Principles of Communication Systems - Part II
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Lecture - 14
Introduction to Signal Space Concept, Orthonormal Basis Signals

Hello, welcome to another module in this massive open online course. So, in today's module this module let us start looking at another concept in digital communication that is a very important concept and in fact also one of the most fundamental frameworks which is termed as a signal space. Slightly abstract but has several applications especially in digital communications.

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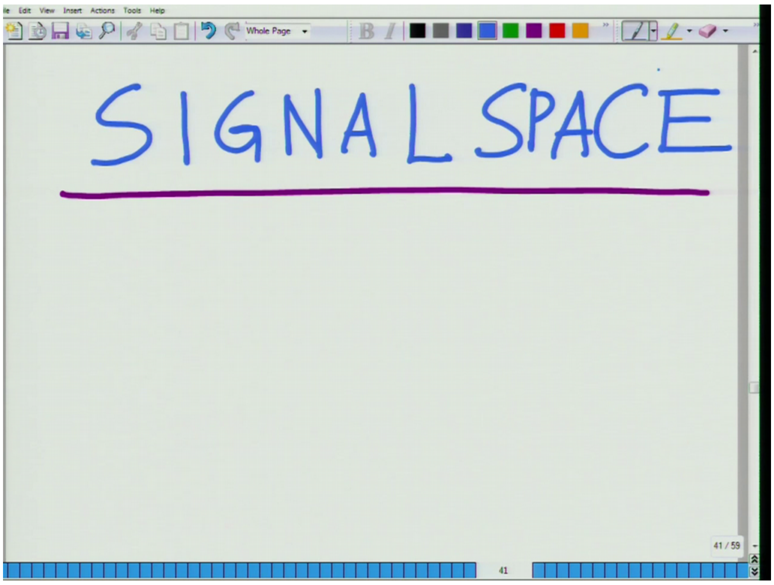

So, let us start looking at what is known as the, start looking at the concept of signal space it is rather a framework, it is a framework for digital.

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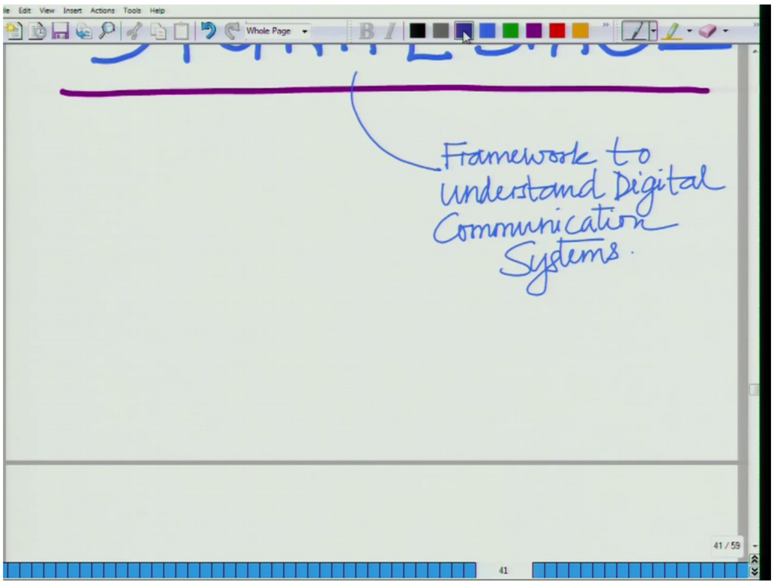

Let us call this as an important framework for to understand, framework to understand digital communication. It is a framework to understand digital communication systems. Now let us start with a simple introduction to the concept of signal space.

Now, consider 2 pulses.

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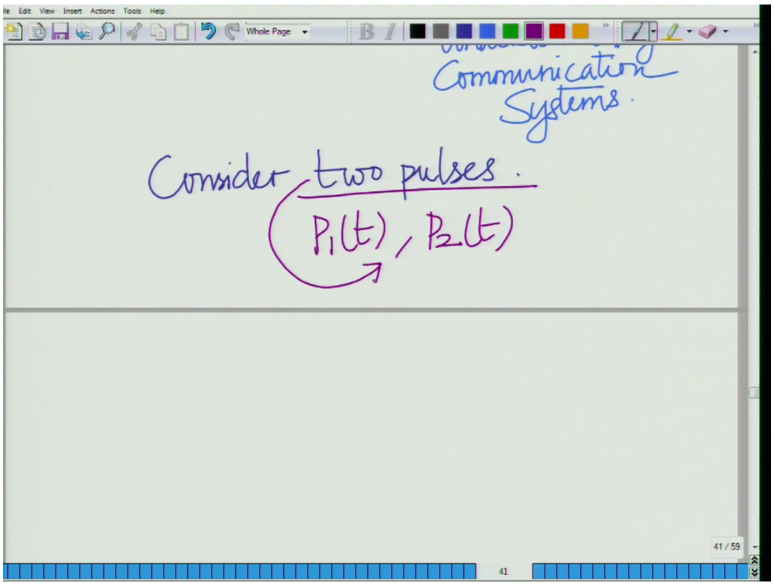

Now, so far what we have been doing is we have been considering a single pulse, now let us consider 2 pulses which we are going to denote by P 1 t and P 2 t these are the 2 pulses that we want to consider.

