

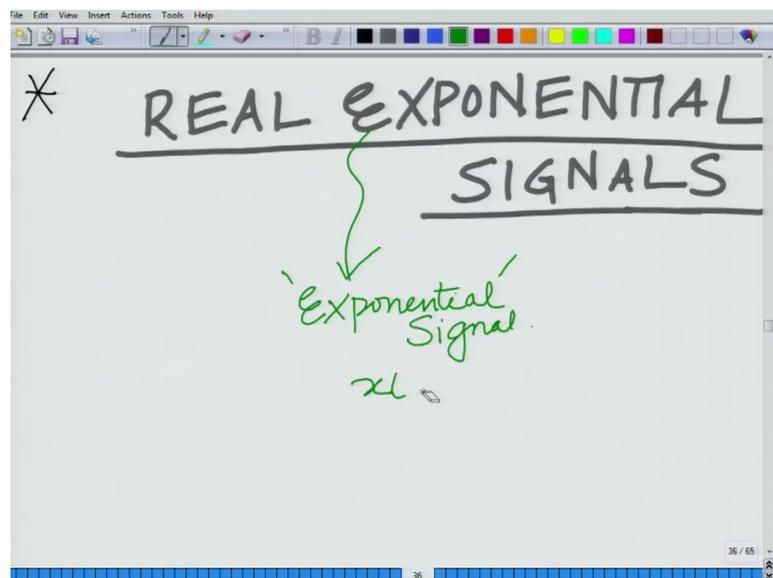
**Principles of Signals and Systems**  
**Prof. Aditya K. Jagannatham**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 04**

**Real Exponential, Sinusoidal Signals, Basic Discrete Time Signals – Unit Impulse/  
Complex Exponential**

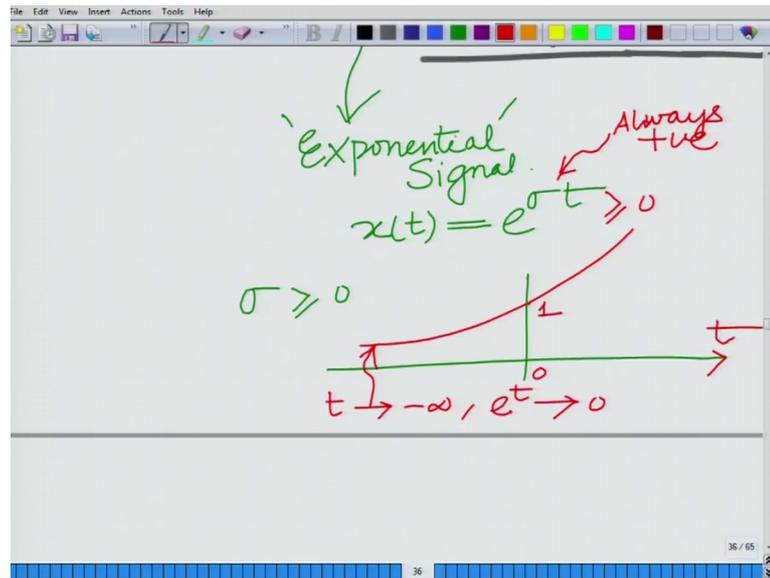
Hello, welcome to another module in this massive open online course. So, we are looking at a classification of signals let us continue this discussion let us look at yet another class of signals which are the exponential the real exponential signals.

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So, what we want to look at is another class of frequently occurring signals which are the a real exponential signals and in real exponential signals well simply refer to and these are simply referred to as exponential signals similar to what we have it is complex then it is referred to as a complex exponential if it is real exponential it is simply referred to as an exponential signal which is of the form  $x(t) = e^{\sigma t}$  and if  $\sigma$  is greater than or equal to 0.

