Principles of Signals and Systems
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Lecture - 03
Energy/ Power Signals, Unit Impulse Function, Complex Exponential

Hello. Welcome to another module in this massive open online course. So, let us continue our discussion on the classification of signals let us look at energy and power signals ok.

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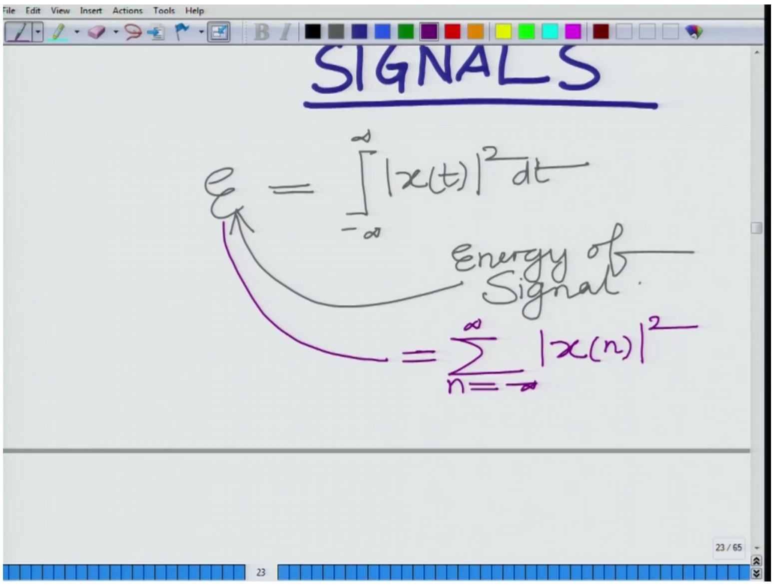

So, what we want to look at is the concept of energy and power signals and the energy of a signal is defined energy of a signal xt continuous signal x t is defined as well minus infinity to infinity magnitude x t square dt. This is the definition of an energy of the signal and the energy of a discrete time signal is well summation n equal to minus infinity to infinity magnitude xn square. This is the energy of a discrete time signal and a signal is an energy signal.

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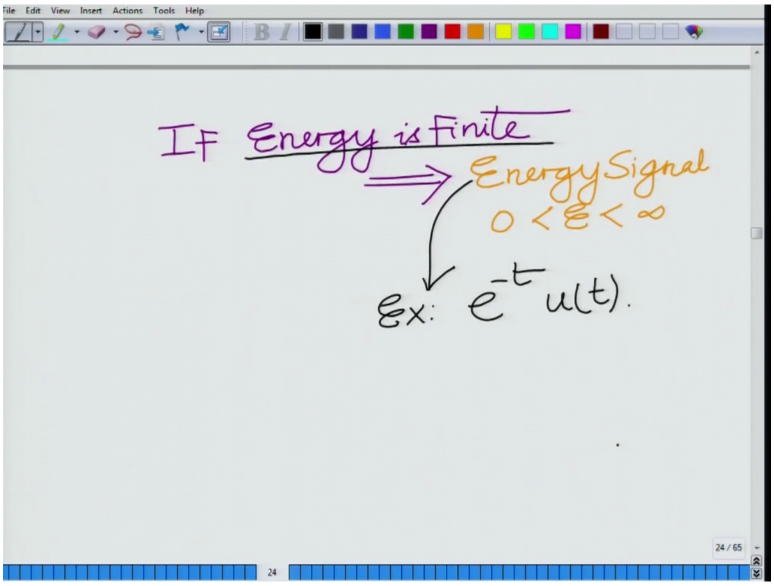

If the energy is finite. So, if the energy is finite this implies that is if it has a finite energy this implies that it is an energy signal. So, this implies that it is a; which basically implies that 0 less than e less than infinity for instance you can check.

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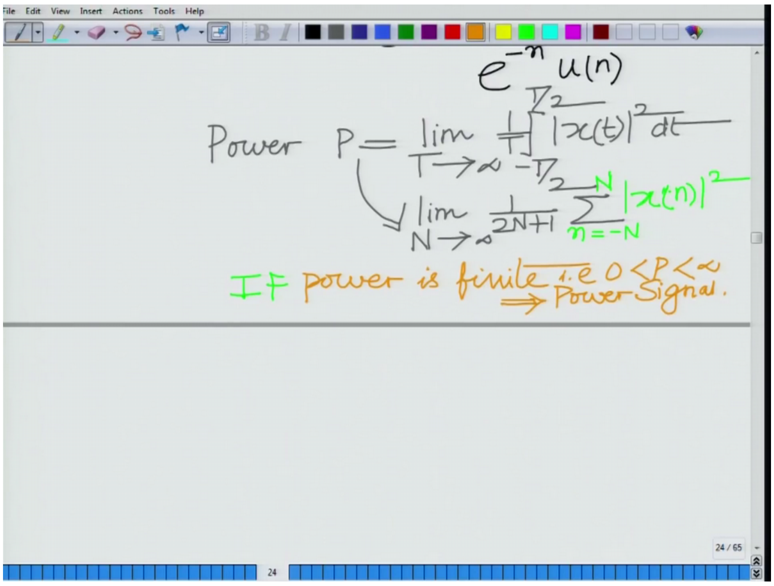

If the energy is finite for instance you can check that a good example of an energy signal is e to the power of minus t ut e to the power of minus e tu t, its energy is finite or for that matter e to the power of minus n or for that matter half to the power of n or e to the power of minus n u of n this is a this is a these are energy signals that is signals whose energy energies are finite ok.

