

An Introduction to Coding Theory
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Module 05
Lecture Number 23
Performance Bounds for Convolutional Codes

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Lecture #12: Performance Bounds for Convolutional Codes



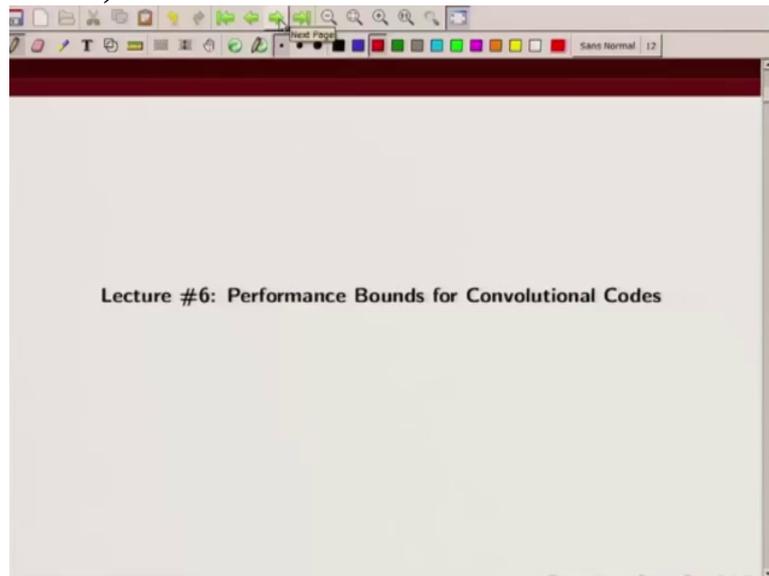
So in this lecture we are going to talk about

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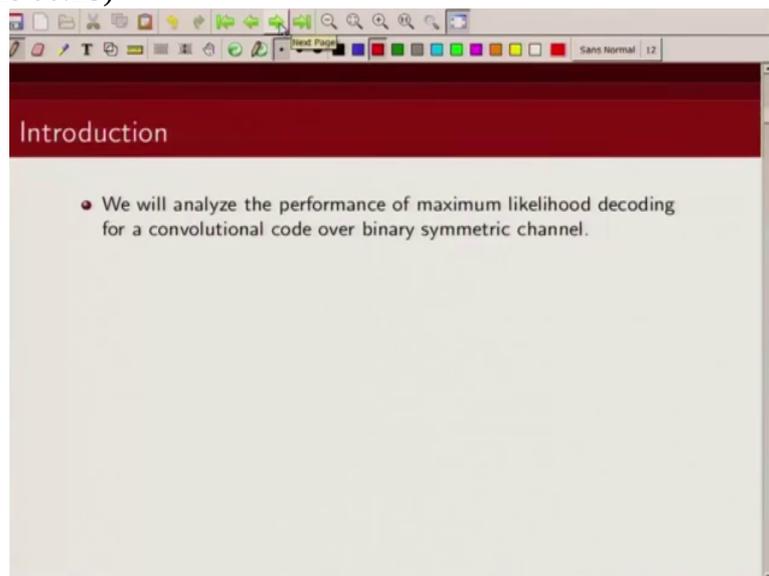
performance bounds for convolutional codes.

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In particular

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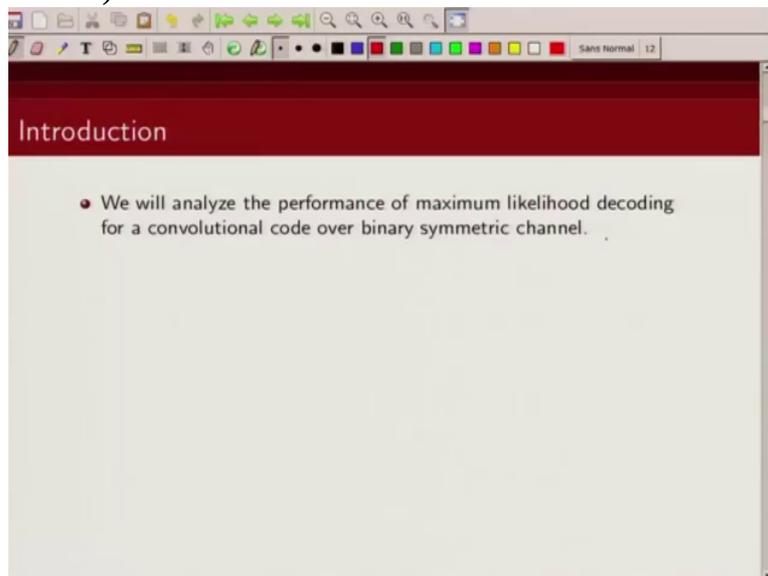
we are going to analyze the performance of

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Viterbi Decoding which is maximum likelihood decoding for convolutional code and as an example we will consider a simple binary symmetric channel. Now if you recall what is a binary symmetric channel,

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so there are 2 inputs, 0 and 1, 0 and 1 and with some probability $1 - p$ we receive the bits correctly and with a crossover probability of p , the bits get flipped. That's our binary symmetric channel.