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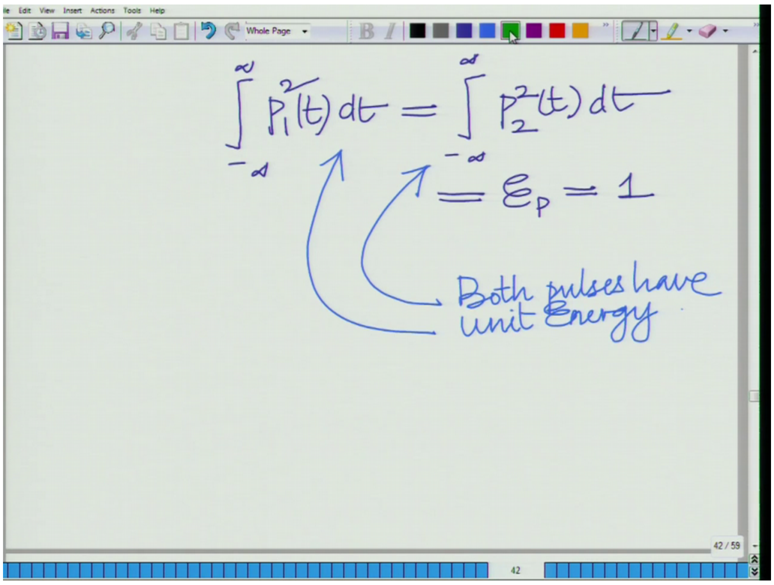

And remember so far we have been considering only a single pulse with thus property that of course, if you look at the energy of the pulses P square t, that is if we look at P 1 square t d t this is equal to minus that is a energy of the second pulse, that is if you look at energies of both the pulses P 2 square d t that is equal to E p, let us set them equal to 1 that is both pulses have unit energy both pulses have unit energy and more importantly that is both pulses are normalized to more important and further let us also note that this is more importantly.

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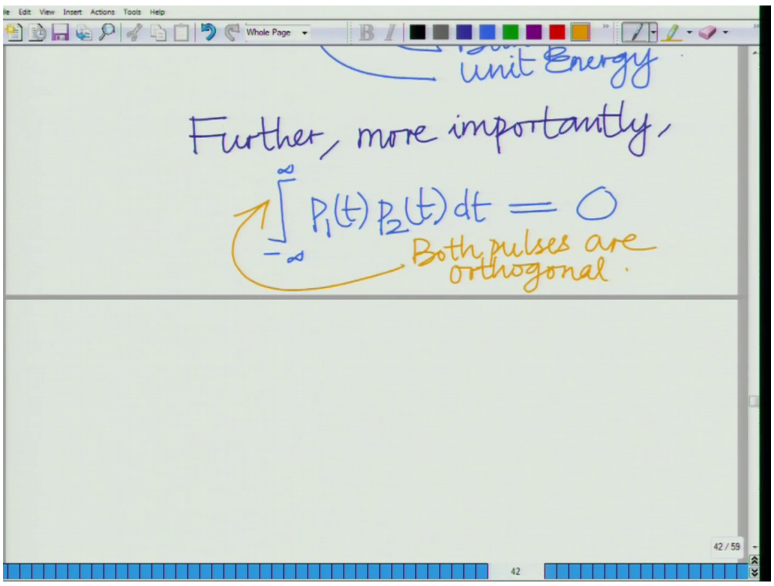

If you look at the inner product between the 2 pulses integral minus infinity to infinity P 1 t, P 2 t this is equal to 0 that is both the pulses are, both pulses are orthogonal. So, what we are saying is that there are 2 pulses each pulse has unit energy and both the pulses are orthogonal that is the inner product if you look at the inner product of these 2 pulses the inner product is 0 for instance.

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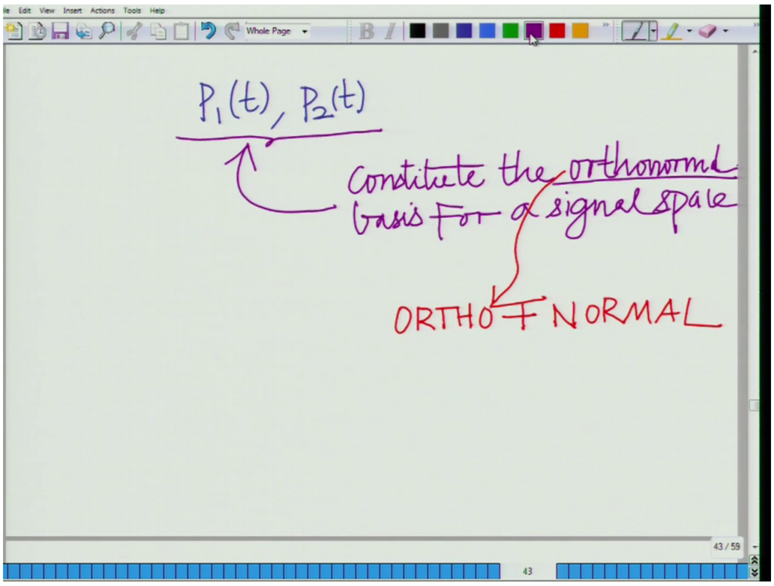

Now what we say is these 2 pulses if you look at P 1 t and P 2 t such pulses P 1 t, P 2 t we said these are the basis functions, these constitute the basis or let us say these constitute not just any basis, these constitute what is known as an orthonormal these constitute what is known as an orthonormal basis for a signal space the key word here is orthonormal. What is the meaning of orthonormal? Remember this is formed from the 2 words ortho plus normal, ortho means both these pulses are orthogonal. That is both pulses are orthogonal and normal means that the pulses are normalized to unit energy.

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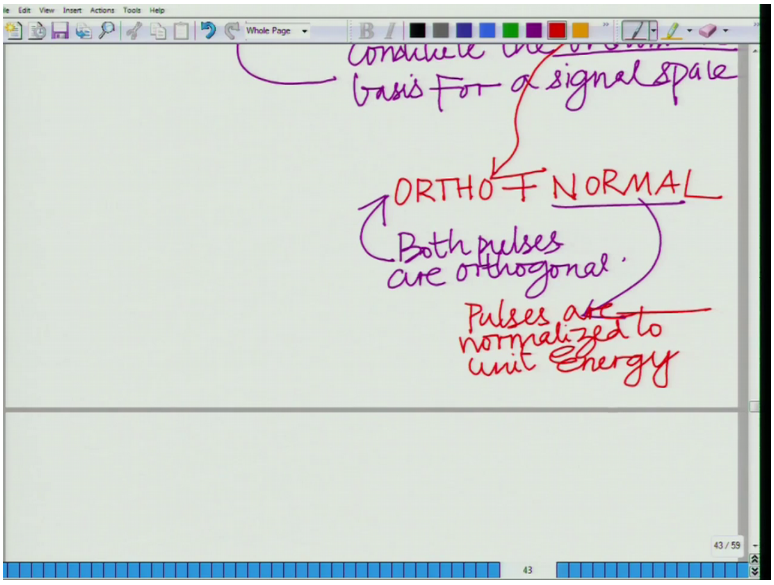

So, they are orthonormal pulses which means the pulses are orthogonal and they are normal that is they are normalized to unit energy and these such to these 2 pulses P 1 to P 2 t form an orthonormal basis for a signal space. Now we have to understand: what is the signal space?

