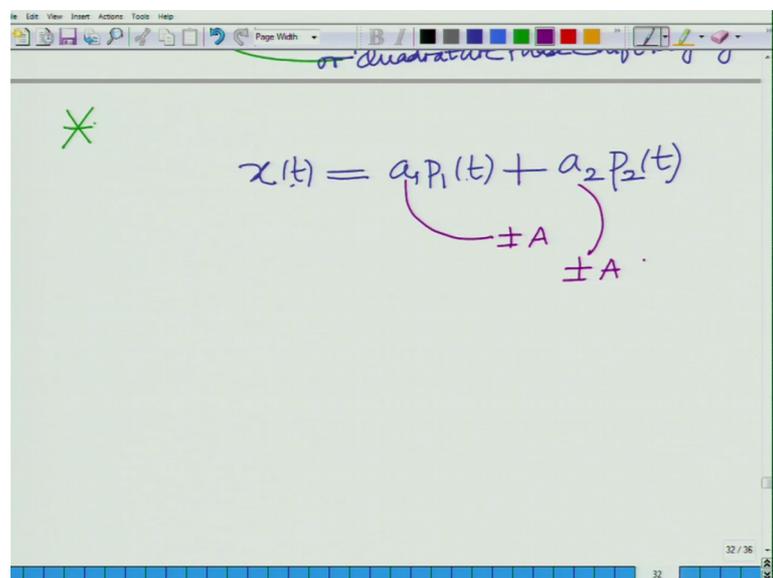


Principles of Communication Systems – Part II
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 18
Waveforms of Quadrature Phase Shift Keying (QPSK)

Hello. Welcome to another module in this massive open online course. So, we are looking at a different digital modulation scheme that is Quadrature Phase Shift Keying.

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$$x(t) = a_1 p_1(t) + a_2 p_2(t)$$

The image shows a digital whiteboard with a toolbar at the top and a status bar at the bottom. The equation $x(t) = a_1 p_1(t) + a_2 p_2(t)$ is written in blue ink. A purple bracket under the a_1 term is labeled $\pm A$, and another purple bracket under the a_2 term is also labeled $\pm A$. A green asterisk is drawn in the top left corner of the whiteboard area. The status bar at the bottom right shows '32 / 36'.

So, let us continue looking at Quadrature Phase Shift Keying, and we have seen that in Quadrature Phase Shift Keying each $x(t)$ equals $A_1 p_1(t) + A_2 p_2(t)$ and each A_1, A_2 is either plus or minus A .

(Refer Slide Time: 00:56)

$$x(t) = \begin{cases} A P_1(t) + A P_2(t) \\ A P_1(t) - A P_2(t) \\ -A P_1(t) + A P_2(t) \\ -A P_1(t) - A P_2(t) \end{cases}$$

4 Possible waveforms

So, $x(t)$ the possible waveforms $x(t)$ there are 4 possible waveforms $x(t)$ that is $A P_1(t) + A P_2(t)$, $A P_1(t) - A P_2(t)$, $-A P_1(t) + A P_2(t)$ and $-A P_1(t) - A P_2(t)$. So, these are the 4 possible waveforms in Quadrature Phase Shift Keying.

(Refer Slide Time: 01:52)

Consider $A P_1(t) + A P_2(t)$

$$= A \cdot \frac{\sqrt{2}}{T} \cos(2\pi f t) + A \cdot \frac{\sqrt{2}}{T} \sin(2\pi f t)$$

$$= \frac{A \cdot \sqrt{2}}{T} \left\{ \cos(2\pi f t) + \sin(2\pi f t) \right\}$$

Now, let us look at these little bit more deeply. Now consider for instance $A P_1(t) + A P_2(t)$. I can write this as A times we note $P_1(t)$ is square root of 2 over T cosine $2\pi f t$, plus A times square root of 2 over T , $P_2(t)$ square root of 2 over T sin $2\pi f t$. I can

simplify this further by taking a square root of 2 over t outside cosine 2 pi f c t plus sin 2 pi f c t.

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$$= \frac{A}{\sqrt{T}} \sqrt{2} \left\{ \cos(2\pi f t) + \sin(2\pi f t) \right\}$$

$$= \frac{A \cdot 2}{\sqrt{T}} \left\{ \frac{1}{\sqrt{2}} \cos(2\pi f t) + \frac{1}{\sqrt{2}} \sin(2\pi f t) \right\}$$

Which is equal to A 2 divided I am going to multiply and divide by square root of 2. So, I take 1 over square root of 2 inside the brackets that gives cosine 2 pi f c t 1 over square root of 2 cosine 2 pi f c of t plus 1 over square root of 2 sin 2 by f c t.

