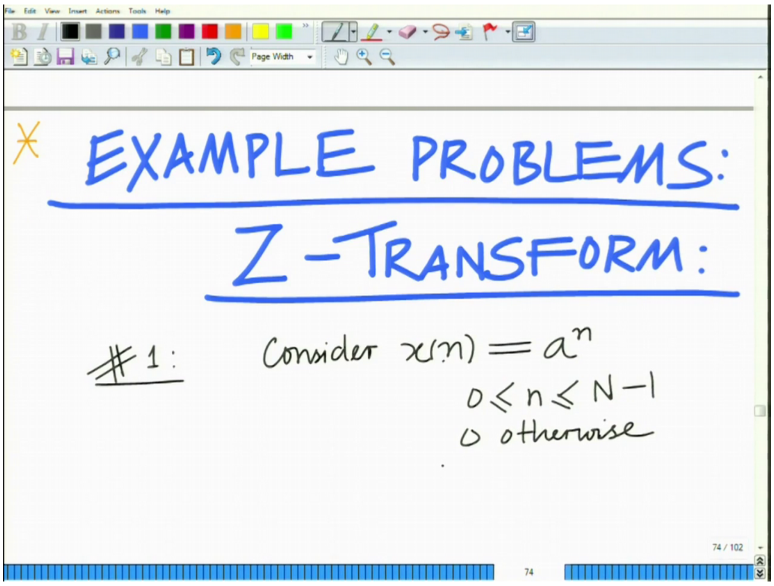
Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 28
Example Problems in z-Transform – Evaluation of z-Transform, ROC

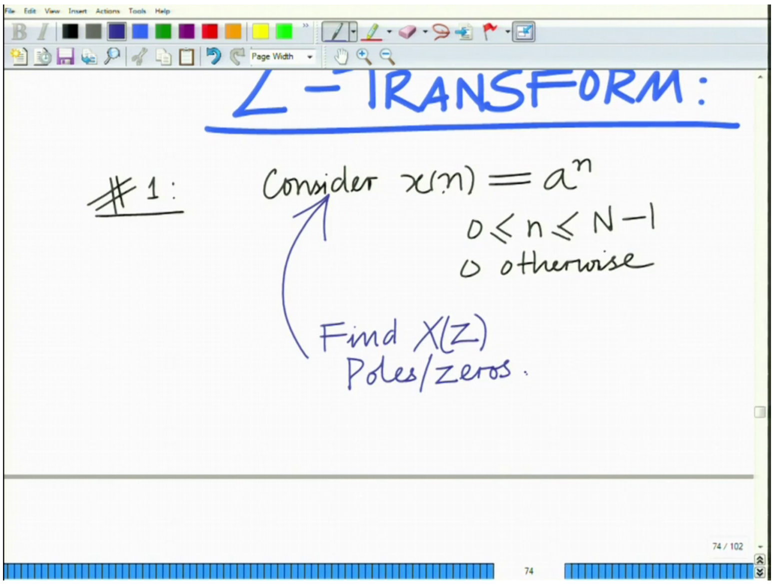
Hello, welcome to another module in this massive open online course. So, we are looking at the z-transform, its various properties. Now let us start looking at some example problems to better understand the applications of z-transpose.

(Refer Slide Time: 00:28)



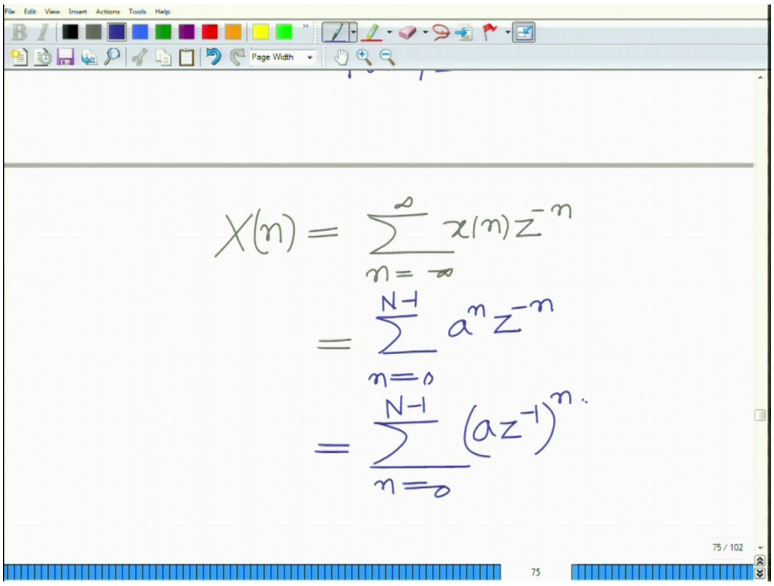
So, what we want to do starting it in this module is we want to look at several example problems to better understand the properties and the applications, several properties and the applications of the z-transform. So, start let us look at the first problem for instance consider x n equals a n zero less than or equal to n less than equal to capital N minus 1 and equal to 0 otherwise. That is x n equals a n for 0 less than equal to small n less than equal to capital N minus 1 and it is 0 otherwise.

