Principles of Signals and Systems
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Lecture - 51
Fourier Transform Problems: Unit Step Response of RC Circuit, Sampling of Continuous Signal

Hello. Welcome to another module in this massive open online course. We are looking of problems in the Fourier transform and specifically the unit step response of the serial RC circuit all right.

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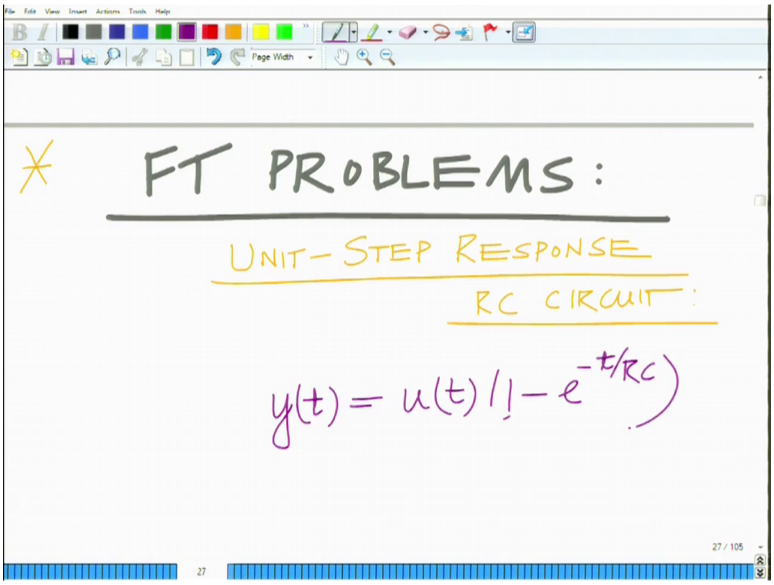

So, we have example problems in the Fourier for the Fourier transform and we are looking at unit response of the RC circuit, unit step response of the RC circuit ok. And what we have seen is the unit step response, is given us the output is y t equals. If you remember y t equals u t into 1 minus e raise to minus t over R C, this is the unit step response ok.

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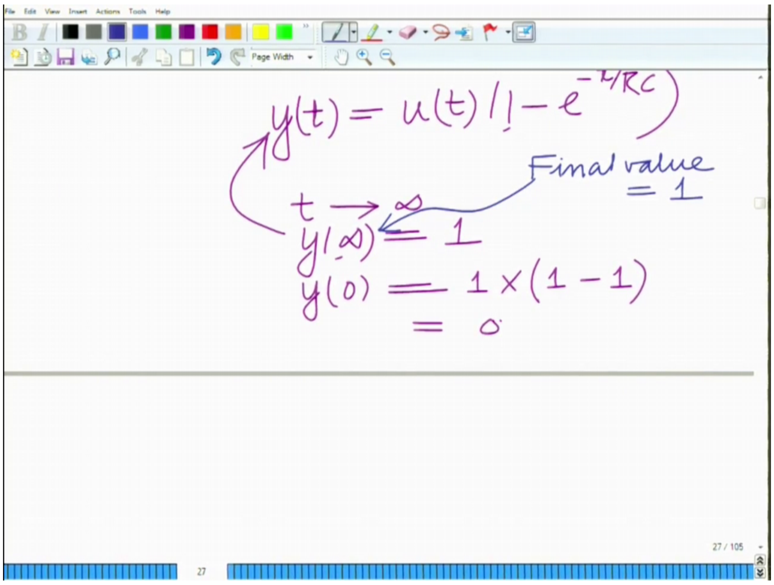

Now, you can see the final value that is if you look at y t equal to infinity or t tends to infinity let us say. We can call that y of infinity equals well, as t tends to infinity e raise to minus t over RC that tends to zero. So, as t raise to t tends to infinity the final output is 1 all right. So, all the voltage will be across the capacitor ok. So, y infinity equals 1. Now if you look at the initial value.

Now y zero that is also fairly clear, y is zero equals well u of zero which is 1 or that is basically u of zero which is 1 into 1 minus e raise to minus t over RC that is 1. So, this is zero. So, it starts from zero and raises to 1; that is what we have seen. Now final value is 1. So, this is the final value y infinity, this is final value is 1. Now, remember the we define the raise time, as the time taken by the RC circuit or the capacitor voltage to go from 90 percent of the final value have to go from 10 percent of the final value to 90 percent of its final value.

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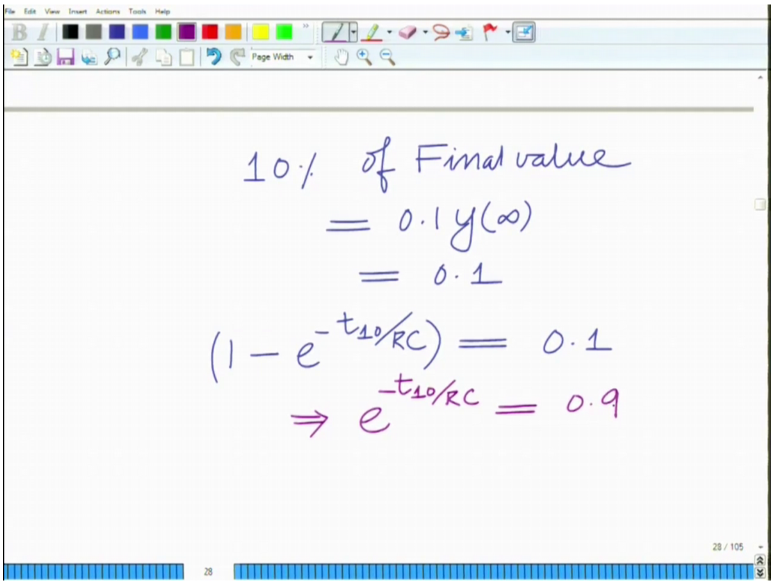

Now 10 percent of its final value that is 10 percent of 1; that is 0.1 into 1 y infinity equals 0.1. Now what is the time we have 1 minus e raise to minus? Let us call this as your t 10; that is time taken for 10 percent of the final value 1 minus e raise to minus t 10 over RC is point 1 implies e raise to minus t 10 over RC equals 1 minus point 1; that is 0.9 implies your t 10 equals minus RC log natural of 0.9.