And we also have the notion of power signals that is the power let us first define the power p of a signal equals well this equals limit t tending to infinity minus t over 2 to t over 2 one by t that is dividing by t magnitude x t square dt and the same thing can be defined for digital signal that is limit n tending to infinity one over a discrete time signal one over 2 n plus one summation n equals or summation n equals minus n 2 plus n magnitude of magnitude of xn square. This is the power of a discrete time signal and if the power is finite if the power of the signal that is continuous time signal or discrete time signal is finite then it is known as a power signal. So, if that is 0 less than p less than infinity this implies; it is a power signal. So, this implies it is a power signal and a classic example of a power signal.

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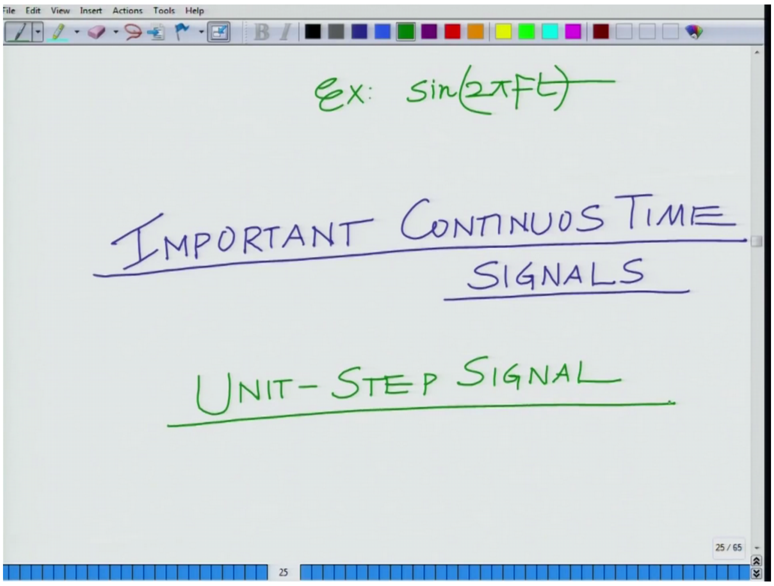

Again is example of power signal is again sin 2 pi f t the sinusoid is a classic example of a power signal now with that we come to the end of the classification of signals. So, we have covered most of the major classes of signals now let us look at some important continuous and discrete time signals which occur very frequently alright its essential to understand some of these signals because these occur very frequently in applications and in the analysis of signals and systems.

So, what we are going to look at next is basically some important continuous important continuous time signals for instance the first important signal is what you must be very familiar with which is the unit step signal which you also been using several points.

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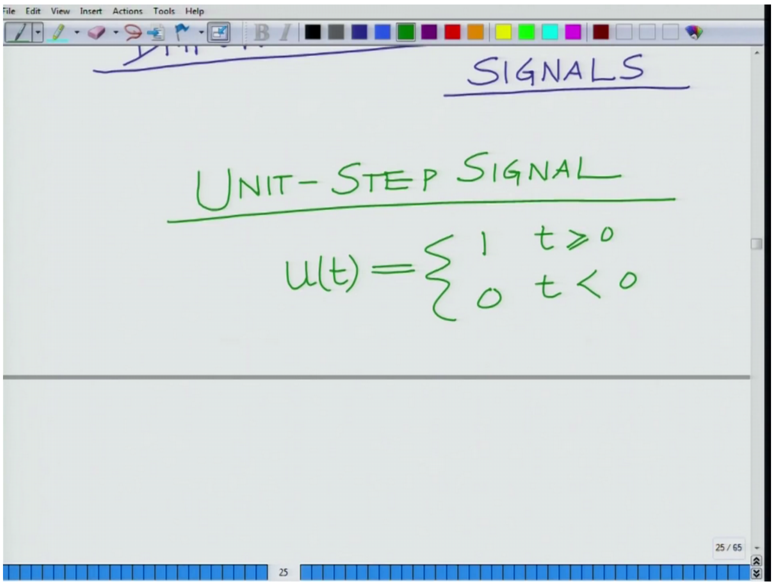

Before the unit step signal is defined as its represented as u t and its defined as one for t greater than equal to 0 0 for t equal to 0 and also for t equal to 0 you can sometimes define it as one half alright. So, it is defined as 1 for t greater than equal to 0; 0 for t less than 0 and; obviously, you can see at t equal to 0. There is a discontinuity there is a jump discontinuity ok.

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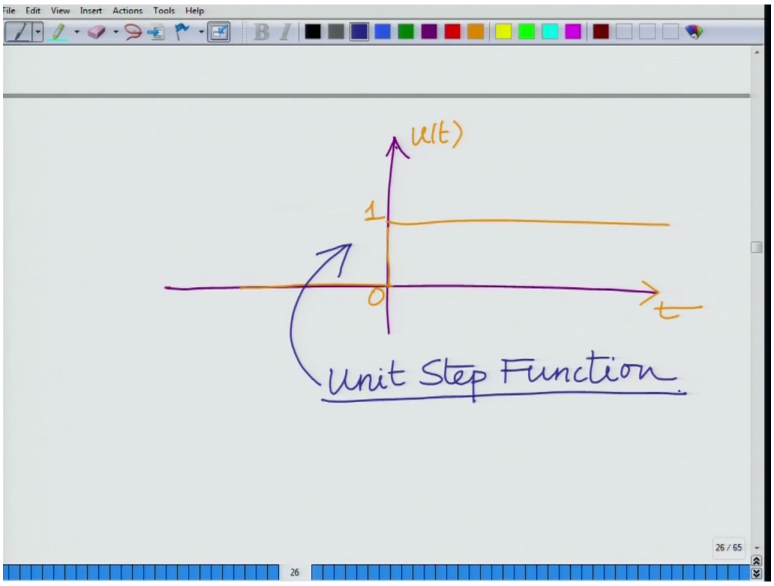

So, this is basically your unit this is basically your unit step signal which looks something like this. So, this is time equal to 0, it jumps from 0 to 1. So, this is your unit step function this is also known as this is the unit step signal or unit step function also known as the unit step also known as the unit step function ok.

