Principles of Signals and Systems
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Lecture – 29
Example Problems in Signals and Systems – Plot, Odd/ Even Components, Periodicity

Hello welcome to another module in this massive open online course. So, we are looking at several example problems to understand the application of z transform. So, let us continue this discussion.

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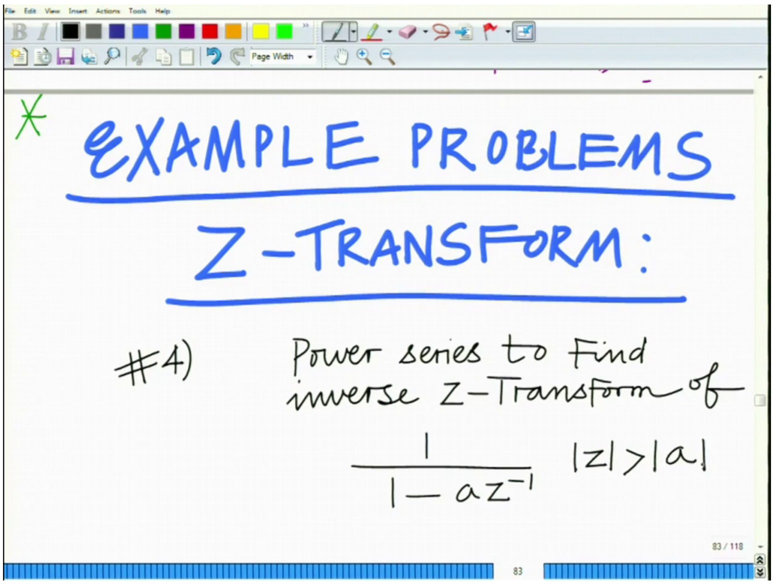

So, we are looking at various example problems for the z transform.

So, we already done 3 examples let us look at the 4th example. So, example, we want to use the power series approach, power series to find the inverse z transform of 1 over 1 minus az inverse magnitude of z is greater than magnitude of magnitude of z is greater than magnitude of a.

So now remember this is of the form magna the ROC is of the form remember, this is your ROC and the ROC is of the form magnitude of z is greater than magnitude of a.

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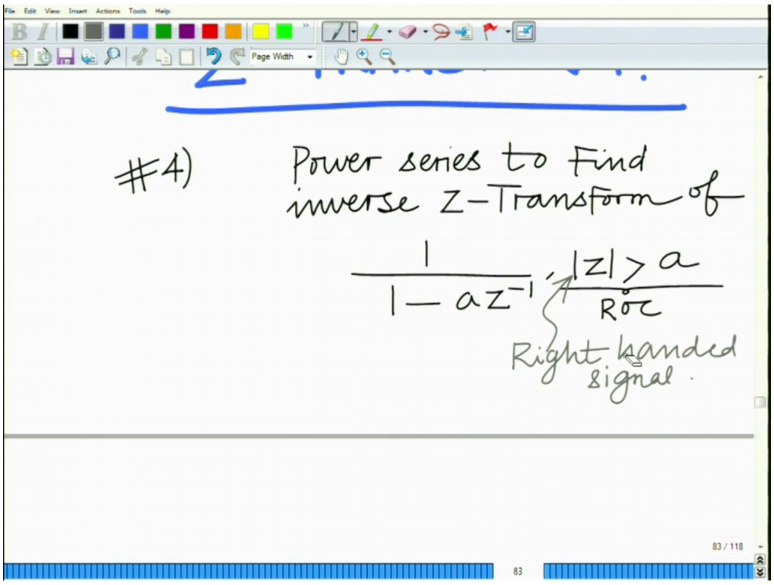

We can even make this as a real number let us look at magnitude of z greater than a real positive number magnitude of z greater than a. So, ROC is of the form magnitude of z is greater than a therefore, this is a right-handed signal.

So, what we are looking at here this is a right-handed signal, or the right sided. This is a right-handed signal or this is a right sided sequence, correct? This is basically the kind of signal that we have here is basically a right-handed signal. And now to use the power series approach, what we are going to do is basically we will look at the numerator the numerator is one and we will divide that by the denominator.

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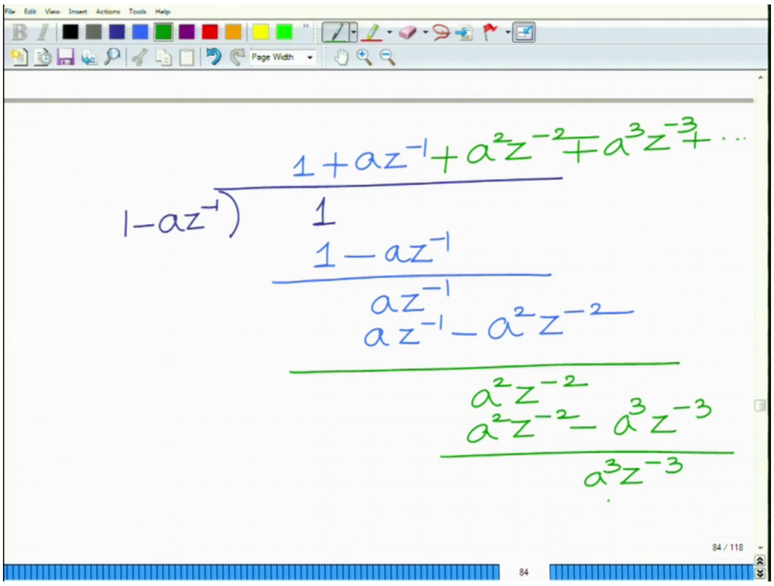

So, similar to the division the long division that is 1 minus a z inverse. So, this is 1 minus az inverse. So, I multiply this by 1 that gives me 1 minus az inverse. So, you subtract this to now get the remainder the remainder is 1 minus that is az inverse. Now, therefore, now you can multiply this by az inverse. So, if you multiply 1 minus az inverse. So now, if you multiply 1 minus az inverse by az inverse, what you obtain is az inverse minus a square z raised to the power minus 2, and now you can subtract this and what this will give is a square z raised to minus 2.