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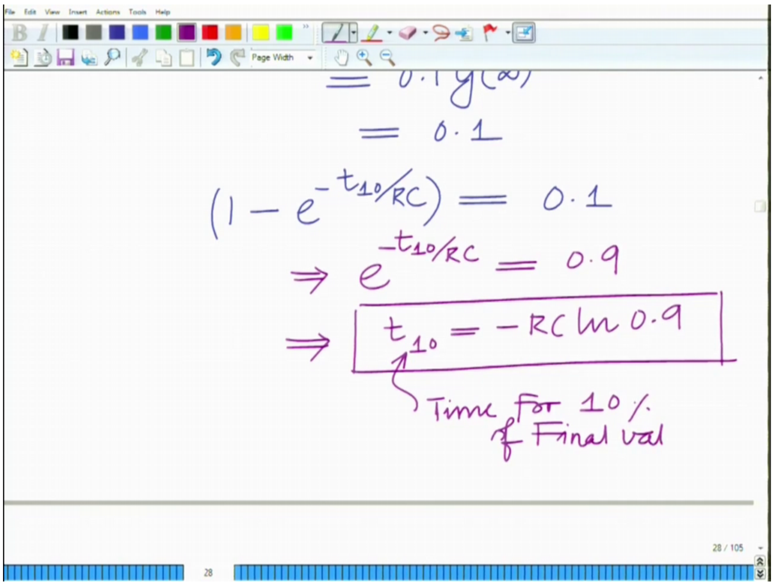

So, this is your time taken for 10 percent of the final value ok, 10 percent of final value. Now, time taken for the, for 90 percent of the final value 0.9 into 1 that is 0.9 ok.

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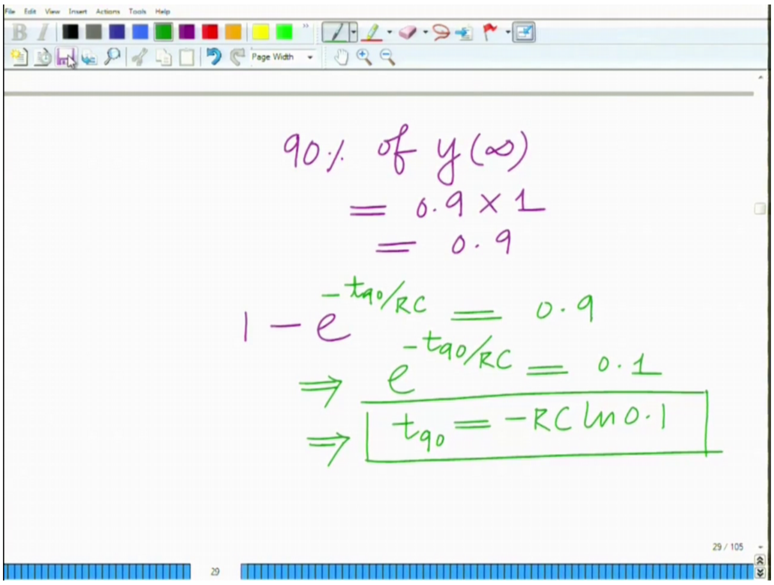

Now time taken for 90 percent of the final value let us call that as t 91 minus e raise to minus t 90 by RC equals 0.9. Now this implies that e raise to minus t 90 over RC equals point 1, which implies t 90 equals minus RC l n or log natural of point 1. So, we have. So, this is basically your time taken for to reach 90 percent

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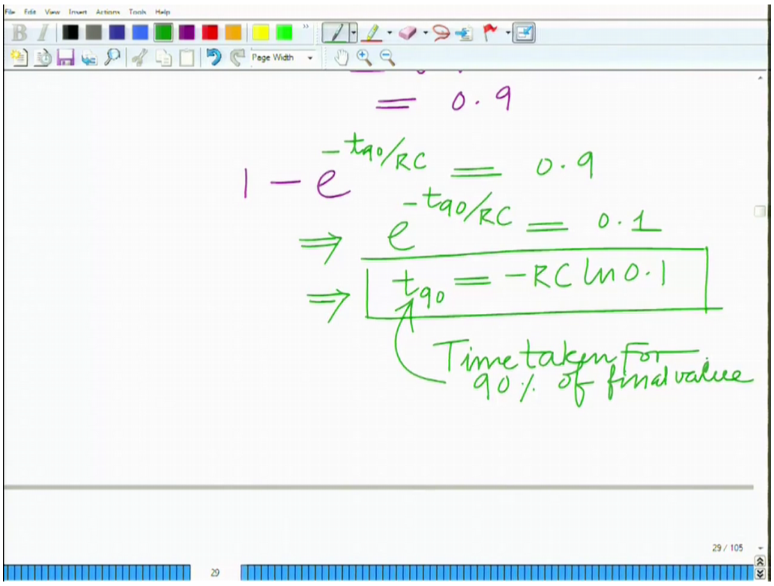

So, this t 90 is basically time taken to reach 90 percent of the final value ok. So, this is basically. So, this is the time taken to reach 90 percent of the final value and therefore the raise time is basically time taken to reach 90 percent of the final value from 10 percent.

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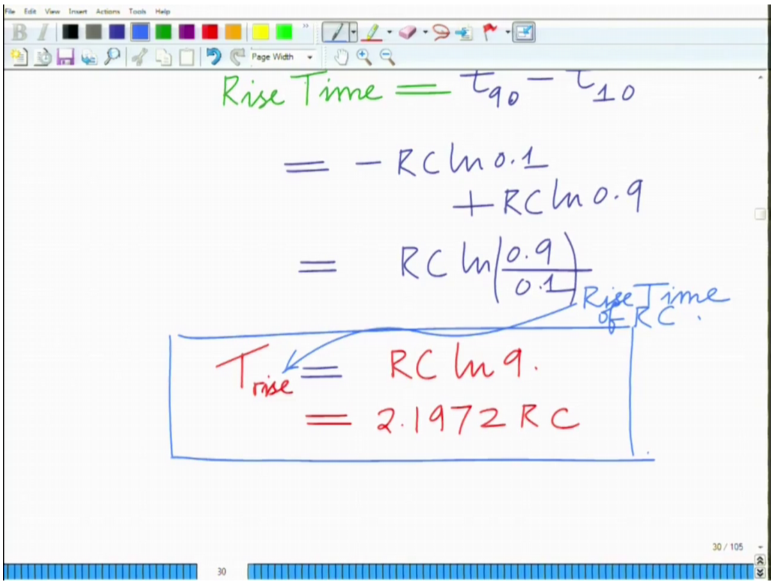

So, that is basically your t 90 minus t 10 which is basically minus RC log natural point 1 plus RC log natural 0.9 which is basically RC log natural of 0.9 divided by point 1 log natural is the log to the base e RC log of 9. So, this is your T raise which is RC log 9, which is basically 2.1972 into RC.

