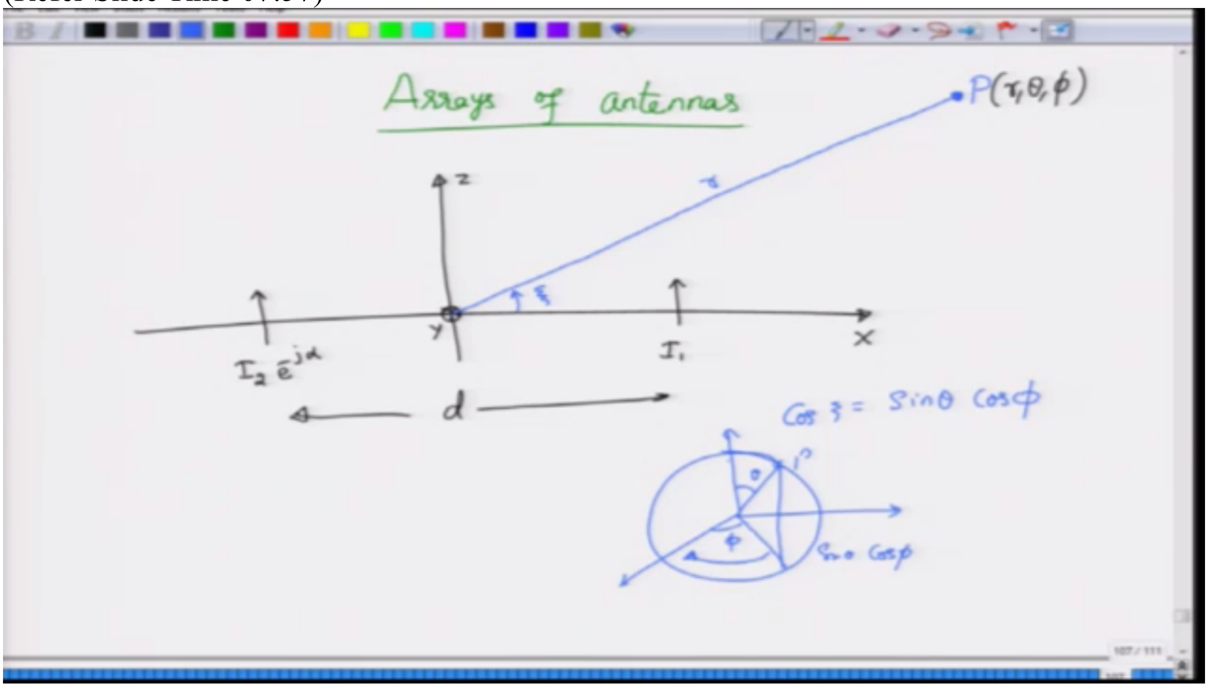

All right. So let's get back to the ideas here. So I have this antenna up at this point. Okay. Now let us suppose that I want to find out the total electric field at point P, which is, of course, my idea, right? I know that there will be a spherical wave that would be launched, okay, which will travel along this particular direction and reach the point P. Okay.

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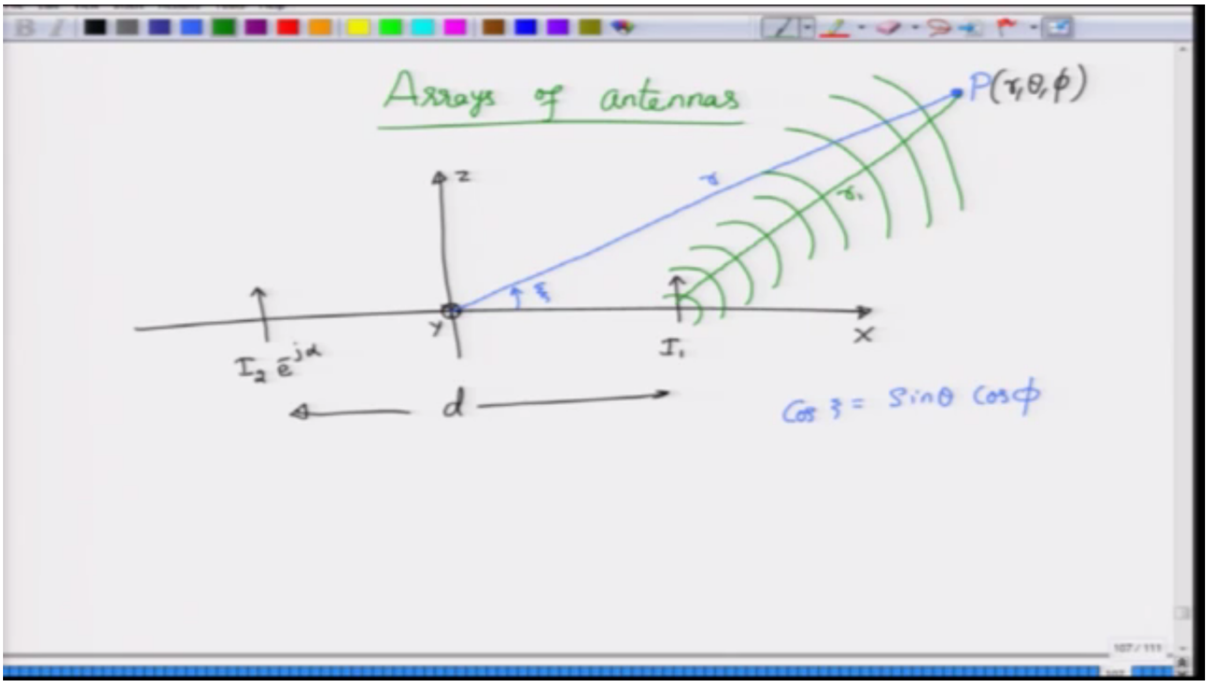

So this is the, you know, the length, sorry, this, this r_1 is the length that is or the distance between the antenna one and the observation point P and this spherical wave would reach.

And what is the electric field that would reach there? Electric field because of only the antenna at I_1 will be I_1 some antenna pattern, which we will call as $F_1(\theta, \varphi)$. Okay. That is the field pattern of the antenna, the first antenna, and then you have e^{-jkr1}/r_1. Okay. That is the spherical wave amplitude that you are going to get. So far so good. Okay.

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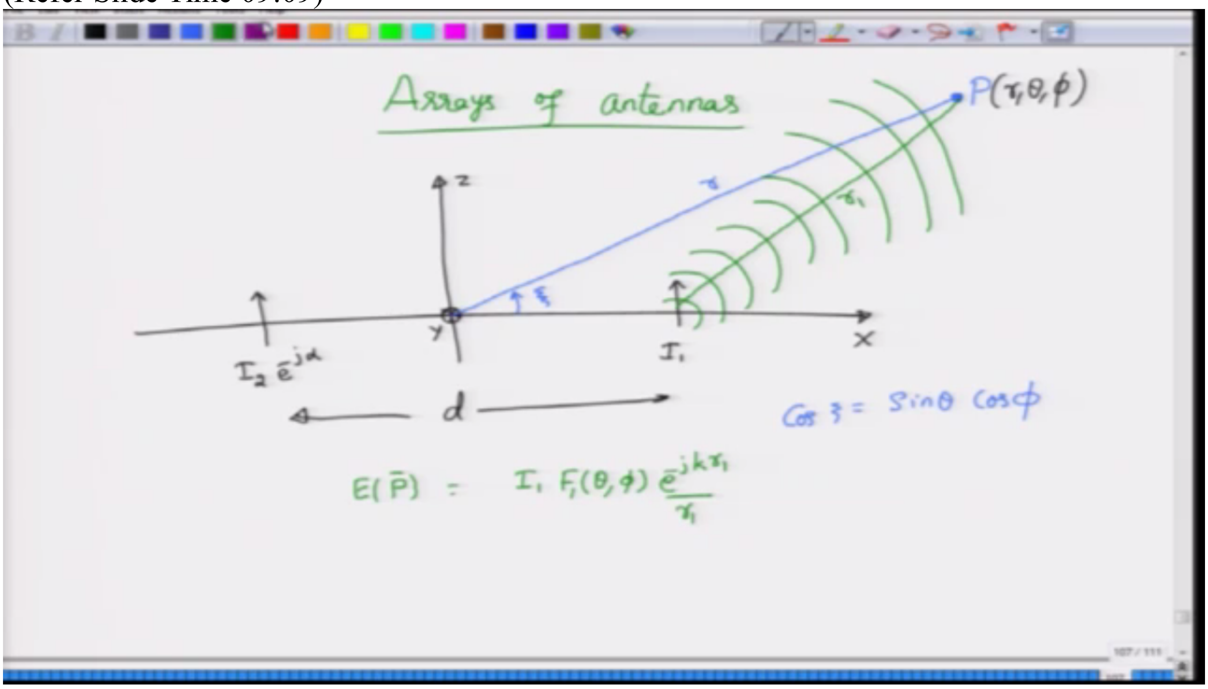

Now let's imagine that the second antenna is also, which is also being fed, will have to also contribute to the field. Sorry, my orientation was not correct, but let's say this spherical wave would also travel. Okay. And that spherical wave would also travel and reach the point P and the distance here is r_2; the current here is $I_2e^{-j\alpha}$. Therefore, the total electric field that I will get is $e^{-j\alpha}I_2F2(\theta, \varphi)$, okay, e^{-jkr2}/r_2.