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Introduction

- We will analyze the performance of maximum likelihood decoding for a convolutional code over binary symmetric channel.

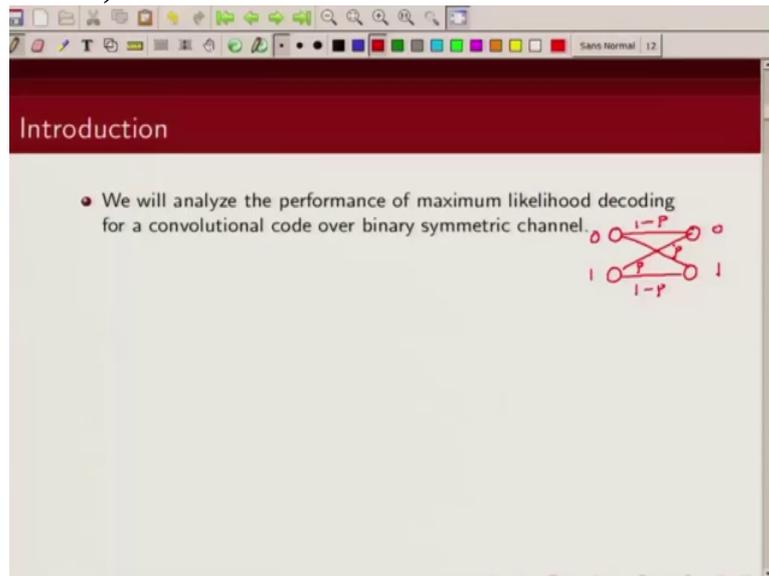
So we are going to analyze

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the probability of error, probability of error when we are doing Viterbi Decoding

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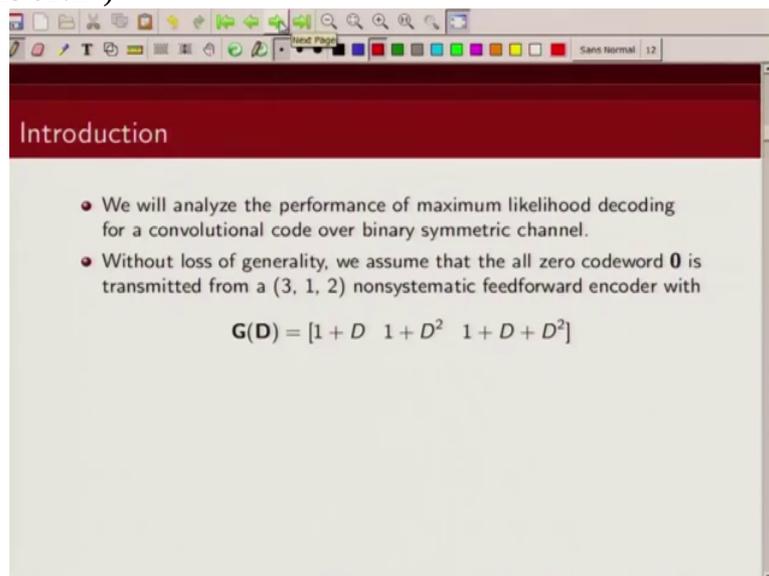
Introduction

- We will analyze the performance of maximum likelihood decoding for a convolutional code over binary symmetric channel.

The diagram shows a trellis with two states at each time step, labeled 0 and 1. Transitions are labeled with probabilities: $1-p$ for transitions that do not change the state (0 to 0 and 1 to 1), and p for transitions that change the state (0 to 1 and 1 to 0).

of convolutional code over a binary symmetric channel.