And another such signal important signal is the unit impulse function this is the unit.

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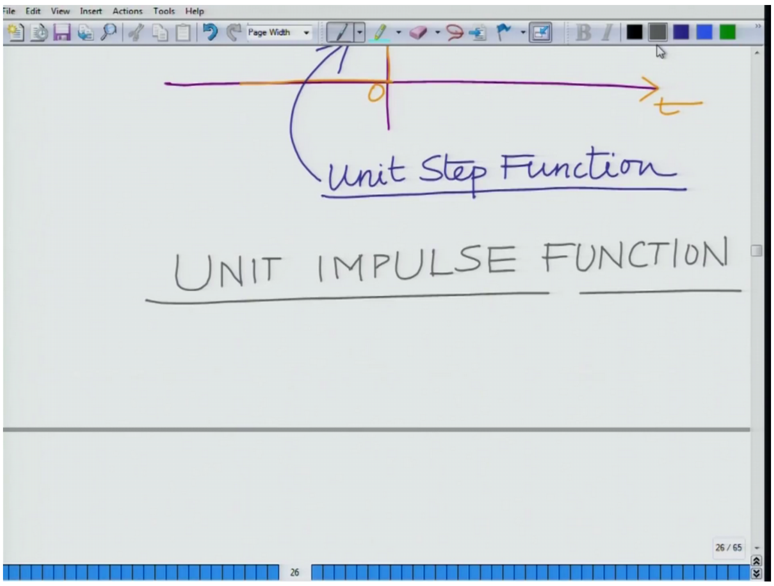

And this is one of the most fundamental one of the most interesting and one of the most; I would say, key signals to understand the various properties or the behavior or understand the properties and model the behavior of system and its definition as is as follows.

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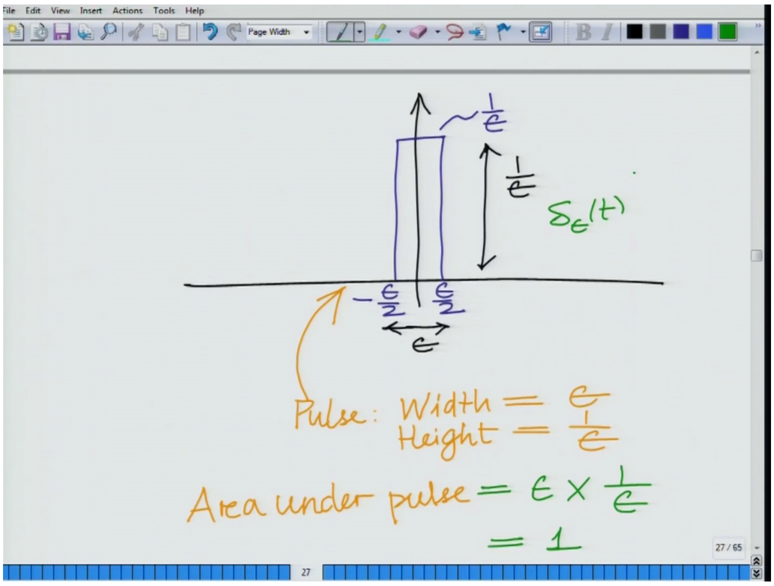

Consider the following signal which is a pulse which is a pulse well which is basically it is a pulse, it looks as follows the pulse from minus e by epsilon by 2 to epsilon by 2 and is of height one over epsilon. So, the pulse has. So, the width equals epsilon and a height equals one over epsilon. So, consider this pulse. So, this is a pulse of width equals well epsilon height equals one over epsilon. So, that the area under the pulse.

Equals will the area under the pulse equals epsilon into 1 over epsilon equals 1. So, this is the area under pulse now consider. So, basically we denote this pulse by delta epsilon t. So, this is a sequence of pulses 1 pulse for each value of epsilon. So, we are considering a narrow pulse with width epsilon from minus epsilon by over 2 to epsilon over 2 and of height 1 over epsilon.

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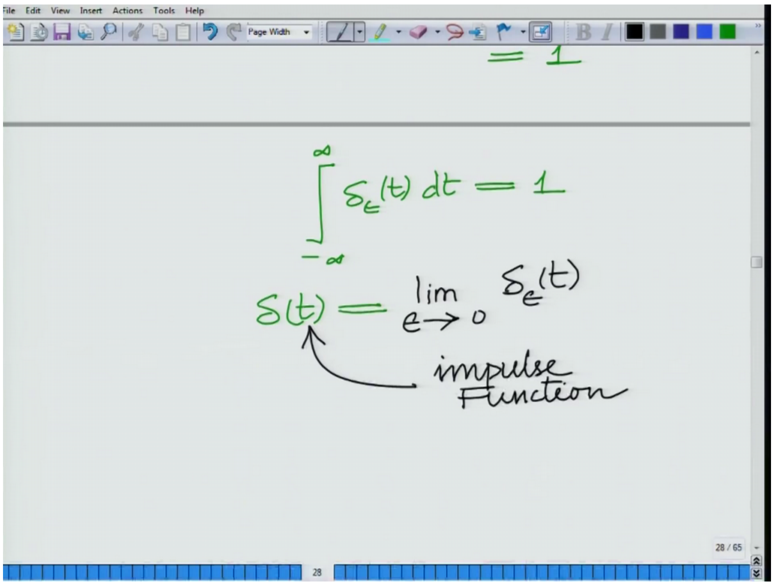

So, the area under the pulse is unity. So, basically what we have is for each pulse delta epsilon t for every epsilon we have the area under pulse equal to 1.