(Refer Slide Time: 01:46)



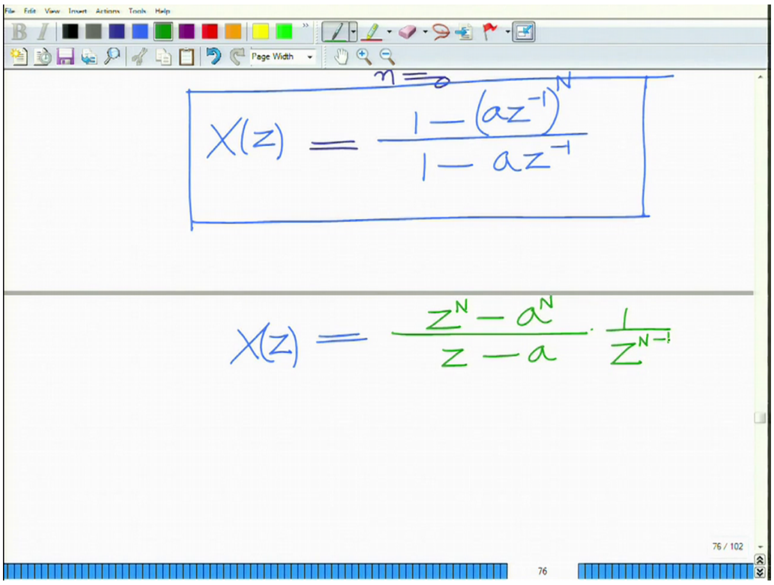
We want to find the z-transform that is fine the z-transform x of Z along with the poles and zeros. Now, the z-transform of this can be evaluated as follows.

(Refer Slide Time: 02:07)



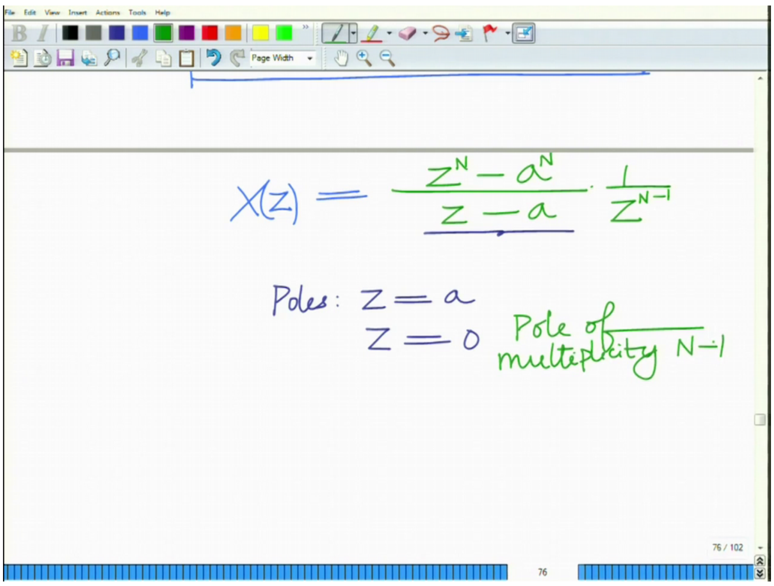
Remember the z-transform x of n equals summation n equals minus infinity to infinity x n Z raise to minus n which in this case will be summation n equal to because this is nonzero only for n lying between 0 and capital N minus 1. So, this is n equals 0 to N minus 1 a n Z raise to minus n which is summation n equal to 0 to N minus 1 a Z inverse raise to the power of n.