So, this is the expression for the raise time. So, this is the raise time of RC; that is 2.1972 RC or log 9 that is log natural log 9 to the base e times RC, this is the raise time all right. And you can clearly see as RC is increasing basically the time constant is increasing, basically the raise time is also increased ok, so no surprise is there. So, as and you can note that as RC is increasing raise time is also increasing as RC increases the raise time increases ok.

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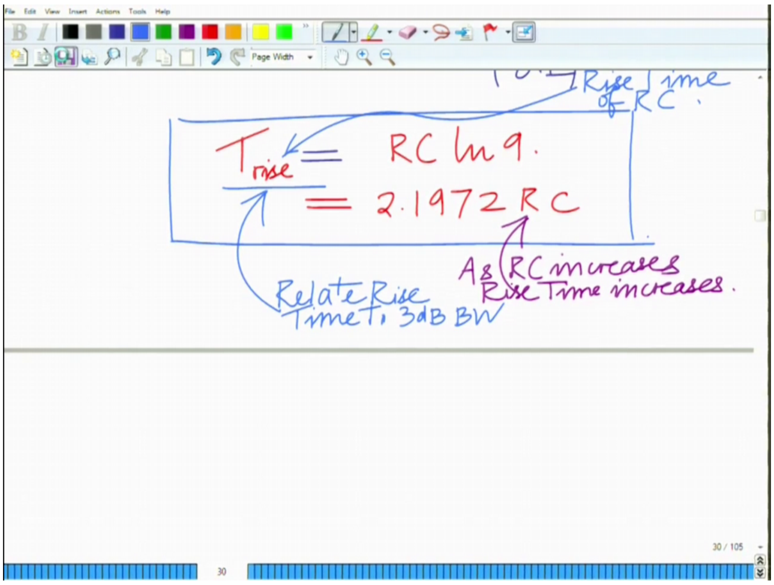

Now, observe that, now we want to relate raise time to the 3 dB bandwidth. So, relate the raise time to the 3 dB bandwidth. Now observe that the response of the RC circuit

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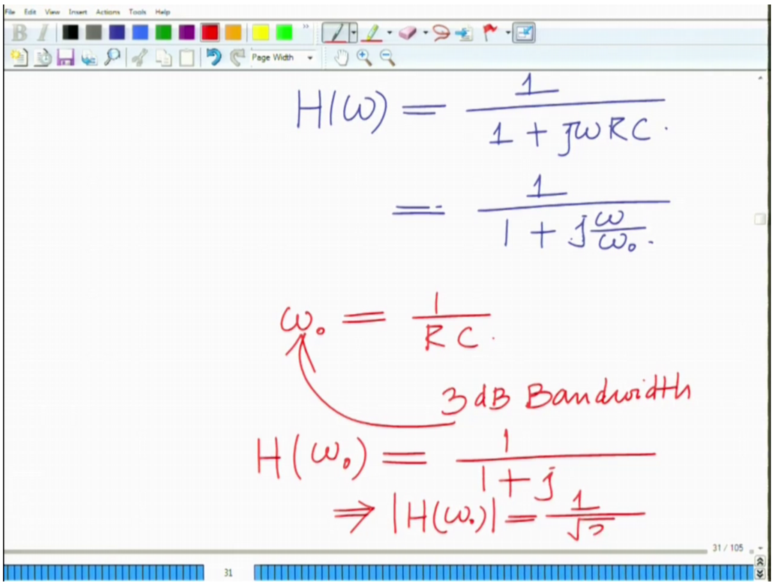

And this is what we already seen the transfer function is 1 over 1 plus j omega RC; this is the transfer function of the RC circuit, which is basically you can see, this is basically 1 over. Well you can write this as 1 plus j omega over 1 plus j omega over omega naught, where omega naught, now if you recall omega naught equals 1 over RC. Now this is termed as the 3 dB bandwidth.

Now, you remember, might also remember that this is equal to the 3 dB bandwidth, because H of omega naught H; that is the transfer function is 1 over 1 plus j which implies magnitude of H of omega naught is 1 over square root of 2, correct.

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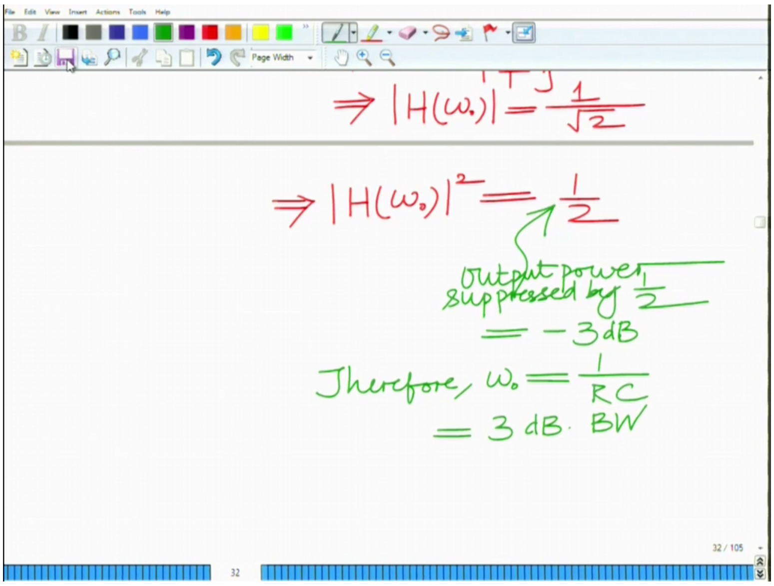

And therefore, the magnitude H of omega naught square equals half. Therefore the power that is transferred, the output power is half corresponding to that frequency all right, which basically corresponds to a 3 dB; 3 dB suppression in the power of the signal that is input to this RC ok. So, the output power is subdued or suppressed by the factor of half, so output power correct. So, magnitude h naught omega square is the, which corresponds to, which equals to minus 3 dB reduction in power. And hence therefore omega naught equals 1 by RC equals the 3 dB band value. And now that gives us does not interesting relation.