Now, we define the pulse delta t as basically limit of epsilon tends to 0 of delta epsilon t and this is your impulse; impulse function or simply known as the impulse that is as epsilon tends to infinity the width epsilon goes to 0 epsilon tends to 0 width.

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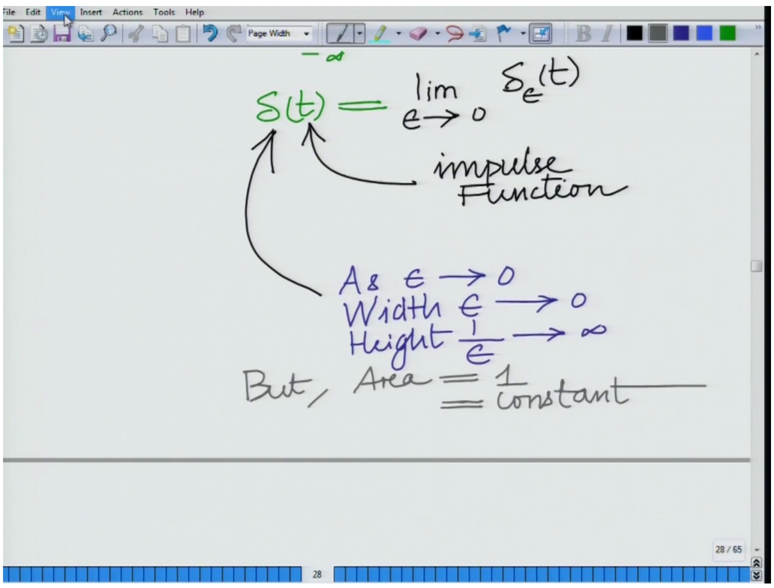

Epsilon goes to 0 height 1 over epsilon tends to infinity, but the area under it still remains constant that is unity. So, as epsilon tends to infinity as epsilon tends to 0 the width of the pulse epsilon tends to 0 the height one over epsilon tends to infinity, but the area is equal to 1 which is a the area is always equal to 1 which is a which is a constant. So, basically what you can see is that the area under this in fact.

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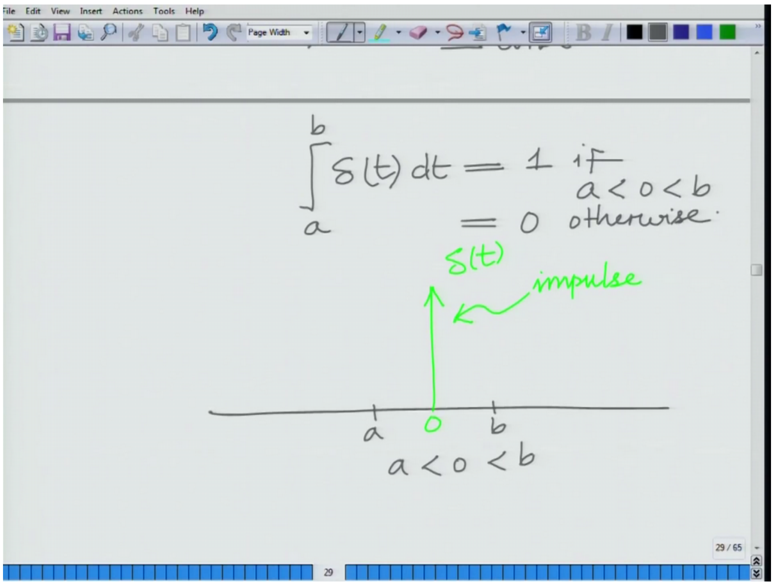

If you look at any integral a to b delta t dt equal to one if 0 if a is less than 0 less than b.

That is you have your impulse which is denoted by let us just draw the impulse at 0. So, this is your impulse this is your impulse that is delta t and if you integrate it over any interval a comma b such that a less than 0 and b is strictly greater than 0 that is if you consider its integral over any interval a to b such that a lies to the left of 0 and b lies to the right of 0 the integral from a to b of the impulse is always the unity; obviously, if either both a and b lie to the left of 0 or both a and b right at the right of 0 that is any interval which does not include the support of the impulse then the integral is 0.

so we have this is this integral is equal to 0 otherwise and if either a or b is equal to 0 then the integral is not defined it is not proper. It is not defined its undefined that is something that is to keep in mind and this has a very interesting as I have already told you it is a very interesting signal and it has several interesting properties for instance one thing that you can show here is that.