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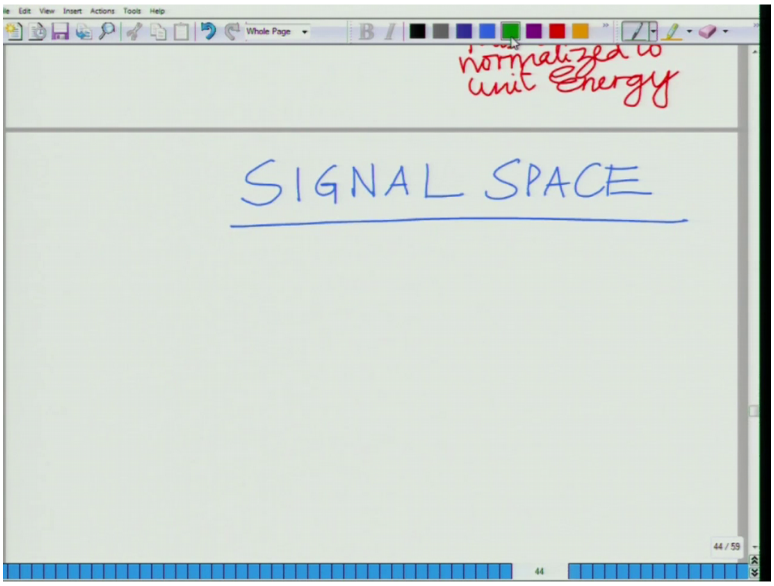

A signal space is simply similar to a vector space is simply a space that is formed by a linear combination.

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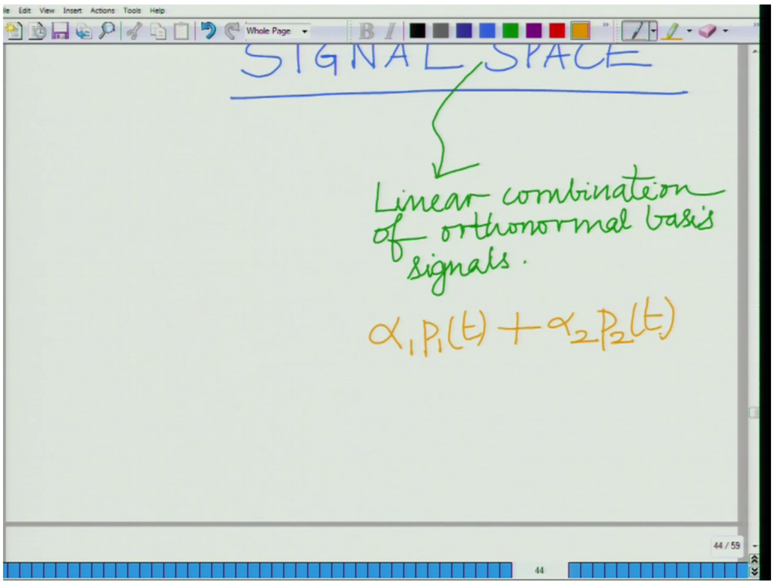

Similar to a linear combination of vectors, we have a linear combination of orthonormal basis signals that is we take alpha 1 times P 1 t plus alpha 2 times P 2 t.

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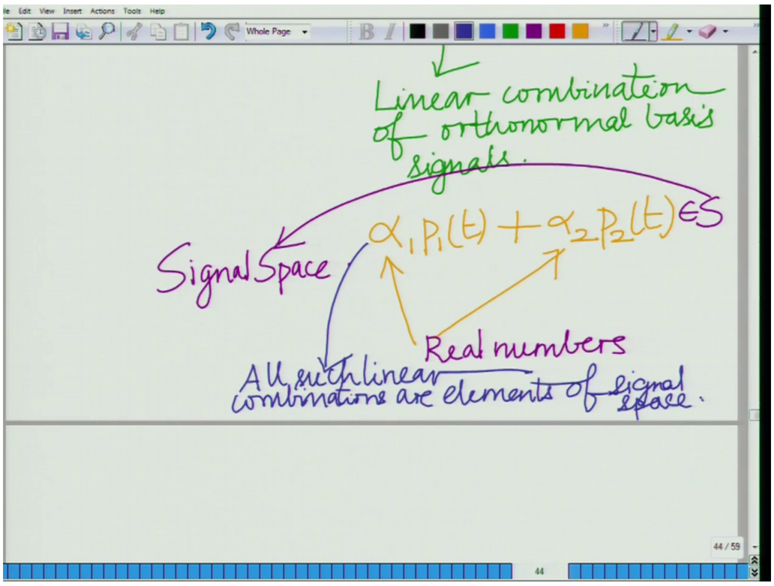

Where alpha 1 and alpha 2 are any, these are any 2 real numbers. So, all such linear combinations form, so all such linear combinations are basically elements of the signal space. So, all these belong to S which is the signal space.

So, all such linear combinations are elements of the signal space. All such linear combinations are all such linear combinations of these 2 orthonormal basis pulses P 1 2 and P 2 t, P 1 t and P 2 t that is alpha 1 times P 1 t plus alpha 2 times P 2 t are elements of this signal space. Now let us take an example.