(Refer Slide Time: 02:55)



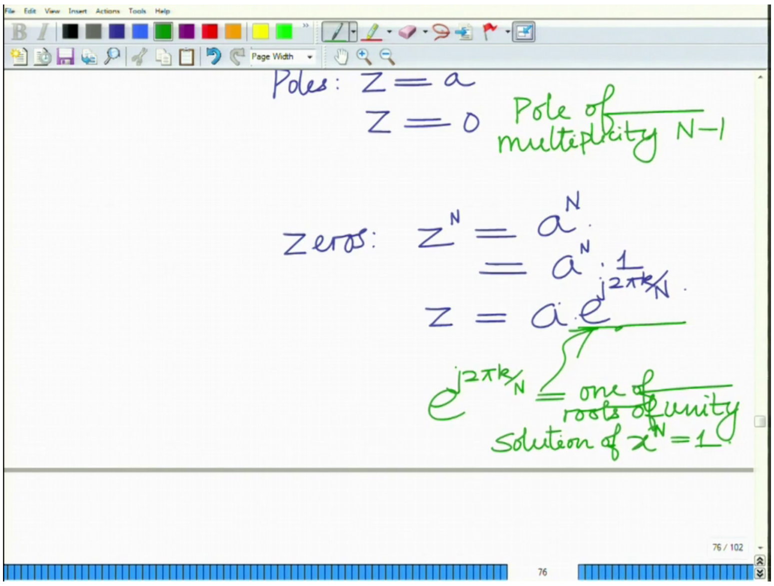
Which is equal to using the properties of the summation is 1 minus a Z inverse raise to the power of n divided by 1 minus a Z inverse that is the z-transform. So, this basically is the this is the z-transform. And I can simplify this as follows. I can simplify this that is X Z, you can also write this as Z power N minus a power N that is multiplying numerator denominator by Z power N Z power N by a power N by Z minus a into 1 over Z power N minus 1, so it also another way to write it. And now you can see the pole the zeros correspond. Now, the pole is at now let us first look at the poles.

(Refer Slide Time: 04:09)



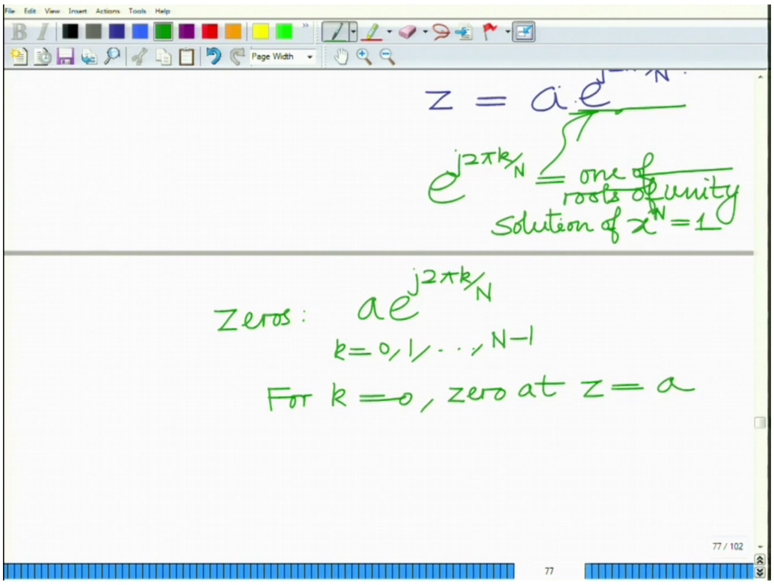
So, this as poles, poles at Z is equal to a and you have this term Z raise to N minus 1 which means this has a pole that is N minus 1 poles or pole with multiplicity N minus 1 because you have Z to the c raise to the power N minus 1 in the denominator. It means that your pole of multiplicity N minus 1 at Z equal to 0. So, Z equal to 0, you have a pole of multiplicity n minus 1. Now, let us look at the zeros; now zeros are very interesting.

(Refer Slide Time: 04:58)



