Principles of Signals and Systems
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Lecture – 33
Fourier Analysis of Continuous Time Signals and Systems – Introduction

Hello. Welcome to another module in this Massive Open Online Course. So, we are looking at the various properties and principles of signals and systems in this course and we have so far looked at various transforms including the Laplace transform and the z transform meant in this module starting this module let us start looking at yet another transform. In fact, one of the most key I would say a critical transforms, that is used overwhelmingly frequently to understand and analyze the properties and signals and systems, which is the Fourier transform. So, we will start by looking at the Fourier transform for continuous time signals and subsequently also for discrete time signals and systems.

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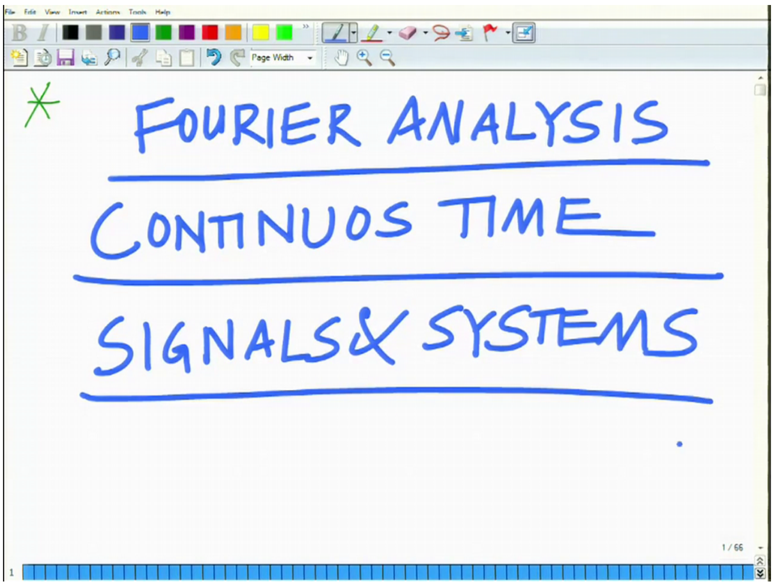

So, starting this module what we want to look at is we want to look at new transform; we want to look at the Fourier analysis. The Fourier analysis the Fourier transform for first for the continuous time signals and system. So, and remember this is basically its transform now if you look at the Fourier transform.

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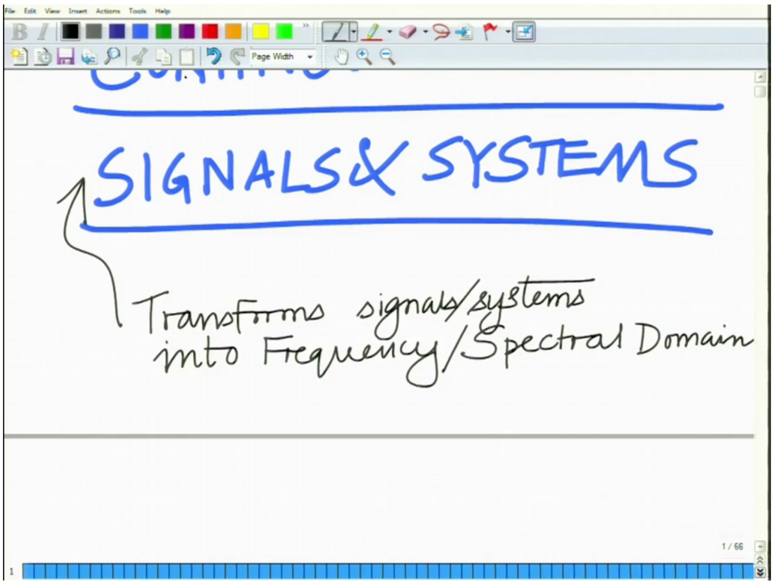

This is basically the transform it transforms signals or systems transform signals and systems into the frequency or spectral domain, and this is or what is also known as the spectral the frequency spectrum ok.

So, this is transforms the signal or a certain system, which can also be represented as we know a system can also be represented in the transfer in the transform domain through that transfer function alright. So, this Fourier transform basically transforms signals and systems all right into the frequency domain; what is known as the frequency domain in terms of its frequency components or also the spectral domain.