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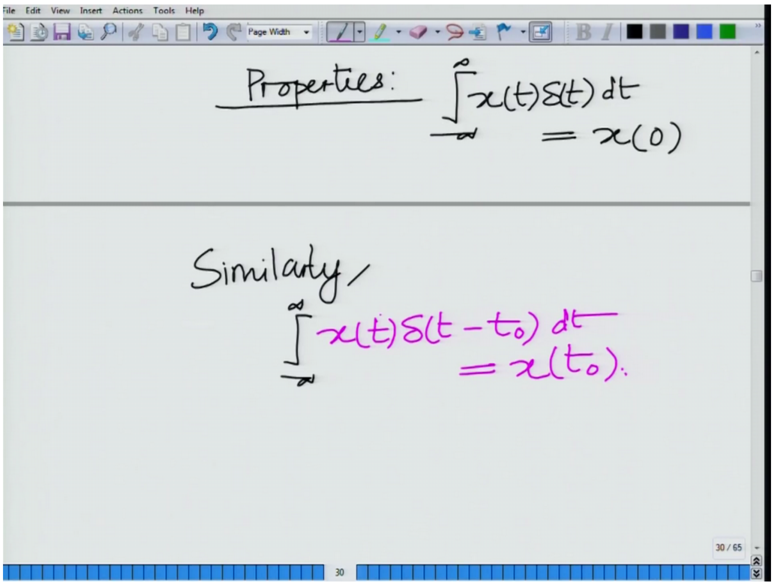

Integral; its properties are as follows integral minus infinity to infinity x t delta t dt. This is equal to x of 0 that is if you multiply delta t impulse function by any signal x of t and integrate it from minus t to t it picks the value of the function x of 0 that is value of the function xt at t equal to 0 now similarly integral minus infinity to infinity x x of x of t times delta t minus t naught into dt equals x of t naught.

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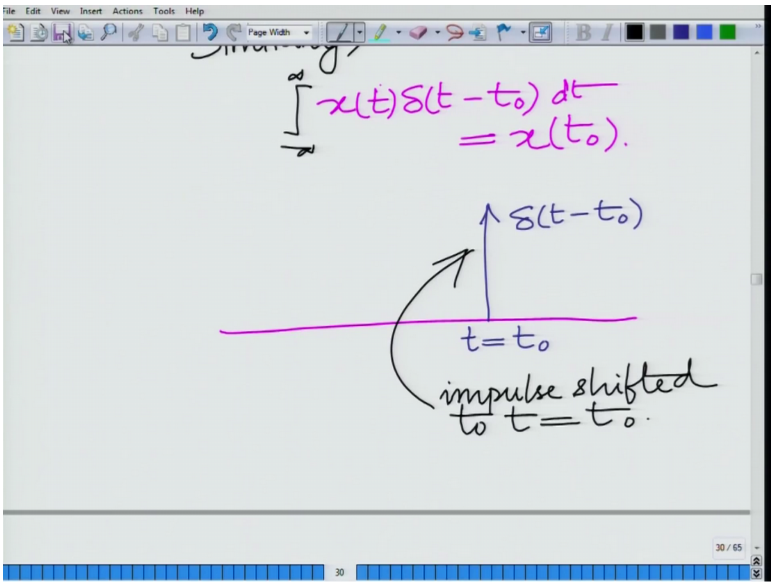

Now, remember delta t minus t naught is nothing, but basically your impulse shifted that is delta t minus t naught equals denotes your impulse that is shifted to t equal to t naught impulse shifted to t equal to t naught. So, if you multiply this by x of t delta x of t into delta t minus t naught and integrate it from minus t infinity to infinity what we have is the signal x at t naught that is x of t naught ok.

Now, there are several other interesting properties of impulse function for instance the other some of the other properties which you can also show for the impulse function.

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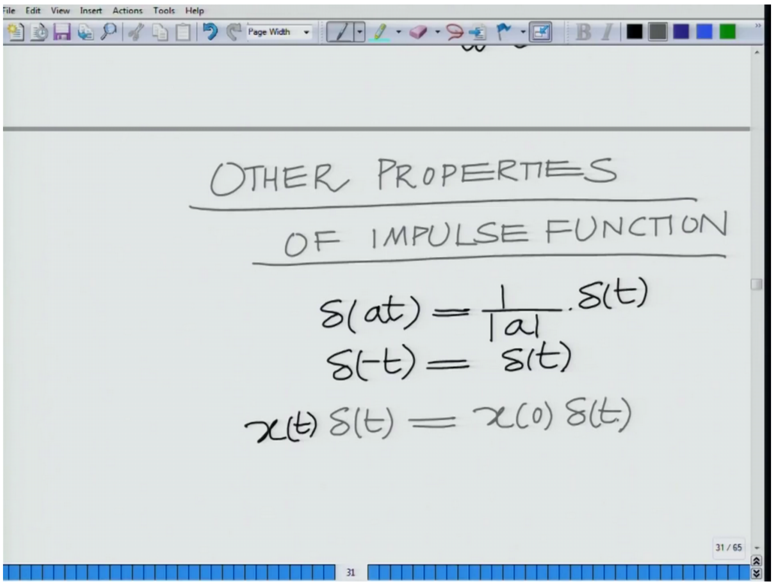

That is other properties of and these are very important properties because the impulse function some of the other properties of the impulse function are delta a t equals one over magnitude a of delta t delta minus t equals delta t and for any signal x tx t times delta t equals x of 0 times delta t. So, these are some of the other properties further.