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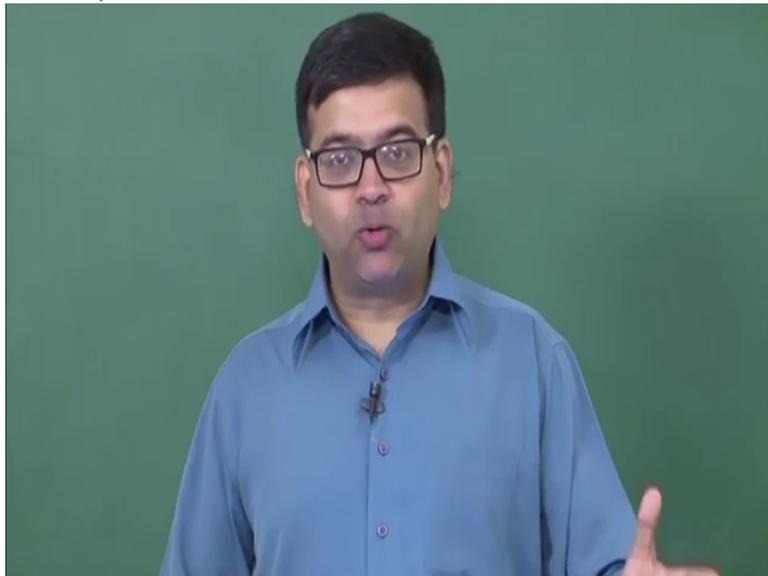
Introduction

- We will analyze the performance of maximum likelihood decoding for a convolutional code over binary symmetric channel.
- Without loss of generality, we assume that the all zero codeword $\mathbf{0}$ is transmitted from a $(3, 1, 2)$ nonsystematic feedforward encoder with

$$\mathbf{G}(D) = [1 + D \quad 1 + D^2 \quad 1 + D + D^2]$$

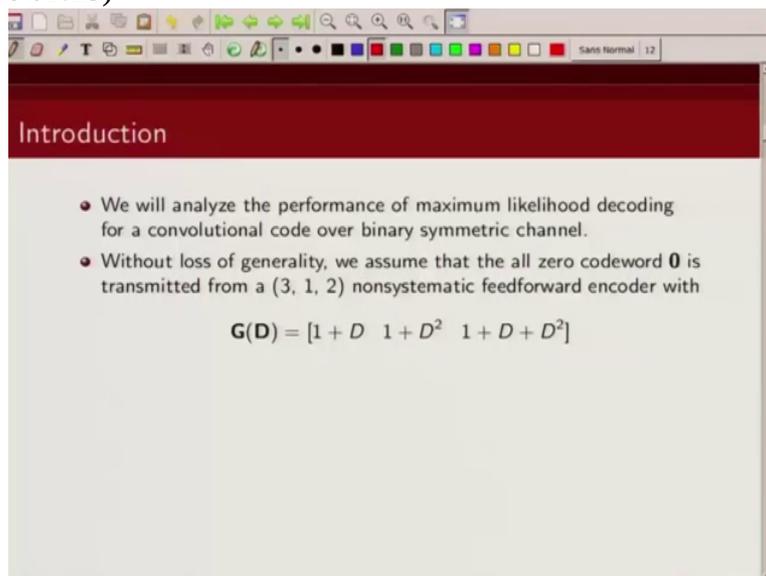
So as an example we will consider one particular

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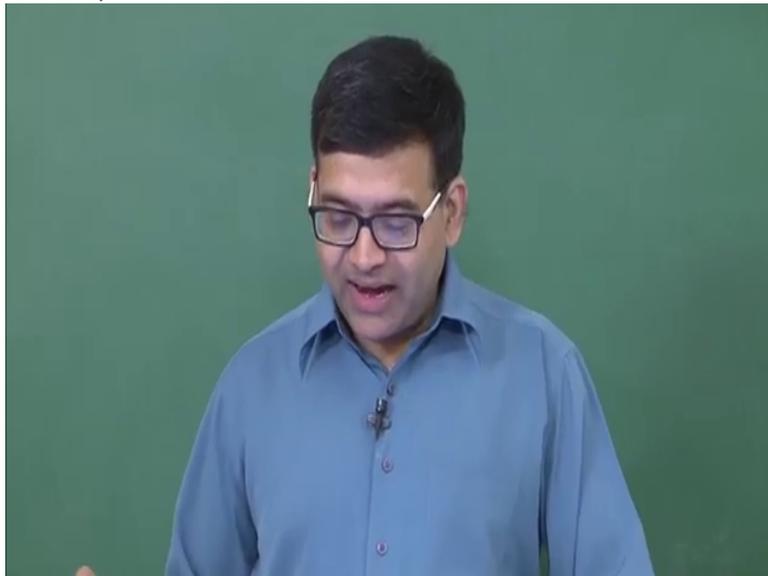
convolutional code. In this case, we are considering a

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rate 1 by 3 convolutional code which has memory 2, so it is a four state convolutional code whose generator matrix is basically given by this. So these are my generators, 1 plus D, 1 plus D square, 1 plus D plus D square and without loss of generality we will assume that all zero codes was transmitted. So these are codewords transmitted over a binary symmetrical channel. And

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we received the bit, we applied maximum likelihood decoding Viterbi Decoding and now we are interested to find, characterize the performance of the Viterbi Decoding algorithm.