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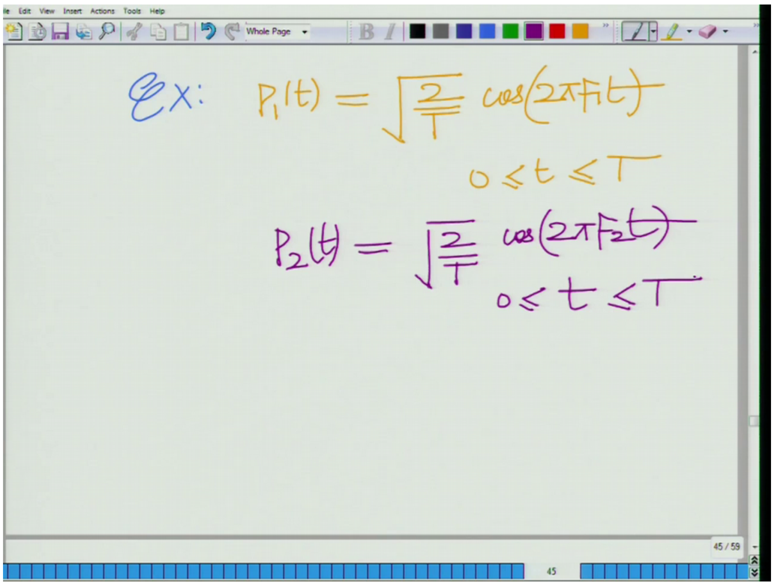

For instance, let us consider a simple example, example: P 1 t equals square root of 2 pi cos this is a pulse that we have already seen and similarly now, let us also take P 2 t equals square root 2 by t cosine 2 pi F 2 t for 0 less than equal to p less than or equal to t, less than or equal to T.

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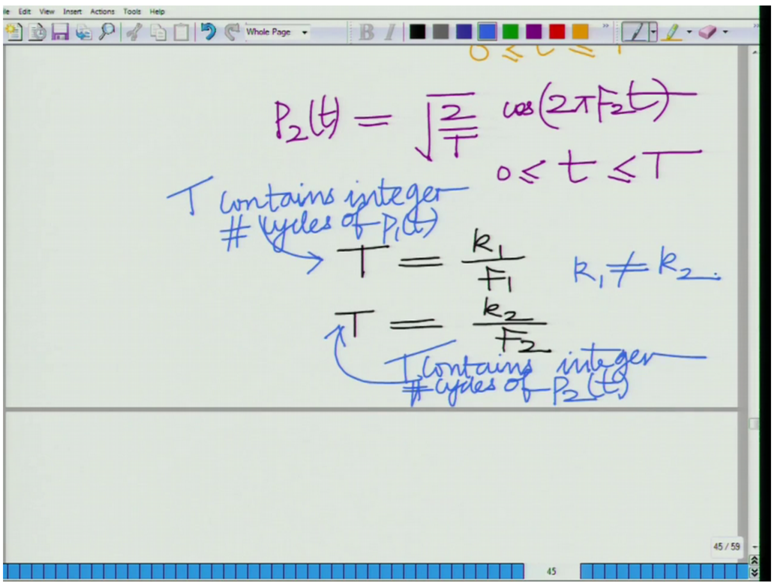

Where can we say, we can say T equals integer multiples that is k 1 over F 1 integer number of cycles of P 1 t also contains integer number of cycles of P 2 that is this condition says T the duration pulse duration T contains integer number of cycles of P 1 t. T also contains integer number of cycles of P 2 t further number of cycles are different that is we are assuming k 1 not equal to k 2 that is the number of cycles of P 1 t the duration T and the number of cycles of P 2 t in the duration T are different. That is these 2 correspond to that is these 2 have a different number of; a different number correct, these 2 have a different integer different numbers of cycles in the symbol duration T.

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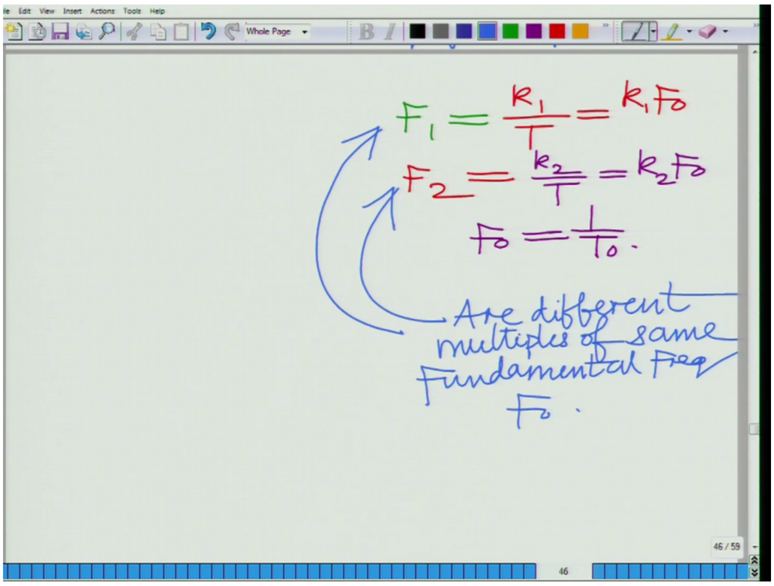

Now, another way to look at this is if you look at F 1 F 1 is equal to now look at it in terms of frequency F 1 is equal to k 1 over T, I can write this as k 1 F naught where 1 over T equals F naught F 2 equals k 2 over T equals k 2 over F naught where F naught equals 1 over T. So, these are different multiples. So, you can see F 1 and F 2 are different multiples these are different multiples of same fundamental frequency F naught. These 2 frequencies F 1 and F 2 are multiples different multiples k 1 times F naught, k 2 times F naught where F naught is 1 over we are considering 2 such pulses alright, cosine 2 pi r square root of 2 or t cosine 2 pi F 1 t square root of 2 or t cosine 2 pi F 2 t.

Now, the reason we are considering square root of 2 over T the scaling factor is obvious that is we want to normalize the pulse to unit energy that is something that we already seen that is if you look at the energy of pulse 1.

