Principles of Signals and Systems
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Lecture – 09
Example Problems in Signals and Systems – Properties of Modulator, Eigenfunction of LTI System

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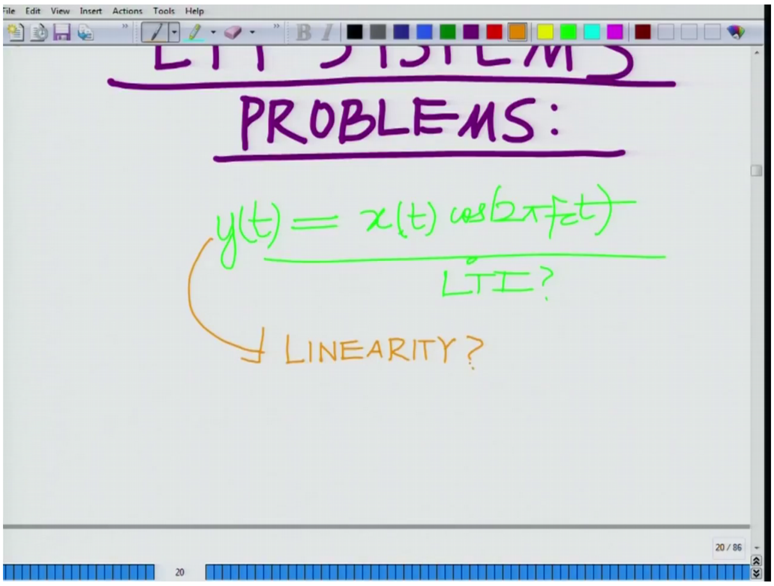

Hello, welcome to another module in this massive open online course. So, we are looking at examples of LTI systems or we are solving problems related to LTI systems. So, let us continue looking at this discussion that is LTI systems. And we are we are solving problems. So, the problem that in currently trying to address is whether this modulation by carrier that is y t equals x t times cosine 2 pi F C t is this a we are trying to address if this is an LTI system. Now, to do that first let us check linearity is this is a linear system. Remember for LTI an LTI system has to be both linear and time invariant.

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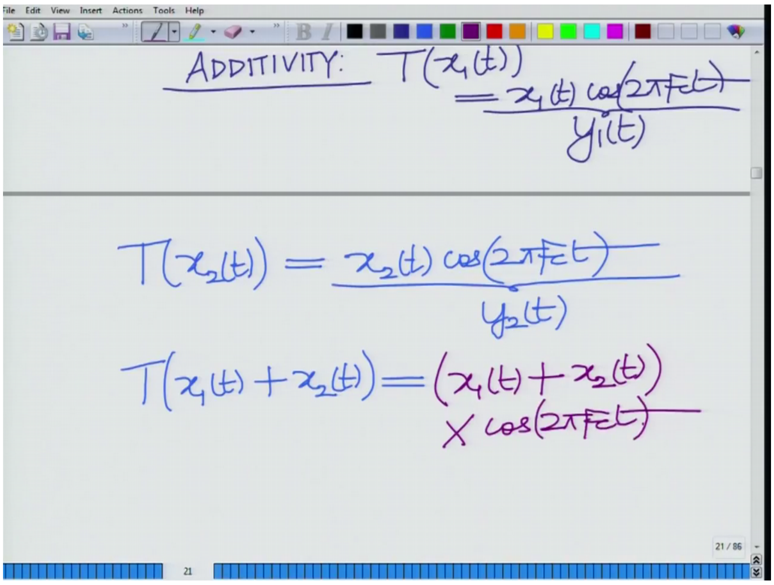

So, let us first check linearity; to check linearity we have to see if it is additive and homogeneous. So, satisfies the additivity and homogeneity property. So, additivity coming now first, to additivity we have let us say t of x 1 t equals x 1 t cosine 2 pi F C t this is your y 1 t. Similarly, T of x 2 t equals x 2 t cosine 2 pi F C t this is your y 2 t. Now, T of x 1 t plus x 2 t, this is equal to you can see that is your modulating signal x 1 t plus x 2 t times cosine 2 pi F C t.