Now, you can multiply by a square z raised to minus 2; that will give you 1 minus az inverse multiplied by a square z raise to minus 2 will give you 1 minus a z inverse multiplied by a square z raise to minus 2 will give you a square z raise to the power of minus 2 minus a cube z raise to the power minus 3. And now once you subtract it you will have a cube z raise to minus 3.

Now, you can multiply by a cube z raise to the power minus 3. And the procedure repeats and now you can see a pattern emerging. So, what you see when you divide this using long division that is the power series approach that gives us.

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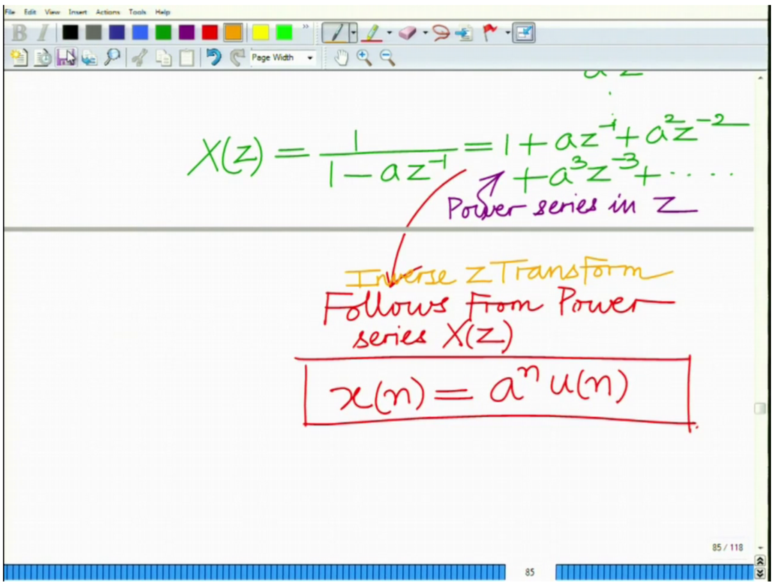

X z equals 1 over 1 minus az inverse that gives us 1 plus a z inverse plus a square z minus 2 plus a cube z minus 3, that is all.

So now what we have been able to do we have been able to express this as a power series in z. All right, we have been able to express this as so, we have been able to express this as x n z raised to minus n. So, here we have been able to express this as a power series in z and now it follows that the sequence xn. So, this is nothing but you can see this is a n z raised to the power minus n for n greater than or equal to 0. And therefore, the sequence is nothing but a n u n that is for n greater than or equal to 0 it is a n.

So, therefore, it follows from the power series x of z that xn equals a raise to n un this is the sequence. So, you have constructed the power series, and from the power series the inverse z transform. So, the inverse z transform follows from the power series as xn equals a raise to n un. So, this is basically the power series approach of converting of constructing the inverse z transform.

Let us now look at another example number 5, let me just start it on a fresh page I think that will make it much more convenient.

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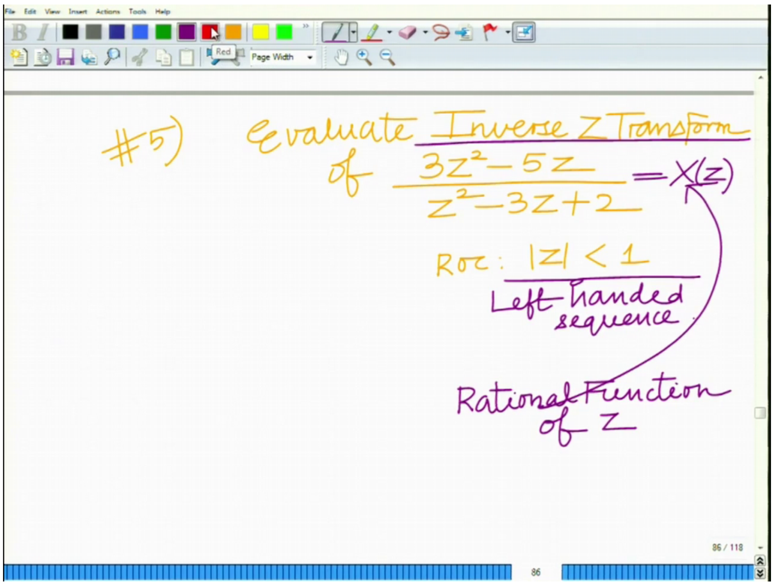

So, we have a rational function of z evaluate inverse z transform 3 z square minus 5 z by z square minus 3 z plus 2 and ROC is given as magnitude z less than 1 magnitude z less than 1. And therefore, what this means is this is a left-handed signals. From the ROC you can see that this is a left sided sequence or a left-handed sequence. This is a left-handed sequence.