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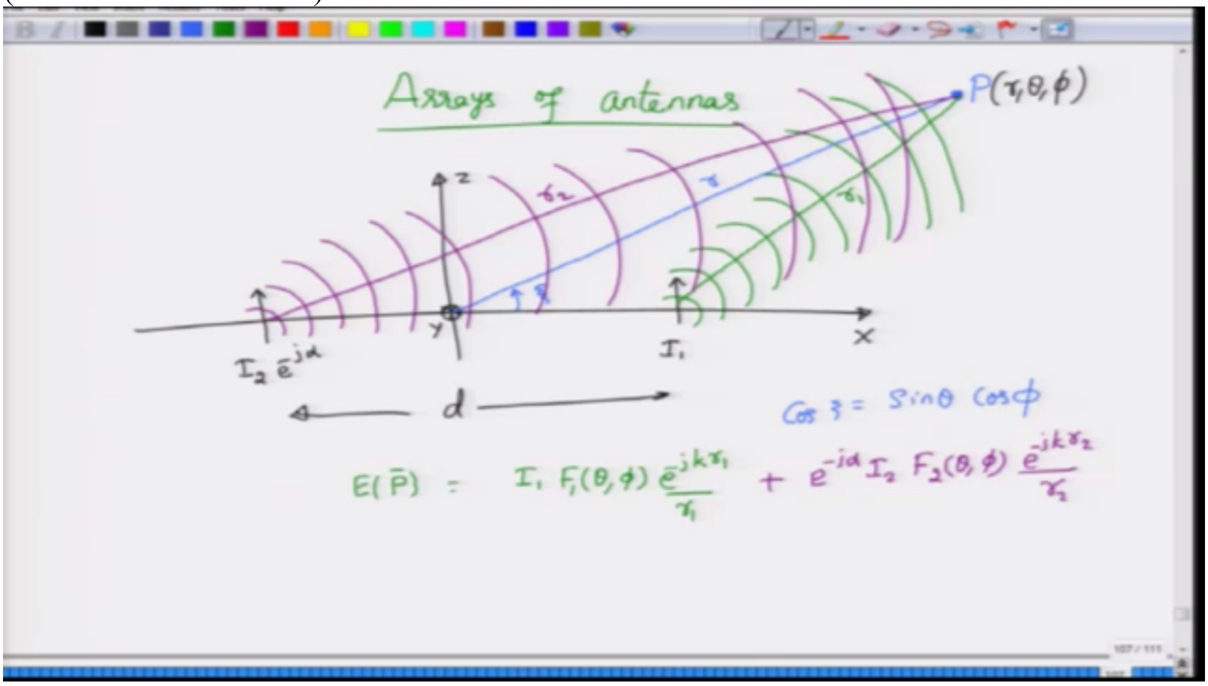

Although we have said that P is in the far field, it is obviously clear that the waves from I_1 that is antenna one reach the observation point P much faster than the waves that would reach or in terms of time they would reach earlier than the waves that come from the, you know, antenna two. Okay. So this is the total electric field that you expect.

What is $F_1(\theta, \varphi)$ and $F_2(\theta, \varphi)$? These are the antenna patterns. So they could be technically any different type of antenna. For example, this could be a loop antenna whereas this could be a short dipole antenna, but what we will assume is that $F(\theta, \varphi)$ for both antennas would essentially be the same. Therefore, I can pull that outside of this summation and then what I will have is $I_1e^{-jkr1}/r_1 + I_2e^{-j\alpha}\, e^{-jkr2}/r_2$. Okay. So this would be the total electric field that you are going to observe at the point P in the antenna.

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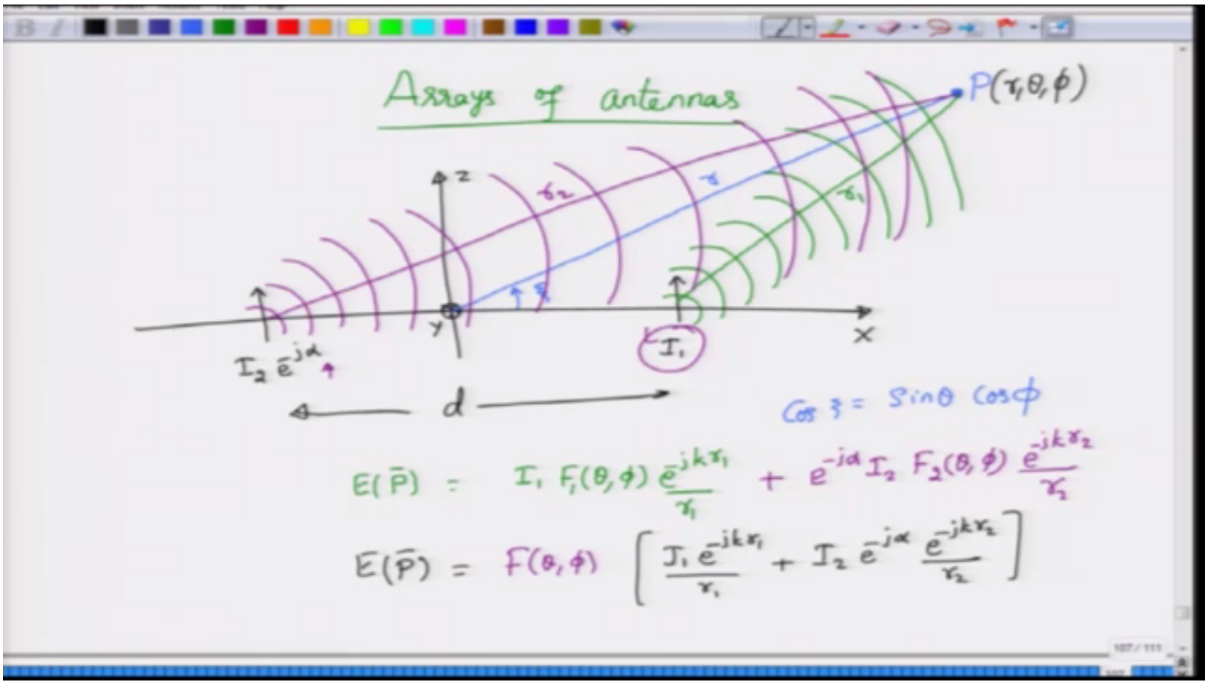

Now when you see look at, look at these spherical waves. Although the drawing is pretty bad, right, you see these spherical waves that I have drawn, they meet the other spherical waves that are coming from the second antenna, right? So the green curves are meeting the magenta curves. Okay.

You can easily imagine a situation where the green curves and the magenta curves kind of overlap or you know they are in parallel with respect to each other. When that happens, we know that these wave fronts are actually coinciding with each other or they are parallel to each other meaning they are constructively interfering. Okay.

There could be a situation where they are not coinciding, but they are 180° apart. In that case, they are destructively interfering. Okay. So if you actually take a compass and a pencil and do this carefully, you can pretty much obtain the antenna patterns with lot of work, of course, by using this geometric sketch. Okay. And you can do it for any type of antenna array. This is something that one can mathematically analyse, but if you place randomly some other antennas, then instead of working out in this particular manner the expressions, we can perhaps just pick up pencil and paper and then, you know, compass, and rulers and other things and then work out what would be the signal level at different points as you plot, as you vary the observation point.

Okay. Anyway, so our problem is not so complicated. Our problem is quite simple. I have this total electric field. If you want to obtain the power, of course, you have to take the magnitude square of it. We will do that magnitude square later on. Okay.

There are two things that we need to understand. We have already made these assumptions earlier. So you probably will understand it better. Here I have r_1 and r_2, and I have e^{-jkr1}, e^{-jkr2}. I am not really interested in r_1 and r_2. Rather I am interested in varying this r, you know, and then seeing how the field would vary. Okay.