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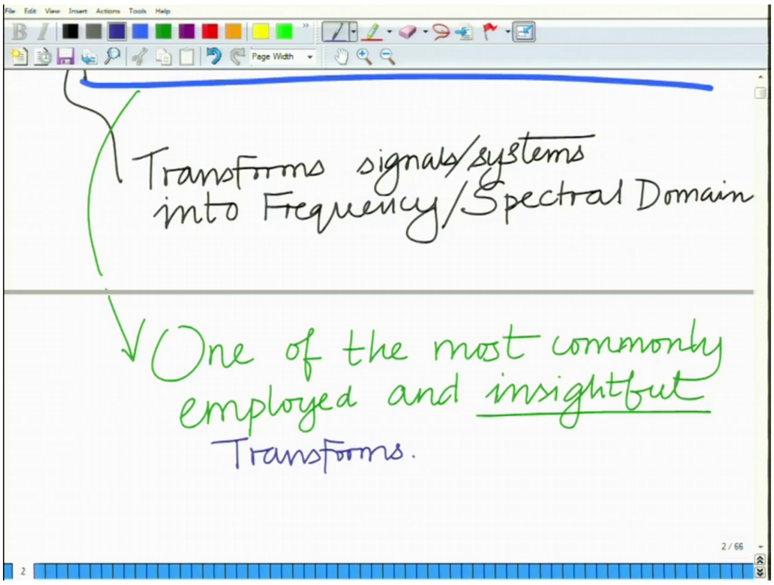

And the Fourier transform needless to say is one of the most commonly employed and insightful transforms, and one of the most commonly nor and not just in electronics and signal processing.

But in various fields as well various fields of science and engineering its one of the most commonly employed and more importantly not just employed, its one of the most insightful that is the key word here one of the most insightful transforms. Inside by insightful I mean that evaluating the Fourier transform and examining the Fourier transform of a signal or a system, yields immediately yields valuable insights which are not possible otherwise for instance by evaluating the transfer function the Fourier transform of a system you can determine, if the signal is a high pass system or a low pass system or a band pass system similarly for a signal, you can see if it is a high frequency signal low frequency signal of or if it contains frequencies in a certain band.

So, it is one of the most convenient and one of the most insightful transforms and it is overwhelmingly and widely employed in across several fields of science and engineering it is one of the most insightful transforms. And the basis for the Fourier transform is the very simple function consider the complex

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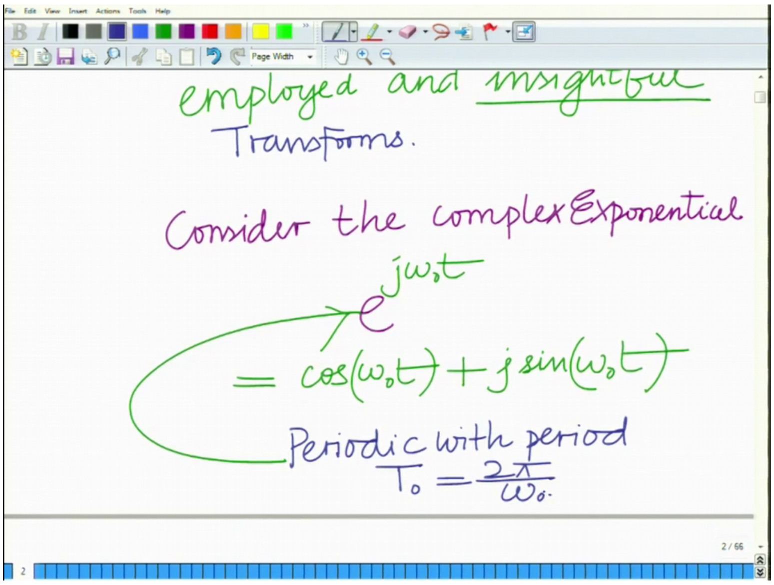

Exponential something that we already seen this forms the basis for the Fourier transform consider the complex exponential signal, that is we have e to the power of the complex exponential remember is something that we have seen very the initial modules, e to the power of j omega naught t, this is cosine omega naught t plus j sine omega naught t, and this is periodic with period and remember this is periodic with period T naught equals 2 pi over omega naught.

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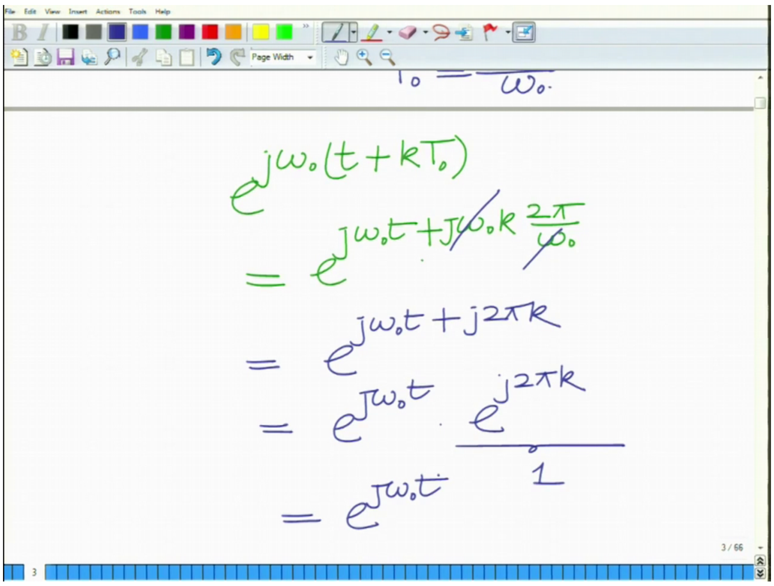