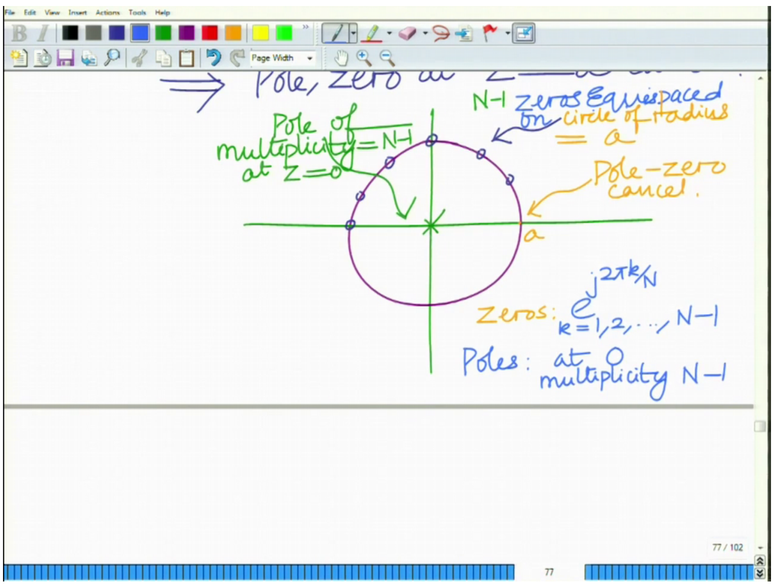
In fact, if you look at zeros, the zeros are obtained by solving this equation Z raise to the power of N equals a raise to the power of N you can write it in fact this is equal to a raise to the power of N into 1. So, Z can be written as a raise to the power of Z equals a times the Nth root of unity. So, times e power j 2 pi k over N correct where e raise to j 2 pi k over N is one of the roots of unity one of the roots Nth roots of unity. It is one of the roots of you know the Nth roots of unity that is a solution to the equation one equals or this is a solution to the equation x raise to the power of N equals 1. Nth roots of unity that is the solution of x raise to the power of N equals 1, so that is e raise to j 2 pi.

(Refer Slide Time: 06:29)



So, the poles are, zeros are e power j 2 pi k over N, where k equals 0, 1 up to n minus 1. Now, if you set k equal to 0, now for k equal to 0, you have zero at Z equal to a. So, basically if you observe you have a 0 at Z equal to a correct remember earlier we have seen that you also have a pole at Z is equal to s. So, the zero and pole at Z equal to a cancel all right. So, what you are left with is basically zero of multiplicity that is a zeros are basically nth roots of a e raise to j 2 pi k over n k ranging from 1 to n minus 1 and multiplied and pole of multiplicity n minus 1 at zero because the pole and zero at Z equal to a cancel.

(Refer Slide Time: 07:30)

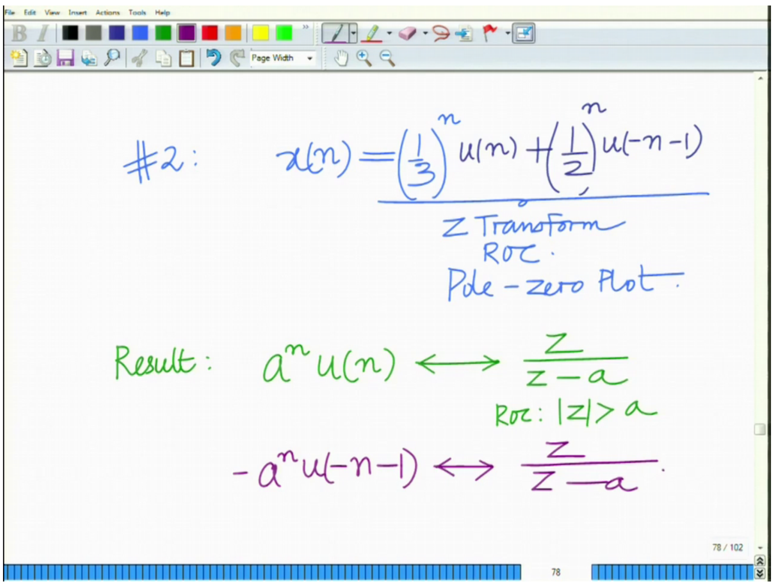


So, what you would have is basically pole and zero at Z equal to a cancel. So, what is remaining is you have the unit circle, and you have the poles which are placed equidistant correct. So, you have the poles which are for instance or these are this sorry zeros which are placed equidistant. So, you have the zeros. So, you have what you have is zeros are equispaced on unit circle zeros are equispaced on the unit circle, and you have a pole of multiplicity n minus 1 at zero.

So, this is n minus 1 zeros, n minus 1 zeros is equispaced on unit circle and pole of multiplicity equals N minus 1 at Z equal to 0. And here mind you at Z equal to 0 you have pole and zero cancellation there you have at Z is equal to a, I am sorry this is a circle of radius a not unit circle. This is equispaced on circle of radius equispaced a circle of radius equal to a. And at this point a you have pole zero cancellation that is the pole and zero, the pole and zero cancel at Z is equal to a, so that is basically how the pole zero plot of this.