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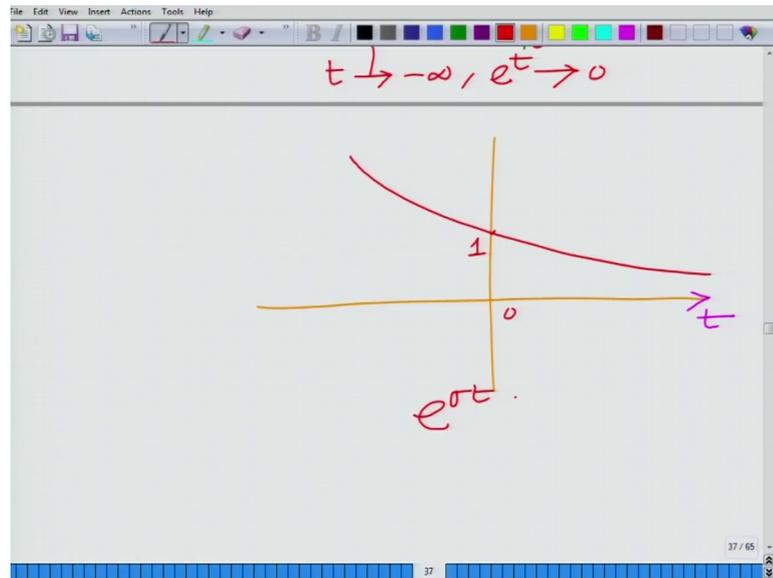


It is increasing at  $t$  equal to 0 it is one at  $t$  equal to 0, it is 1 and as  $t$  tends to minus infinity  $e$  to the power of  $t$  tends to 0, all right.

So, it is always positive. So,  $e$  to the power of  $\sigma t$  for any  $t$  is always greater than or equal to 0 always greater than always this is. In fact, a positive signal this is. In fact, a positive signal  $e$  to the power of  $\sigma t$  for  $\sigma$  greater than or equal to 0 this is increasing at  $t$  equal to infinity it tends to infinity at equal to minus infinity it tends to 0.

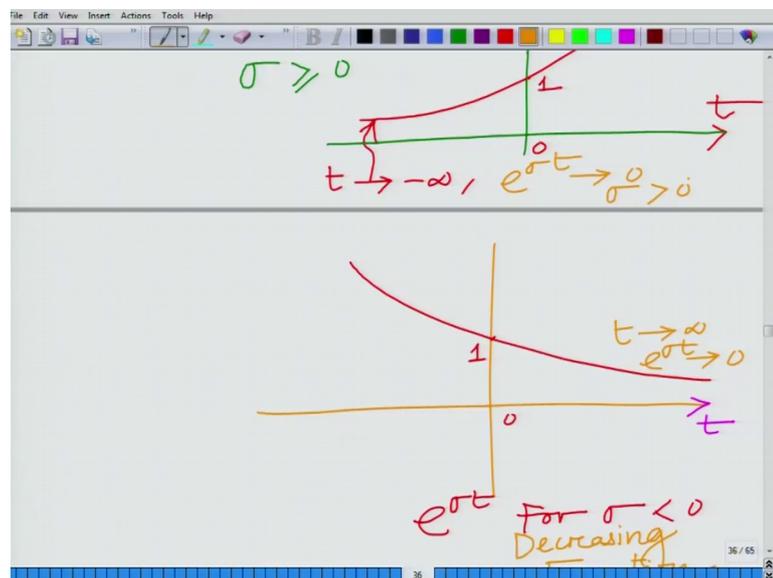
And similarly when  $\sigma$  is less than 0 it is a decreasing signal alright. So, when  $\sigma$  is less than 0.

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It is a decreasing signal. So, once again at  $t$  equal to 0 it is one.