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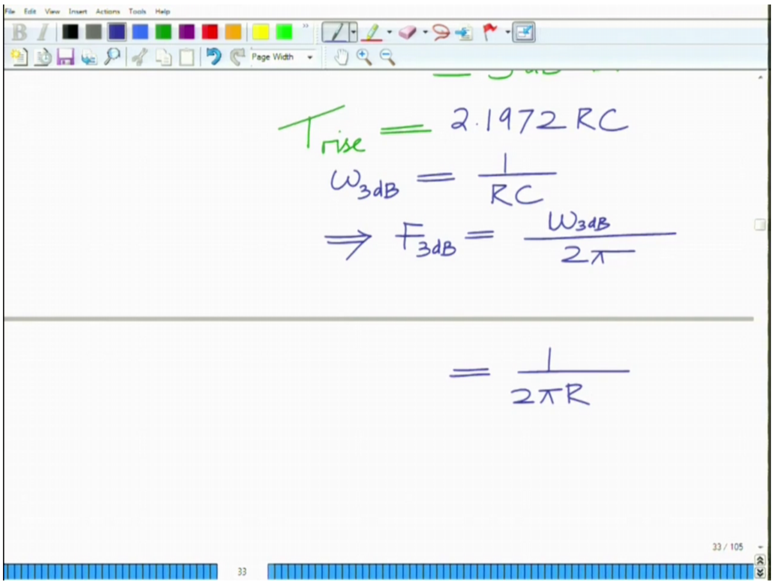

So we have remember raise time equals, the raise time equals, we have just seen that 2.1972 times RC and 3 dB bandwidth omega 3 dB equals 1 over RC implies the 3 dB frequency equals omega 3 dB by 2 pi which is equal to 1 over 2 pi R C, which implies your RC equals 1 over 2 pi times the 3 dB frequency.

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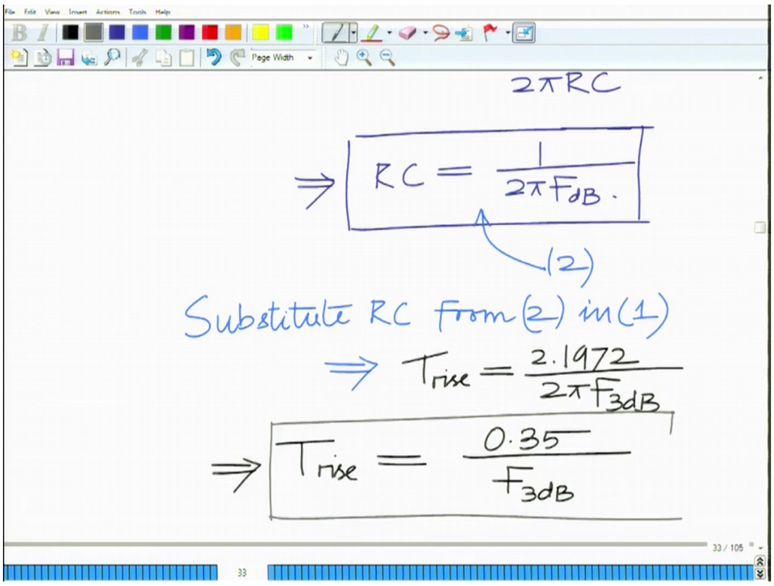

Now what the only thing that is remaining is basically if you call this, let us say two and you call this let us say 1 and all that is remaining is substituting this value of RC. So, substituting RC from 2 in 1 this gives us the result that T raise equals 2.1972 times RC, but RC is 1 over 2 5 f of 3 dB, the 3 dB frequency implies the raise time is 2.1972 over 2 pi; that is basically 0.35 over the 3.

So, basically that gives us the relation that raise time is point. So, raise time is 0.35 over the 3 dB frequency (Refer Time: 13:32) which is very interesting what this tells us, is that the raise time is inversely proportional to the 3 dB frequency, which means of the 3 dB frequency is large right, the filter has the large bandwidth correct, the raise time is much smaller. On the other hand in the three different frequency is small, there is a filter has a very small band width; the raise time is much larger ok.

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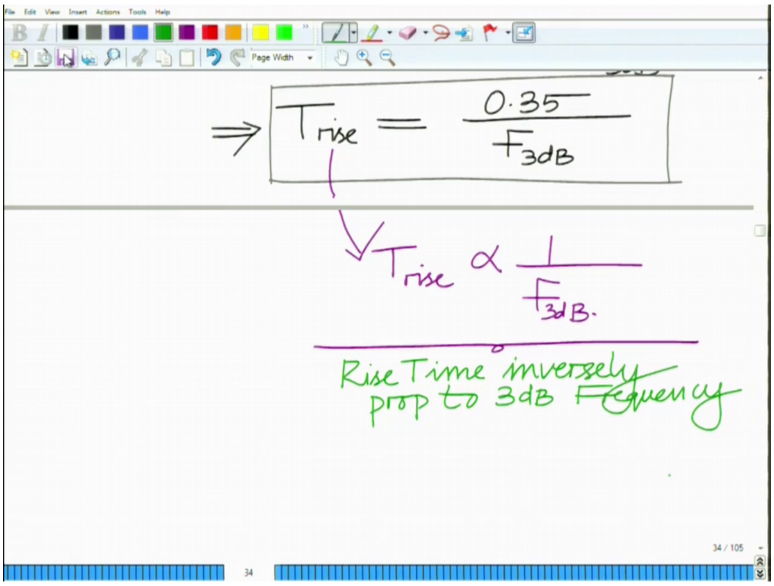

So, what this tells us is the raise time is, the raise time is basically inversely proportional to 1 over 3 dB frequency ok. So, raise time is inversely proportional, let us also note that the raise time is inversely proportional to the 3 dB frequency which is a very interesting thing. So, basically this problem illustrates the concept of this 3 dB frequency, how to derive the concept of raise time, how to derive the raise time all right and finally, its relation to the 3 dB frequency at the RC circuit all right.

Let us now look at another very important aspect, one of the very, one other very important application, one other very important concept of the Fourier transform is known as the sampling which is related to the sampling of a continues time and a log signal. So, let us talk about.