So, net we have poles. So, net we have zeros e raise to j 2 pi k over n k equals 1, 2 up to N minus 1 and poles you have ah at zero pole at zero of multiplicity N minus 1. So, you have pole at zero of multiplicity N minus 1 that is of the pole zero diagram of this z-transform of the given signal looks which is a raise to n for zero less than or equal to n less than or equal to capital N minus 1 all right.

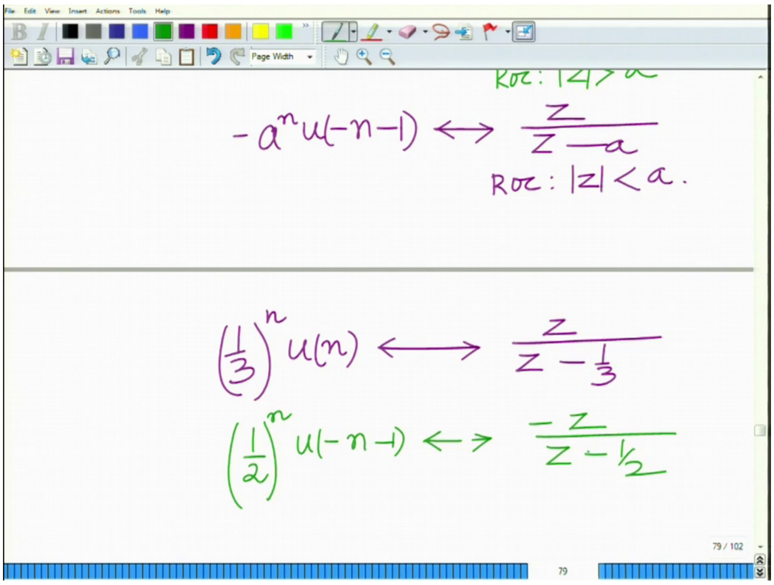
(Refer Slide Time: 11:06)



And let us look at in the next problem that is x n equals 1 over 3 raise to the power of n u n plus half raise to the power of n u of minus n minus 1. We are required to find the z-transform of the given signal and naturally also the associated naturally also the associated ROC. And for this remember the z-transform and also sketch the pole zero plot, find the pole zeros and pole zero plot also we want to the we want the pole zero plot. So, we required to find the z-transform of this given signal and also find the pole zero plot all right.

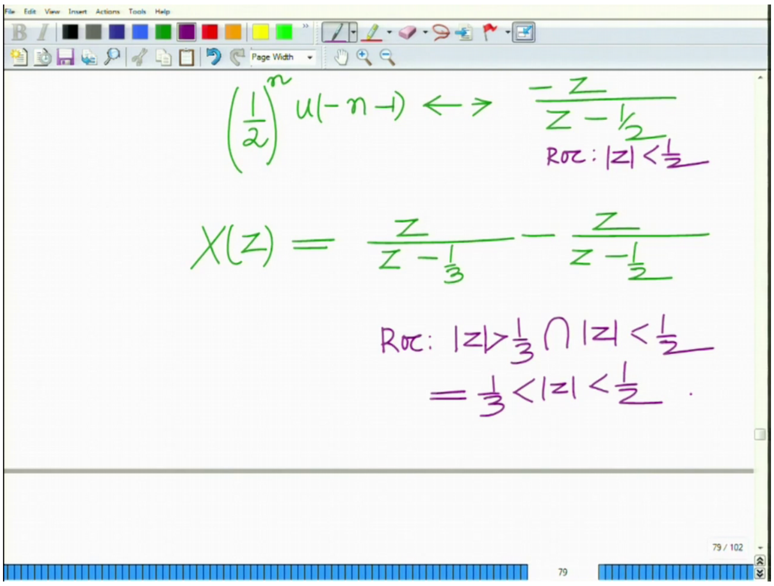
So, now, to do this we are you going to use the following result we will use the result that what are the result that we are going to use we are going to use the result. So, these are the result a n un this has a z-transform Z over Z minus a, but the ROC is magnitude Z is a right handed signal. So, ROC is magnitude Z greater than and a to the a raise to n minus u u of minus n minus 1 or minus a raise to n u of minus n minus 1 this as the z-transform Z over Z minus a. And the ROC is this is a left-handed signals, so the ROCs of the form magnitude Z less than a.