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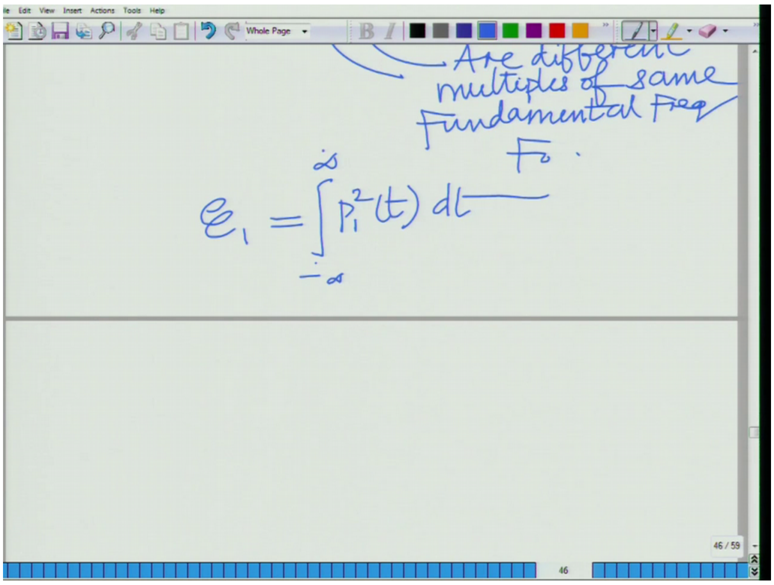

That is if you look at E 1 that is equal to E 1 square T integral 0 to T or integral minus infinity to infinity actually, but it is 0 only from 0 to T.

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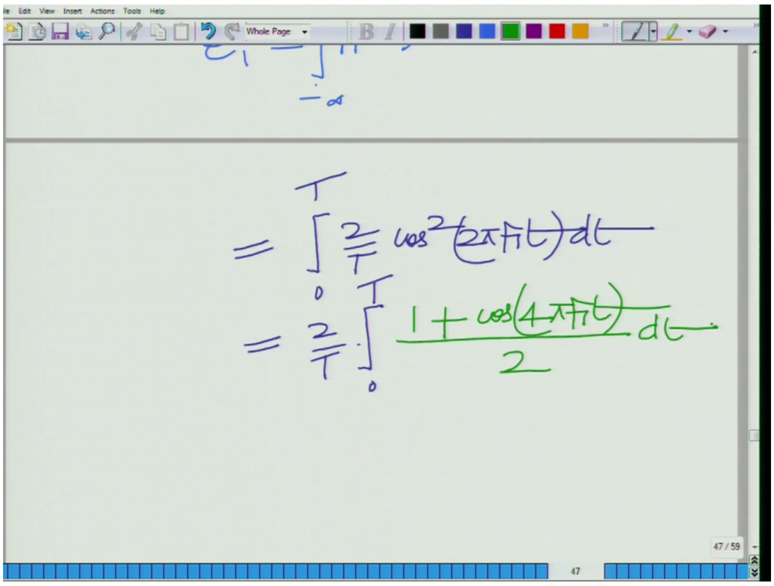

This is equal to 0 to T well 2 over T cosine square 2 pi F 1 t which is equal to 2 over T integral 0 to T cosine square 2 pi F 1 t is 1 plus cosine 4 pi F 1 t divided by 2 times d t.

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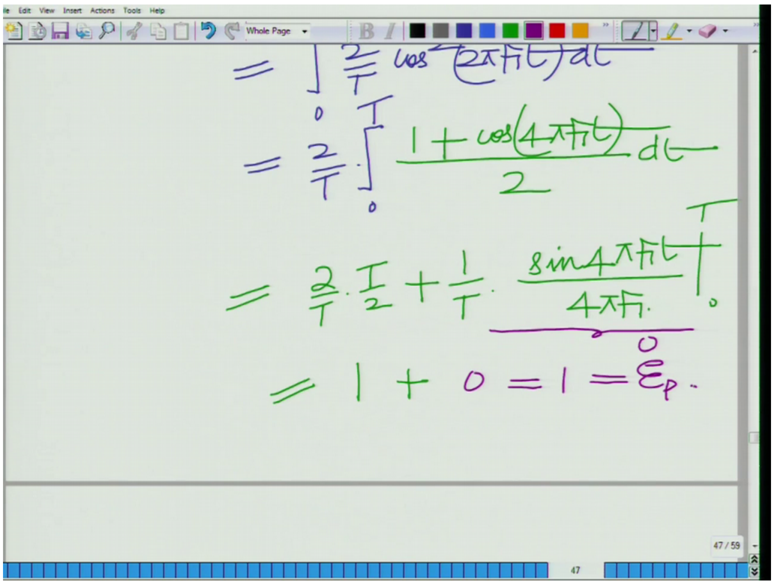

Which is equal to 2 over T times T over 2 plus 1 over T integral cosine 2 pi 1 T is 4 sin 4 pi F 1 t divided by 4 pi F 1 between the limits 0 to t this is equal to 1 plus of course, this quantity we have seen before also this quantity is 0. So, this is equal to 1 which is equal to (Refer Time: 13:47).

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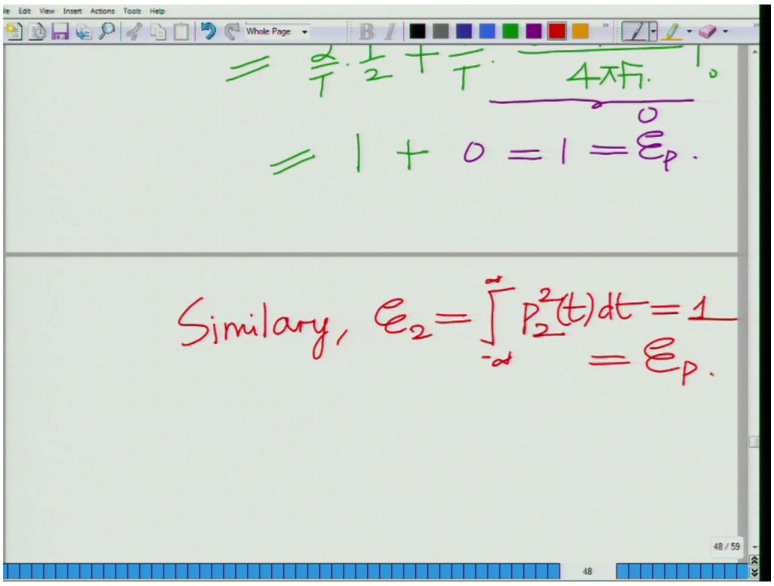

Similarly, if you look at energy of pulse 2 naturally that is also similarly E 2 equals minus infinity to infinity P 2 square d t that is equal to 1 that is equal to E p. Now let us consider the inner product, consider now the inner product.