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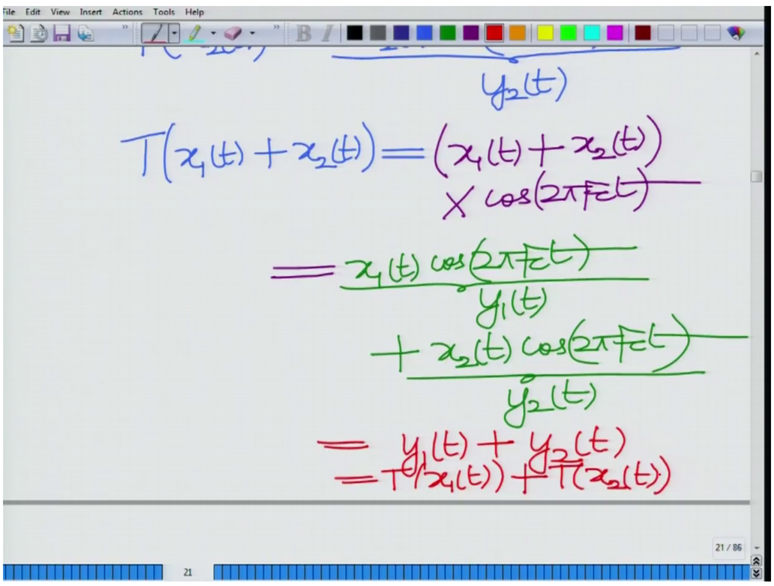

Which is again x 1 t cosine 2 pi F C t that is y 1 t plus x 2 t cosine 2 pi F C t which is y 2 t. So, this is t of x 1 t plus x 2 t equals y 1 t plus y 2 t that is basically T of x 1 t plus T of x 2 t. So, therefore, it satisfies the additivity property. Therefore, it satisfies the additivity property. So, we have seen that it satisfies the additivity property.

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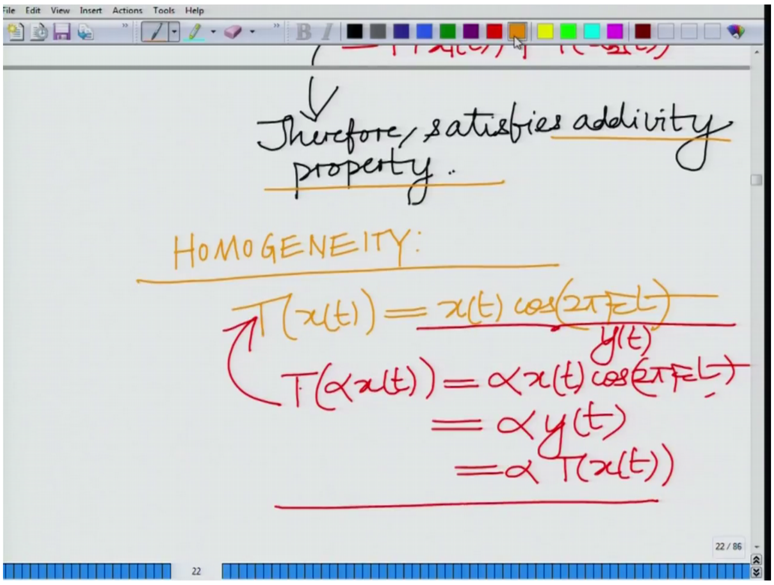

Now, how are the other property in linearity is homogeneity all right. So, we have to check if it satisfies the homogeneity property. So, what we want to check next is homogeneity. So, let us say again T of x t equals x t cosine 2 pi F C t. Now T of alpha, so this is your y t, T of alpha times x t naturally this is equal to alpha times x t cosine 2 pi F C t, which you can clearly see it is alpha times y t which is basically alpha times T of x t. Therefore, system is again T of alpha times x t is alpha times T of x t. Therefore, system satisfies homogeneity property therefore, system satisfies homogeneity.