And we have to evaluate the inverse z transform of this. This is your x of z remember, this is a rational function of z you can clearly see and whenever we have a rational function of z we can x try to express this as a partial, you can use the for partial fraction approach. So, this is a rational function, this is a rational function in z. And now this can be done as follows I can express x z.

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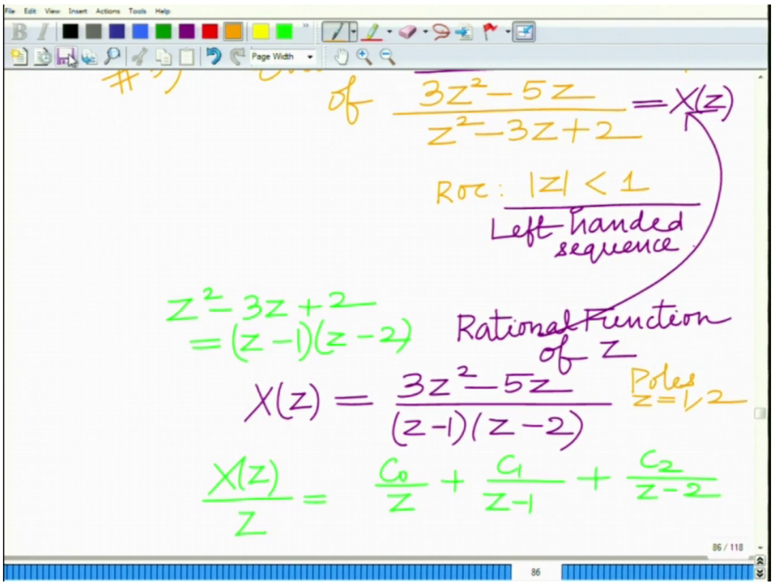

Remember we have x z equals 3 z square minus 5 z divided by z minus 1 into z minus 2. I can express this as follows now, remember we said x z over z can be expressed as a partial fraction expansion is of the form c naught over z plus c 1 over z minus 1 plus c 2 over z minus 2. By the way I forgot to mention that z square minus 3 z plus 2 can be expressed as z minus 1 into z minus 2. Now remember there is one more step here we have z square minus 3 z plus 2 equals z minus 1 into z minus 2.

So, the poles of this r is equal to 1 and z equal to 2. So, poles or it is equal to 1 comma 2 there is a rational function. And obviously, there are 0s the 0s will be at 3 z square minus 5 z.

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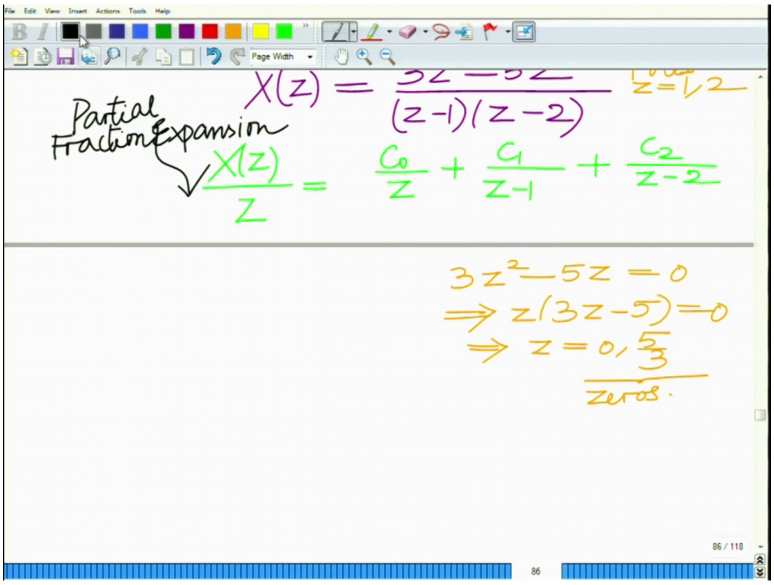

So, 3 z square minus 5 z equal to 0 implies z into 3 z minus y equal to 0 implies either z is equal to 0 or z is equal to 5 by 3. So, these are the 0s, these are the 0s of the 0s of the z transform.

So, the poles are at one comma 2 0s are at 0 comma 5 by 3 anyway the 0s are not important. For the partial fraction expansion remember the partial fraction expansion depends only on the I mean the terms depend only on the poles that is to look at the structure of these different terms we need knowledge of the poles. And so, basically what we are using here is we are using the partial fraction approach. So, this is the partial fraction expansion.

We are using the partial fraction expansion and what the various terms can be found as follows.