And this can be seen as follows for instance if you consider e to the power of j omega naught into t times ej e raised to j omega naught times t plus k T naught, this is equal to e raised to J omega naught t plus J omega naught k times T naught is 2 pi by omega naught.

So, these omega naught cancel and what is remaining is e erased to J omega naught t plus J 2 pi k which is e raised to J omega naught t times e raised to J 2 pi k remember he raised to J 2 pi k equals 1 which is basically e raised to J omega naught t. So, this is periodic with period t naught, T naught equals 2 pi over omega naught.

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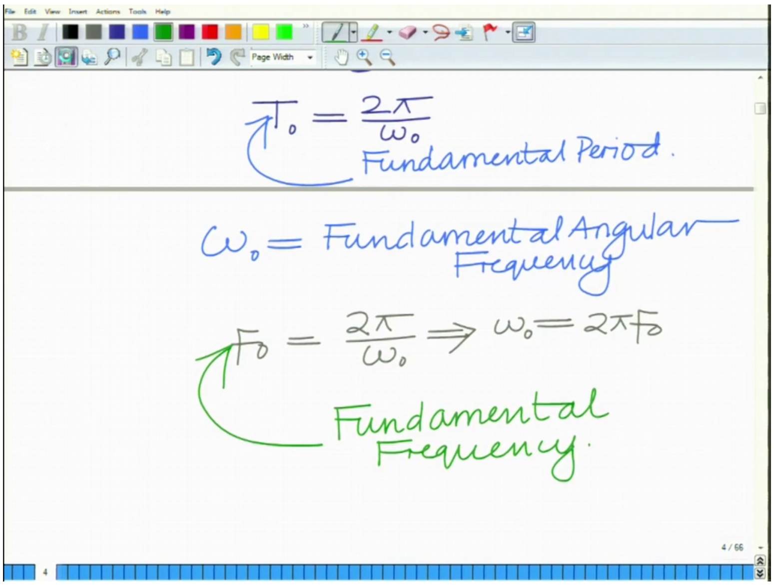

So, T naught equals 2 pi omega naught this is the period or the fundamental period of the signal also known as the fundamental period ok.

This is the period of the fundamental period and omega naught now this quantity, omega naught is the angular frequency in fact, this is the fundamental. This nomenclature will become clear slightly later, this is the fundamental angular frequency. And if you consider F naught remember F naught equals 2 pi by omega naught or basically this implies omega naught equals 2 pi F naught, this is known as the fundamental frequency ok.

So, we have a complex exponential e raised to j omega naught t that is speed that is periodic with T naught the fundamental period T naught equals 2 pi over omega, omega naught omega naught is the fundamental angular frequency if not equals omega naught over 2 pi or omega naught equals 2 pi f naught f naught is the fundamental frequency all right. And now let us look at the complex exponential let us look at what is known as the complex exponential or the Fourier series representation.

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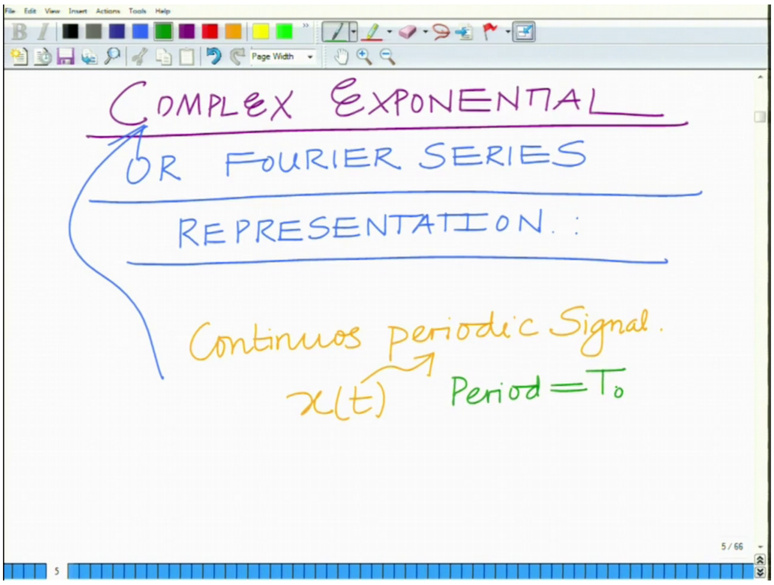

So, what we want to look at now is the complex exponential or basically a Fourier series representation. Complex exponential or Fourier series representation and this is defined for continuous periodic signal remember this kind of representation for a continuous.