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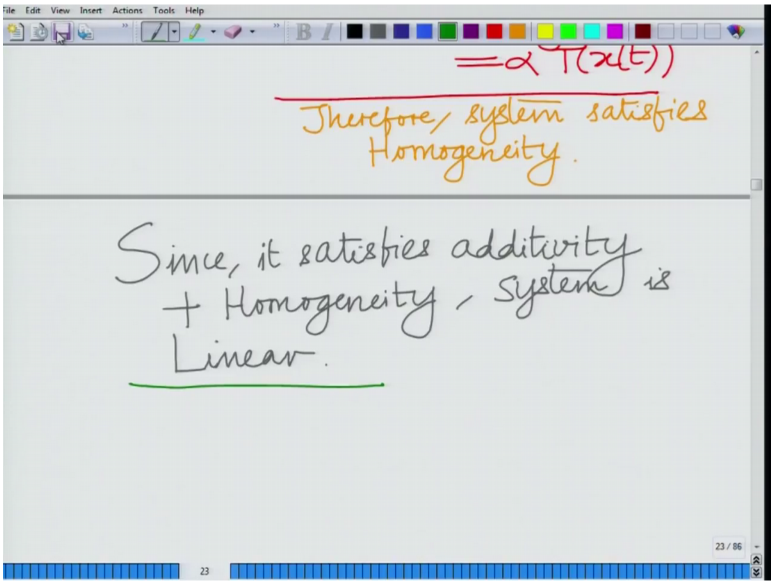

So, we have additivity, homogeneity. So, system satisfies both additivity and homogeneity properties, therefore the system is linear. So, the modulator, the modulation system is a linear system. Therefore, since additivity plus homogeneity system is linear system; it is a linear system.

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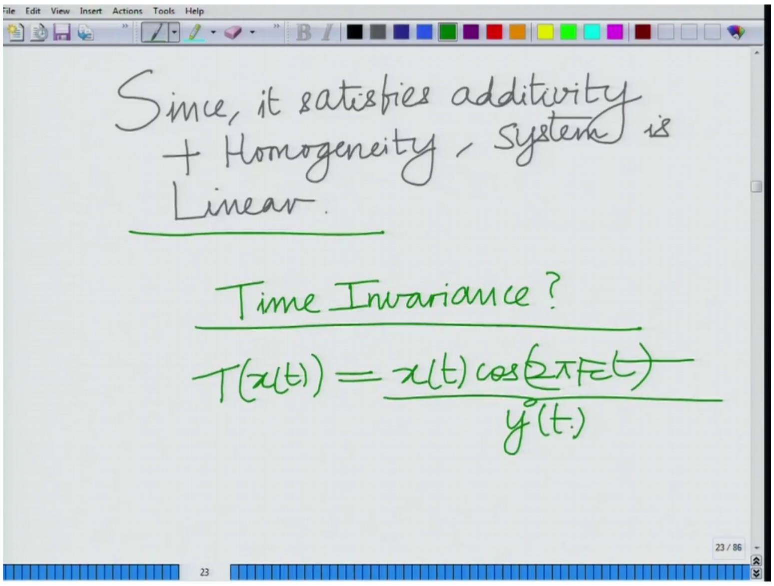

Now, how about time invariance, it is a time invariant. How about the time invariance property? Let to address that let us look at T of x t that is equal to x t cosine 2 pi F C t this is your y t.

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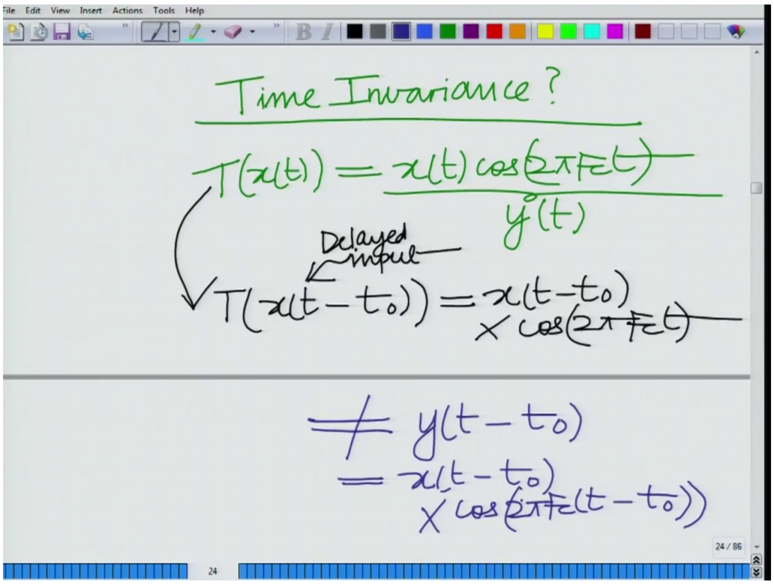