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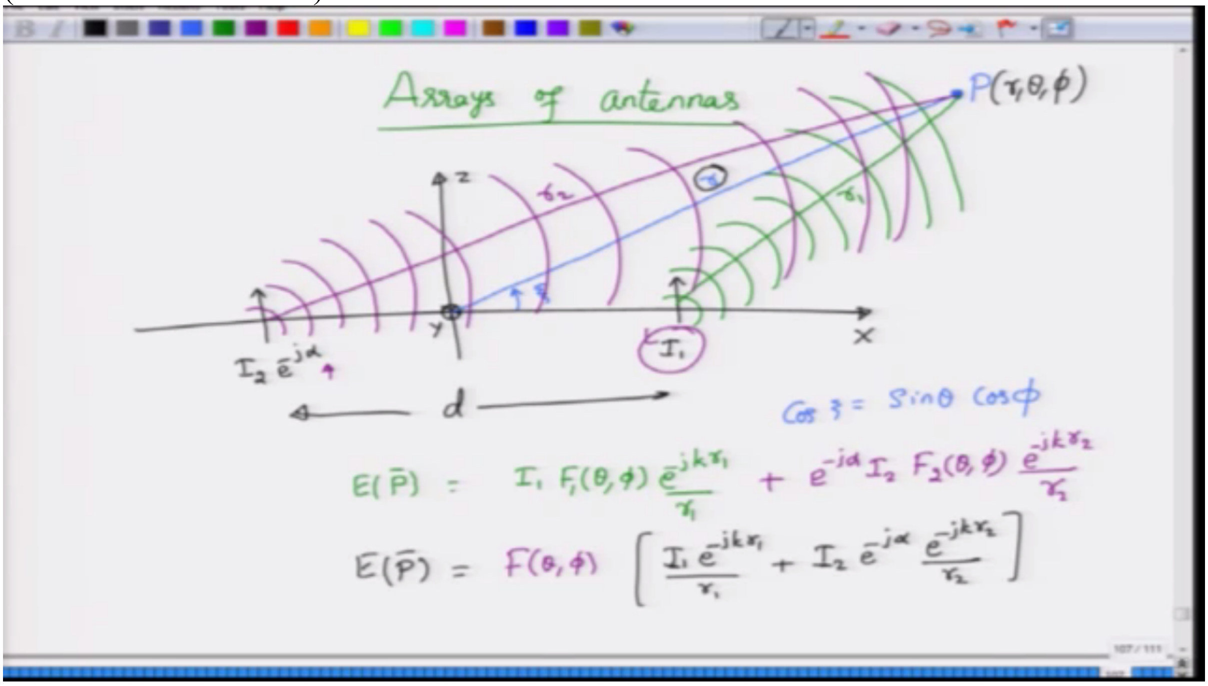

So I have to find the relationship between r, r_1 and r_2 and that relationship comes rather easily because if I draw a line here and here, I see that this is the extra part of the length that the wave would have travelled had it originated from the origin.

Since it did not originate from the origin and this angle from the x-axis to the blue line is ζ, the extra distance can be easily given as ζ or rather cos ζ is basically this adjacent, this one to the extra length, right? So the extra length can be written as, I'm sorry. Hold on.

So this is the hypotenuse. I am sorry. So the adjacent length is this fellow so that adjacent side or the extra length, which we will call as delta, right, divided by the hypotenuse, which is basically d/2 here, that extra length is, of course, given by d/2 cos ζ. In terms of magnitude, this extra length that you get here will also be the same. That would be (d/2) cos ζ except that r_2 is larger than r and r is larger than r_1, right?

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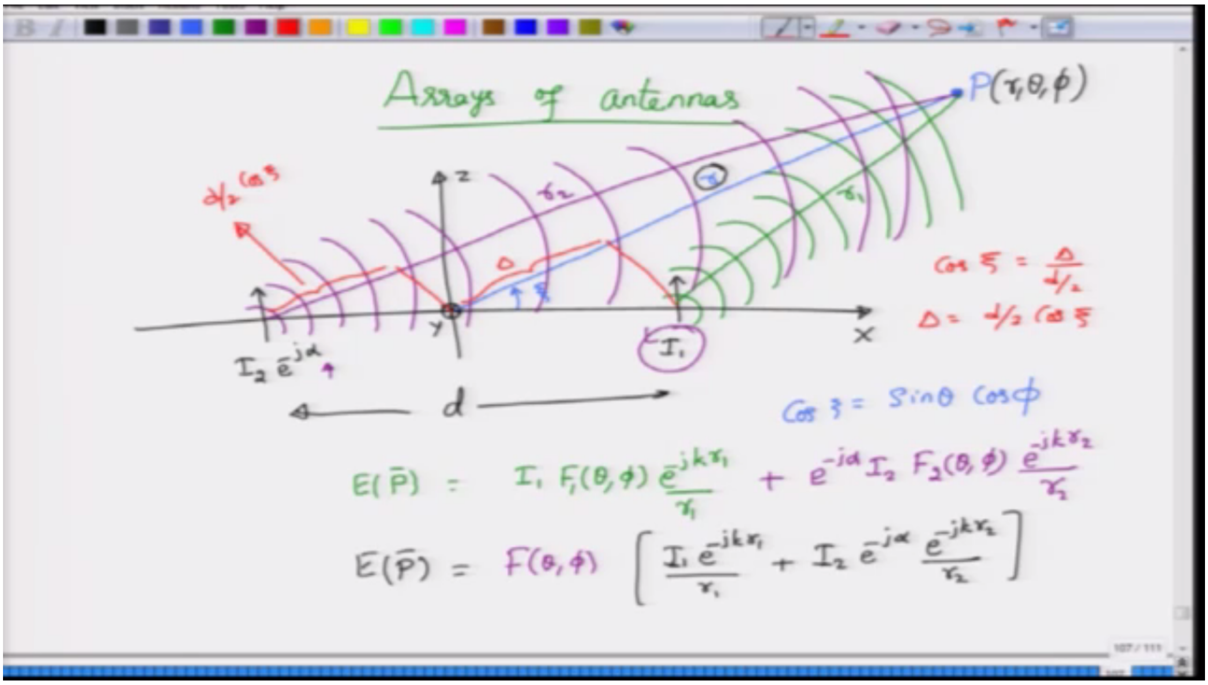

So I can in these expressions write r_1 as r - (d/2) cos ζ and r_2 as r + (d/2) cos ζ, and then you can go back and substitute into the expression there and then write down. So the antenna pattern is F(θ, φ) and then you have e^{-jk}, r_1 is basically r - (d/2) cos ζ, I_1 current here, $I_2 e^{-j\alpha} e^{-jkr+(d/2)\cos\zeta}$, okay, divided by r - (d/2) cos ζ divided by r + (d/2) cos ζ.

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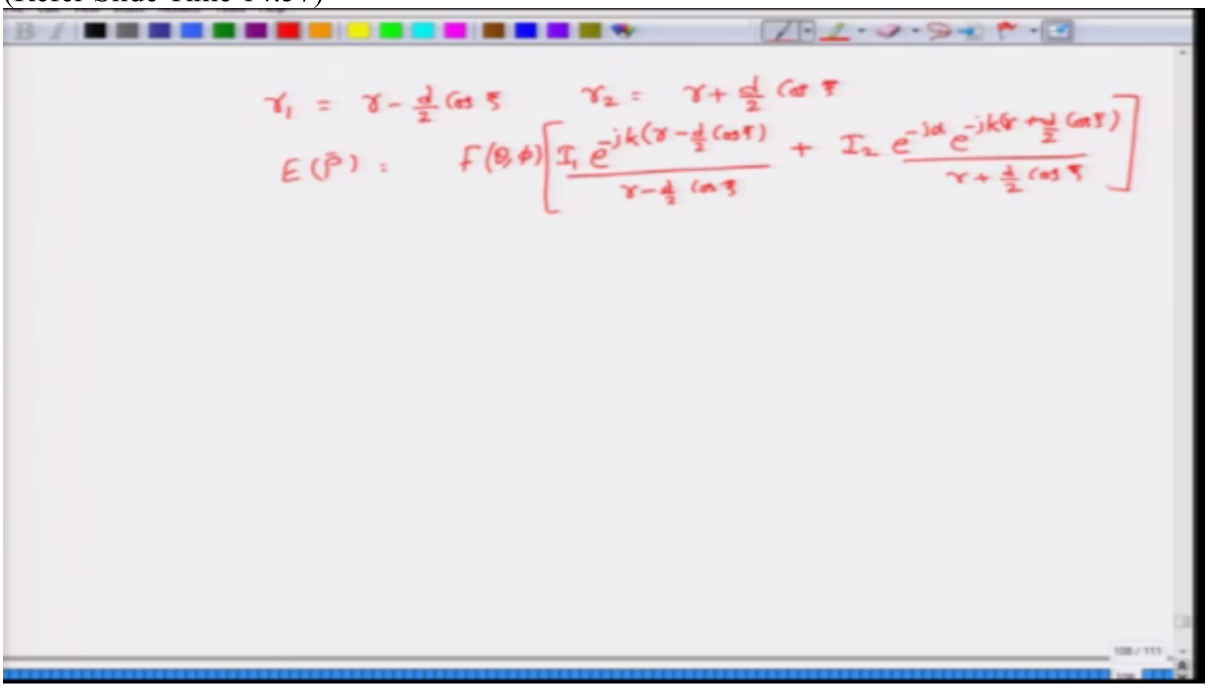

So this is the expression, right?