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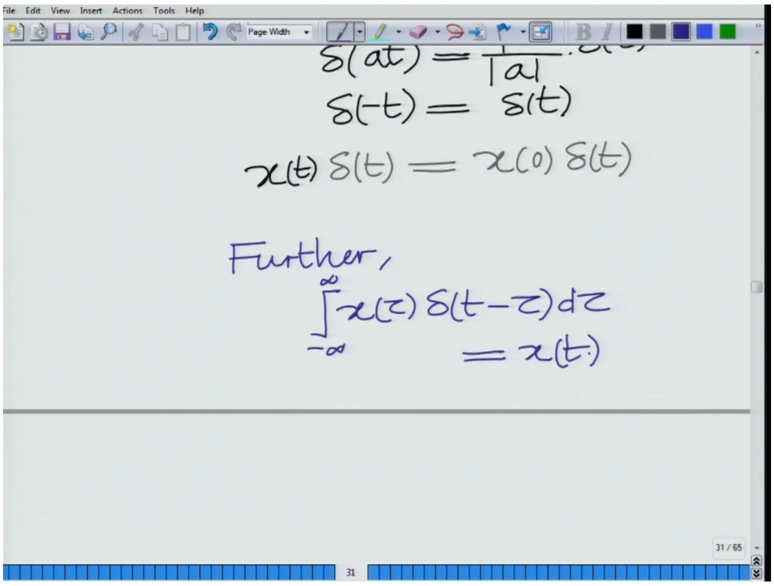

What we also have is another very interesting property if you consider integral minus infinity to infinity x of tau delta t minus tau d tau this can be shown to be equal to x of t this is a very important property what you can show is x of x of tau integral minus infinity to infinity x of tau delta t minus tau d tau is x of t; let us try to see; how we can demonstrate this or prove this property.

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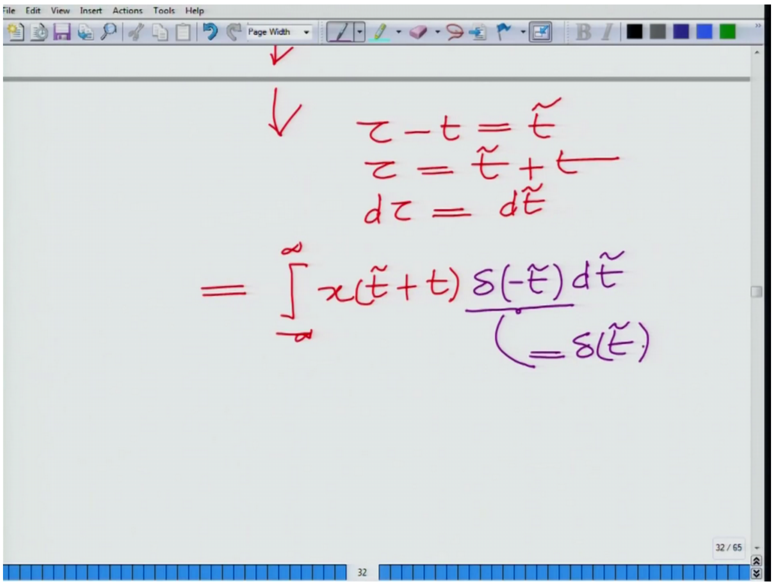

So, here what we are going to do is we are going to set tau minus t equals t tilde which means tau becomes equals becomes equal to t tilde plus t and d tau will be equal to dt tilde. So, the integral simplifies as integral minus infinity to infinity xt tilde plus t x tau is x tau is t tilde plus t xt tilde plus t times delta tau minus delta t minus tau, but t minus tau is minus t tilde d t tau equals dt d tau equals d t tilde.

Now, here we use the property that delta tau delta minus t tilde equals delta t tilde.

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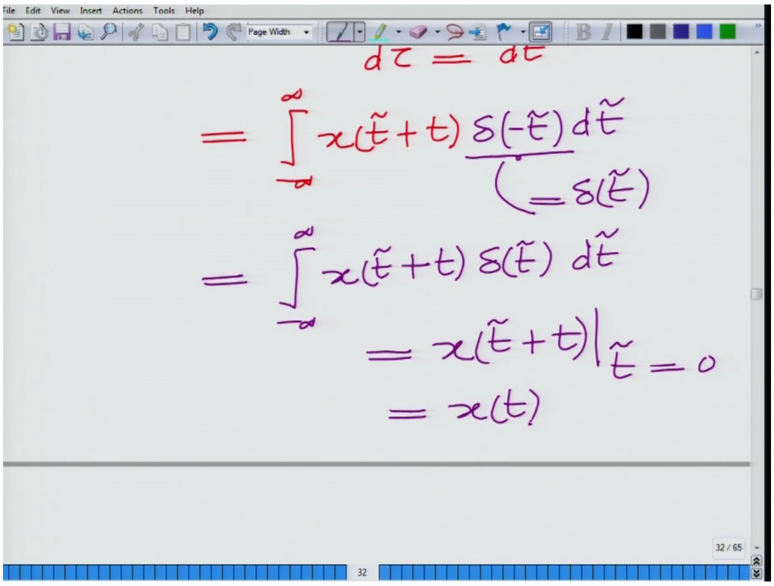

Which implies this becomes minus infinity to infinity x of t tilde plus t into delta tilde times d tilde, but x of t tilde plus t delta t tilde is nothing, but x of t tilde plus t evaluated at t tilde equals evaluated at t tilde equals 0. So, this is equal to simply x of t.