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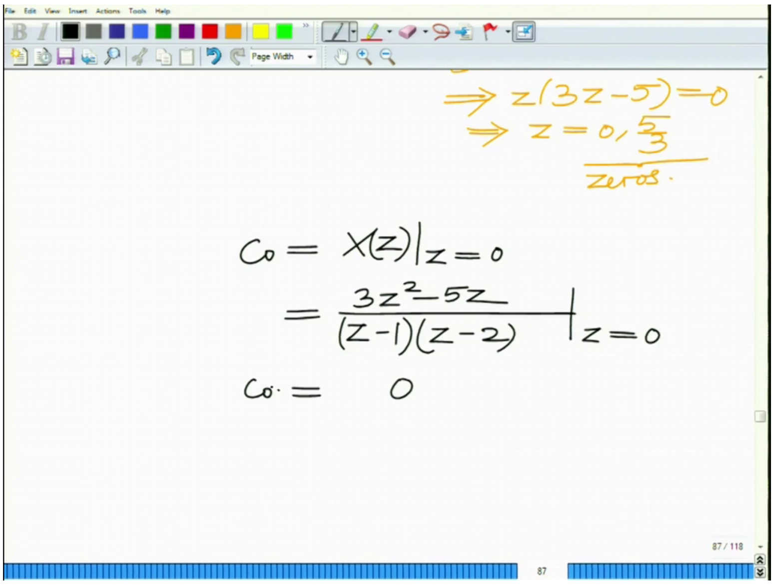

Now, we know c naught is simply that is x z evaluated at z equal to 0. That is basically your 3 z square minus 5 z divided by z minus 1 into z minus 2 evaluated at is equal to 0 and this is basically you can clearly see this is 0.

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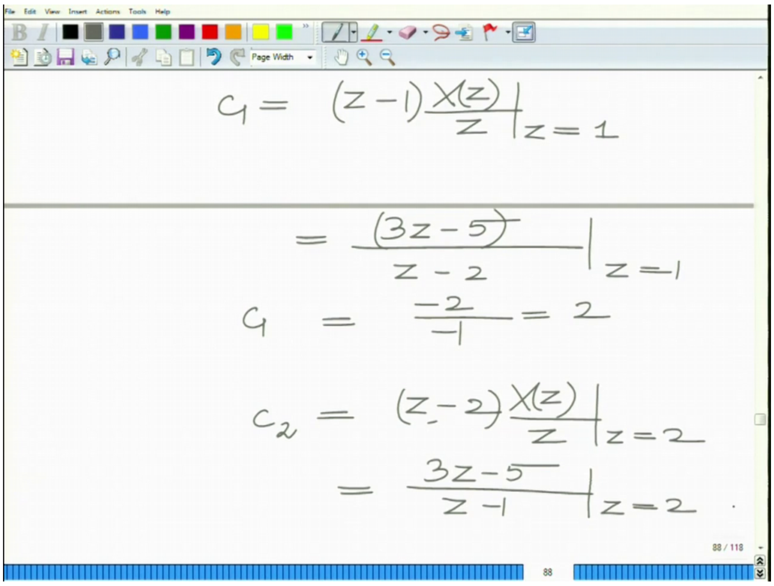

So, c 0 is 0. Now, what about c 1? Now c 1 remember, c 1 is z minus 1 into x z over z evaluated at z equal to 1. That is basically 3 z minus 5 divided by z minus 2 evaluated at z equal to 1. So, this is 3 z minus 5 at z equal to 1, this is minus 2 divided by z minus 2 is minus 1. So, minus 2 divided by minus 1 is 2. So, c 1 equals 2, and similarly we have c 2 equals z minus 2 into x z over z evaluated at z is equal to 2.

So, this is your 3 z minus 5 divided by z minus 1 evaluated at z is equal to 2. So, this is basically your 1 divided by 1 equals 1.

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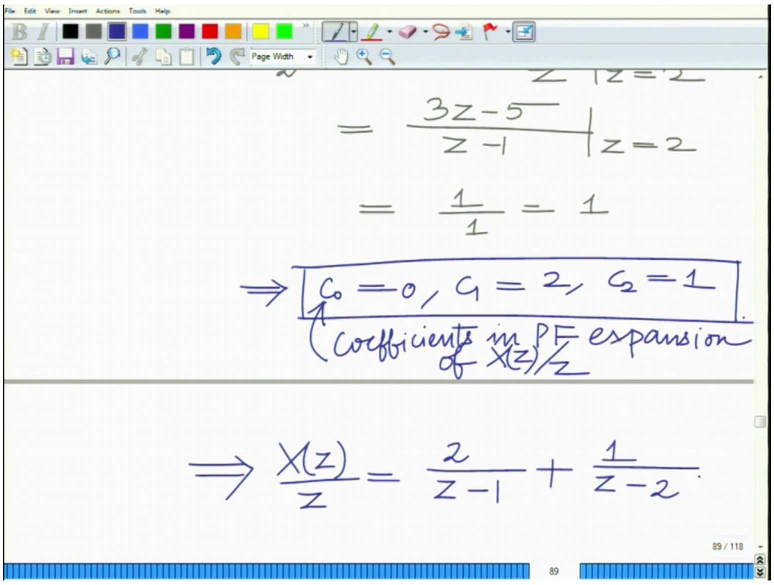

So basically, what we have at the end of this is basically that c naught equals 0, c 1 equals c 1 equals 2 and c 2 equals 1. So, these are the coefficients in the partial fraction expansion.

So, these are basically your coefficients in the partial fraction expansion of x z over z, partial fraction expansion of x z or z. Which implies so now, this implies that basically your xz over z can be written as x 0 over z can be written as twice over z minus 1 plus 1 over z minus 2, which basically implies that x z equals twice z divided by z minus 1 plus z divided by z minus 2, and remember the ROC remains that is magnitude of z less than 1.