Now in the denominator if you look at it, you have terms like r - (d/2) cos ζ and r + (d/2) cos ζ, right? The maximum value that you can have there for the denominators would be r - (d/2) or r + (d/2). Okay. And we have assumed that r is very large compared to d.

So let's say r is about 30 km, d is about 1 m. Okay. These are not exact numbers. I am just giving you the order of magnitude. So for complete, I mean, considering 30 m, 1 m would hardly make a difference in the denominator, right? The amplitude would be almost the same, right?

So I can in the approximation neglect this $(d/2) \cos \zeta$ in the denominator and then have only r written there, and there is an e^{-jkr} in both terms, which can be thought of as common factor and pulled it outside of the integral, so, I mean, summation. So I have e^{-jkr}/r, which is a spherical pattern that you expect and $F(\theta, \varphi)$, which is the antenna pattern, here I have assumed them to be the same antenna pattern. Therefore, I am writing this separately times $I_1 e^{-jk(d/2)\cos \zeta} + I_2 e^{-jk(d/2)\cos \zeta} + \alpha$. I have included this α into the second exponent itself. Okay.

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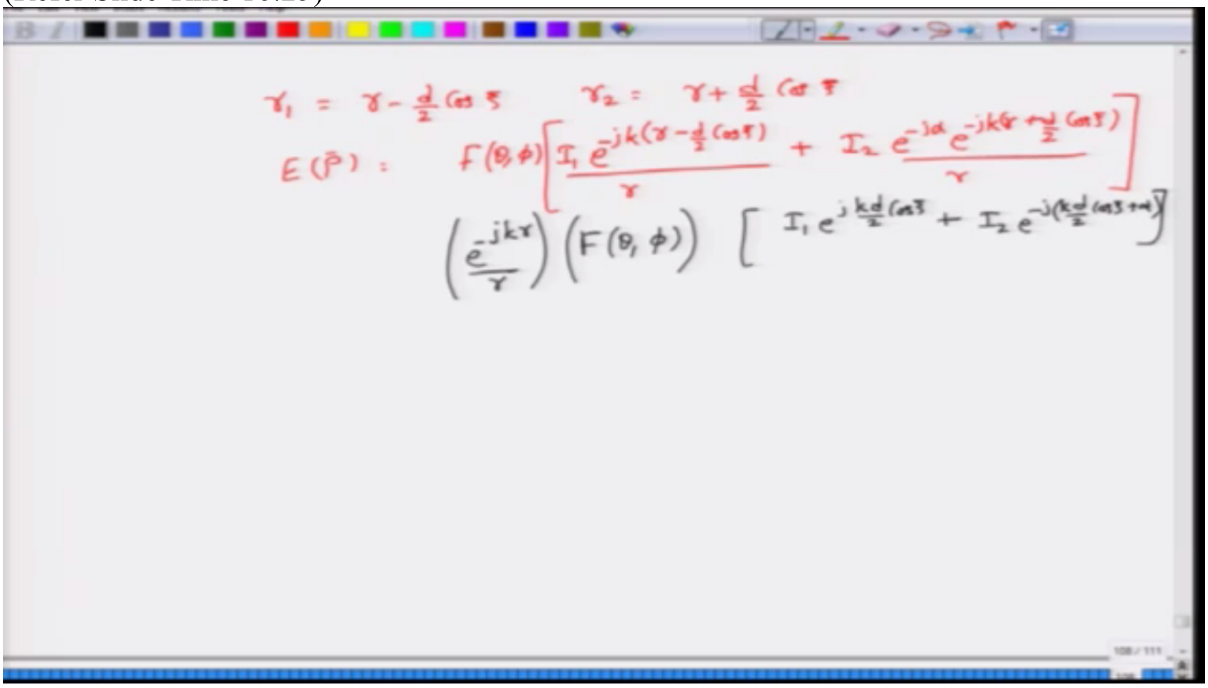

Now, of course, we have told you why I cannot make that $(d/2) \cos \zeta$ in the numerator to be equal to 0. That is I can't neglect $(d/2) \cos \zeta$ in the numerator or in the exponential function simply because that $(d/2) \cos \zeta$ will be in the order of the wavelength. Okay. The antennas are let's say $\lambda/2$ antenna. So this is $\lambda/2$. This is $\lambda/2$. And if you place them in, you know, distance of λ or let's say 2λ, their lengths and the distance are comparable to each other. Okay. So because of that, I am not making this approximation in the exponential function.

So I just have $I_1 e^{-jk(d/2)\cos \zeta} + I_2 e^{-jk(d/2)\cos \zeta} + \alpha$. And I will leave the further simplification by moving $e^{j\alpha}$ on to the other side for you to work out and also assume that I_1 and I_2 are equal. They are equal to I_0. Then you can rewrite the question as $I_0 e^{-jkr}/r\ F(\theta, \varphi)$, there will be some additional phase that is going to come up here, which we will call as some δ. So you can neglect that additional phase because it's an overall global phase and what you get here will be cos kd/2, sorry, $(kd \cos \zeta + \alpha)$ whole thing divided by 2, okay, because you would have taken $e^{-j\alpha}/2$ outside and this is what you are going to get. Okay.

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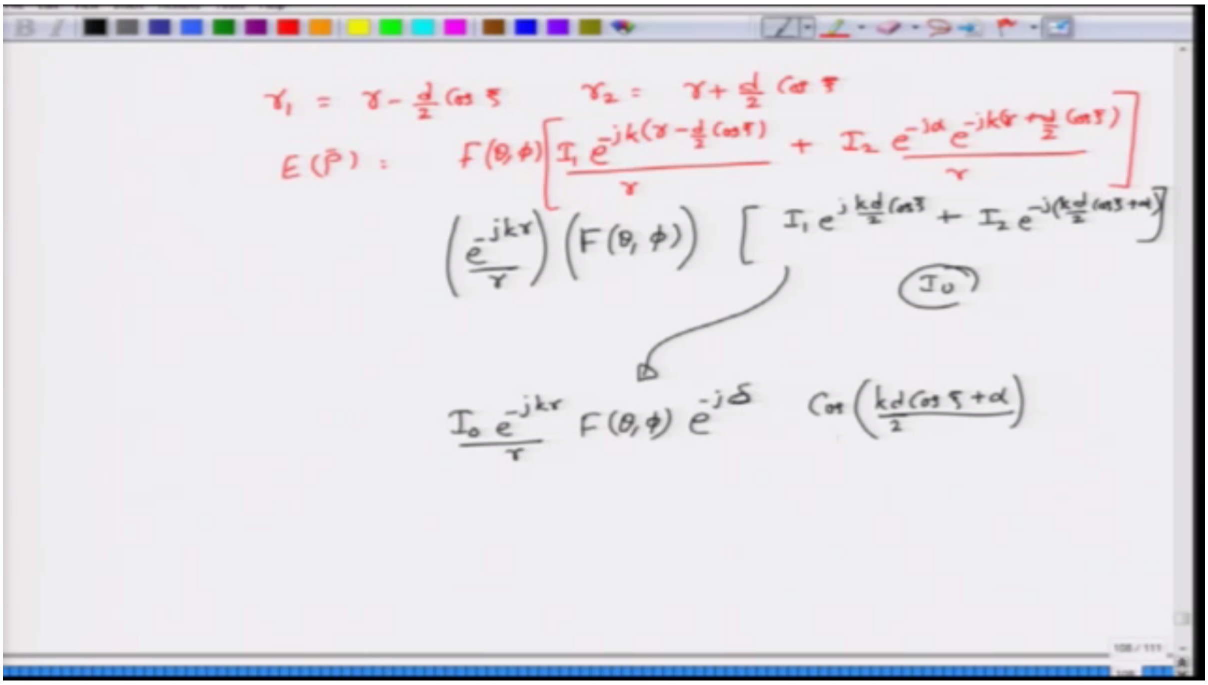

The power, of course, will be proportional to the magnitude of this fellow, which would be F(θ, φ)/r2, this magnitude. Of course, there is I_0^2, which I have assumed to be real. This $e^{-j\delta}$ will cancel out and then you have |cos (kd/2) cos ζ + α|, okay, or magnitude square of it.