(Refer Slide Time: 03:29)

$$= \frac{A \cdot 2}{\sqrt{T}} \left\{ \cos\left(\frac{\pi}{4}\right) \cos(2\pi f t) + \sin\left(\frac{\pi}{4}\right) \sin(2\pi f t) \right\}$$

$$= \frac{2A}{\sqrt{T}} \cdot \cos(2\pi f t - \pi/4)$$

Now, I can now realize that this is nothing, but A times 2 over square root of t 1 over root 2 is cosine pi by 4 that is cosine 45 degrees cosine pi by 4 cosine 2 pi f c t plus sin pi by

4 into $\sin 2\pi fct$ which is equal to well. Now you have something interesting this is $2A$ over square root of t cosine a cosine b plus $\sin a$ $\sin b$ is basically $\cos a$ $\cos b$ plus $\sin a$ $\sin b$ is $\cos a$ minus b . So, this is cosine $2\pi fct$ minus π by 4 this is basically your waveform A times $P_1 t$ plus A times $P_2 t$. So, this is the wave form A times $P_1 t$ plus A times $P_2 t$.

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$$= \frac{2A}{\sqrt{T}} \cdot \cos\left(2\pi fct - \frac{\pi}{4}\right) \quad \text{waveform 1.}$$

$A_{P_1}(t) + A_{P_2}(t)$

$$A_{P_1}(t) - A_{P_2}(t)$$

$$= A \cdot \sqrt{\frac{2}{T}} \left\{ \cos(2\pi fct) - \sin(2\pi fct) \right\}$$

$$= A \cdot \frac{2}{\sqrt{T}} \left\{ \cos\left(\frac{\pi}{4}\right) \cos(2\pi fct) - \sin\left(\frac{\pi}{4}\right) \sin(2\pi fct) \right\}$$

Now, how about the waveform let us look another waveform that is A times $P_1 t$ minus A times $P_2 t$. So, let us look at the waveform A times $P_1 t$, minus A times $P_2 t$. Well this will be A times square root of 2 over t cosine $2\pi fct$ minus $\sin 2\pi fct$ again multiplying and dividing by square root of 2 I can write this as A times 2 over root t times.

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$$= A \cdot \frac{2}{\sqrt{T}} \left\{ \cos\left(\frac{\pi}{4}\right) \cos(2\pi f c t) - \sin\left(\frac{\pi}{4}\right) \sin(2\pi f c t) \right\}$$
$$= A \cdot \frac{2}{\sqrt{T}} \cos\left(2\pi f c t + \frac{\pi}{4}\right)$$

↑
waveform 2.

Well, $1/\sqrt{2}$ or basically I can again write this as something similar. I can directly write this as $\cos(\pi/4) \cos(2\pi f c t) - \sin(\pi/4) \sin(2\pi f c t)$ which is equal to $A \cdot 2/\sqrt{2} \cos(2\pi f c t + \pi/4)$, $\cos a \cos b - \sin a \sin b$ is $\cos(a+b)$. So, this is $\cos(2\pi f c t + \pi/4)$. And remember the phase difference.

Now, if you look at these 2 waveforms let us call this as waveform one, and let us call this as waveform let us complete the rest of the thing let us call this as waveform 2. Let us call this as waveform 2 and what we have here is basically. So, what we have here if you look at this is basically.

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$$\text{Similarly, } -A P_1(t) + A P_2(t) = A \cdot \frac{2}{\sqrt{2}} \cos\left(2\pi f c t + \frac{5\pi}{4}\right)$$

$$-A P_1(t) - A P_2(t) = \frac{2A}{\sqrt{2}} \cos\left(2\pi f c t + \frac{3\pi}{4}\right)$$

Now, let us look at another. Now similarly, if I look at if you look at minus A times P 1 t, plus A times P 2 t that is equal to well A times 2 over root t times cosine 2 pi f c t plus pi by 4 5 pi by 4. You can easily show this also, and finally if you look at minus A times P 1 t minus A times P 2 t this will be equal to 2 A divided by root 2 root t times cosine 2 pi f c t plus 3 pi by 4.