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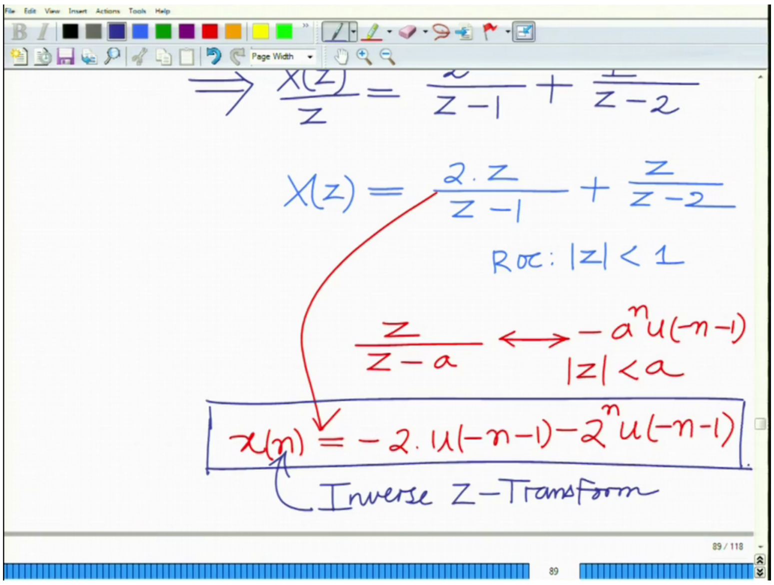

And therefore, now if you look at the left-handed sequence corresponding to this, that is remember we use the property z by z minus a, I think z by z minus a for a left-handed signal that will be minus a raise to n u minus n minus 1. That is magnitude of z, less than ROC is magnitude of z less than a. So, this would be 2 z. So, this would be minus 2 the pole is one. So, this would be a to the power of n would be whole to the power of n which would be 1. So, this will be minus 2 u of minus n minus 1 minus z or z minus 2 that is 2 to the power of 2 raise to n u of minus n minus 1.

So, this is basically your sequence x of a and x of n is minus 2 u minus n minus 1 minus 2 raise to n u of minus n minus 1. So, this is basically your solution. This is basically the inverse z transform. This is basically the inverse z transform. So, we are able to evaluate using the partial fraction expansion of x z over z we are basically able to evaluate the inverse z transform of the given rational function of z.

Let us look at another example again to find the inverse z transform.

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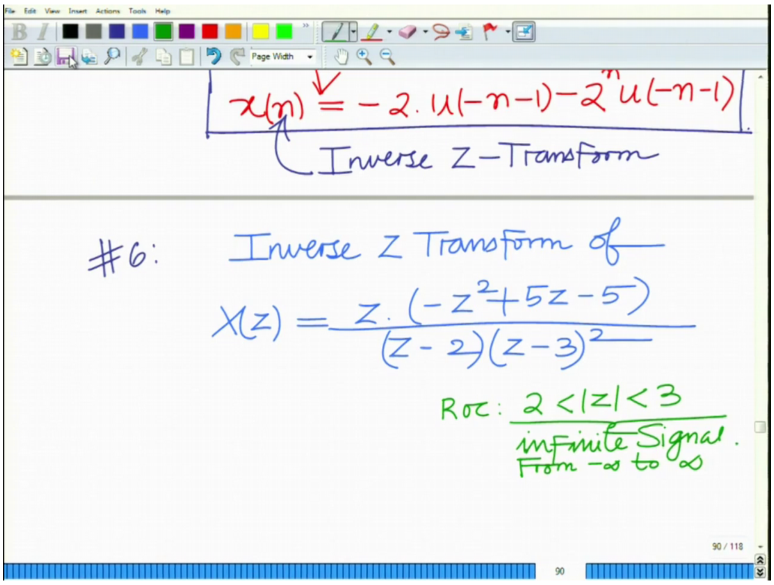

So, once again we have to find the inverse z transform find the inverse z transform of x z equals z times minus z square plus 5 z minus 5 divided by z minus 2 times z minus 3 whole square the ROC is 2 less than magnitude of z less than 3. So, remember this is a infinite signal. Because, remember the ROC of the form is it lies between 2 poles 2 and 3 all right. So, you can see the poles are 2 and 3 ROC lies between 2 and 3 correct. So, this is an infinite signal from minus infinity to infinity.

So, from the ROC itself we can see this is an infinite signal that is spans minus infinity to infinity. And now you can see there is a multiple now previously, you had simple poles right now you have repeated poles now you have multiple poles of multiplicity greater than one for instance the poles here.

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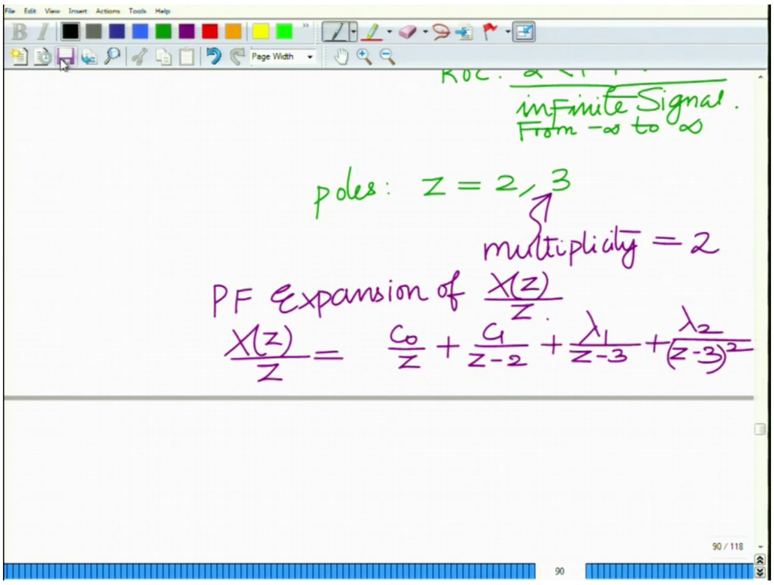

You can clearly see the poles are z is equal to 2 and 3, but at z equal to 3 you have pole of multiplicity 2, multiplicity equals 2.