Now, for time invariance, we have to considered a delayed input or shifted input x of t minus t naught this is your delayed input, this is equal to x of t minus t naught into cosine 2 pi F C t. And now you see you have a problem here this is not equal to y of t minus t naught, where y of t minus t naught you can see is x of t minus t naught into cosine 2 pi F C t minus t naught, so that is the problem.

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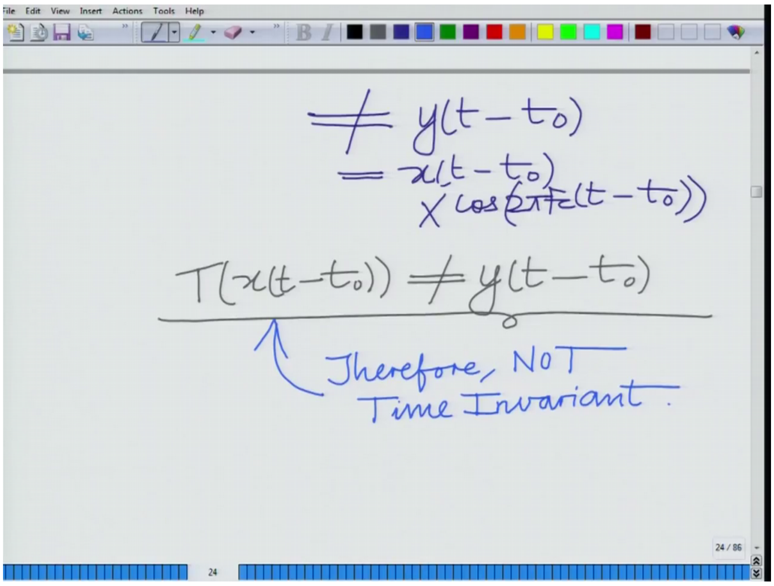

What you can see here is that interestingly T of x of t minus t naught is not equal to y of t minus t naught that is if you delay the input, the resulting output is not the output corresponding to the previous that is not the output corresponding to x t delayed similarly. These two things are different that is T of x t minus t naught is not equal to y of t minus t naught, therefore the system is not time invariant that is an important observation. Therefore, hence it is not an LTI system, because system is LTI only if it is both linear and time invariant, this system is linear, but not time invariant. Hence, it is not an LTI system.

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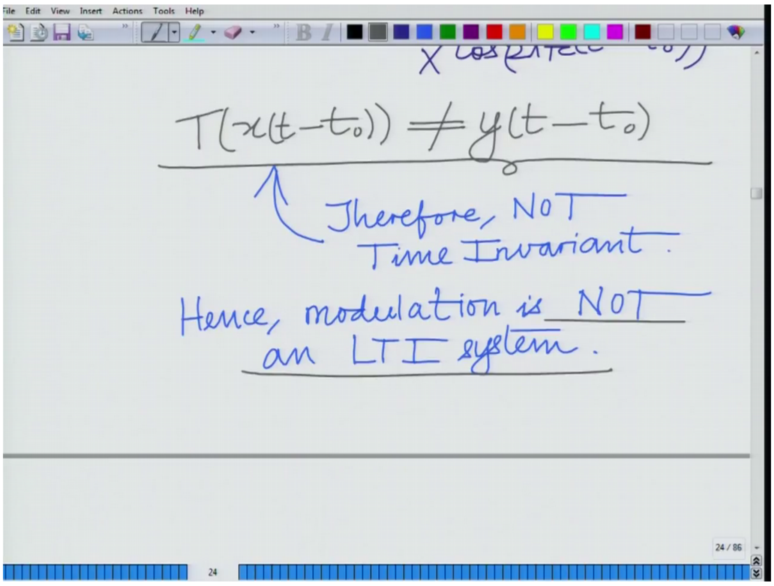

It is linear, but not time invariant hence the modulation operation or modulator subsystem is not modulation is not linear time invariant system, this is not an LTI system, so that is the important aspect that you have to understand. Sometimes, it is not obvious. So, it is a linear system, but it is not a time invariant system.