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$$x(t) = \begin{cases} \frac{2A}{\sqrt{2}} \cos\left(2\pi f c t - \frac{\pi}{4}\right) \\ \frac{2A}{\sqrt{2}} \cos\left(2\pi f c t + \frac{\pi}{4}\right) \\ \frac{2A}{\sqrt{2}} \cos\left(2\pi f c t + \frac{3\pi}{4}\right) \\ \frac{2A}{\sqrt{2}} \cos\left(2\pi f c t + \frac{5\pi}{4}\right) \end{cases}$$

So, now if you if we rewrite these corresponding waveforms, if we rewrite this x of t equals well we have the 4 possible waveforms, if we rewrite them. We have x of t equals

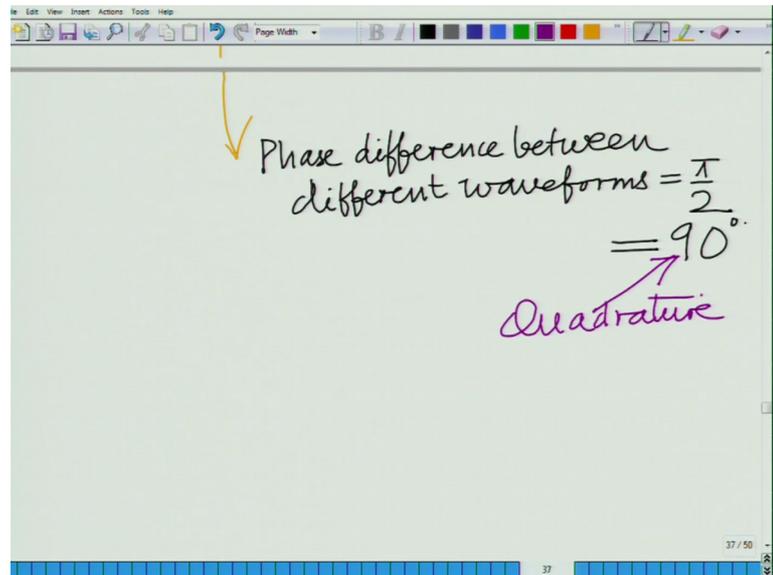
well $2A$ the same waveforms, we write them cosine this is $2A \cos(2\pi fct - \pi/4)$. This is $A \cos(2\pi fct - \pi/4)$ plus $A \cos(2\pi fct - \pi/4)$. Similarly the other wave form will be $2A \cos(2\pi fct + \pi/4)$. Another waveform is $2A \cos(2\pi fct + 3\pi/4)$. I am not writing them in that order, but you can clearly see there is a certain order that I am following and this is $2A \cos(2\pi fct + 5\pi/4)$.

Now, what you will see is very interestingly is that if I shift this by a phase of $\pi/2$ first waveform by a phase of $\pi/2$, I get the next waveform I shift this by $\pi/2$ that is $\pi/4 + \pi/2$ that is $3\pi/4$, I get the next waveform $3\pi/4$ if I shift this I get $3\pi/4 + \pi/2$ that is equal to $5\pi/4$. And now $5\pi/4$ I shift by $\pi/2$ that is $5\pi/4 + \pi/2$ that is $7\pi/4$, modulo 2π that is minus $\pi/4$. So, I get back $\pi/2$.

So, what you observe is that interestingly what you observe is if you can look at this carefully, that all the waveforms there are different waveforms there are 4 different waveforms and this is interesting point. There are 4 different waveforms and these waveforms, differ from each other by a phase difference of $\pi/2$. So, the phase difference between first and second is $\pi/2$ second and third is $\pi/2$ third and 4th is $\pi/2$ and 4th and first is $\pi/2$ sorry these differ by phase difference of $\pi/2$.

So, the difference between the successive waveforms is $\pi/2$. So, I can, so the phase shifts between the first waveform and the next waveform and the waveform after that is $\pi/2$. And $\pi/2$ which correspond when 2 waveforms have a phase shift of $\pi/2$, such as $\cos(2\pi fct)$ $\sin(2\pi fct)$. Remember they are said to be in quadrature. So, we are shifting the phase by ninety degrees or by a quadrature therefore, this is known as Quadrature Phase Shift Keying that is the point.