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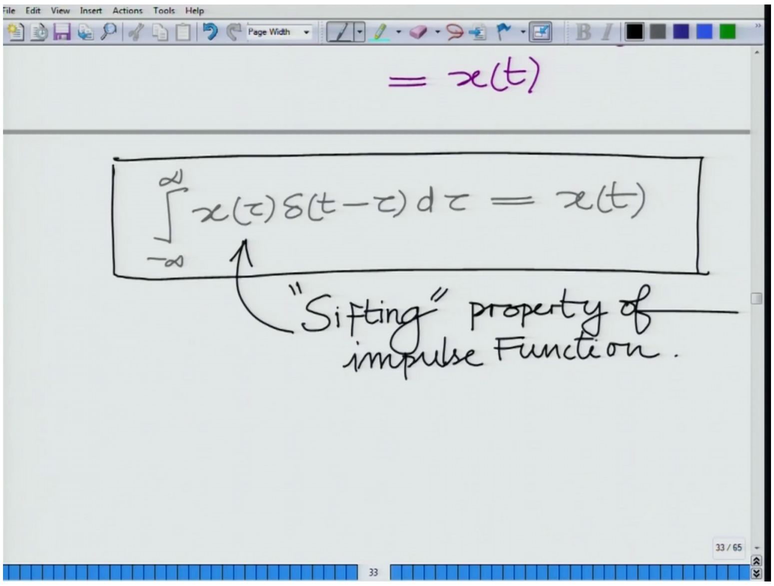

So, what we have as a result is that integral minus infinity to infinity x of tau delta x of tau delta; delta t minus tau d tau equals x or. So, what we have is let me just write it integral minus infinity to infinity x of tau delta t minus tau d tau equals x of t and this property is known as the; this property is known as the this is known as the sifting property of the impulse function this is the sifting property. So, this property is known as the; this property is known as the sifting property of the impulse function or the sifting property of the delta function.

So, this is; so, the delta function we have looked at the delta function it has it has a very interesting at a complicated definition; it is not easy to understand. So, I urge you to take a look at this again and it has a lot of interesting properties. So, it would be good to examine and understand the properties of the impulse functions because this arises very frequently in the analysis of signals and systems and its very important to both understand the behavior and also model and understand the behavior of systems and we will see several applications of this impulse function as we go through the rest of this course ok.

Let us come to another interesting function that occurs frequently which is the complex exponential.

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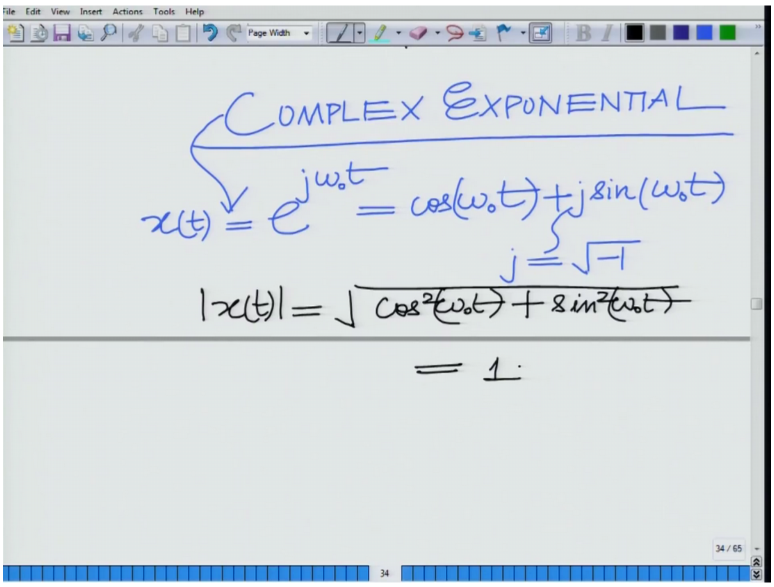

The complex exponential function is defined as e to the power of j omega naught t or e to the power of j 2 pi f t which is basically cosine omega naught t plus j sin omega naught t where j equals square root of minus one is the imaginary number square root of minus one and if you can look if you look at if you xt equals e to the power of j omega naught t you have magnitude of xt which is equal to square root of cosine square omega naught t plus sin square omega naught t equals one. So, magnitude of x t is always one that is a complex exponential always has unit magnitude e to the power of j omega naught t its magnitude is 1 and further it is a periodic signal.

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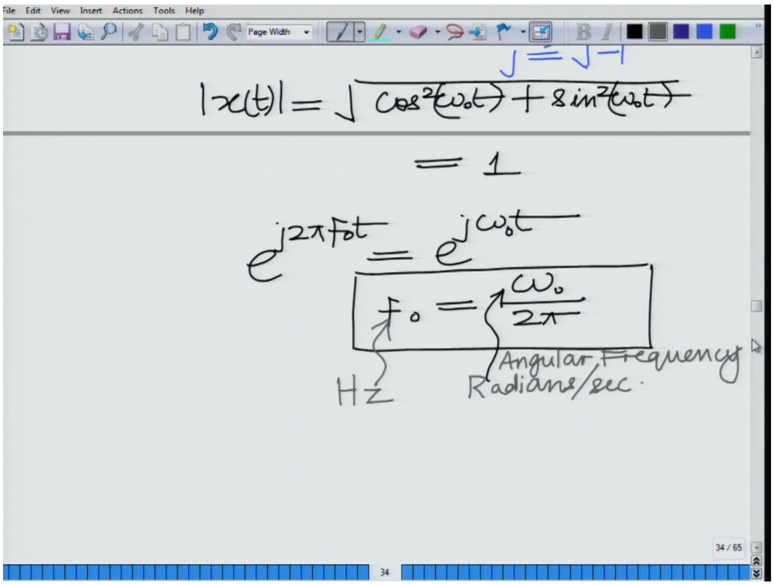

It can also be represented as e to the power of j 2 pi f naught t equals e to the power of j omega naught t where f naught equals omega naught divided by 2 pi this is an important relation omega naught is the circular frequency this is also known as the angular frequency this is in radians per second the unit of this and the unit of f naught frequency is hertz and this is your circular cover angular frequency.