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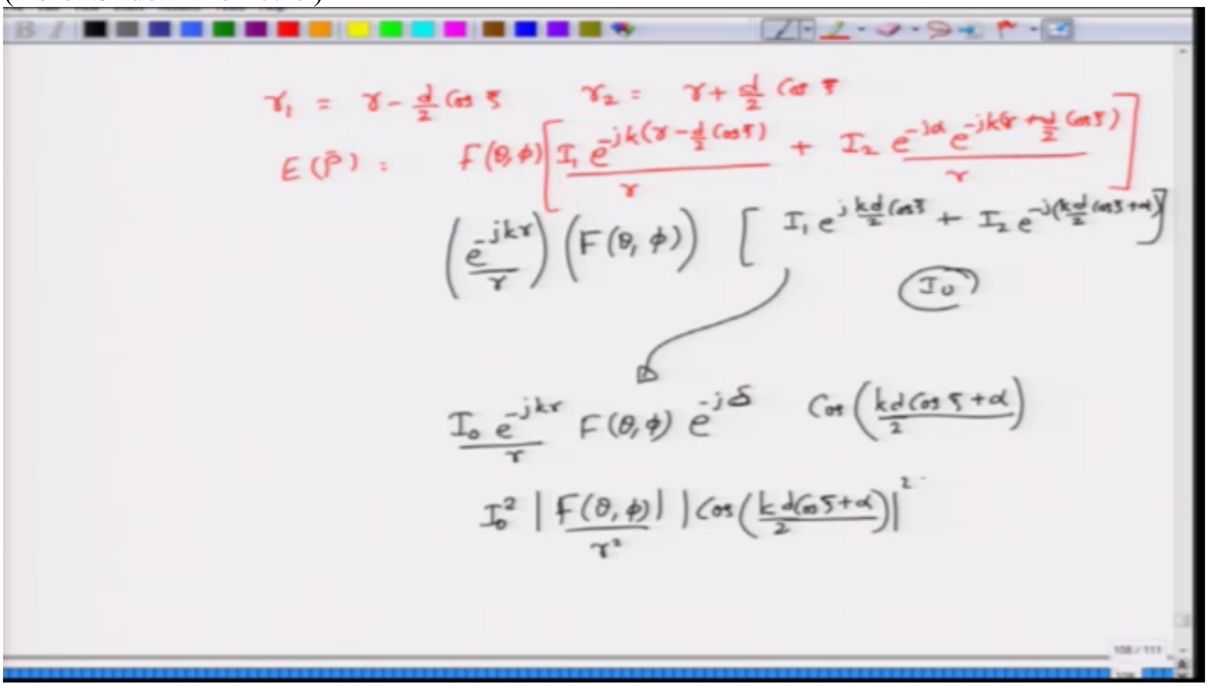

Now this kind of a thing if you fix r to be a constant and I_0 is usually a constant, the power pattern is proportional to the magnitude of the field pattern or magnitude of the filed patter square times this additional cosine, you know, term. This term can be called as array factor. Okay. That is the factor that needs to be multiplied to the pattern in order to obtain the overall antenna pattern. Okay.

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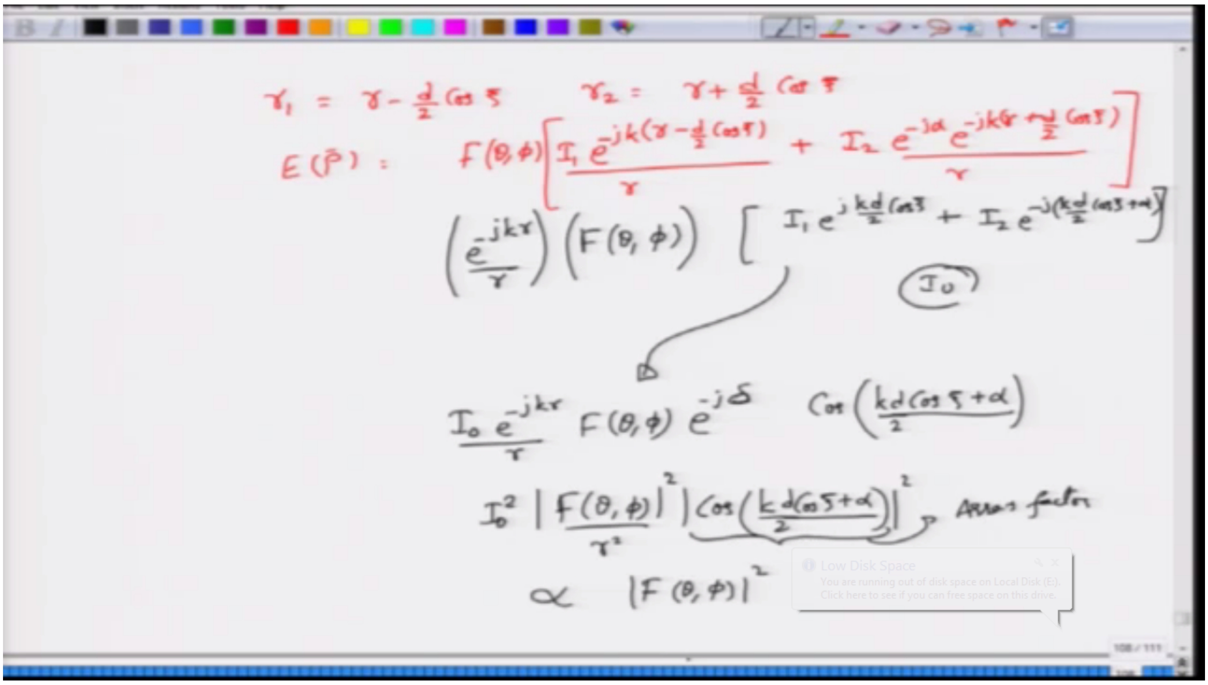

But if you are interested only in the field amplitude pattern or the field magnitude pattern, then can drop the squares and just take the magnitude of it. Okay.

So what we have found is that the antennas may have identical pattern, but the overall field that you get at the observation point will not simply be the sum of these field patterns, right? There will also be of the fact that there is an, you know, array factor that is getting multiplied and because it is a cosine function, when the argument is such that it is π/2 or its multiple, then, or multiple, that that cosine function can go to 0 and you can see that you may get destructive interference at the observation point. Okay.

So that is the main point of this antenna array. So you can trim the edges or trim the lobes by successfully placing nulls in those directions, and you can maximise energy in a different direction by somehow placing the maxima in those directions and you can do that by controlling the ζ, by controlling, sorry, by controlling the power pattern or rather the field pattern of the antenna plus the current plus the phase shift between the two antennas.

Let us take the simple case. Assume that we are looking at $\alpha = 0$ and then we place d to be equal to $\lambda/2$. Okay. d of $\lambda/2$ means that the distance between the two antennas is $\lambda/2$ and what will happen to the array factor? This array factor you can see will be cos $\pi/2$ cos φ sin θ. Okay.

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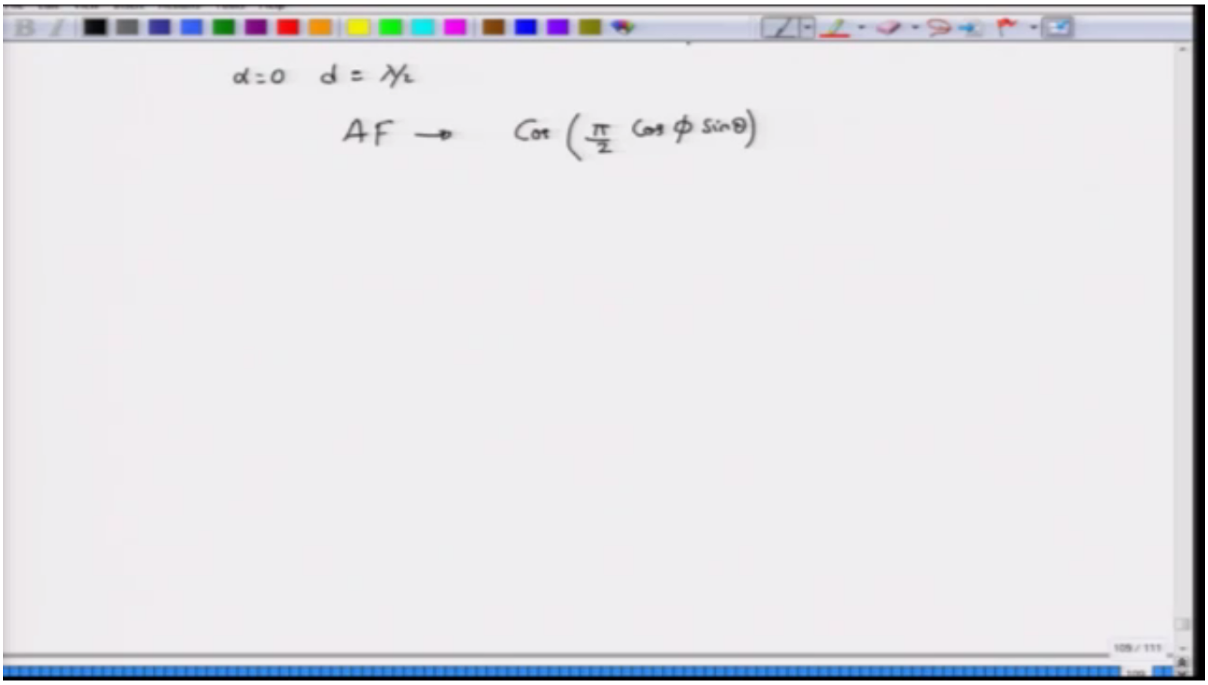