And therefore, if you look at the partial fraction expansion of x z over z, that will be of the form x z over z equals c naught by z plus c 1 by z minus 2. Plus, we will have 2 corresponding to pole that is equal to 3 of multiply a multiplicity 2. We will have 2 terms lambda 1 by z minus 3 plus lambda 2 by z minus 3 by z minus 3 whole square.

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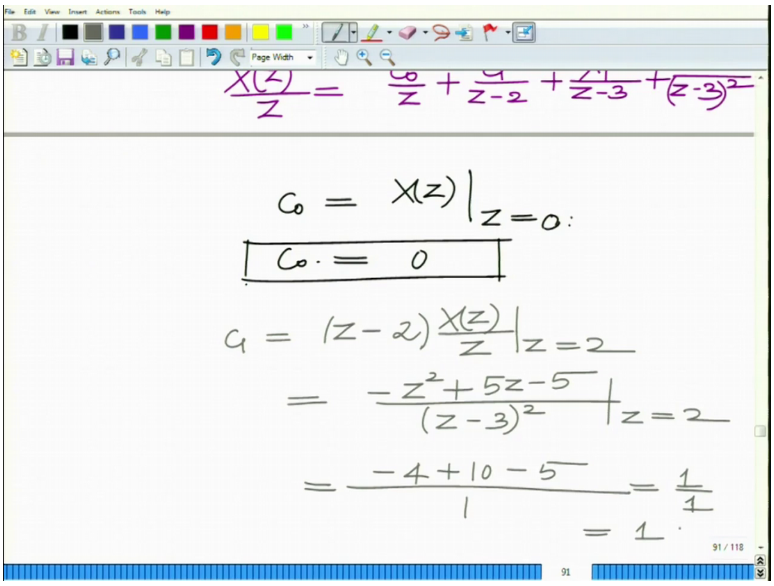

Now, coming to z naught c naught is x z evaluated at z is equal to 0. And this is easy c naught is 0, x z evaluated it is equal to 0 this is 0. So, we have c naught is basically equal to 0. So, the coefficient corresponding to z that is c naught by z the coefficient c naught in the partial fraction expansion of x z by z c naught is 0.

Let us look at c 1 now c 1 is also relatively simple to find c 1 equals z minus 2 this corresponds to the simple pole that is pole of multiplicity one z minus 2 x 0 or z at z equals 2. This is basically your minus z square plus 5 z minus 5 divided by z minus 3 whole square evaluated at is equal to 2. So, this is equal to what is equal to? This is equal to basically you will have minus 4.

So, I evaluate the numerator at 2 minus 4 plus 5 2s are 10 minus 5 divided by 3 minus 3 whole square one square that is 1. So, this will basically be 10 minus 4 10 minus 9. So, this is one divided by one which is equal to 1. So, c 1 equals 1.

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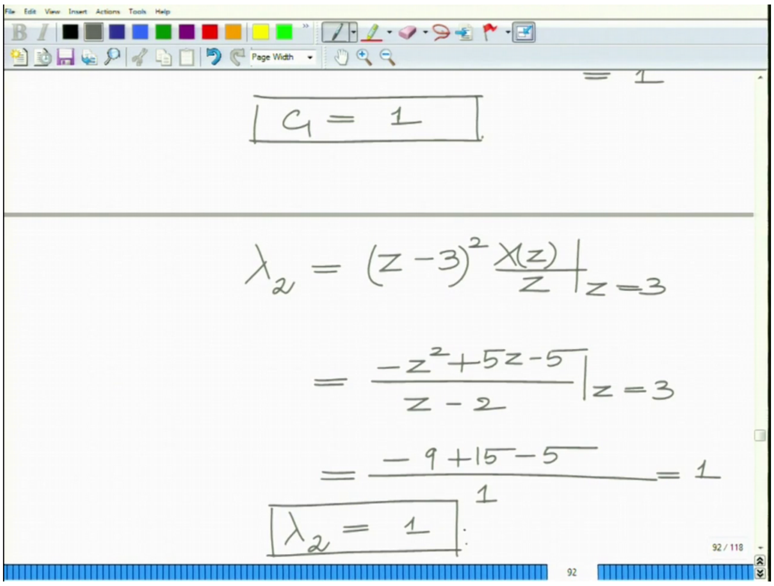

So, what we are able to establish is that c 1 is equal to 1. Now what about lambda 2? Now lambda 2 that is the coefficient corresponding the corresponding to z minus 3 whole square, so that can be evaluated as z minus 3 whole square x z over z evaluated at z is equal to 3. So, that will simply be minus z square plus 5 z minus 5 over z minus 2 evaluated at z is equal to 3. So, this will be equal to minus 9 plus 15 minus 5 divided by 1. So, this will be this will be basically 1.

So, lambda 2 equals lambda 2 equals 1.