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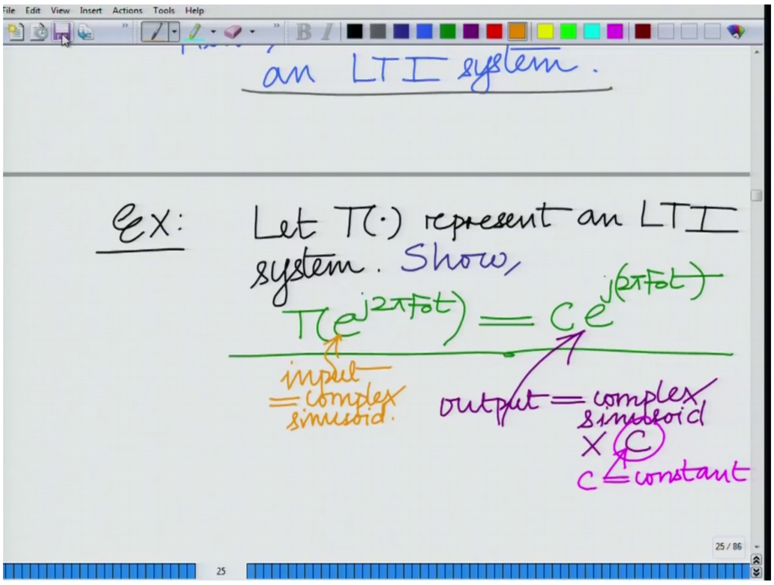

And let us do one final example to understand something. So, let us do another final example for an LTI system or consider let the transformation represent, let this represent a LTI system. Now, for this if this represents an LTI system, then show that T of e to the power of j 2 pi F naught t for any frequency F naught equals that is if you consider any input which is a complex sinusoid. And this is a very interesting property for a pure for a complex sinusoid, the output is some constant C times e to the power of j 2 pi F naught n. This is a very interesting property that is basically output to any complex that is if you consider an LTI system output, if the input is a complex sinusoid, the output is also a complex sinusoid simply scaled by another constant c. And this is a very interesting result. Output is a complex sinusoid scaled by X, where C equals this C equals a constant. And input is a complex sinusoids then output is also a complex sinusoid scaled by C, this can be shown as follows.

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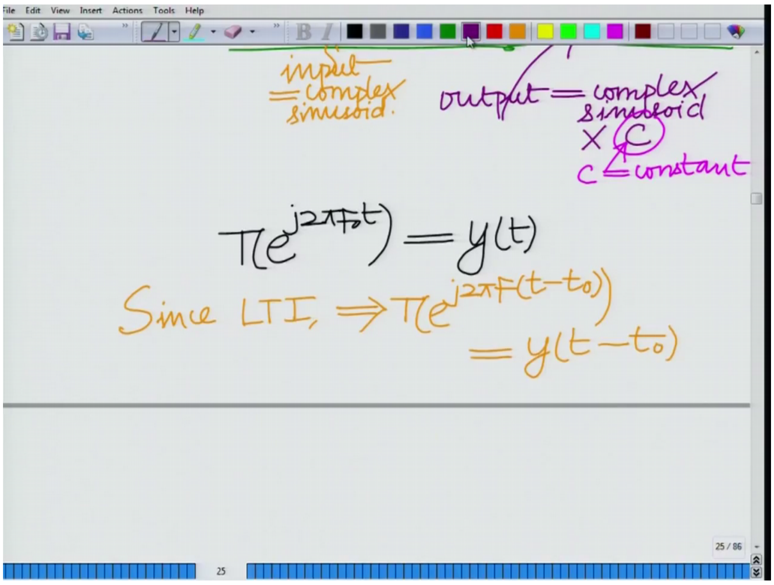

Since we have let t to the power of j 2 pi F naught t let this be equal to y t. Since, LTI since the system is shown to be LTI, it is a linear time invariant system, this implies T e to the power of j 2 pi F t minus t naught equals y of t minus t naught, but t e to power of j 2 pi F t minus t naught. This is equal to T e to the power of j 2 pi F t e to the power of minus j 2 pi e to the power of minus j 2 pi f naught t naught, which is equal to y of t minus t naught.