So, this is defined for a signal x t, which is basically continuous and periodic and let us say the period is T or let us say period equals T naught, it is also called this as fundamental period.

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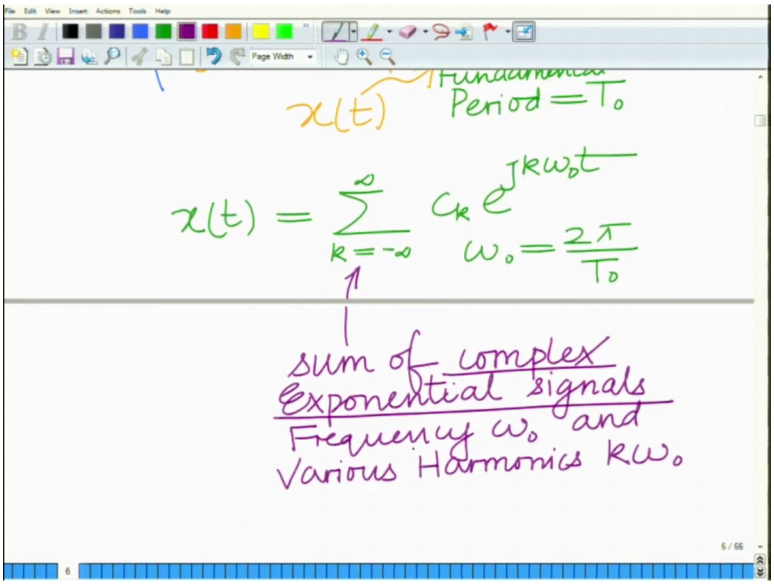

Let us call this as a fundamental period the fundamental period equals T naught. So, we have a periodic signal x t with fundamental period T naught, then it says that, this x t can be represented as the sum of complex exponentials. That is if x t is a continuous signal and periodic with fundamental period t naught, then it can be represented as the sum of complex exponentials as follows and that is basically the Fourier series representation. So, this x t can be represented as summation k equals minus infinity to infinity, C k e raised to J k omega naught t.

So, it is can be represented as the sum of complex and remember omega naught equals 2 pi by T naught. So, what we have is basically remember x t can be expressed as a sum of complex exponentials. So, we are expressing it as some of complex exponentials, at the fundamental frequency omega naught that is exponentials e raise to j k omega naught r at the fundamental frequency omega naught and k a times omega naught, these are known as the harmonics that is their frequencies are basically multiples of the fundamental frequency.

So, they are at the fundamental frequency omega naught, and multiples of the fundamental frequency omega naught that is k times omega naught or the various harmonics corresponding to omega naught; and x t can be expressed as sum of such complex exponential signals. So, x t can be expressed as the sum of complex some of complex exponential signals come up at the frequency omega naught and the various harmonics at k omega where k is; obviously, an integer that is harmonics at frequency omega equals k omega naught ok.

So, it can be expressed as a sum of complex exponential signals, at the fundamental frequency omega naught and the various harmonics, which correspond to the frequencies that are integer multiples of omega naught it is k omega naught; and x t can be expressed as the sum of an infinite number of such complex exponentials. And this is a very important property and as we will see this yields a very convenient, and as a very tractable is a very tractable and a convenient method for representation of a periodic signal that yields various insights about its properties and behavior. Now these coefficients are the C ks these are the Fourier series coefficients.

So, these are the Fourier coefficients the C ks.