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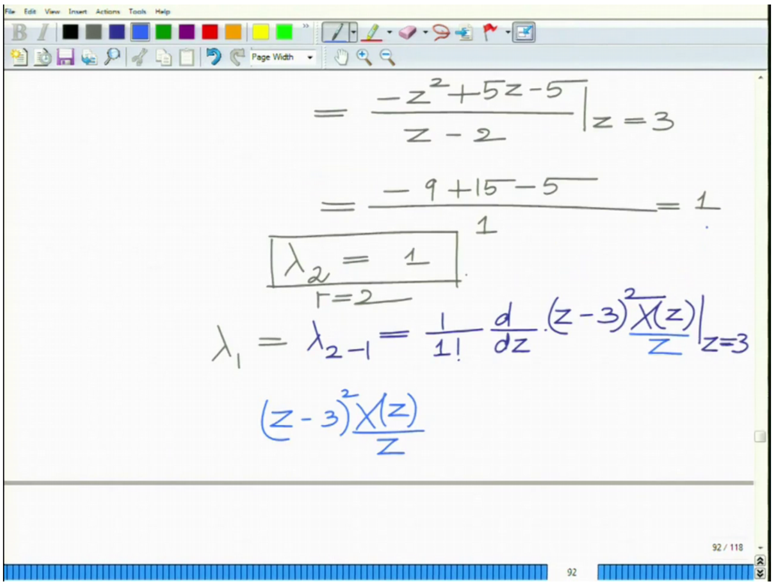

Now what about lambda 1 remember we can use this find this using this formula to go all the way back, let me just refresh your mind refresh your memory. Regarding the formula that we are going to use that is we are going to use the formula that lambda r minus k, remember I look at this formula lambda r minus k is 1 over k factorial.

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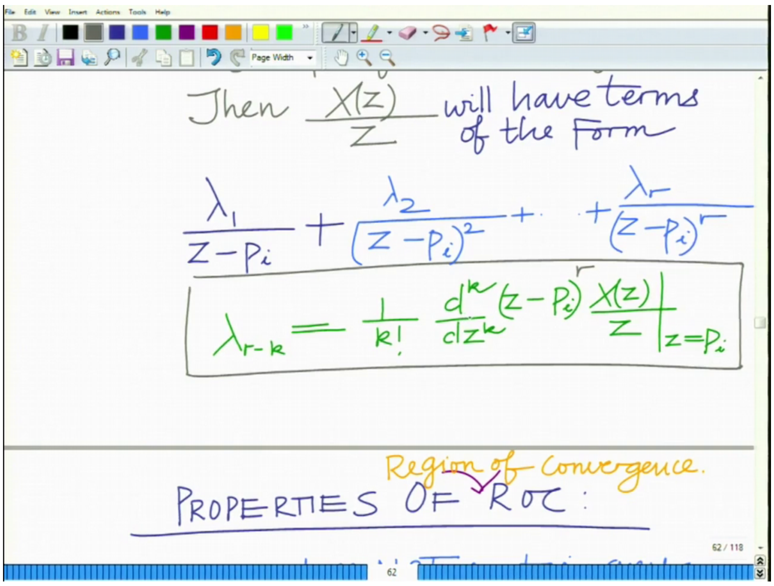

The k th derivative dk or d z k z minus p i raised to the power of k x z over z. I am sorry, this should be I think z minus p i raise to the power of r. I think that should be z minus p i raise to the power of z pi raise to the power of r.

Xz minus pi raise to the power of r x z over z at z is equal to pi. So, this is basically what that turns out to is remember r is equal to in this case r is equal to 2. We have r is equal to 2. So, I have lambda 1 equals lambda r 2 minus 1. So, k equal to 1. So, r minus k equals 1 over k factorial. So, 1 over one factorial times d over the k th derivative k equals 1 in this case. So, d by simply d by dz times z minus 3, z minus 3 whole square into x z evaluated at z equals 3.

Now, let us evaluate what this quantity is z minus 3 whole square x z is z minus 3 whole square x z I am sorry, z minus 3 whole square x z over z minus 3 whole square x z over z.

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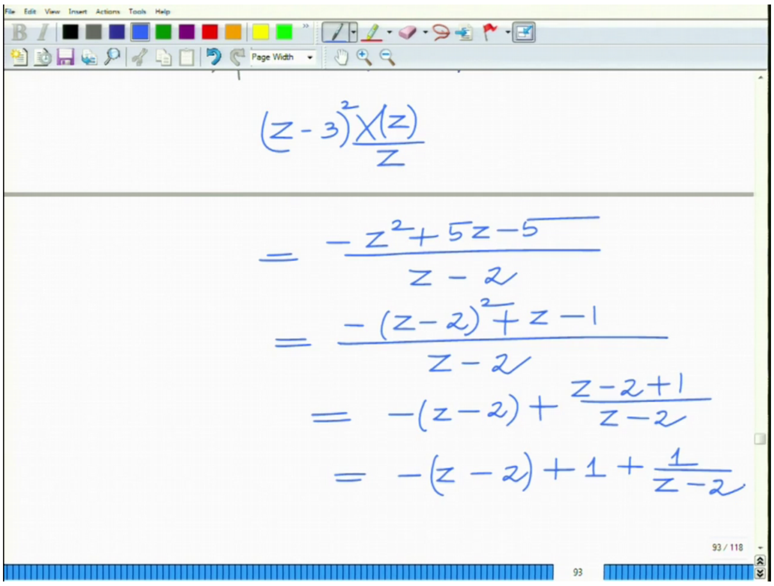

Now this quantity If you look at this the z minus 3 whole square x z over z is minus z square plus 5 z minus 5 divided by z minus 2, which is basically you can write this as minus z minus 2 whole square plus z minus 1, right; divided by z minus 2 which is equal to minus z minus 2 plus now, z minus 1 can be written as z minus 2 plus 1 z minus 2. So, this will be minus z minus 2 plus 1, minus of z minus 2 plus 1 plus 1 by z minus 2; this is equal to minus z plus 3 plus 1 by z minus 2.