(Refer Slide Time: 13:10)



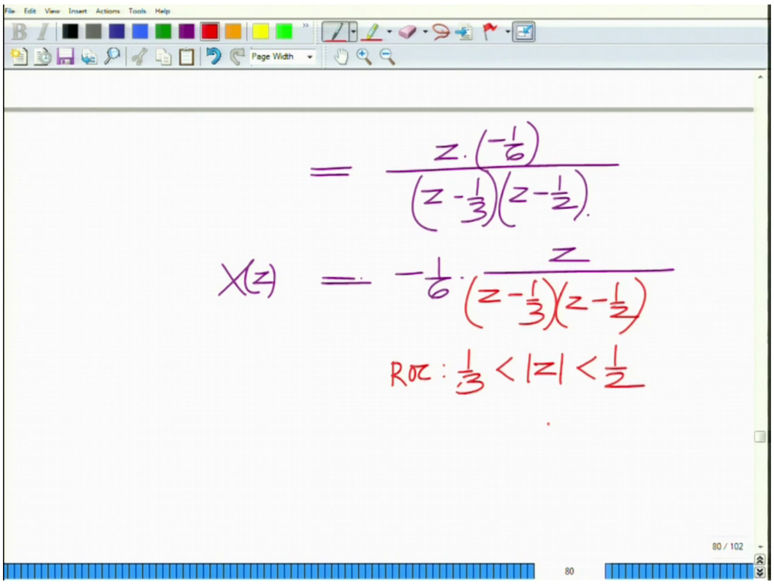
So, now, if you look at this you will have 1 raise to 1 over 3 u n this has z-transform Z over Z minus 1 over 3 and the half to the power of n u minus n minus 1 you can see it has z-transform minus Z over Z minus half. Because minus half raise to n u minus n minus 1 as z-transform Z over Z minus half half raise to n u minus n minus 1 has z-transform that is minus of Z over Z. And the net z-transform of the u n signal is the sum of these two components.

(Refer Slide Time: 14:09)



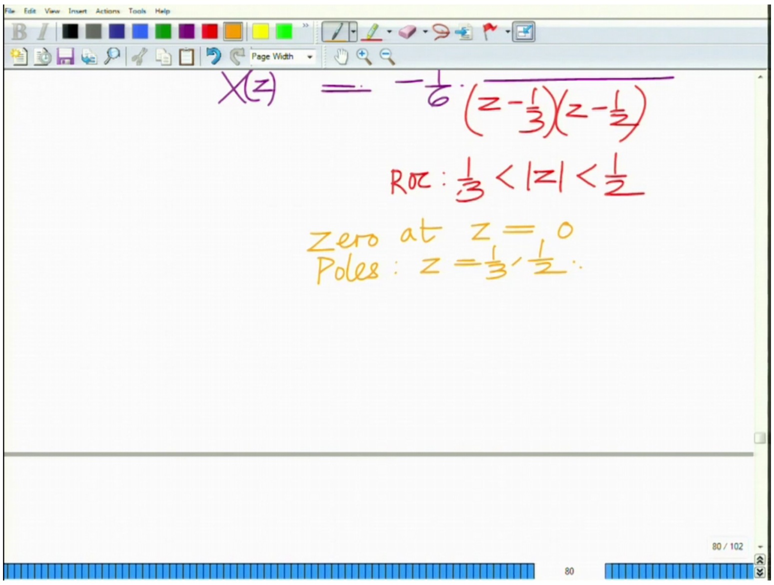
So, the z-transform that is your signal x of Z is therefore Z over Z minus 1 by three minus Z over Z minus half. Now, the ROC of this is ROC of the top one is magnitude of Z greater than 1 by 3, ROC of the bottom one is magnitude of Z less than half. So, the ROC of this is basically the intersection of these two that is magnitude of Z greater than 1 by 3 intersection magnitude of Z less than half which is basically equal to the region half or 1 by 3 less than magnitude of Z less than half this is the ROC.