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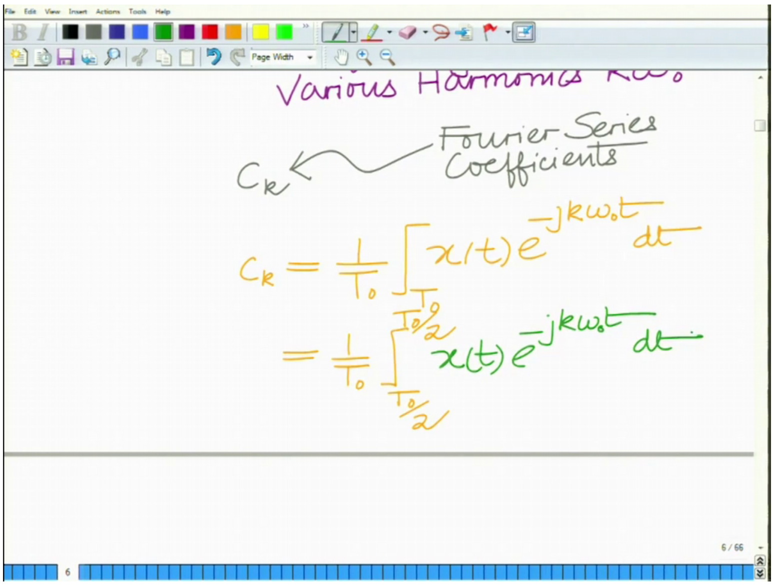

These are the Fourier series coefficients these are the Fourier series coefficients corresponding to of course, the harmonic that is e raise to j k omega naught. And this cks can be obtained as follows C k equals 1 over T naught integral over any fundamental period T naught over any duration that spans T naught contiguous duration, x t e raised to minus j k omega naught dt. And this can be any interval of length T naught for instance it can be minus T naught by 2 to T naught by 2.

So, this can be minus T naught by 2 to T naught by 2 x t e raised to minus j k omega naught t d t or this can be 0 to t naught in fact, it can be any contiguous duration.

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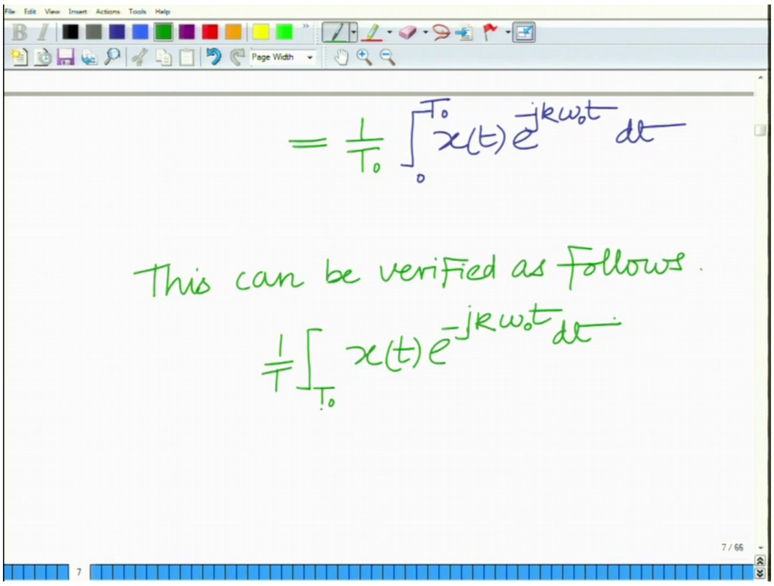

These are just some things that you can typically employ for contain for the sake of convenience x t e raise to minus g k omega naught t dt. So, this can be integral from 0 to T naught minus T naught by 2 to T naught by 2 or any contiguous interval of spanning of width t naught. In fact, this can be T naught to 2 T naught or it can be T naught by 2 to three T naught by 2. So, it can be any continuous duration spanning the spanning of length T naught and this can be seen as follows this can be readily seen as follows this can be verified as follows now consider C k or consider integral over t not or consider let us say 1 over T naught x t e raised to minus j k omega naught dt, now I can.

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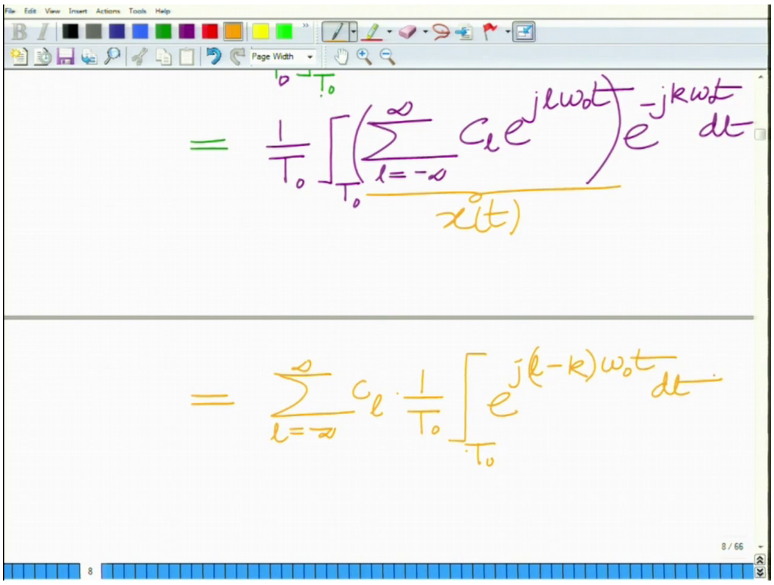