That is something that you can easily verify, and if you plot this one, right, and call this, you know, plotting this at, well, okay, where should I plot this? Suppose I consider the sphere and consider the cutaway in this plane, okay, I am basically plotting the pattern in this plane, which corresponds to a θ of 90°, right? So θ of 90°.

In that θ of 90°, if you assume that the field pattern is maximum or the field pattern is normalised to its unity, that is what happens for a short dipole antenna or for a linear antenna, then in that plane, in the horizontal plane, as you move along the horizontal thing, you know, like you are moving along in this way, right, as you move along the φ direction, what you will see is that with θ = 90°, the sin 90° will be 1 and you get cos π/2 cos φ.

So at φ = 0, cos φ will be equal to 1. Cos π/2 will be 0. These are the antenna locations. So let's put down the antenna locations. Now you are in the xy plane. So this is x and y because that's where I am plotting this, right?

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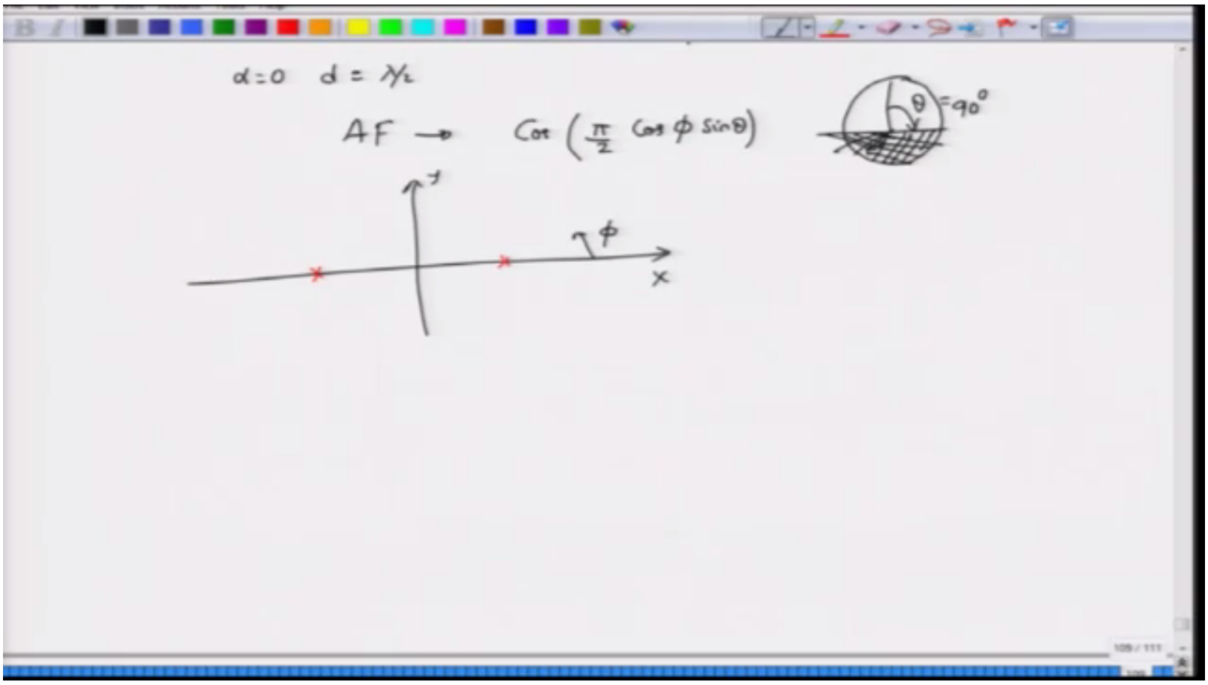

So what you will actually see is that along this x-axis, whether it is on this direction or on this direction, the power will actually be minimum or the array factor will be minimum, and what you get is an array factor that looks like this, which is actually maximum along the y direction.

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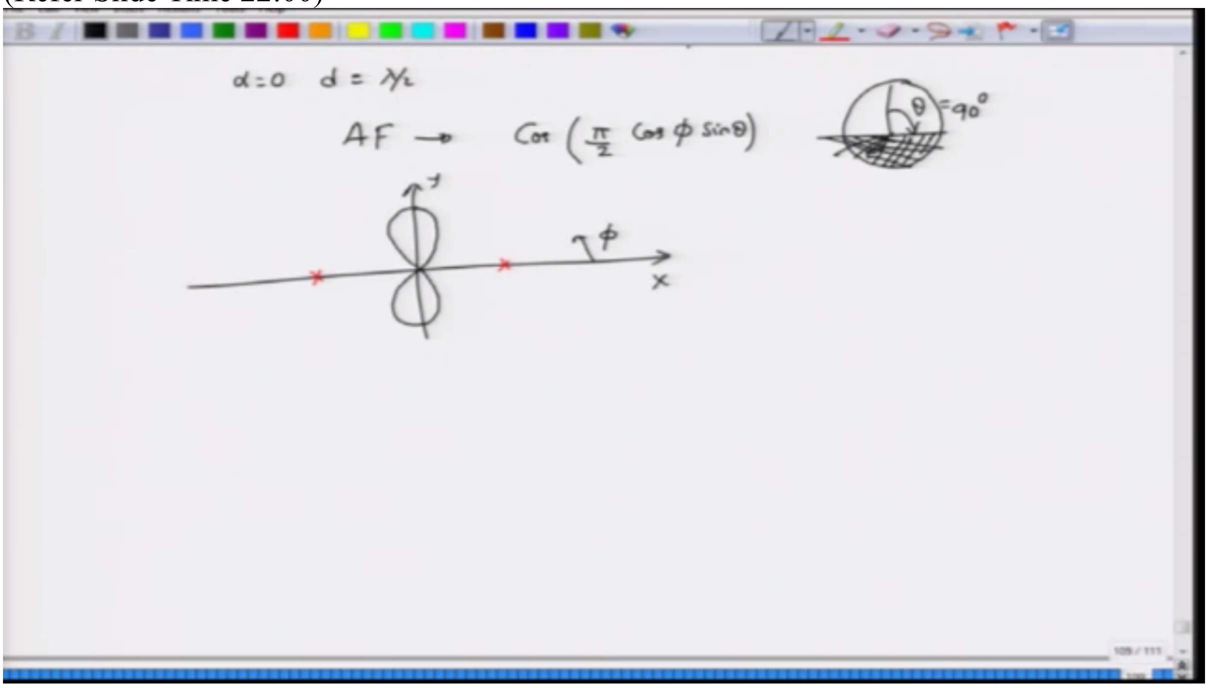

Does it makes sense physically? Yes, it does makes sense physically, like just take two antennas here. These two antennas are at the same, you know, are at the same phase. The currents are in the same phase and the length between these two is d = λ/2.

Now if you keep this, you know, on the z-axis, on the axis, if you now or rather on the y-axis in case, if you look at the field radiated, that field radiated will have travelled the same distance as the field that has travelled here so that there is a constructive interference along this axis or along this axis, which is what we have captured on the graph here. Okay.