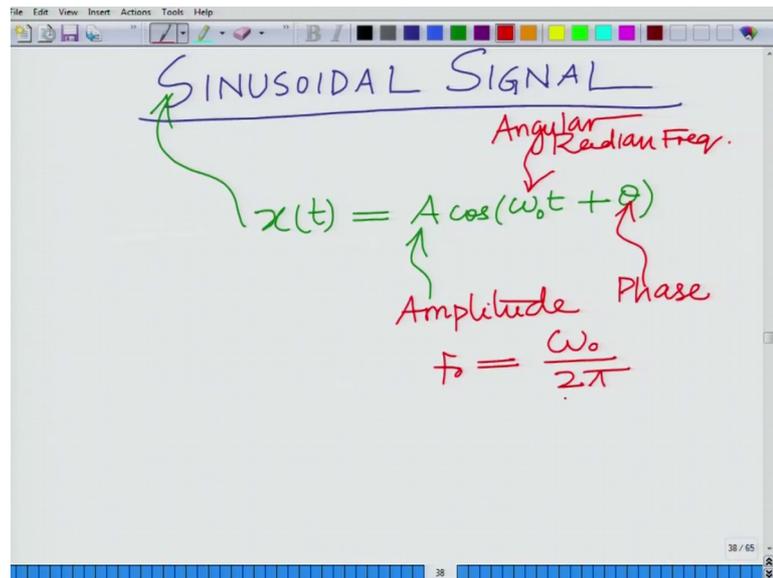
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So, let me just draw it correct it is decreasing at 0 it is one  $e$  to the power of  $\sigma t$  for  $t$  less than equal to 0 for  $\sigma$  less than equal to 0 or for  $\sigma$  less than equal to 0. This is a decreasing signal, this is a decreasing function and that equal to as  $t$  tends to infinity  $e$  to the power of  $t$  tends to  $e$  to the power of  $\sigma t$  tends to 0 similarly here also at  $t$  tends to minus infinity  $e$  to the power of  $\sigma t$   $e$  to the power of  $t$   $e$  to the power of  $\sigma t$  tends to 0, but this is only for  $\sigma$  greater than 0 for  $\sigma$  equal to 0;

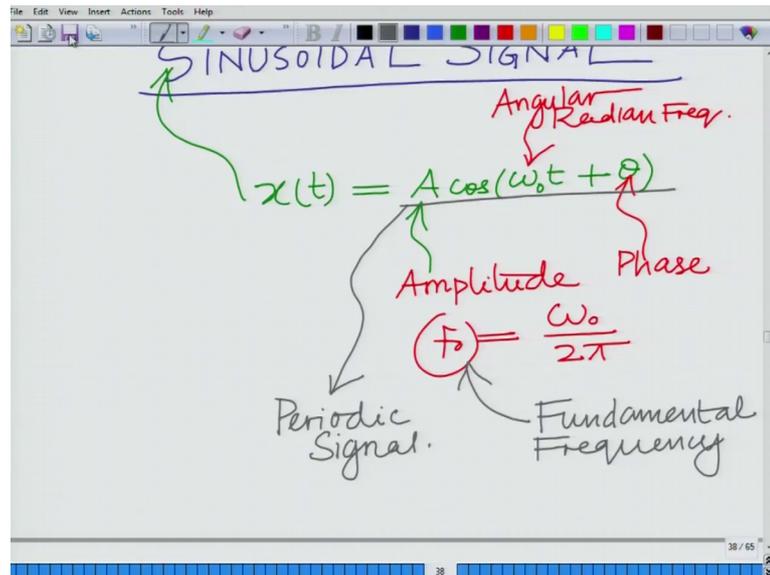
obviously,  $e$  to the power of  $\sigma t$  is going to simply be  $e$  to the power of 0 which is one that is for the case when  $\sigma$  is 0  $e$  to the power  $\sigma t$  is unity. So, for  $\sigma$  greater than 0 it tends to 0 as  $t$  tends to minus infinity for  $\sigma$  less than 0 it tends to 0 as  $t$  tends to plus infinity all right.