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A screenshot of a presentation slide. The slide has a red header with the word "Introduction" in white. Below the header, there are two bullet points. The first bullet point discusses analyzing the performance of maximum likelihood decoding for a convolutional code over a binary symmetric channel. The second bullet point states that without loss of generality, an all-zero codeword is transmitted from a (3, 1, 2) nonsystematic feedforward encoder with a generator polynomial G(D) = [1 + D, 1 + D^2, 1 + D + D^2]. Below the bullet points, the Input Output Weight Enumerating Function (IOWEF) is given by a mathematical formula: A(W, X, L) = (X^7 W L^3) / (1 - X W L (1 + X^2 L)), which is then expanded to X^7 W L^3 + X^8 W^2 L^4 + X^9 W^3 L^5 + X^10 (W^2 L^5 + W^4 L^6 + ...). The slide is shown within a software window with a standard toolbar at the top.

Now we would require one more concept which we discussed in lecture 2 c which is input output weight enumerating function. So what does input output weight enumerating function tells us? It tells us about

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what is the input that will cause or what is the corresponding output that it will cause and what's the length of that particular sequence. So as you can see, all zero paths to the Trellis are all valid codewords. So this weight enumerating function essentially enumerates all non zero codewords.

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A screenshot of a presentation slide. The slide has a red header with the word "Introduction" in white. Below the header, there are three bullet points. The second bullet point includes a mathematical expression for the generator polynomial G(D). The third bullet point includes a mathematical expression for the Input Output Weight Enumerating Function (IOWEF) A(W, X, L).

Introduction

- We will analyze the performance of maximum likelihood decoding for a convolutional code over binary symmetric channel.
- Without loss of generality, we assume that the all zero codeword $\mathbf{0}$ is transmitted from a (3, 1, 2) nonsystematic feedforward encoder with
$$\mathbf{G}(D) = [1 + D \quad 1 + D^2 \quad 1 + D + D^2]$$
- The Input Output Weight Enumerating Function (IOWEF) of this encoder is given by
$$A(W, X, L) = \frac{X^7 W L^3}{1 - X W L (1 + X^2 L)}$$
$$= X^7 W L^3 + X^8 W^2 L^4 + X^9 W^3 L^5 + X^{10} (W^2 L^5 + W^4 L^6 + \dots)$$

So for this particular example the input output weight enumerating function is given by this expression

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and which we can expand, we can divide this by this. So this we will get our terms. Now here x is my, the exponent of x will give me the overall weight, the exponent of w will give me the information sequence weight and this length will give me the length of this path like of this so the time it diverges to all zero state to the time it remerges to all zero state. So $x^7 w^1 l^3$ means there exists a codeword of distance 7 which has information weight 1 and the time it diverges from all zero state to the time it merges back to all zero state, that is 3. And there is one such codeword. This can be interpreted as there is a codeword of weight 8 and corresponding input weight is 2 and it has length 4. Length 4 meaning, again this l denotes, so you have your codeword which diverging from all zero state and then after staying in some all zero state it is merging back into all zero state. So this l denotes the time, the time from which

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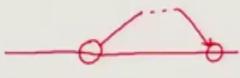
Introduction

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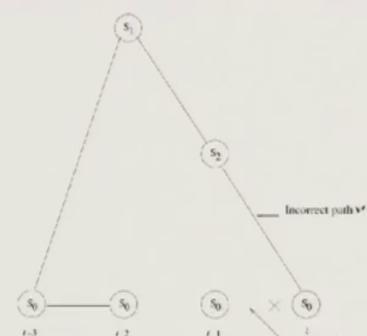


it moves away from the all zero state to the time it takes to come back to all zero state.

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First Error Event

- A first event error happens at an arbitrary time t if the all zero path is eliminated for the first time in favor of an incorrect path.



So let us talk about what do we mean by error here. So since we consider an all zero sequence we expect that our correct path should be the one which goes through all zero state

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First Error Event

- A first event error happens at an arbitrary time t if the all zero path is eliminated for the first time in favor of an incorrect path.

The diagram illustrates a trellis code with states S_1, S_2, S_3, S_4, S_5 . A red path is labeled "Correct path v " and a black path is labeled "Incorrect path v^* ". A timeline below the states shows time steps $t-3$, $t-2$, $t-1$, and t .