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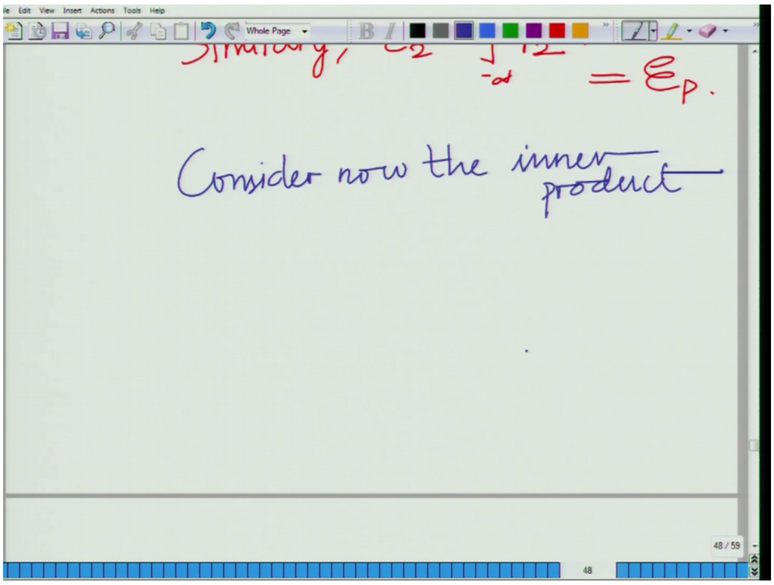

Consider now the inner product if you look at the inner product what is going to happen is we have integral minus infinity to infinity P 1 t, P 2 t d t that is equal to integral minus infinity to infinity substituting P 1 and P 2 square root of raise to over T times cosine 2 pi F 1 t cosine 2 pi F 2 t d t which is equal to integral minus infinity to infinity 1 over T.

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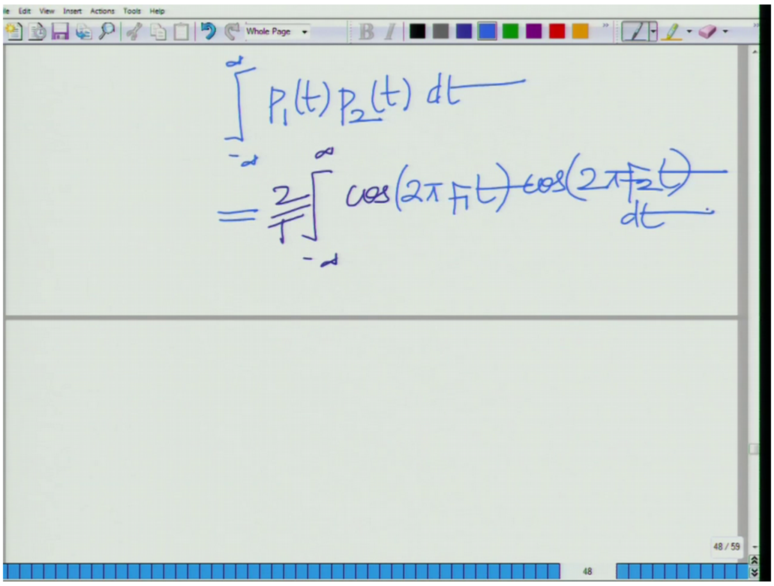

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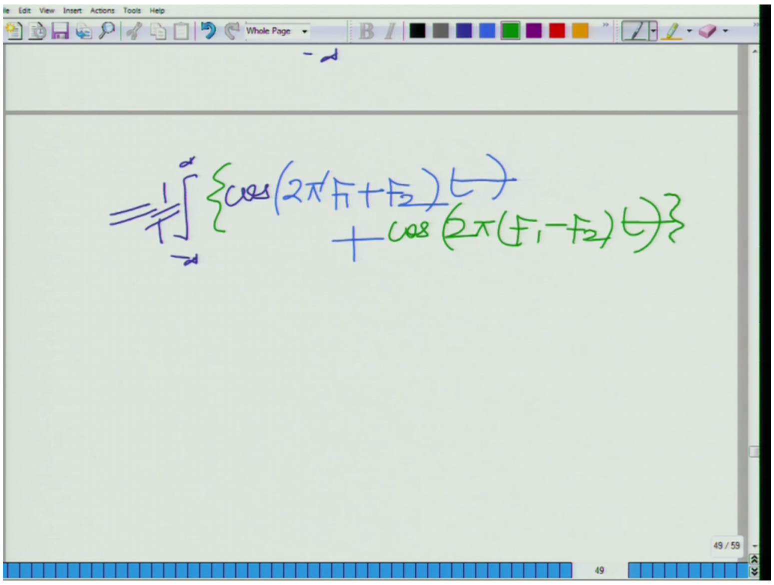

Now, we are going to use the result 2 times cosine a cosine b is cosine a minus b minus cosine a plus or plus cosine a plus b. So, this is going to be cosine 2 pi F 1 plus F 2 t plus cosine 2 pi F 2 F 1 minus F 2 t d t.

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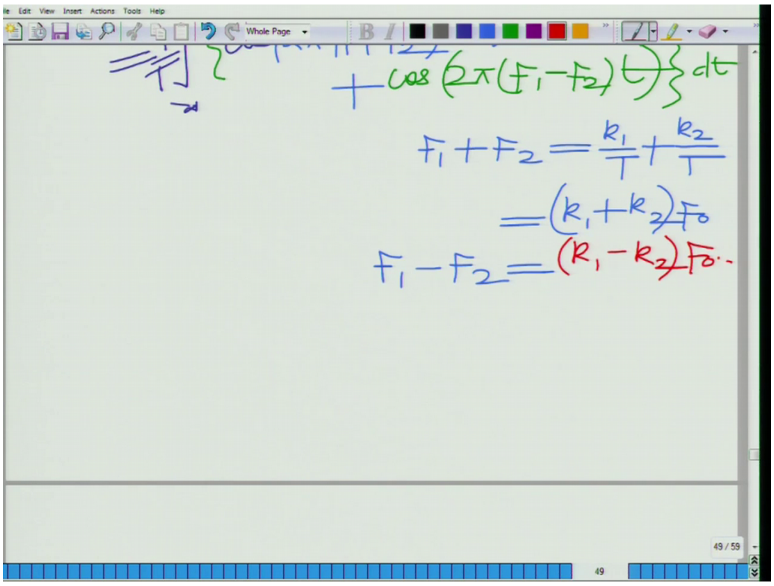

Now, look at this if you look at this F 1 plus F 2 equals k 1 over T plus k 2 over T which we can say this is k 1 plus k 2 times F naught and F 1 minus F 2 is similarly k 1 minus k 2 times F naught. And therefore, the integral is basically if you look at this integral cosine 2 pi F 1 plus F 2 times T this is sin 2 pi F 1 plus F 2 times T.