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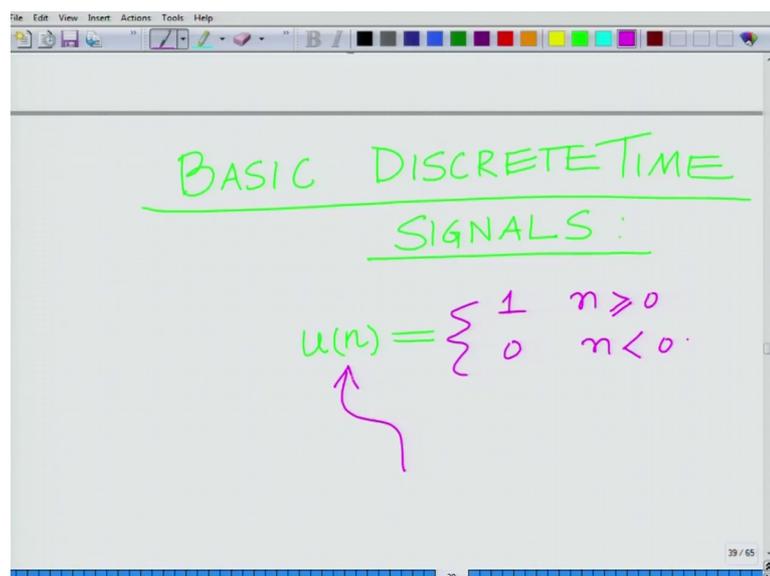
And the other the last class of signals which are also very important and one of the most fundamental classes of signals is the sinusoidal signal simply a real sinusoidal signal we have already seen a complex sinusoid. So, it is a sinusoidal signal or a real sinusoid which is frequently referred to as simply as a sinusoidal signal and this is  $x$  of  $t$  equals  $A \cos(\omega_0 t + \theta)$   $A$  is the amplitude of the sinusoid alright  $\omega_0$  is the angular frequency or radian frequency and  $\theta$  is the phase of the phase offset  $\theta$  equals this is the phase of the sinusoid and the frequency  $f_0$  as we already seen is given as  $\omega_0$  divided by  $2\pi$  and  $\omega_0$ .

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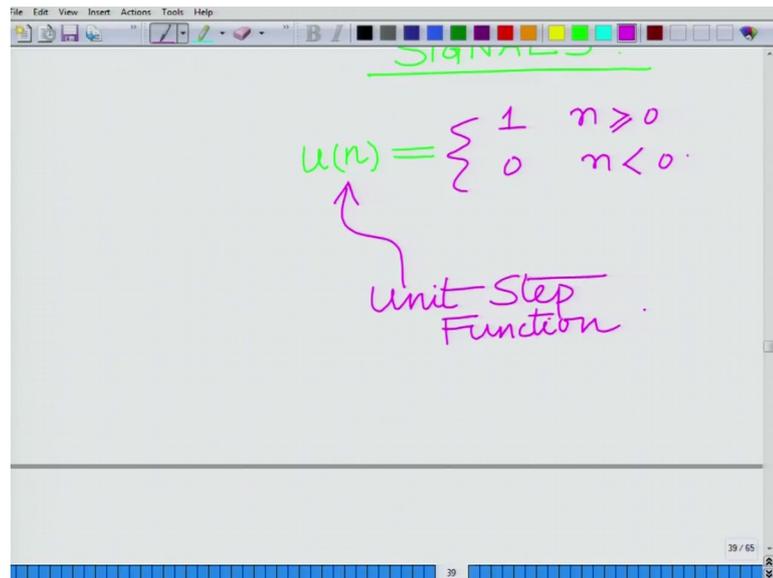
Naught is the  $f$  naught is a fundamental frequency of this the quantity  $f$  naught right correct we have also seen that it can be represented a cosine it can be represented as a cosine  $2\pi f$  naught  $t$  plus theta  $f$  naught is the frequency or it is a fundamental frequency this is a periodic signal of course, the sinusoid it is a periodic signal all right I think anyone should be familiar to that one of the most classic examples of a periodic signal. So, this is periodic signal and  $f$  naught is it is fundamental frequency.