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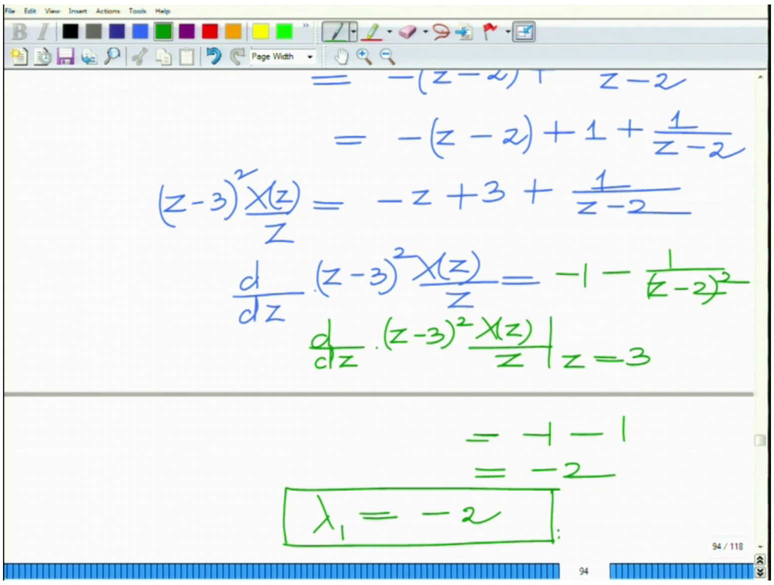

This is your z minus 3 whole square z minus 3 whole square x z over z.

Now, the derivative of this d over dz z minus 3 whole square x z over z this will be the derivative of this will be; well, the derivative of minus z is minus 1 derivative of 3 is 0 minus 1 over z minus 2 whole square. And this evaluated at z is equal to remember, we have to evaluate this derivative d over dz z minus 3 whole square x z over z it has to be evaluated at z is equal to 3. So, that will be minus 1 minus 1 over z minus 2 square minus 1 minus 1 minus 1 that is equal to minus 2. So, what we have is lambda 1 equals minus 2.

So, we have basically lambda 1 equals minus 2.

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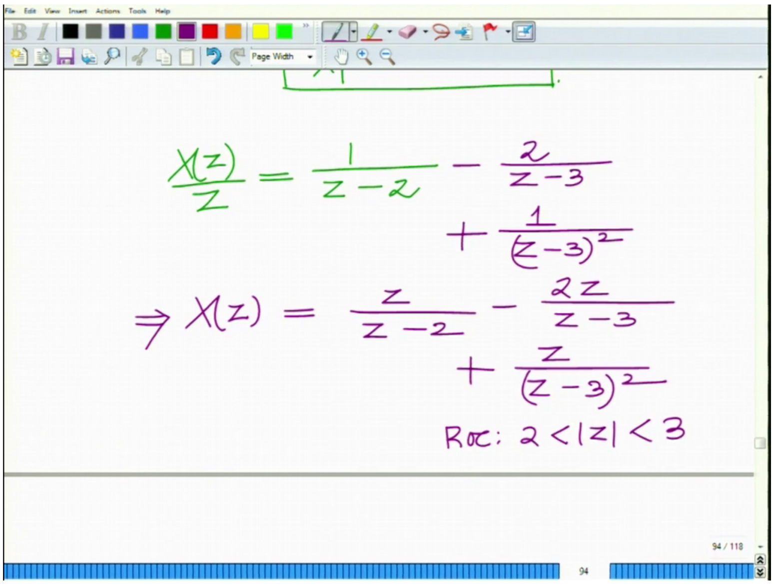

And therefore, we have x z over z x z over z equals 1 over z minus 2 that is your c 1 which is 1 plus, lambda 1 lambda 2 remember, lambda 2 is 1. So, 1 over z minus oh lambda plus lambda 1 over z minus 3 lambda 1 is minus 2. So, this will be minus 2 over z minus 3 plus lambda 2 over z minus 3 whole square lambda 2 is 1.

So, this is 1 over z minus 3 whole square. And this is the partial fraction expansion of x z over z. So, xz therefore, this implies that xz equals z over z minus 2 plus twice z over z minus 3 plus z over twice z over z minus 3 plus z over z minus 3. So, this is basically the partial fraction expansion of now xz.

And remember the ROC still remains the ROC is given as 2 less than magnitude of z less than 3. That is ROC, all right. So, basically, we are now able to derive the partial fraction expansion of x z over z what we have we are going to now do is basically compute the inverse z transform of each of the individual terms, all right. And from that we can reconstruct the individual the inverse and putting them together we can reconstruct the inverse z transform of the given rational function of c that is x so, c all right. So, we will stop this module here and we will continue with this example and look at also other examples in the subsequent module.

Thank you very much.