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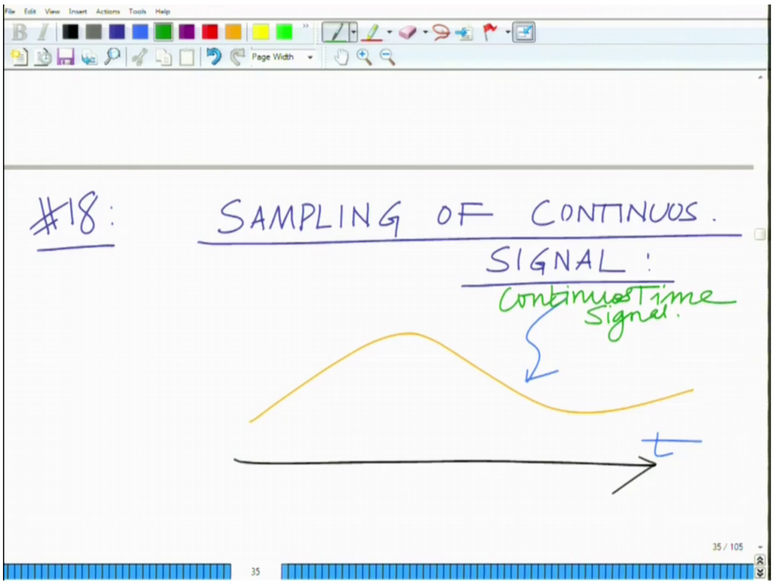

Now, let us come to another yet another important concept; that is the concept of sampling ok. So, this is let us treat this as problem number 18 and this is although its an example problem, this is a very important concept by itself, this is sampling of continues signal. Now consider a signal, a continues time signal which looks something like this ok. And now this is a continues time signal ok, this is t, this is the continues time signal, this is a continues time signal.

Now before converting it to digital signal we would like to convert it to a discrete time signal. So, to convert this continues time signal into a discreet time signal, we have to look at the values of the signal at discreet time instants or we have to extract the values of this signal values at discreet time signal, discreet time instants which are equispaced. This process is termed as sampling; that is converting this continues time analogue signal into a discrete time signal, this is termed as sampling.

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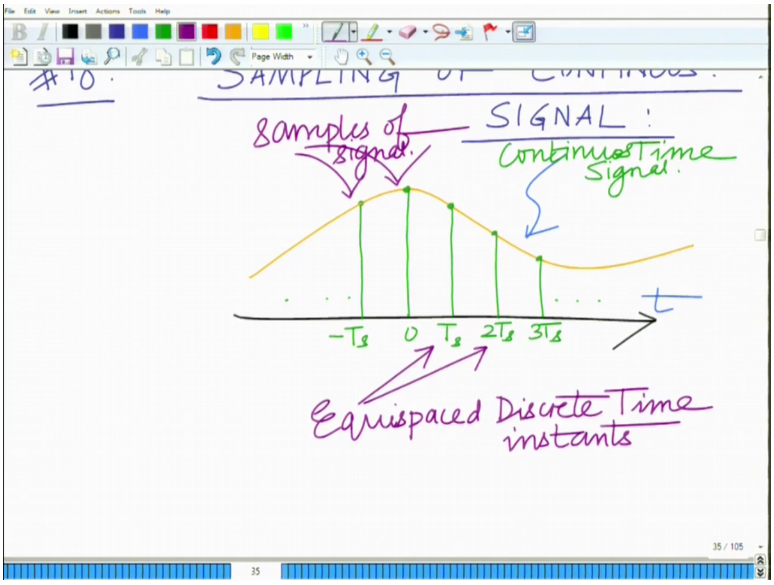

So, we place different time instants at which we extract the values of the. So, let us say this is zero, this is T s, this is 2 T s, 3 T s minus T s and so on. So, these are the discrete time instants ok.

So, these are the various equispaced; equi-space discrete time instants and these are the samples, samples and extracting the samples of the signal at this discrete time instances all right, this process correct is termed as sampling ok. So, sampling is a very interesting concept and one of the most fundamental concepts which has a very interesting theoretical framework, which is basically analogue to or continuous time, continues time; that is C T to D T; that is continuous time.

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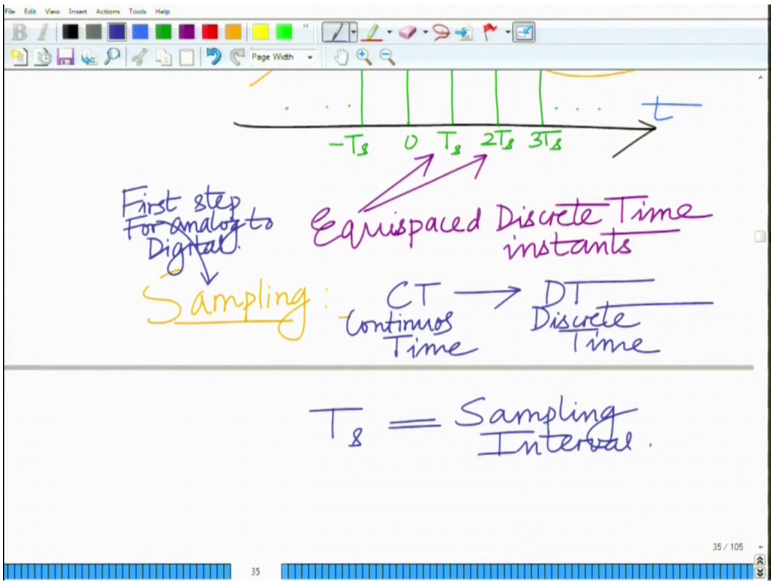

This is what sampling does, you have a continuous time signal converted into a discrete time signal by extracting samples at equispaced instance or equispaced intervals of time and this interval of time which is of duration T s. So, we are extracting samples at zero T s twice T s and minus T s minus 2. So, we are extracting samples at multiples of T s, this quantity T s is known as the sampling interval ok.

So, this is the sampling duration T s. This quantity T s is termed as a sampling interval and this is the first step. So, sampling is basically to convert, this is first step conversion of analogue to for analogue to digital. Eventually to convert this analogue signal into a digital signal, sampling is the first step and it is. Therefore, one of the, one of the most important term, because frequently the natural signals are analogue and to store and transmit signals, we convert them into digital signals.

Alright, which is the most popular technology both for storage and communications of sampling plays a key role in that and it is very important to have understand the concepts, various concepts and the various aspects of sampling in good detail ok. Now if you look at the sampling. Now this T s is as we said is sampling interval. Now this quantity omega s equals 2 pi over T s, you can call it call this as a sampling frequency or basically the angular frequency of sampling.