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Now, let us look at some basic discrete time signals which again occur frequently in the analysis of systems some basic some basic discrete time signals again the unit impulse  $u[n]$  equals one for  $n$  greater than equal to 0 for  $n$  less than 0 this is termed as the unit function the unit step function.

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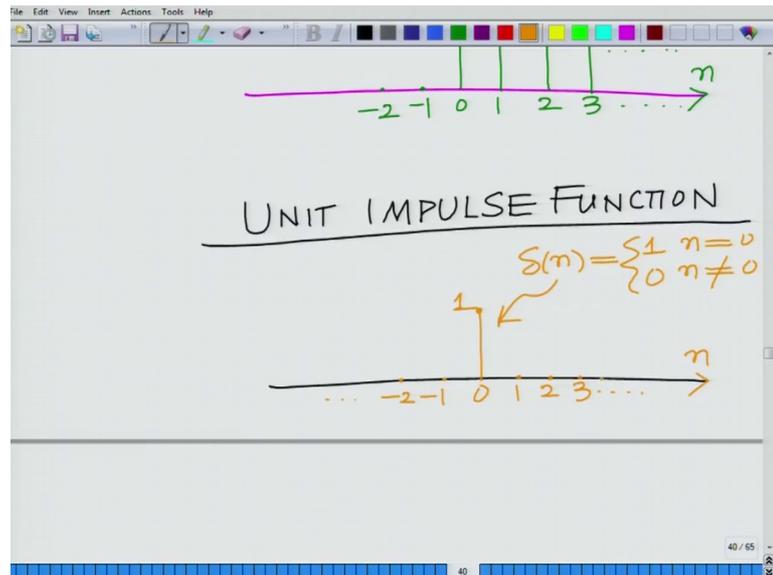
Discrete unit; this is the step function the unit step function alright which is basically if it is a discrete time signal.

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Which means it is defined only at discrete time instants therefore, it is for  $n$  greater than equal to 0 it is one and otherwise it is 0. So, this is 0 for  $n$  less than or equal to 0. So, greater than and less than 0 it is 0 for  $n$  greater than or equal to 0 it is equal to one. So, so. So, this is your unit step function and similarly we have the unit impulse function, all right.

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Delta  $n$  which is one if  $n$  equal to 0 and 0 otherwise. So, we have the unit impulse function similar to the impulse function for the continuous time we have the unit impulse function.

Or the unit impulse signal which is basically the following thing it is one at  $n$  equal to 0 this is  $n$ . So, one at all other values of  $n$  it is 0. So, this unit impulse function which is represented as delta  $n$  equals one for  $n$  equal to 0 for  $n$  not equal to 0. So, this is one for  $n$  equal to 0 for  $n$  not equal to 0 this is the unit impulse function all right.

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$$\sum_{n=-\infty}^{\infty} \delta(n) = 1$$
$$\sum_{k=-\infty}^{\infty} x(k) \delta(n-k) = x(n)$$

And as can be seen, this has also some interesting properties for instance if your sum from minus infinity to infinity delta n this is equal to 1 and further, we can also have the sifting property for this unit impulse function, similar to what we had for the continuous time scenario that is k equal to minus infinity to infinity x of k delta n minus k equals x of n.

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$$\sum_{k=-\infty}^{\infty} x(k) \delta(n-k) = x(n)$$

Sifting property for Discrete Time Signals.