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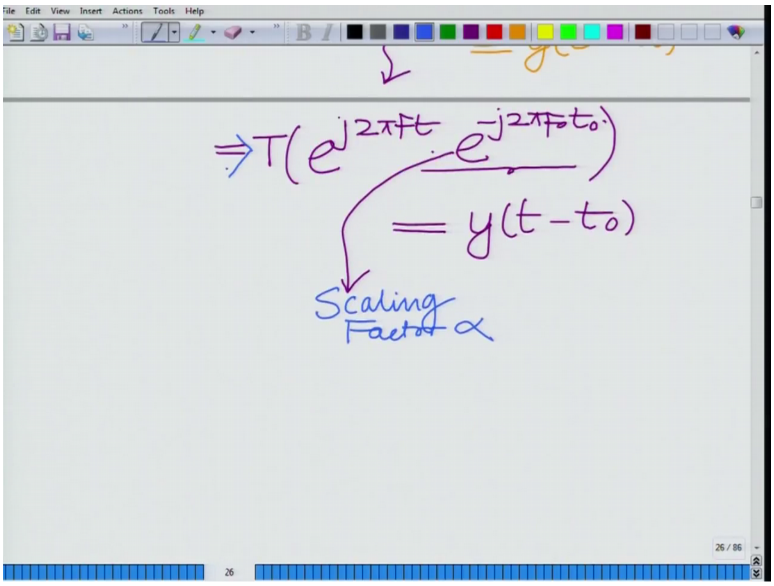

Now, look at this e to the power of minus j 2 pi F naught t naught this is simply a scaling factor because F naught is a constant, t naught the delay is a constant I mean we are assume. So, this is simply a scaling factor you can see this is similar to your scaling factor alpha.

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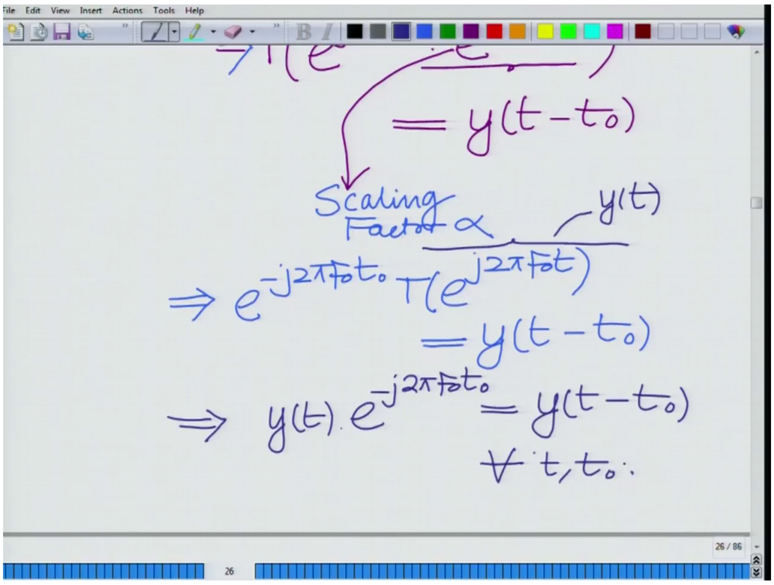

So, using now homogeneity this implies from LTI this implies e to the power of minus j 2 pi F naught t naught into T of e to the power of j 2 pi F naught t equals y of t minus t naught, but t e to the power of j 2 pi F naught t this is nothing but y of t. So, what we have, so this is y of t. So, what we have is basically this implies y of t into e to the power of minus j 2 pi F naught t naught because y of t minus t naught and this holds for all t naught of course, this also all t comma t naught.