(Refer Slide Time: 15:04)



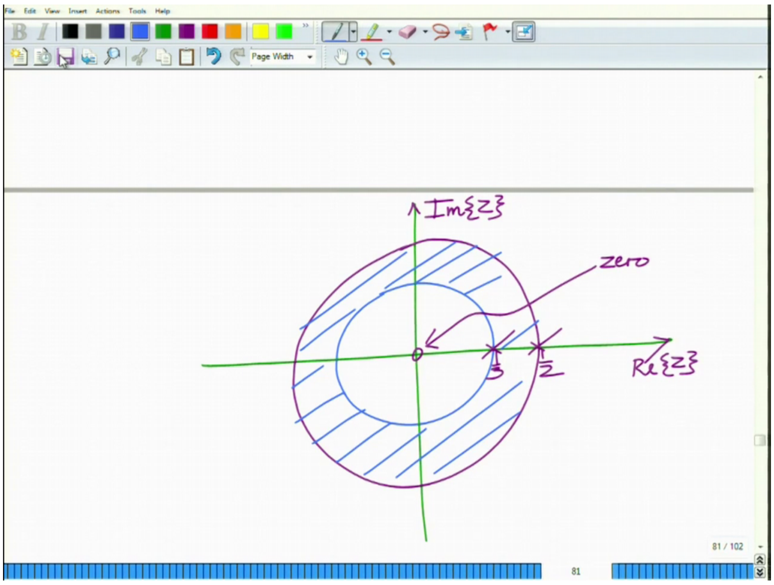
And the z-transform can be simplified as that is Z over Z minus 1 by 3 minus Z over Z minus half that is Z into minus 1 by 6, so minus half Z plus 1 by 3. So, Z into minus 1 by six divided by Z minus 1 by 3 into Z minus half and therefore, this is equal to the net z-transform x of Z equals minus 1 by 6 divided by Z Z minus 1 by 3 into Z minus half. And therefore, we have the ROC is 1 by 3 less than and then ROC this is the z-transform of course, the z-transform is incomplete with of the ROC. So, the ROC is 1 by 3 less than magnitude of Z less than R. And therefore, if you draw the region of convergence on the Z plane, it looks something like this. So, the poles are at so zero at you can see it is Z over Z minus 1 by 3 in to minus Z minus 1 by half.

(Refer Slide Time: 16:14)



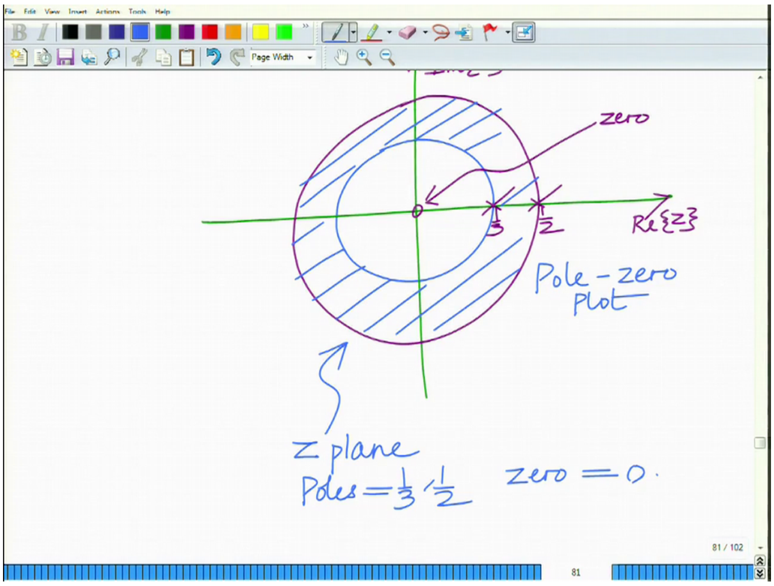
So, zero at Z equal to 0 the poles are at Z is equal to the Z equal to 1 by 3.

(Refer Slide Time: 16:29)