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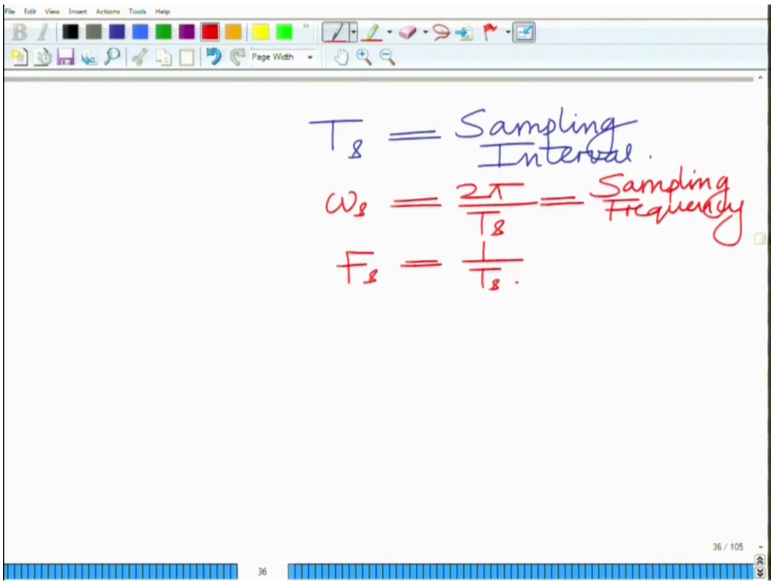

So, this is equal to sampling frequency. This is the angular sampling frequency domain. The sampling frequency can also be denoted by F s equals 1 over T s ok. So, one is the frequency, the other is the angular frequency all right 2 pi over T s 1 over T s all right, the frequency with which samples are being taken for this continues time signal ok.

And now there are various techniques of sampling, one of the most simplest methods of sampling, is basically have an impulse scaled by the value of the signal at each multiple of T s; that is basically what you do is you have impulses which are periodically spaced or equispaced at T s at multiples of T s, and you scale the amplitude or you scale each impulse by the value of the signal at that particular point. This is known as impulse train sampling. In other words you have this impulse train of impulses right which are of unit scaling and you multiply it with the signal ok.

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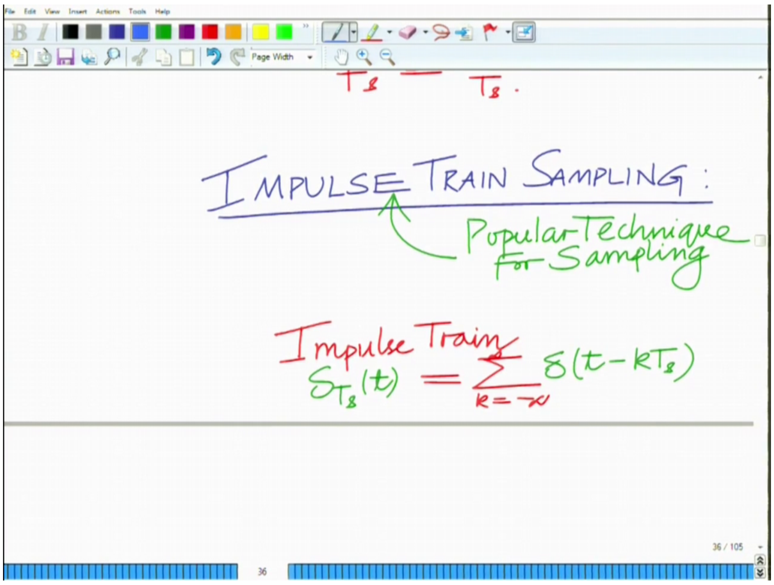

So, what you have, is one of the most popular techniques for sampling is what is known as impulse train. So, this is one of the most popular technique; one of the most popular technique for sampling is impulse train sampling. Now the impulse train is basically you can write this as summation k equals minus infinity to infinity delta t minus k T s; that is delta t minus k T s; that is impulses of unit scaling at each multiple of T s. Unit impulses at, unit impulses at each multiple of T s. So, this is basically termed as your impulse train.

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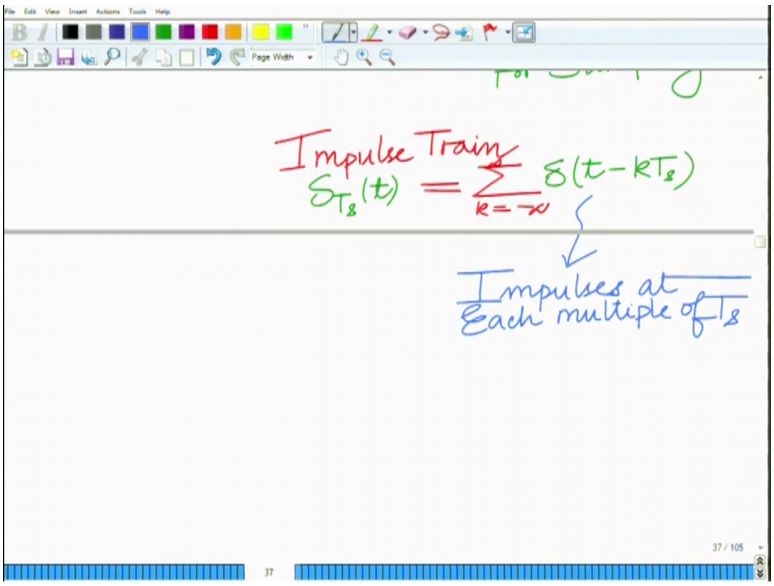

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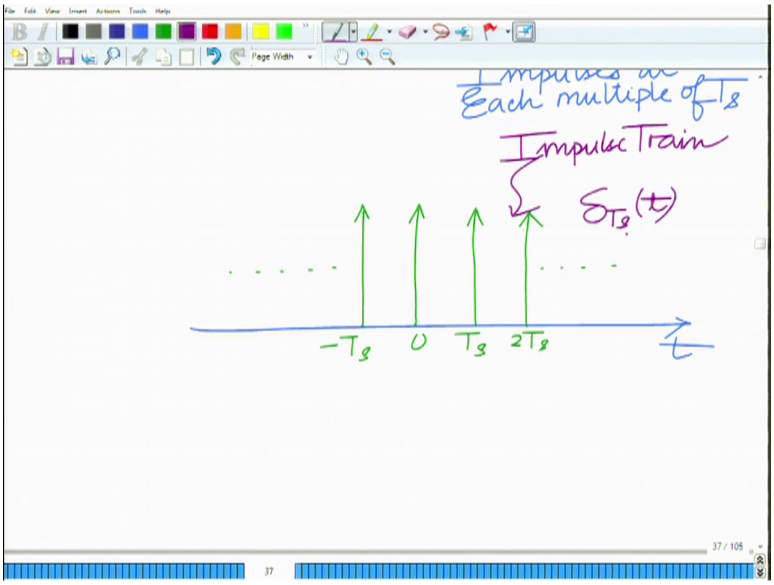

So, what you have is, you have that time axis and at each multiple of T s you have unit impulses. So, this is at zero T s twice T s minus T s. So, this is basically your impulse train or your train of impulses, ok.