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because our transmitted code sequence that we assume, you recall,

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The slide is titled "Introduction" and contains the following text:

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we assume that an all zero codeword was transmitted. So what is the correct

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- The Input Output Weight Enumerating Function (IOWEF) of this encoder is given by

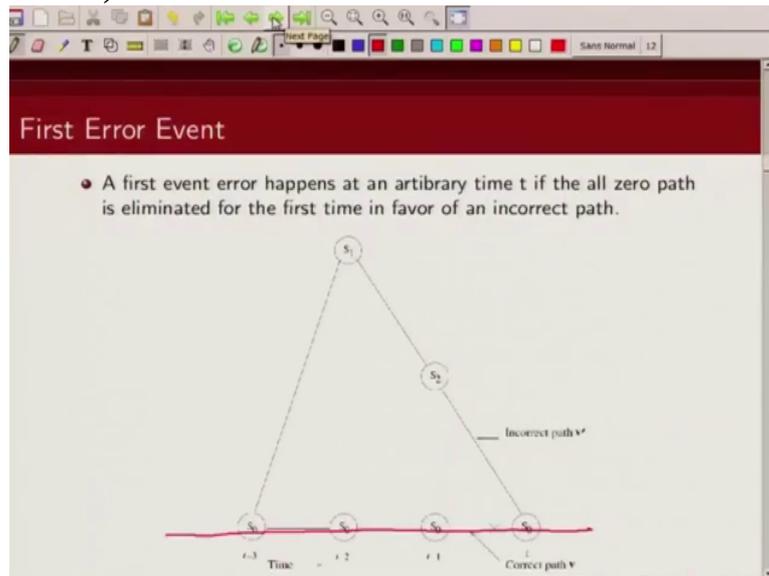
$$A(W, X, L) = \frac{X^7 W L^3}{1 - X W L (1 + X^2 L)}$$

$$= X^7 W L^3 + X^8 W^2 L^4 + X^9 W^3 L^5 + X^{10} (W^2 L^5 + W^4 L^6) + \dots$$

Handwritten annotations include a red box around the fraction in the first equation, red arrows pointing to the terms $X^7 W L^3$ and $X^8 W^2 L^4$ in the second equation, and a red state transition diagram showing two states with a self-loop and a transition between them.

path?

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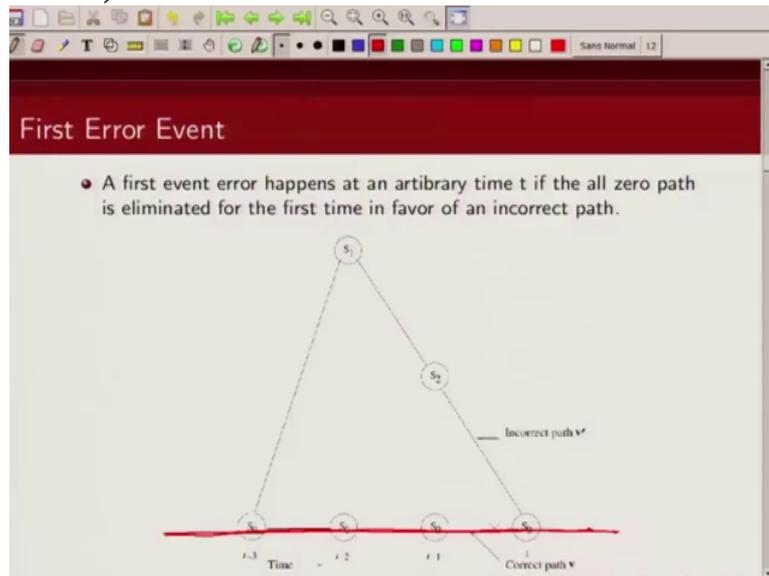
Correct path is one that goes through the all zero states. So this is the correct path. Now when will error happen?

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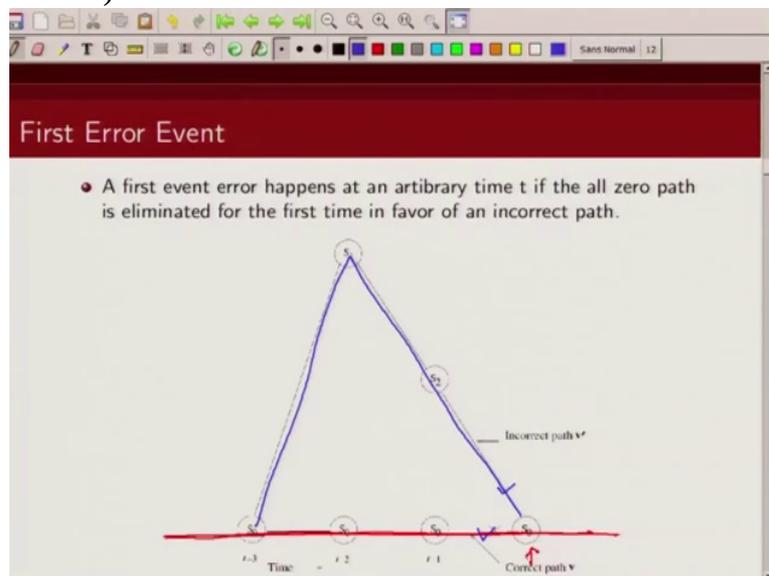
An error will happen if