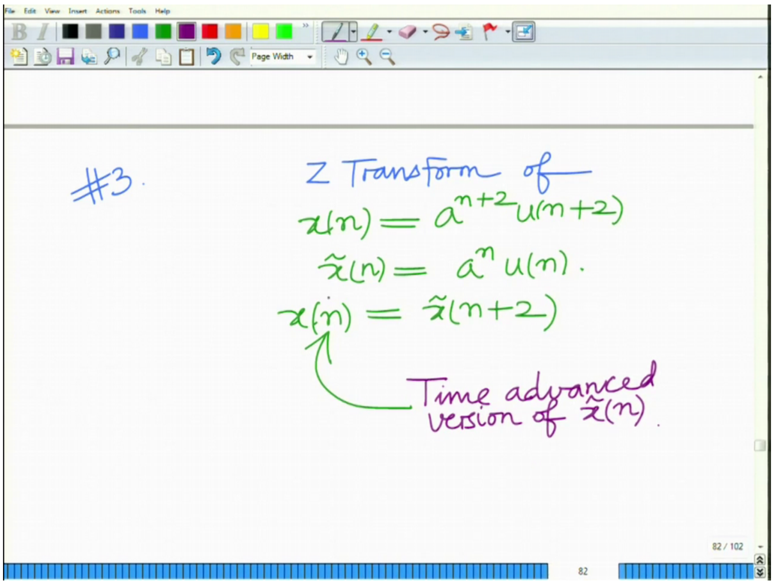
And the roc lies in between these two poles. Remember this is a two sided signal. So, the ROC is of this form r 1 from your properties of the roc you will realize that roc is of the form r 1 less than magnitude of Z less than r 2. So, I am not able to draw this concentric circles you just apologize for this I think that we will have to do. So, what we have over here is let us say we call this 1 by 3 is equal to, so this is the real part of Z, this is the imaginary part of Z. This there is a zero at zero and there are two poles one at 1 over 3, one at one over half. And ROC is basically between the two poles that is between the concentric circles corresponding to the concentric circles corresponding to the two poles Z plane.

(Refer Slide Time: 17:51)



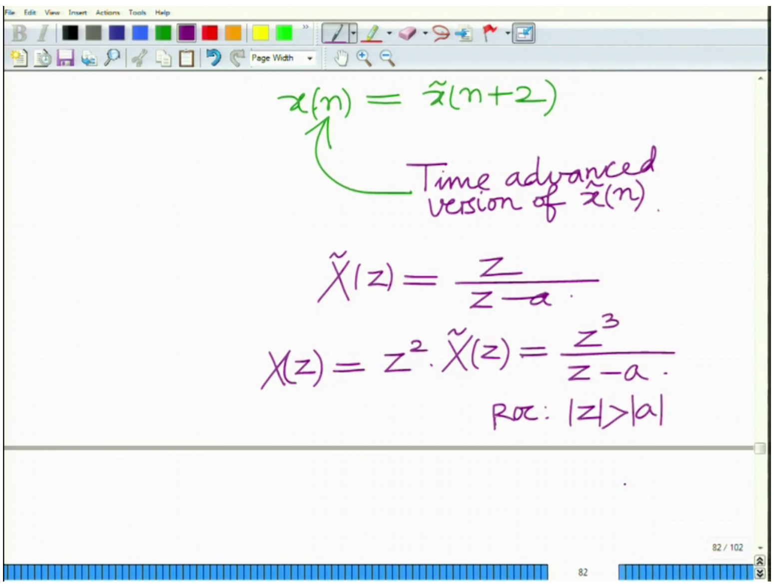
Poles equal to 1 by 3 comma half, zero at zero. So, this is what the z-transform. So, ROC, so this is the pole zero plot. So, this is the pole zero plot this is what the ROC looks like and this is basically the Z plane diagram. In the ROC it comprises of basically the region between these two concentric circles, one is a concentric circle corresponding to a magnitude of Z equals 1 by 3, the other is the circle corresponding to magnitude of Z equals half and ROC basically corresponds to the region between these two concentric circles all right. So, this is basically how the ROC corresponding to this problem looks like.

(Refer Slide Time: 19:07)



Let us do one more problem in this module and then we can continue later. So, this is the z-transform. So, find the z-transform of you know this basically it illustrates a property a power n plus 2 u power u power n plus 2. Now, consider to find the z-transform of this consider x tilde z x tilde n a different sequence that is a power n into u n you can see that x n is x tilde of n plus 2 that is time advanced version of x tilde that is x tilde advanced by 2. So, x n obtained by time advanced version of x tilde n time advanced version of x tilde n and now if you take the z-transpose of x tilde n, we know the z transpose x tilde n that is x tilde equals z over z minus a.

(Refer Slide Time: 20:29)



Now we have x n is x tilde n plus 2, so X of Z we know is Z square by the time shifting property Z square x tilde Z that is Z square into Z over Z minus a that is Z cube by Z minus a. And the ROC will be since this is a right handed signal ROC will be magnitude of Z greater than a or magnitude of Z greater than magnitude of a. So, ROC is magnitude of Z greater than magnitude of a and the z-transform is X Z equals Z cube over Z minus a. So, that is the z-transform of this signal. So, that completes this problem all right. So, we will wrap up this let us wrap up this module with this problem, and we look at other problems in the subsequent modules.

Thank you very much.