Now, first what I am going to do here is I am going to substitute evaluate over interval of duration T naught, I am going to substitute the expression for x t, which is I am using a different index l equals minus infinity to infinity, C l e raised to j l omega naught t, I am using a different index l. So, that I do not confuse with k times e raised to minus j k omega naught t dt remember all I am doing is substituting the expression for x t. Now I am going to interchange the integral and summation.

So, I will bring the summation outside, I am going to take the integral inside. In fact, this 1 over T naught this 1 over T naught is also constant factor. So, I can also move it inside and outside the integral all right. So, that does not create any problem. So, this is going to be summation l equals minus infinity to infinity C l times 1 over T naught integral 1 over T naught integral, now you can see I can combine these 2 exponential terms integral over T naught e raise to j l minus k omega naught t dt.

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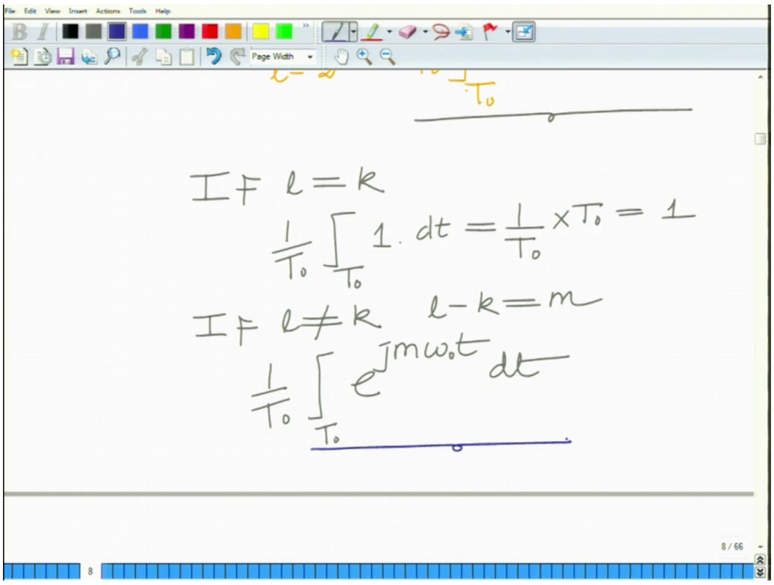

And now, consider look at this integral, I want to draw your attention towards this integral. Now 1 over T naught e raised to J l minus k omega naught t dt, integral over duration T naught. Now if l equals k this integral becomes 1 over T naught l, l minus k 0 if l equal to k. So, this is e raised to 0 which is one 1 over T naught or integral of duration T naught 1 dt, which is 1 over T naught into T naught, which is equal to 1. Now if l is not equal to k then this becomes 1 over T naught let us say l minus k equals m. So, this becomes 1 over T naught over integral of duration T naught correct e raised to J m omega naught t dt, this is a integral over integral of a harmonic over one fundamental period. So, this is integral this integral of a harmonic.

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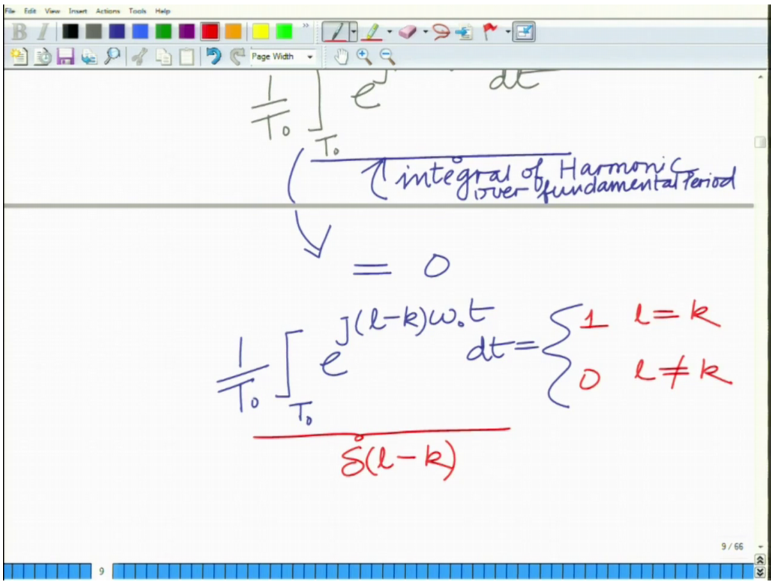