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at a particular time instance let's just say this is the time instance I am looking at. If at this time instance, instead of deciding for this path, instead of deciding for this path, if I go for this path then error will happen. Why? Why is this an incorrect path? Why is this an incorrect path? This is an

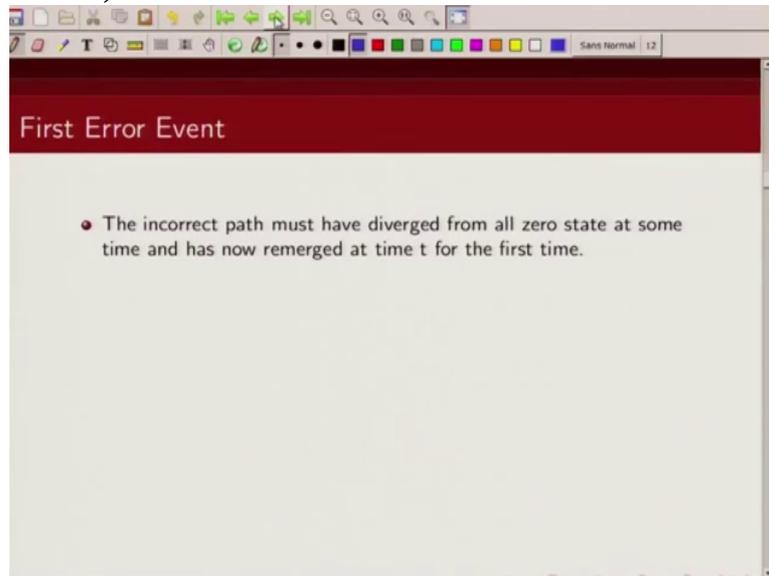
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incorrect path because my transmitted codeword was all zero sequence. So if I decide in favor of this path instead of this path then I make an error. So I say a first event error happens at some arbitrary time t if the all zero path is eliminated in favor of a non zero path which is our incorrect path. So the first instance when I do that, so the instance when I do that, that's basically my event has error because I should have chosen all zero path. But at this instance what happened was the metric corresponding to this path was better so I chose this and I

eliminated this path. But this is a wrong path because my transmitted codeword was all zero sequence.

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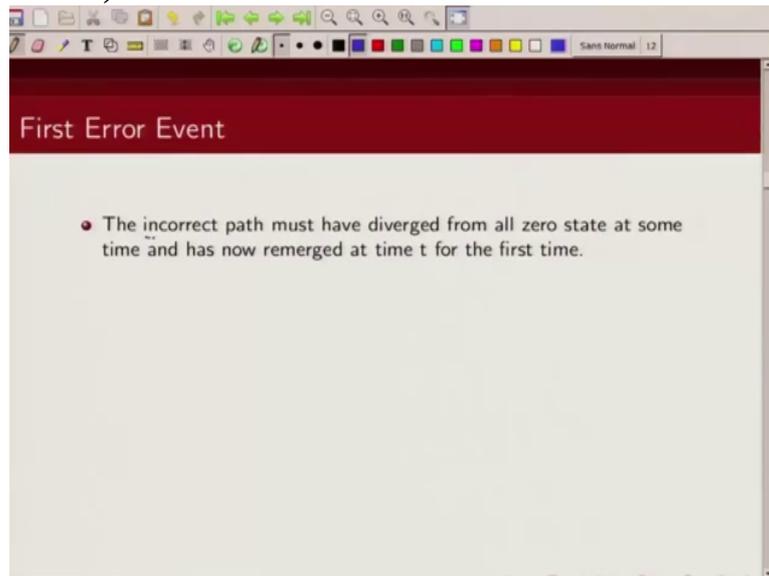
Now what does,

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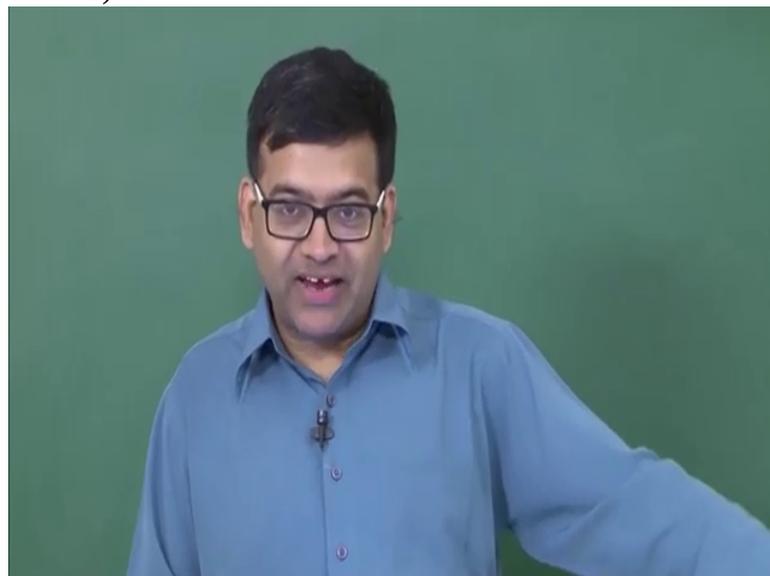
we know that all paths to Trellis are essentially valid codeword. Now what does