. So, this is your impulse train, that is impulses space at equal intervals on the time axes in multiples of T s, where T s is the sampling intervals ok, and this is your delta T s t ok, this is your impulse train. Now what we are doing is we have our continues time signal.

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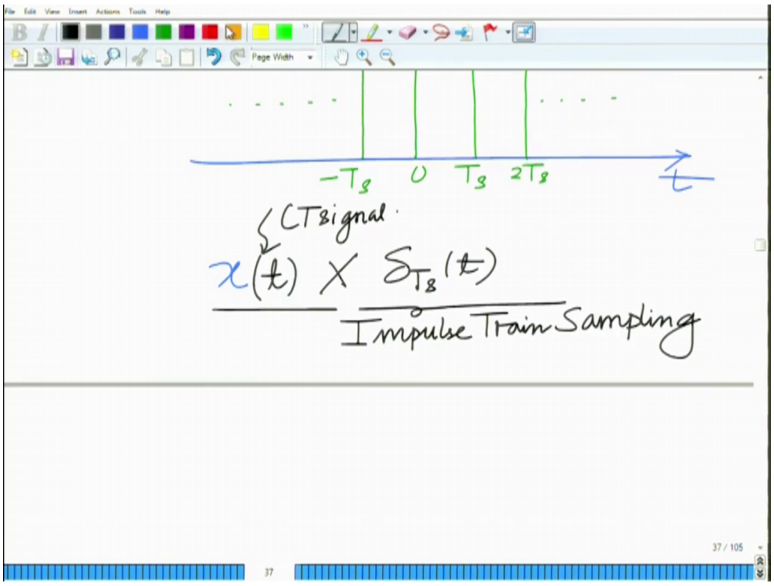

So, impulse train sampling is simply you take the continues time signal x t, this is your C T signal and you multiply it with the impulse train. This is termed as your impulse train sampling. This is your impulse train sampling which is basically

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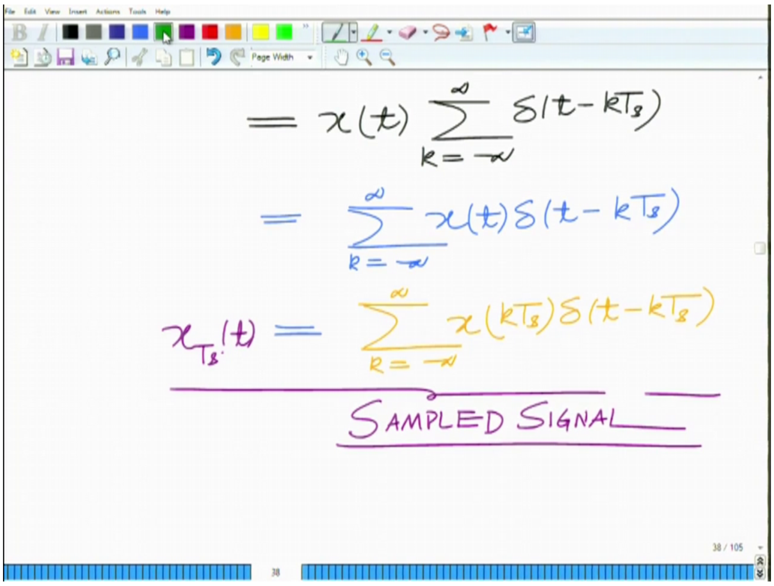

Now if you look at this, this is x of t summation k equal to minus infinity to infinity delta t minus k T s which is equal to summation k equals minus infinity to infinity x of t into delta t minus k T s. And now you can see delta t minus k T s multiplies by x t the property of the impulse function, basically extracts the value of the signal at the sampling instant corresponding sampling instant is k T s ok.

So, for instances you use the property that x t multiplied by delta t minus t naught is equal to essentially x t naught into delta t minus t naught. This is the impulse space t naught and multiplied by x t, extracts the samples, extract the samples of that value t naught ok. So, this is equal to your k equals minus infinity to infinity x of k T s delta t minus k T s and, and this you can denote this as the sampled signal ok. So, this is basically your x of, this is basically your. So, this is basically the sampled signal ok. And this is basically you can see the sample value of the signal at sampling instant k T s, sample value, this is the sample value of the signal at the sampling instant k T s.

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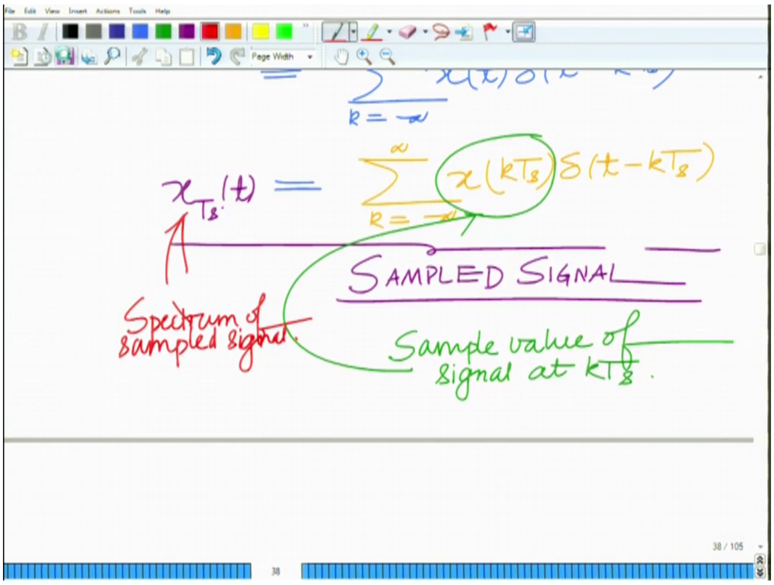

Now, what you want to do as basically we want to study this sampling process in detail, and a very interesting view point or a very int and lot of insides what the sampling process can be gained by looking into the Fourier transform, as what happens to the spectrum of the signal that is sampled. So, what you want to do to understand this twice process better, is to look at the spectrum, we want to look at the spectrum of the sampled signal; that is if we want to ask the question, if x t has the spectrum, if x t has the spectrum x of omega then what is the spectrum of this quantity x T s that is the sampled signal.