Now, remember corresponding to the fundamental frequency, any harmonic is also periodic 50 naught. That is if the complex exponential e raise to j omega naught t has fundamental period T naught, then T naught is also a period although not a fundamental period T naught is also a period of e raised to j m omega naught t, m omega naught t alright. So, when you integrate this over one fundamental period, when you integrate this complex sinusoid over one fundamental period the integral vanishes it is like taking a sinusoid or taking a sinusoid integrating over a multiple of its period, the integral vanishes therefore, this integral is 0 if l not equal to k which is equal to 0.

And therefore, as a result what you will have is that this integral 1 over T naught over a duration T naught e raised to J l minus k omega naught t dt, this is equal to 1 if l equals k 0 if l is not equal to k this is the integral over fundamental period this is vanishes and therefore, you can also say that, this this quantity is basically nothing, but delta l minus k. That is this is equal to 1 if l equal to k, l minus k is 0 there is a discrete delta function or this is equal to basically this is equal to 0 if l is not equal to k therefore, what we are going to have here

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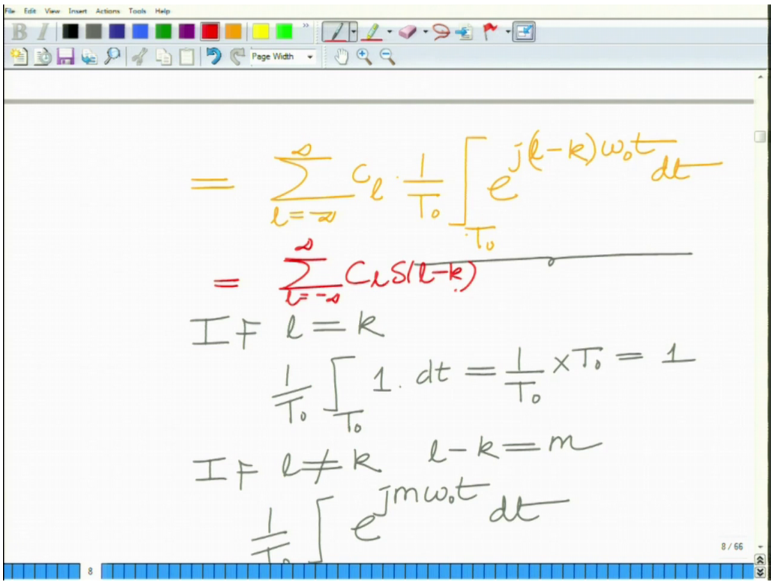

is basically if you substitute this here you will have summation l equal to minus infinity to infinity C l delta l minus k.

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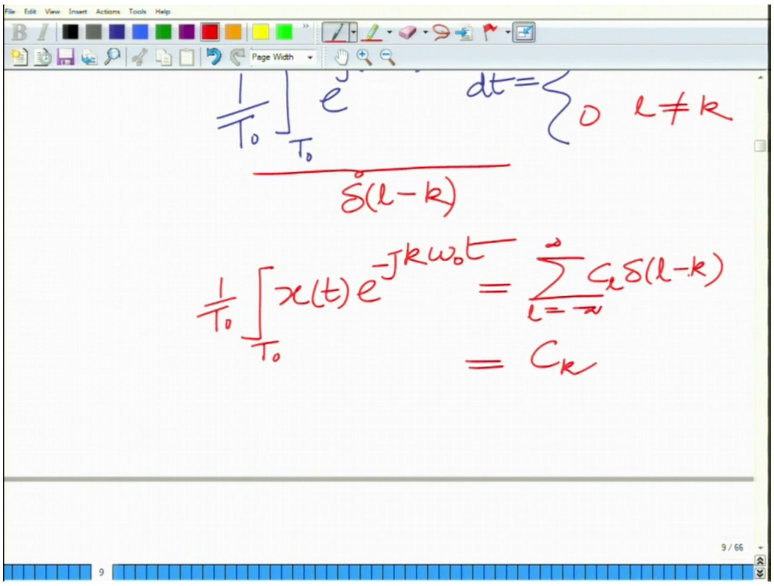

So, this is so, we have that we just write it 1 over T naught x t e raise to minus J omega or e raise to minus J k omega naught t dt, e raised to minus jk omega naught t dt, this is equal to summation l equal to minus infinity to infinity C l delta l minus k well delta l minus k is 1. If l equals k is 0 if n is not equal to k therefore, the result of this summation is C k because for all other l delta l minus k equal to 0 it only survives when l is equal to k.