On the other hand, if you are on the x-axis, the antennas are, you know, along my thumb, which is along the z-axis, but now you connect it in this way, right? Now if you are looking for the antenna array factor here, you know that the radiation from this antenna onto this one will reach earlier than this fellow, and what is the phase shift that you are going to get because of this path length difference of $\lambda/2$? $\lambda/2$ will give you a phase shift of 180°. So along this direction as well as along this direction, the overall field will be equal to 0, right? So that is what is captured here. And this is called as an end fire array. Sorry, this is called as a broadside array. Okay.

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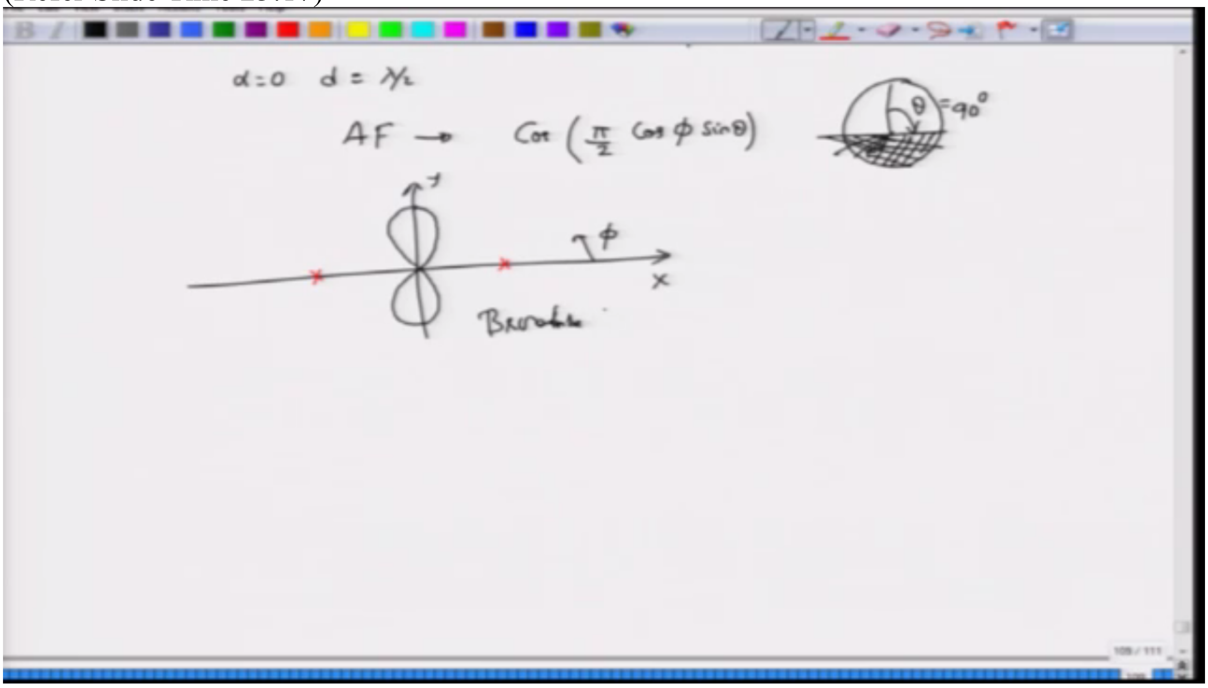

And if you want to switch the direction, that is if you want to make the fields go maximum along the x direction, what you have to do is to take $\alpha = \pi$ and then the distance to be equal to $\lambda/2$. So if you simply make the currents out of phase with respect to each other, then that extra phase shift of π that was coming along the x-axis will be compensated by an additional π.

So there will be a phase shift. I mean, there will be a total phase shift of 2π, which means the fields will add along the x-axis, but on the y-axis, unfortunately, this, they will arrive in, you know, at the same time and because the currents are out of phase by π, that overall, you know, interference will be 0, make that this one will be 0 and the field component along the y-axis will be 0. Okay.

So as before if you have x and y plane, the field pattern that you are going to get now will be or rather the array pattern that you are going to get will be maximum here. Okay. This is called as end fire array. Okay.

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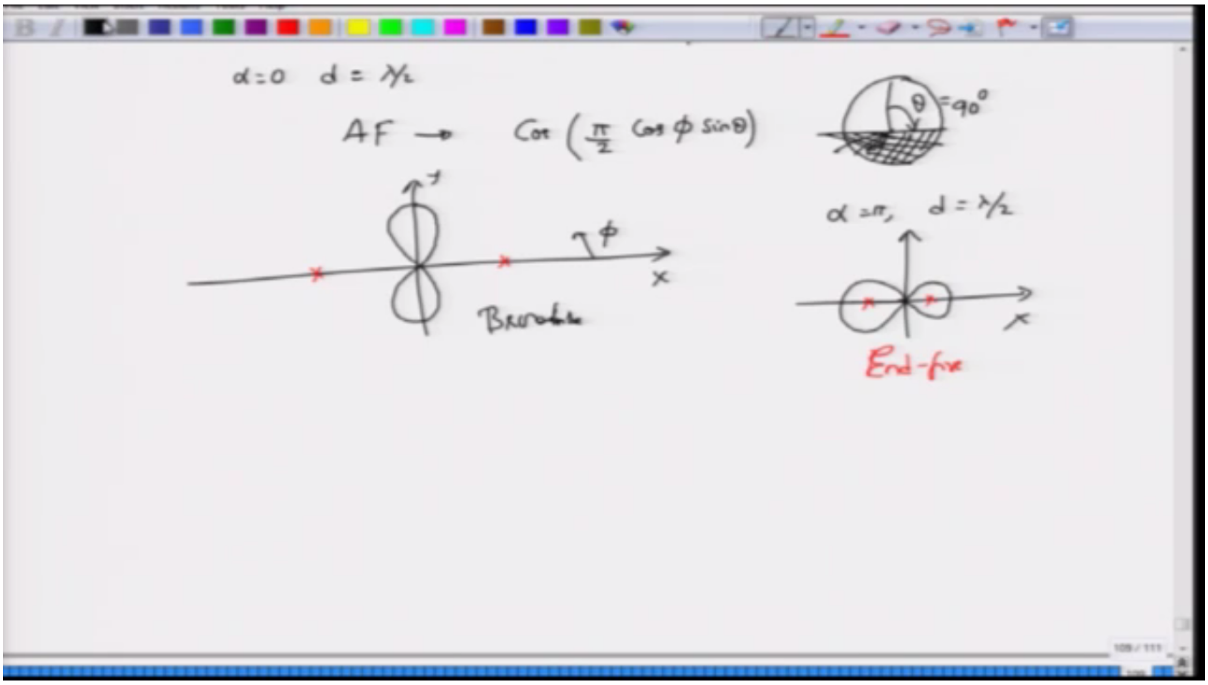

As an again, you can take what would happen when α = 0 and d = λ, okay, and you can also take as an exercise α = π, d = λ. You can also try what is α = π/2 and d = 3λ/2. Okay.

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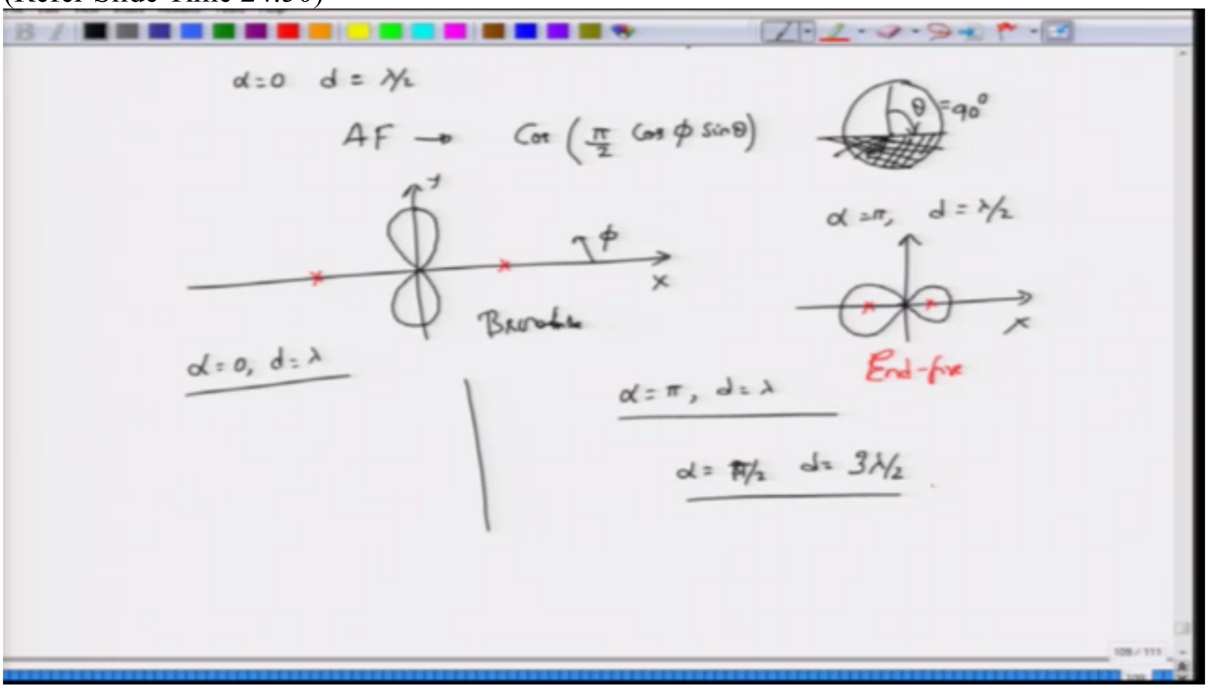

I leave all the patterns that you are going to get from making these substitutions as an exercise. You can, in fact, get an antenna pattern that would essentially almost look at along this direction. Okay. So this would be there, but it will be very small, and you can have maximum energy along the x-axis here, okay, by adjusting the phase shift and the distance as you can see later on and this one would radiate only in one direction. In fact, you can get nice radiation pattern. I did not draw it correctly. You can get nice radiation pattern like this, okay, with almost no side lobes here. Okay.