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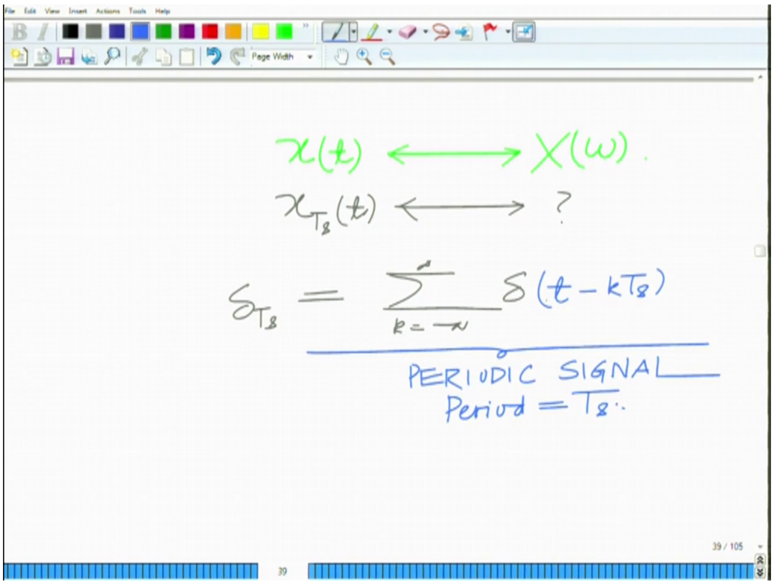

What is the spectrum of the sampled signal that is the question that we would like to addressed. Now so we would likely find the spectrum of the resulting sampled signal to do that first let us start with the spectrum of the impulse train ok.

So, you know the impulse train, now if you take a look at this impulse train equals summation k equals minus infinity to infinity unit impulse functions at each multiple of T s, each integer multiple of T s. Now you can see this is the periodic signal. This is the periodic signal and the period is nothing, but T s, the period is basically T s and you can see that this is basically. If you look at minus T s if you look at this here, you will see that this is basically periodic signal.

So, you have one impulse in every interval of T s, at every duration of after every duration of T s all right; so impulse at 0 T s 2 T s minus T s minus 2 T s of. So, this is periodic and the time period, fundamental time period is basically T s and. In fact, the fundamental frequency of this is 1 over T s the fundamental angular frequency is 2 pi over T s ok. So, that is the first thing that we note ok. So, this is periodic period is T s fundamental frequency was 2 pi over T s which is equal to, which is equal to omega s.

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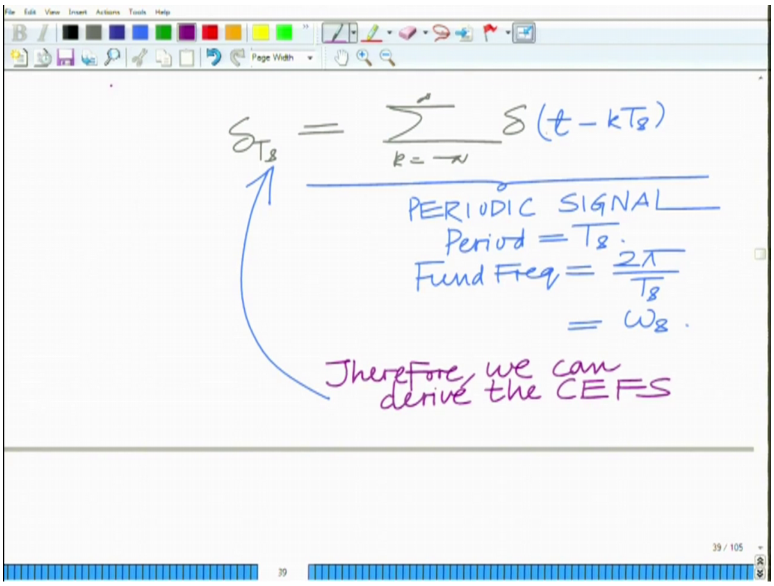

Now there for this has a complex exponential Fourier series, because it is a periodic signal. Therefore, we can develop or derive the C E F S; that is the complex exponential Fourier series that is. I can express this as delta T s of t equals summation k equals minus infinity to infinity C k e raise to j 2 pi k or e raise to j, simply e raise to j k omega s t,

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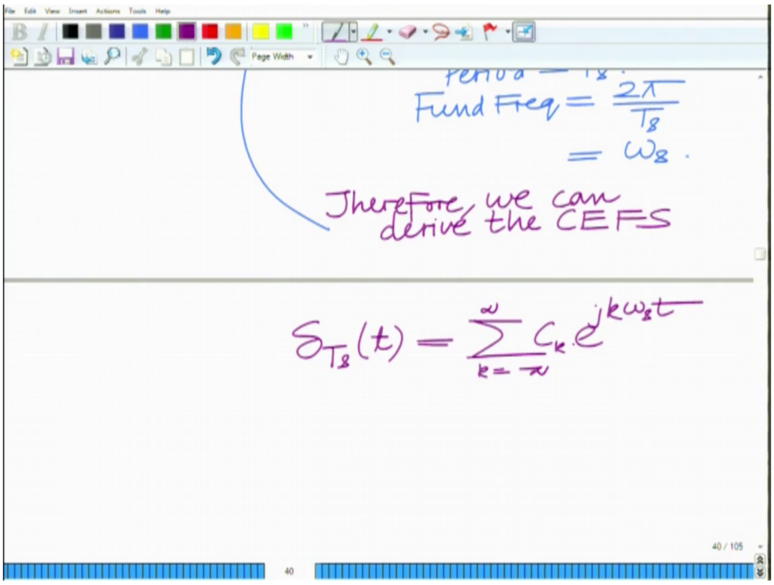

Where omega s is the fundamental frequency; that is basically linear combination, there is a combination of an infinite number of basically sinusoids, harmonics, corresponding to the fundamental frequency omega s and multiples of the fundamental frequency; that is harmonics right multi, harmonics at multiples of the fundamental frequencies that is k omega s ok. And we can derive this coefficients of the C E F S as the complex exponential furrier series C k and from that we can basically one can derive where is the resulting spectrum of the sampled signal.

So, I will stop this module here and continue in the subsequent module.

Thank you very much.