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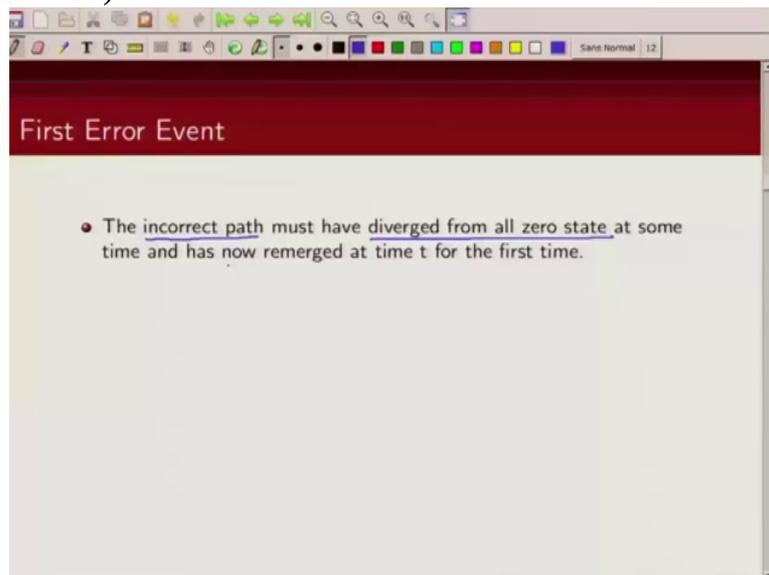
this incorrect path does? It diverges from all zero state at some other time before this time t

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and has merged back

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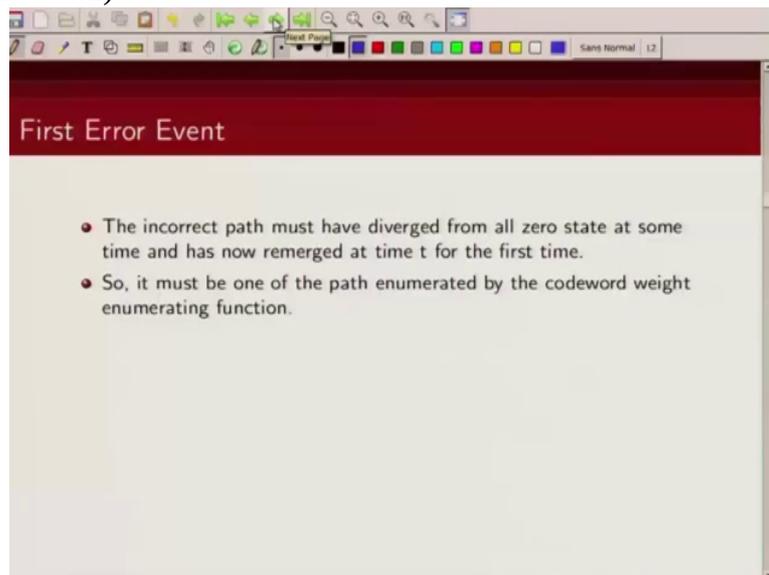
into all zero state at time t for the first time. So that's what we call first error event. So at a time t when we are deciding in favor of some other path other than the all zero path so that means this is the time

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when this other path, incorrect path has merged into all zero state. So at some time in the past it would have diverged from all zero state and now at time t it is merging back into

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all zero state. Now can we find out what is the weight corresponding to this incorrect path?