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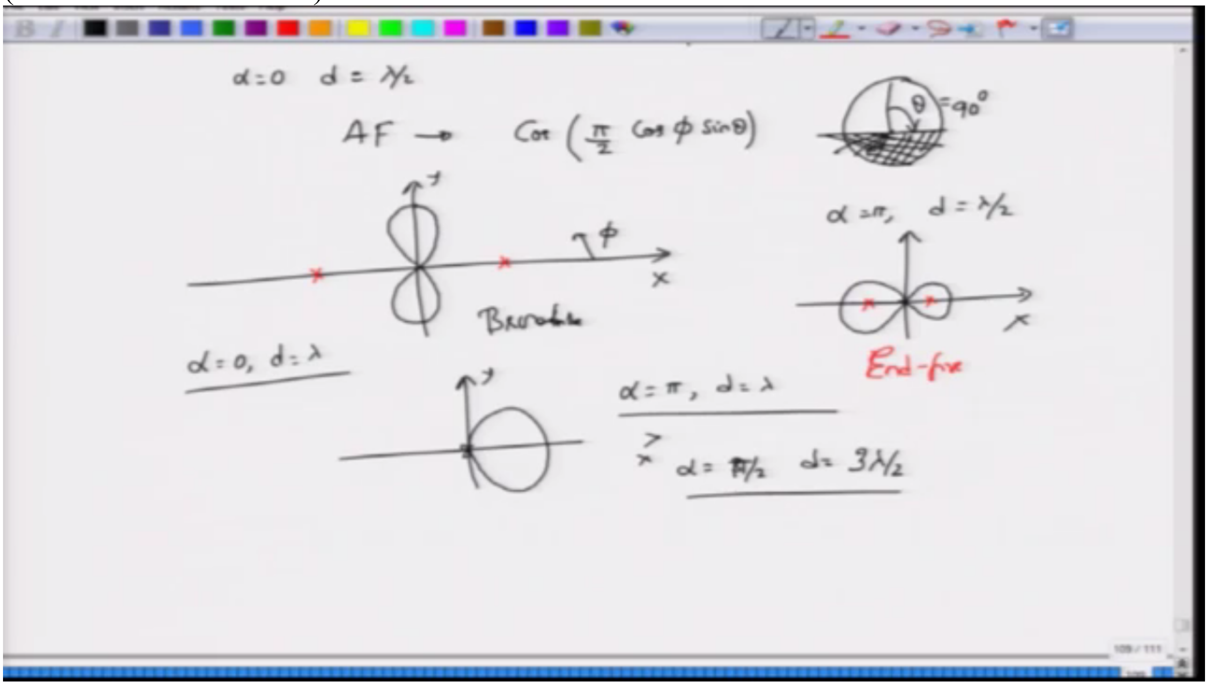

What is the use of such antenna? For example, if you are, you know, in building houses or building a tower near the sea, right, so you want all the radiation from the tower not to go to the sea, but to the shore side that is where people are actually present. So you place the antenna in such a way that the antenna array would transmit nothing in the sea direction because there is no one to receive signals there, but transmit with everything back onto this one. So such antennas are, you know, bread-and-butter of people who are working in the near the seashore. We can, of course, turn the other way around also.

So the point I am trying to make is that antenna arrays offers you flexibility to generate variety of patterns and most importantly by adjusting the current phases, you can even get exotic patterns changing in time. Okay. So you have a time varying field patterns that can be generated by this array antennas.

What happens when you have an n number of antennas? Okay. All of them in phase located here. Then you can show a factor would actually reach a maxima along a direction and then the side lobes would actually be very, very small. Okay. So this, you know, amplitude of the array factor or the magnitude of the array factor, which essentially looks like a sync function and this can be done is the basis of what is called as beam forming. Okay.

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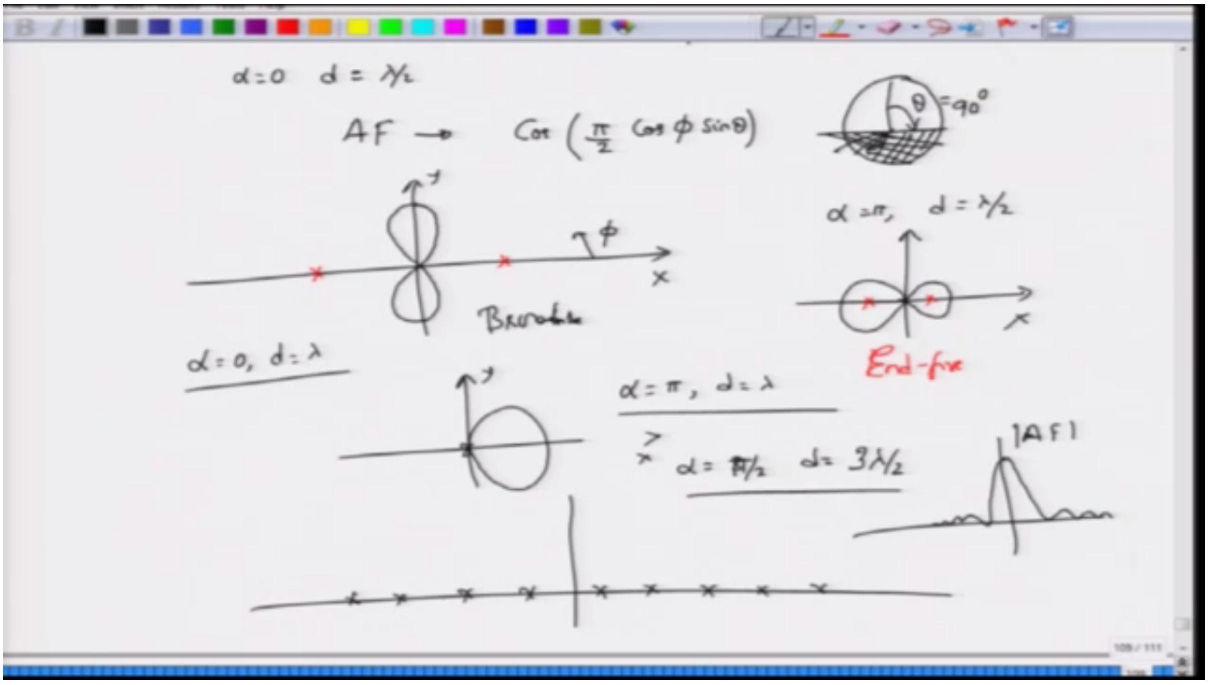

We are basically taking all these partial waves in such a way that the maximum energy is oriented in a particular direction and the side lobes can be minimised, you know, as much as possible. Okay. By adjusting the phases differently, you can also get different types, but this would be a nice focused antenna that you can actually have, right?

So the point is overall antenna pattern should be multiplied by the array factor as well as the antenna pattern itself. For example, you may have an antenna pattern, which at θ = 90°, the one that we considered here may actually be not radiating at all, right?

So if you consider such an antenna, then if you buy a broadside, then it still works because in the broadside case, you know, you are going to get, I mean, it actually doesn't really work, I am sorry, with the broadside or the end fire because there is no antenna, antenna itself is not radiating in this direction. Okay. So there is no point in making arrays out of that and expecting some radiation in the horizontal plane when the original antenna pattern itself shows a null there. Okay.

So these are considerations that you will, you know, study in detail in case you do further studies or take other courses in antennas. Thank you very much.