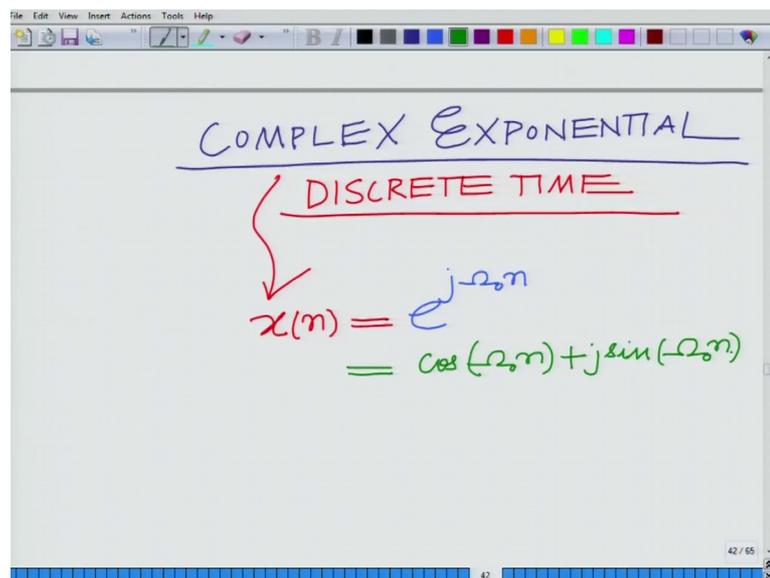
$$\delta(n) = u(n) - u(n-1)$$
$$u(n) = \sum_{k=-\infty}^n \delta(k)$$

This is basically your this is basically the sifting property of the unit impulse function or sifting property for sifting property for discrete time signals the sifting property for

discrete time signals also you can see that  $\delta[n] = u[n] - u[n-1]$  is of course, the unit step function the discrete time unit step signal and  $u[n-1]$  is the unit step signal shifted by 1 all right. So,  $u[n] - u[n-1]$  gives us the discrete time the discrete time impulse of the unit impulse function and similarly  $u[n]$  also alternately  $u[n]$  can be written as summation you can verify this  $k$  equals minus infinity to  $n$  equal to minus infinity to  $n$  delta of  $k$ . So, delta of  $k$  if summation  $k$  equal to minus infinity to  $n$  is the unit function if  $n$  is greater than or equal to zero; obviously, this is going to be equal to one and if  $n$  is less than 0 then summation  $k$  equal to minus infinity to  $n$  of delta  $k$  will be 0 ok.

So, this is again. So, this is again another representation of the unit step function and finally, we have the complex exponential discrete time signal.

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The image shows a whiteboard with the following handwritten text:

COMPLEX EXPONENTIAL

DISCRETE TIME

$x(n) = e^{j\Omega_0 n}$   
 $= \cos(\Omega_0 n) + j \sin(\Omega_0 n)$

We have a or discrete time a discrete time complex exponential which is defined as follows  $x[n]$  is  $e^{j\Omega_0 n}$  where  $\Omega_0$  is again the frequency which is equal to  $\cos(\Omega_0 n) + j \sin(\Omega_0 n)$  and of course.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, a red arrow points to the equation  $x(n) = e^{j\omega_0 n}$ . Below this, it is expanded to  $= \cos(\omega_0 n) + j \sin(\omega_0 n)$ . A green arrow points to the question "Periodic?". To the right, the frequency is defined as  $f_0 = \frac{\omega_0}{2\pi}$ . Below the question, the derivation shows  $e^{j2\pi f_0 n} = e^{j2\pi f_0 (n+N)}$ , which is then split into  $e^{j2\pi f_0 n} \cdot e^{j2\pi f_0 N}$ . A red arrow points to the term  $e^{j2\pi f_0 N}$ , which is underlined and labeled "1.", indicating that the signal is periodic only if this term equals 1.

Now this signal the periodicity of this signal let us look at what is the periodicity of this signal in particular is it periodic we would like to ask the question is this discrete time exponential complex exponential periodic and the answer is slightly complicated although we have seen that the continuous time complex exponential is always periodic and the frequency is the fundamental period is given as one over f not correct or  $2\pi$  over  $\omega$  the discrete time complex exponential is slightly tricky the reason being as follows so.

Let us look at this  $e^{j2\pi f_0 n}$  where  $f_0 = \frac{\omega_0}{2\pi}$  or  $f_0 = \frac{\omega_0}{2\pi}$  this is equal to  $e^{j2\pi f_0 (n+N)}$  where  $N$  is the period equal to  $e^{j2\pi f_0 n} \cdot e^{j2\pi f_0 N}$  now this is periodic only if this quantity equals one that is  $e^{j2\pi f_0 N} = 1$ .