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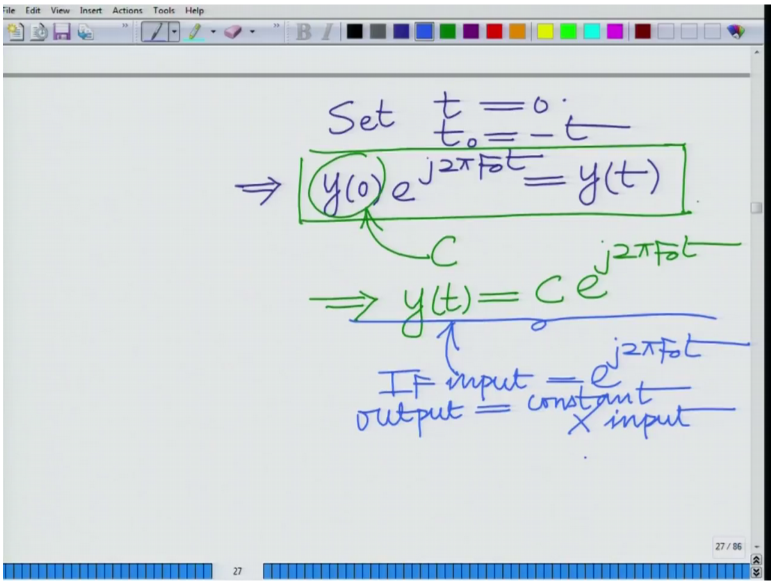

Now, set t equal to 0, and t naught equals minus t this implies y of 0 e to the power of minus j 2 pi F naught t is equal to so we are setting t naught equal to minus t e to the power of j 2 pi F naught t equals y of t. So, what we have been able to show is that y of t equals y of t equals now this is y naught you can call this as your constant C. So, what we have shown is that y of t this implies y of t equals some constant C e to the power of j 2 pi F naught t. So, output is simply scaled version of input that is this shows if input equals e to the power of j 2 pi F naught t, output equals some constant factor times the input. So, output is simply some constant times the input such an input is known as an eigenfunction. So, e to the power of j 2 pi F naught t is an eigenfunction of the LTI system, because output is simply a scaled version of the input.

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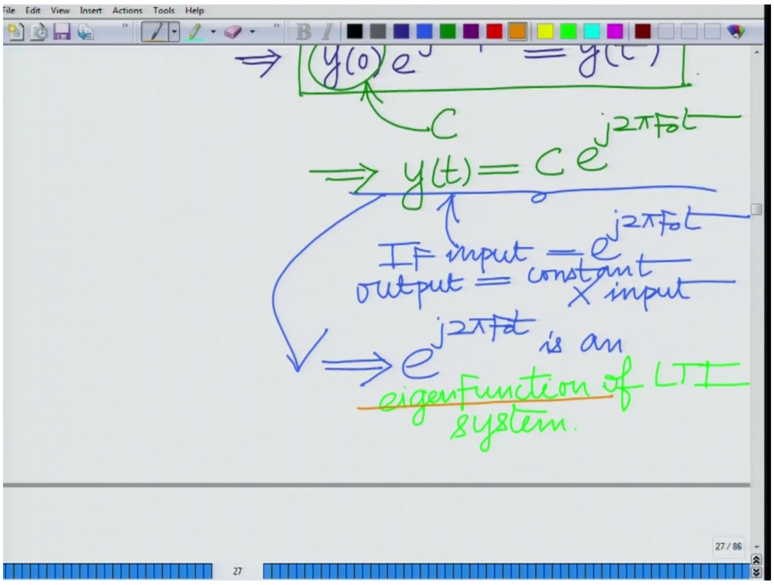

Hence this implies, this is a very interesting property, this implies e to the power of j 2 pi F naught t is an eigenfunction. This is a very interesting property is an eigenfunction of an LTI system for that matter is an eigenfunction of any LTI system e to the power of j 2 pi F naught t this has a very special relevance of course, we are going to explore it further through the various modules. But it is important to realize that this function this complex sinusoid has a has very important relevance correct, in the context of analysis of LTI systems because this is an eigen function of any LTI system all right.

So, with that we will wrap up our example section. So, hopefully we have done several examples which are hopefully to some extent have covered various aspects of the principles of signals and systems or the various introductory let us say introductory principles of signals and systems classification of signals and systems that we have seen so far, and we will look at other aspects in the subsequent modules.

Thank you very much.