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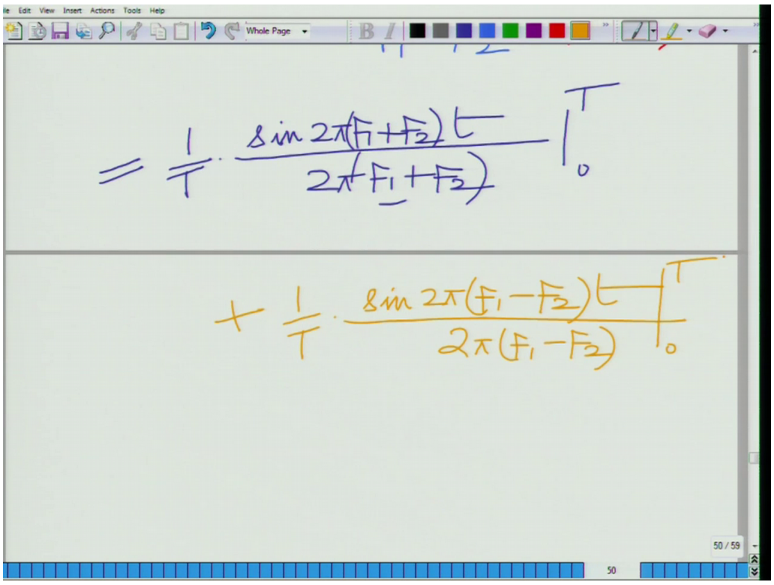

So, this is equal to I can write this as 1 over T sin 2 pi F 1 plus F 2 times T divided by 2 pi times F 1 plus F 2 evaluated between the limits 0 to T plus 1 over T sin 2 pi F 1 minus F 2 times T divided by 2 pi times F 1 minus F 2 0 to T. We already seen F 1 plus F 2 is k times F naught.

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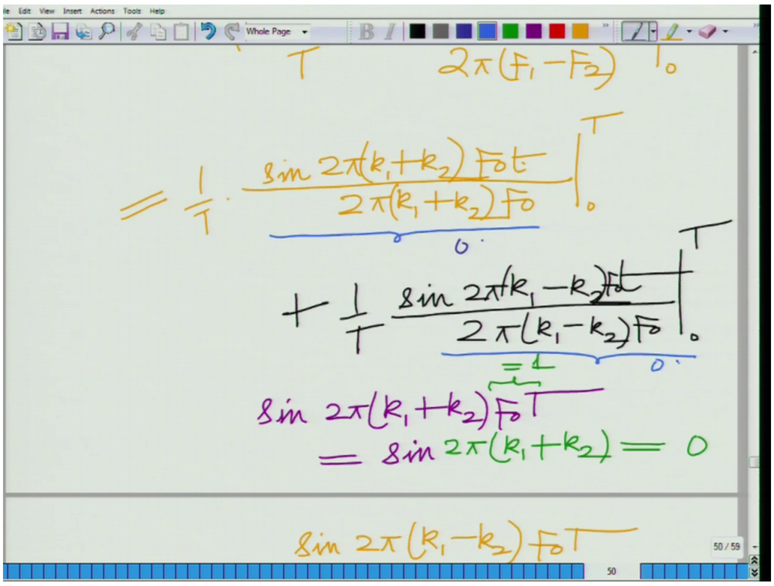

So, that is basically 1 over T sin 2 pi k 1 plus k 2 F naught divided by 2 pi k 1 plus 2 pi F 1 minus F 2 F 1 plus F 2 is 2 pi k 1 plus k 2 F naught between the limit 0 to T plus 1 over T sin 2 pi k 1 minus k 2 into T, into F naught t 2 pi k 1 minus k 2 into F naught 0 to T.

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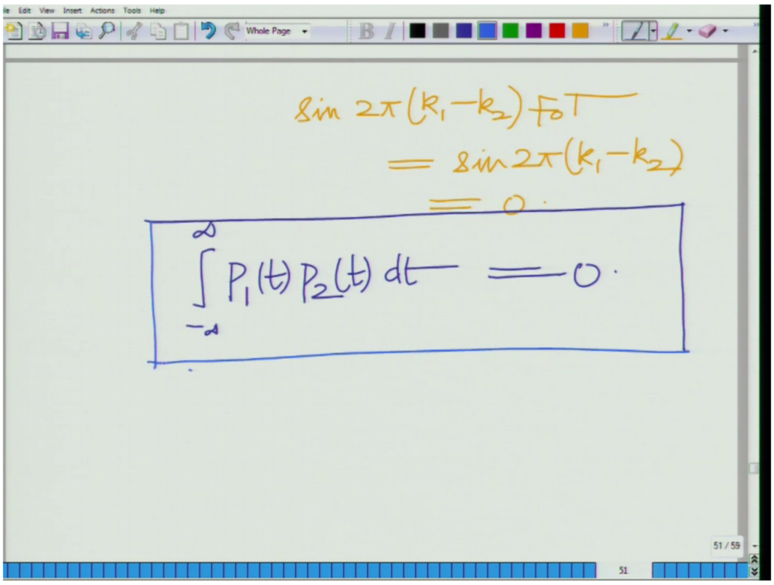

Now, you can see of course, sin of 0 is 0, sin of 2 pi k 1 plus k 2 F naught times T, F naught times T is 1 this is equal to well this F naught times T, F naught equals 1 more t. So, this is equal to 1. So, this is sin 2 pi k 1 plus k 2 sin of 2 pi integer multiple of 2 pi this is equal to 0. Similarly it sin of 2 pi k 1 minus k 2 F naught t equals sin of 2 pi k 1 minus k 2 this is equal to 0. So, therefore, if you look at both these terms these are 0 that is if you look at both these terms these terms are 0 and therefore, this integral we have, the integral the inner product P 1 t, P 2 t d t this is equal to 0.