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$$e^{j2\pi f n} = e^{j2\pi f n (\dots)}$$
$$= e^{j2\pi f n} \cdot \frac{e^{j2\pi f n}}{1}$$
  
$$e^{j2\pi f n} = 1$$

Periodic if

$$f n = k$$
$$\Rightarrow f_0 = \frac{k}{n}$$

Equals one. So, this is periodic if this quantity equal to one which means  $f$  naught  $n$  must be equal to an integer  $f$  naught  $n$  must be equal to integer in which case this will be  $e$  to the power of  $j 2 \pi k$  which is equal to unity. So,  $f$  naught  $n$  must be equal to any integer  $k$  which means  $f$  naught equals  $k$  over  $n$ . So, it must be a rational number. So, the condition for periodicity.

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The image shows a digital whiteboard with the following handwritten content:

$$e^{j2\pi f n} = 1$$

Periodic if

$$f n = k$$
$$\Rightarrow f_0 = \frac{k}{n}$$

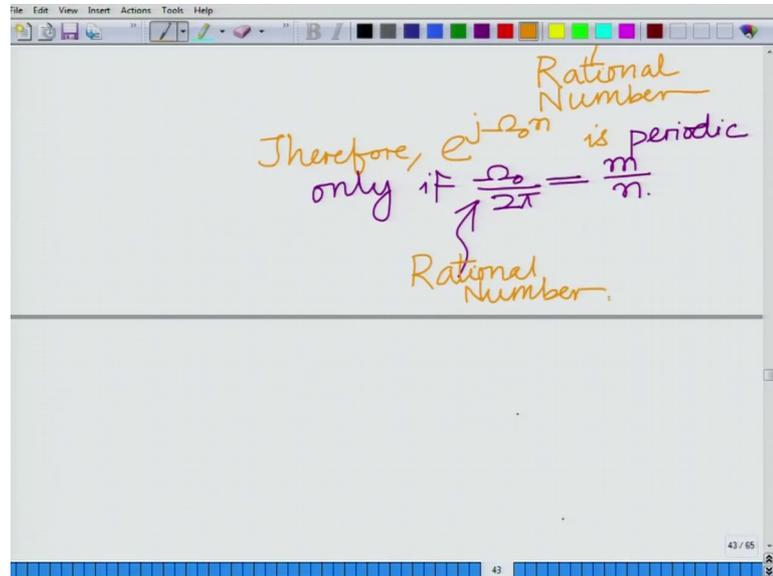
Rational Number

Therefore,  $e^{j\omega_0 n}$  is periodic only if  $\frac{\omega_0}{2\pi} = \frac{m}{n}$ .

Of the discrete time impulse is that  $f$  naught must be equal to  $k$  over  $n$  which is a rational number therefore,  $e$  to the power of  $j \omega_0$  naught  $n$  is periodic only if  $f$  naught that is

$\omega$  naught by  $2\pi$  equals some  $k$  over  $n$  or some  $m$  over  $n$  some  $m$  over  $n$  that is this has to be.

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A rational number. So, that is an interesting aspect of the comb of the discrete time complex exponential while the complex exponential for the continuous time is always periodic the discrete time complex exponential is periodic only if its frequency is a rational number it is frequency is rational number or the angular frequency  $\omega$  naught divided by  $2\pi$  is a rational number ok.

So, this basically completes our characterization of classification of signals where we have looked at a broad set of signals which frequently arise from practice of course, there are several other signals because it is not possible to categorize entire set of signals; however, we have managed to classify a fairly large set of frequently occurring signals or frequently or very frequently occurring signals very frequently occurring signals in practice the properties of the properties of these signals are important to understand because these arise frequently in practice and these are important to understand the principles of signals and principles of analysis of signals and systems alright. So, we will stop this module here and look at other aspects in the subsequent modules.

Thank you very much.