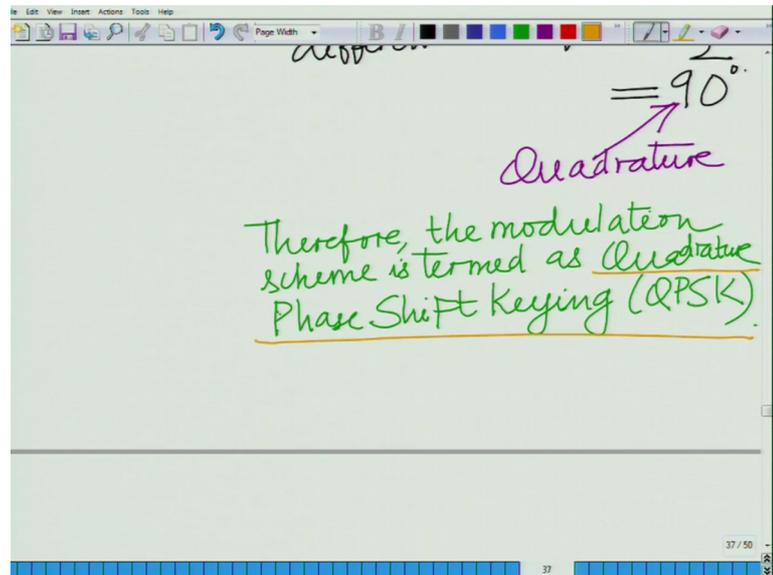
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So, if you look at these waveforms it is slightly subtle, but it is important to understand why this is known as Quadrature Phase Shift Keying.

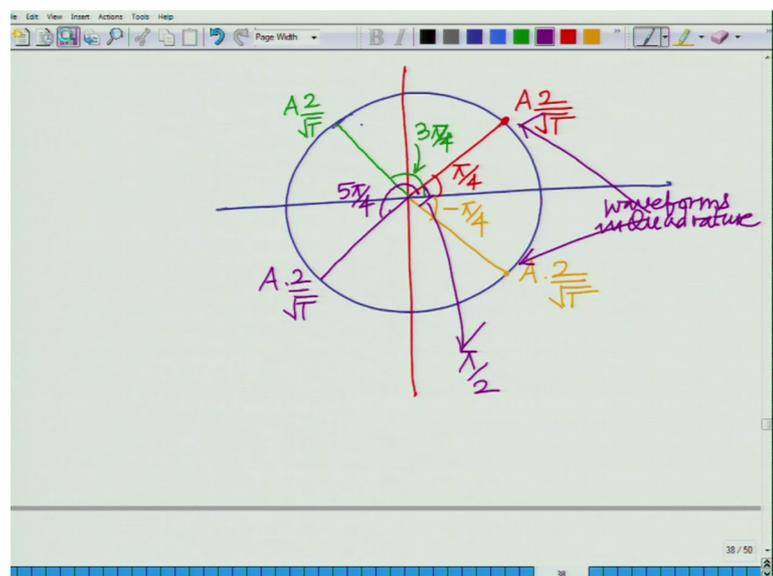
So, we see that phase difference between different waveforms is pi by 2 that is each waveform differs from the previous one that is I can order them, successively with a phase difference of pi by 2 between each successive waveforms right. And the phase difference between waveform is pi by 2 which is equal to basically if you look at this which is equal to ninety degrees. And this is termed as ninety degrees is termed as quadrature, when 2 waveforms have phase difference of ninety degrees they are said to be in quadrature.

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Therefore, the modulation scheme is termed as Quadrature Phase Shift Keying. Modulation scheme is termed as termed quadrature phase or Quadrature Phase Shift Keying. This is the different waveforms, since these can be arranged as since the waveforms can be arranged in order such that each successive pair of successive each pair of successive waveforms differ by a phase difference of π by 2. That is, they are in quadrature therefore and this modulation scheme is termed as Quadrature Phase Shift Keying.

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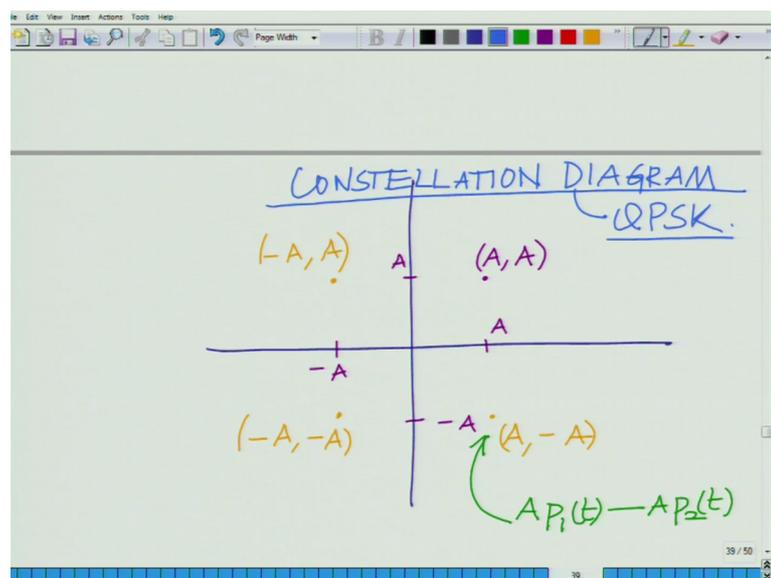