So, we have inner product is equal to 0.

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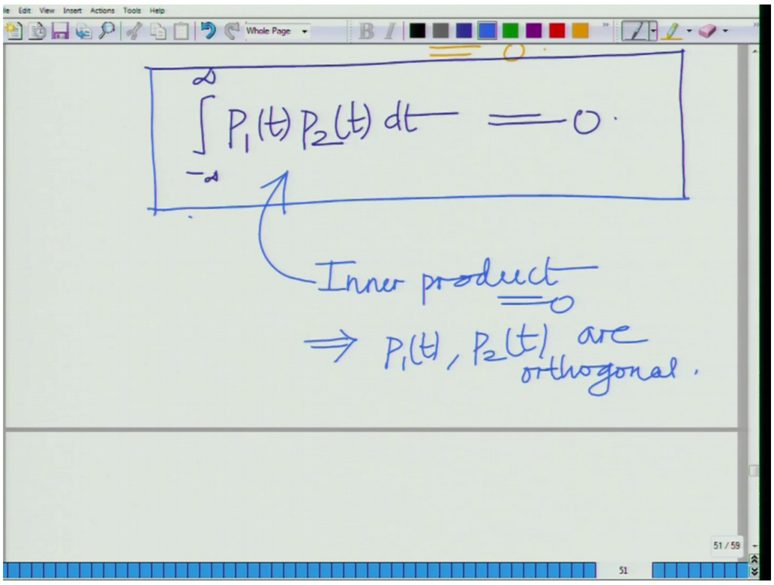

So, we have inner product equal to 0 implies P 1 t, P 1 t, P 2 t are orthogonal. So, we have unit norm. So, we have 2 pulses P 1 t, P 2 t which are unit norm. Also they are normalized have unit energy both have unit energy and they are orthogonal. So, these 2 properties are satisfied.

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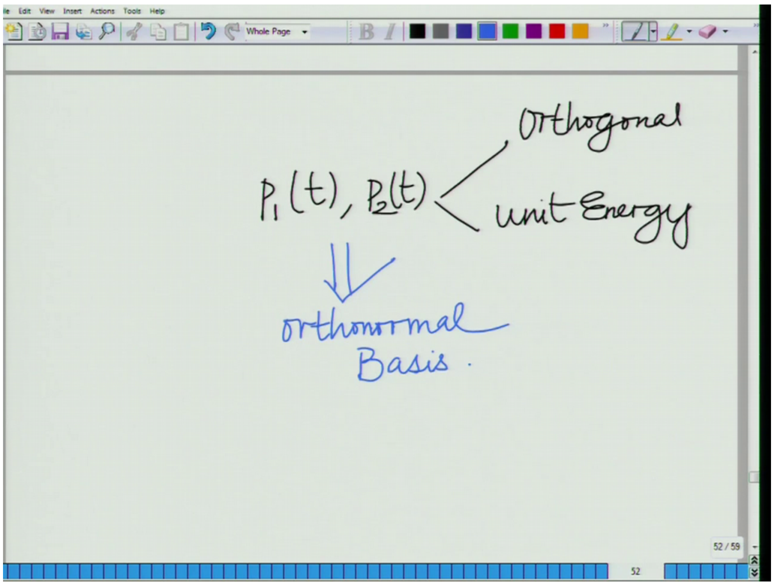

Let us again write that clearly we have P 1 t, these are orthogonal and they have unit energy as well implies they form an orthonormal basis, so these to form an orthonormal basis for the signal space.

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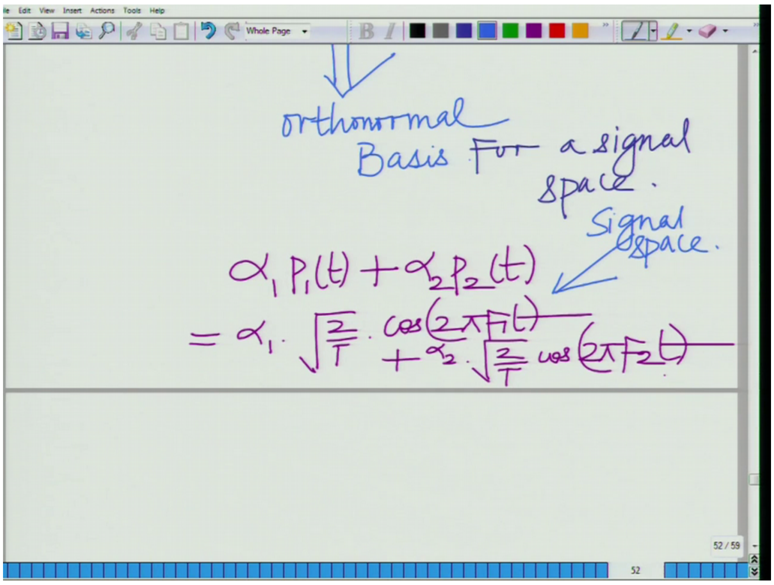

And what is the signal space? The signal space is from the all linear combinations alpha 1 P 1 t plus alpha 2 P 2 t that is alpha 1 square root of 2 over t cosine 2 pi F 1 t plus alpha 2 square root over T cosine 2 pi F 2 t. So, this is basically your, this is basically your signal space. So, they form an orthonormal basis. They form an orthonormal basis for the signal space.

And therefore now, we are going to employ this signal space to divide or to design modulation schemes various modulation schemes, describe the performance of those modulation, describe the various describe how to construct the modulation scheme, describe how to construct the demodulation scheme corresponding to each modulation scheme and also see what is the performance that is a bit error rate performance of each such digital modulation scheme all right, such new digital modulation scheme which is constructed using the employing or based on this concept or based on this framework of signal space. And we are going to look at several interesting digital modulation schemes which rely in this on this abstract concept what is known as a signal space which is generated by multiple orthonormal that is orthogonal and normal basis signal, basis signals or basis pulses what we have seen in this case. So, let us stop here.

Thank you very much.