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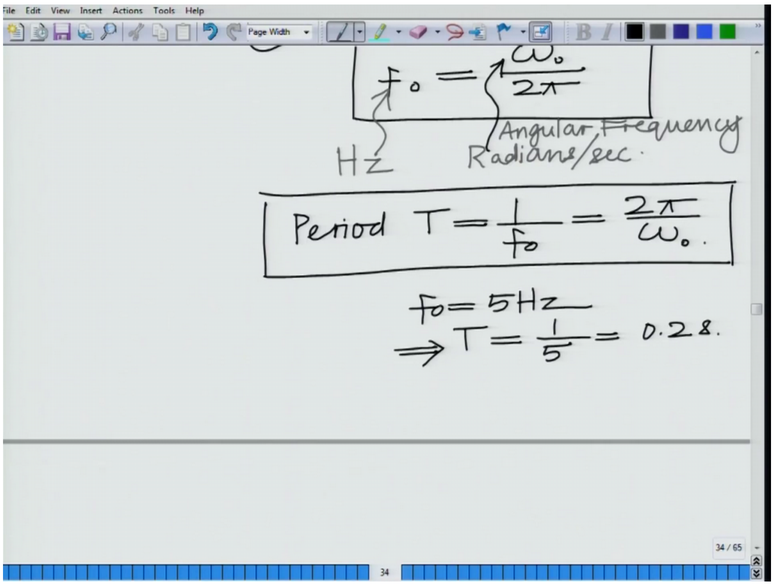

And period this is a periodic signal its period t equals one over f naught equals 2 pi over omega naught. So, the period for instance if f equal 2 or f naught equals five hertz implies f naught equals five hertz implies the period t equals one over five seconds which is equal to point 2 seconds that is the meaning. So, the period is basically the reciprocal of the frequency ok.

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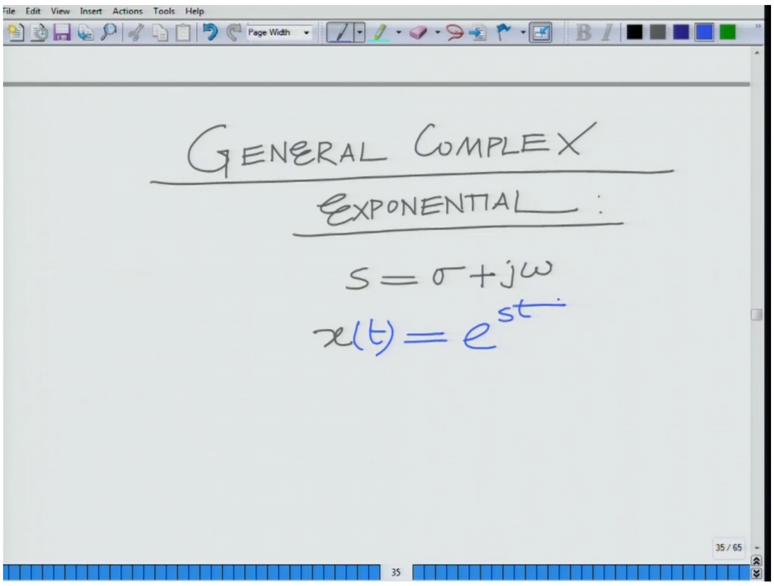

And a general complex exponential is now defined as something a general complex exponential a general complex exponential is defined as follows s equal to sigma plus j omega and xt equals e to the power of s t which is basically equal to this as a general complex exponential.

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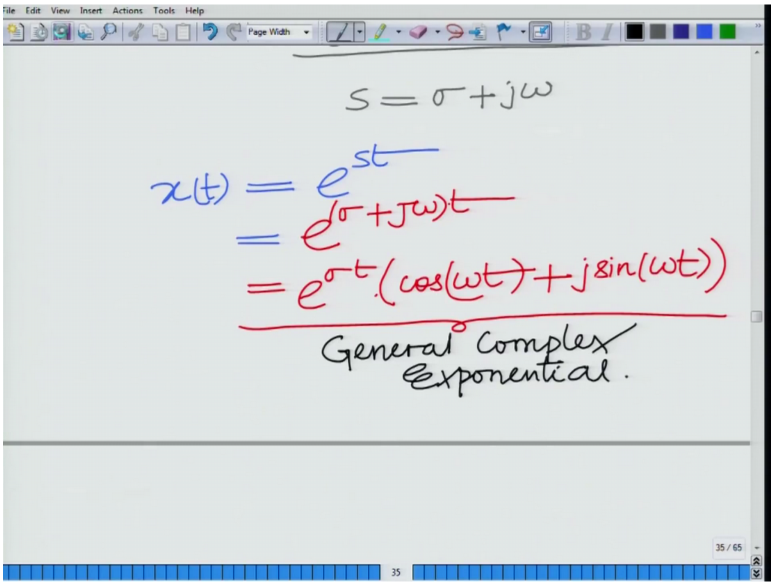

Which is e to the power of x t equals e to the power of s t which is equal to well e to the power of sigma plus j omega t which is e to the power of sigma t into cosine omega t plus j sin of omega t. So, this is your general complex alright. So, what we have seen in this module is we have seen yet a different other different classes of signals such as energy and power signals etcetera we have also seen some very commonly arising and important continuous time signals such as the unit impulse or such as the unit step signal or the unit step function the unit impulse function and we have also seen the complex exponential and the general complex exponential signals. So, we will stop here and continue with other aspects in the subsequent modules.

Thank you very much.