And now if you look at these different waveforms, I can draw them or I can arrange them in a circle correct. So, these different waveforms corresponding to Quadrature Phase Shift Keying, this corresponds to a phase of $\pi/4$. So, this is the point one that corresponds to A; this is basically if you look at this this corresponds to a phase of $\pi/4$ and radius is A, times this is $A \times 2$ over square root of t. Similarly I have something at minus $\pi/4$ $A \times 2$ over square root of t.

Similarly, I have something at this is basically if we look at this angle, this is $3\pi/4$ $3\pi/4$ the radius is the same $A \times 2$ over square root of t and if you finally, look at this this is well this is $5\pi/4$, and again the radius is A times square root of this. So, these and now if you look at the phase difference between successive waveforms for instance the phase difference between this and this a for instance the phase difference between these 2 waveforms this is equal to the phase difference between these 2 waveforms for instance this is equal to $\pi/2$ the phase difference between successive waveforms is $\pi/2$.

So, the waveforms for instance these; so waveforms are in quadrature. Since the phase difference is $\pi/2$, by since the phase difference is $\pi/2$, and interestingly I can also represent this using A 2 dimensional constellation diagram.

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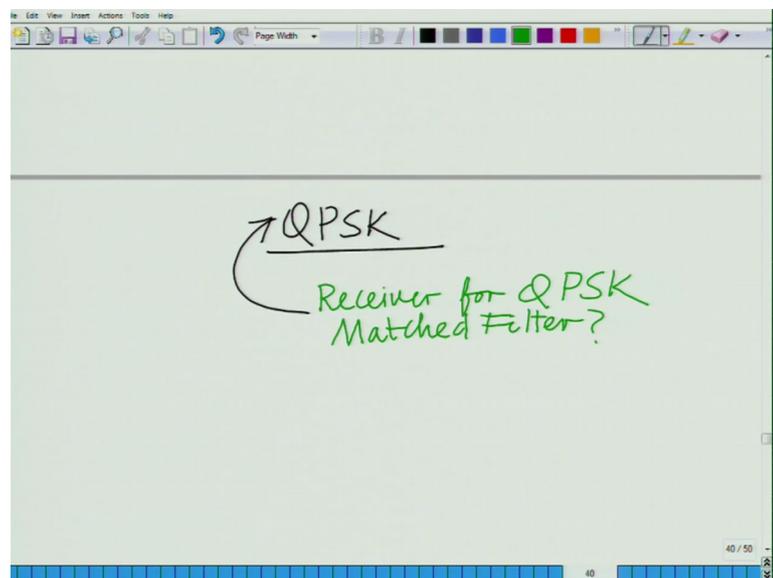
And this also something that you will frequently see in a digital communication system the point one A times $P_1(t)$ plus A times $P_2(t)$ that can be represented as the point a

comma a this corresponds to let us say this is a this is minus A similarly A minus A. So, this point is a comma a this point is minus A comma A this point is minus A comma minus A this point is a comma minus A for instance this point corresponds to A times P 1 t minus A times P 2.

The first coordinate represents the component along the signal space direction P 1 the component along P 1 t and the second coordinate that is minus A comma minus A the second coordinate represents the component or the scaling or the symbol that is transmitted along the signal space component P 2 t, this is known as the constellation diagram because you can see these are points in a plane 2 dimensional plane this is termed as a constellation diagram, and this is the constellation diagram for QPSK that is Quadrature Phase Shift Keying.

So, this is the constellation diagram for QPSK Quadrature Phase Shift Keying and now what we can do is we can look at the matched filter.

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So, what we can do is QPSK what is the receiver and how do we design how do we design the matched filter. So, basically we still need to look at QPSK that is; what are the different operations that need to be carried out at the receiver. How do we perform matched filtering and what is the probability resulting probability of error? So, this we will look at in the subsequent week.

Thank you very much.