So, this is equal to C k and therefore, we have demonstrated that basically we have C k the kth Fourier coefficient can be evaluated as 1 over T naught, integral over T naught xt e raised to minus J k omega naught dt.

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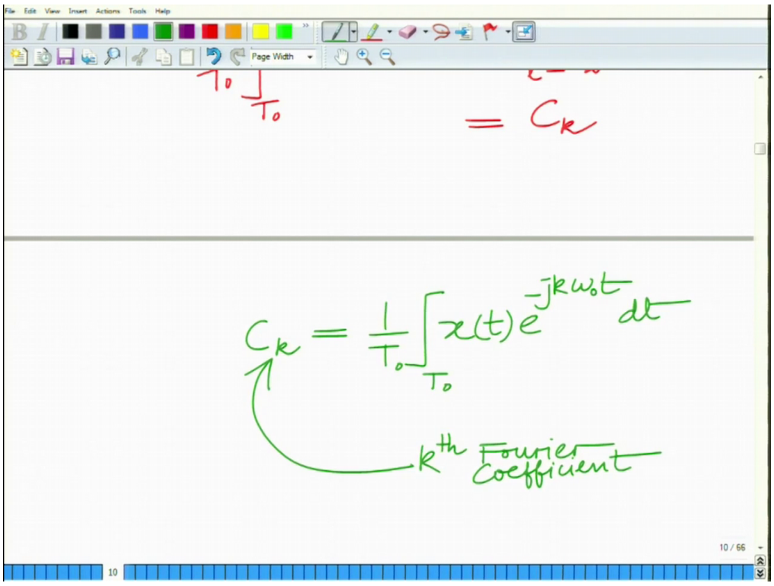

This is the kth Fourier coefficient corresponding to the complex exponential, e raise to j corresponding to the complex exponential e raise to j k times omega naught t that is basically corresponding to the kth harmonic at the frequency k times omega naught, where omega naught is the fundamental angular frequency and of course, if you look at c naught let us say k equal to 0 here.

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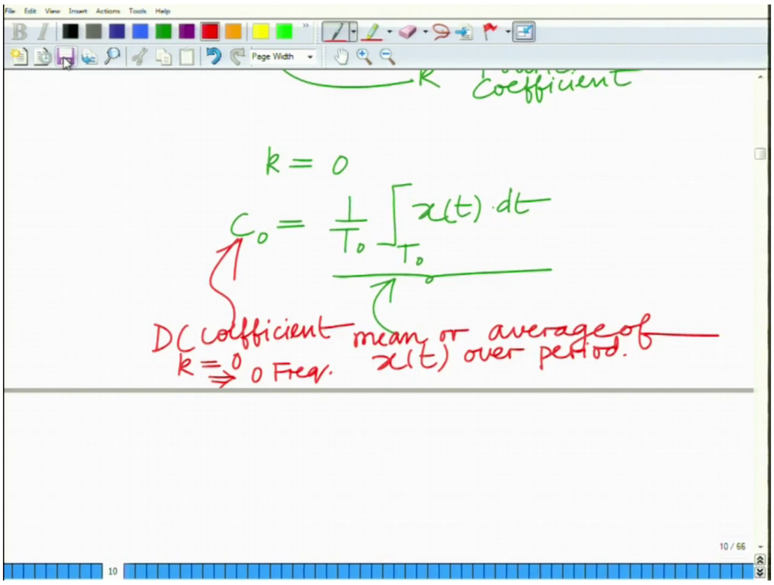

C k or C 0 in fact, this is equal to C oequals 1 over T naught integral of duration T naught x t e raised to j k omega naught is simply e raised to 0 that is 1 times dt and if you can see this, this is simply the mean or average of xt over the interval t naught.

This is mean or average of x t or one period, that is T naught and this C naught corresponds to zero frequency, that is k equal to 0 this is also known as the DC coefficient corresponds to the zero frequency. So, that corresponds to the zero frequency. So, this is also known as the dc coefficient and basically that is your C naught all right.

So, what we have done in this module is, we have introduced this new concept or we are starting our discussion on Fourier analysis the Fourier analysis of continuous time signals and continuous time signals and systems are in particular we are starting with the continuous time with a continuous time periodic signal with fundamental period, T naught and we have looked at the Fourier series expansion of that and how to derive the coefficients in the Fourier series of x alright. So, we will stop here and continue in subsequent modules.

Thank you very much.