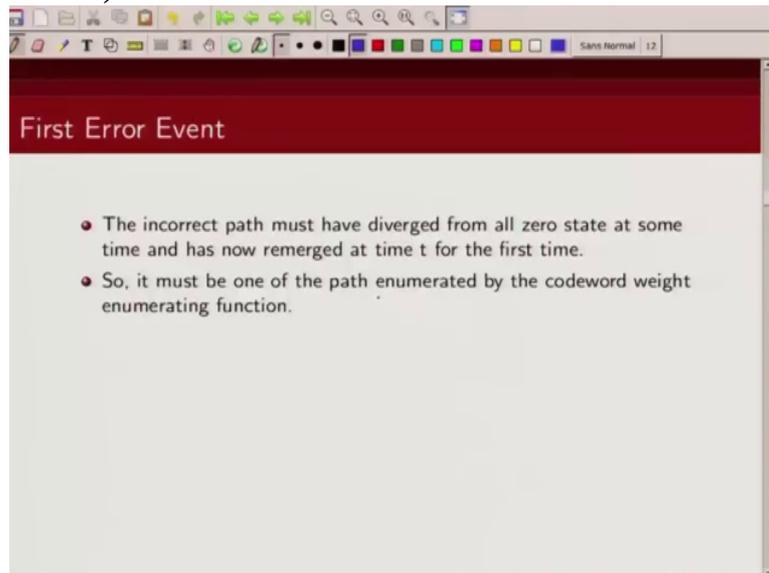
Yes we can because

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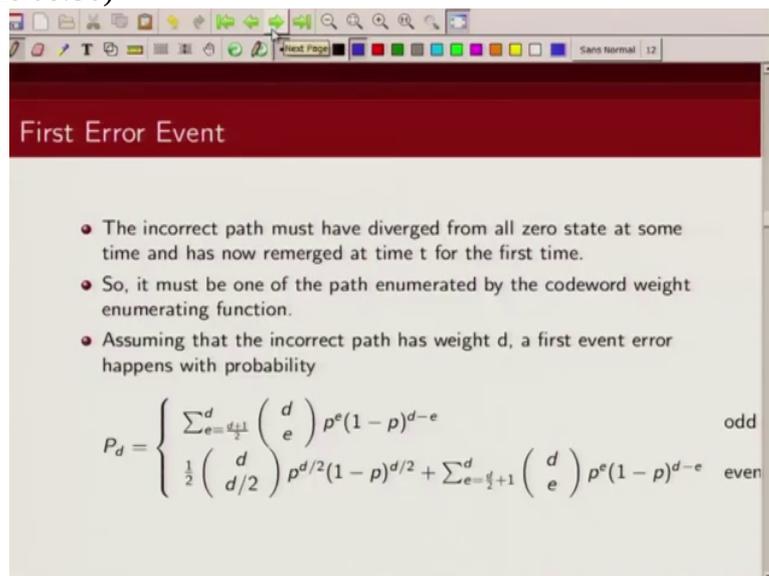
when we write the weight enumerating function of the convolutional code it enumerates weight distribution of all valid codewords. Now any path through this Trellis diagram is a valid codeword. So we can easily

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enumerate what is a code weight corresponding to any path through this Trellis.

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Now when will we decide in favor of this incorrect path and not the all zero path?

We can take the example of the same code that we are considering. Let's

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just this code as d free 7 then it has one

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codeword of weight

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Handwritten annotations: A red box highlights the fraction $\frac{X^7 WL^3}{1 - XWL(1 + X^2L)}$. Red arrows point to the exponents 4, 8, and 9 in the expansion. A red trellis diagram shows a path from state 0 to state 0 with a weight of 10.

8, one codeword of weight

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9, there are 2 codewords of weight 10 like that, so minimum distance d_{free} is 7 here. So let's say, let's

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First Error Event

- A first event error happens at an arbitrary time t if the all zero path is eliminated for the first time in favor of an incorrect path.

assume that this incorrect path has weight 7. Now when will you decide in favor of this path rather than this path? When the number of errors are such that when there are more than 4 errors such that your received sequence is closer to

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First Error Event

- A first event error happens at an arbitrary time t if the all zero path is eliminated for the first time in favor of an incorrect path.

this sequence rather than this, right then you will make an error. Or let's say if this path was weight 8 then of course when your error is more than 4 bits you will decide in favor of this path and whenever error has happened in 4 bit location there is equal probability, you can either choose this path or you can choose this path.

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The slide is titled "First Error Event" and contains the following text:

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d, a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

So we can calculate the probability of this first event error. We can find out its probability. What is its probability? Let's say this incorrect path has weight d. So d is the weight of this incorrect path which has diverged from all zero state at some other time and has merged back at all zero state at time t. Now if d is odd then this probability is given by sum of this probability and

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- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d, a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{d odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

Handwritten annotations include a blue box around the binomial coefficient $\binom{d}{e}$ in the odd case and a blue underline under the variable d in the text above the equation.

where summation takes place over all error pattern which are greater than d plus 1 by 2 up to d. So in the example we just considered, let us say if we had incorrect path of weight 7 then

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability $\frac{d}{7}$

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

we should look at error pattern of 4, 5, 6 and 7. If 4

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First Error Event

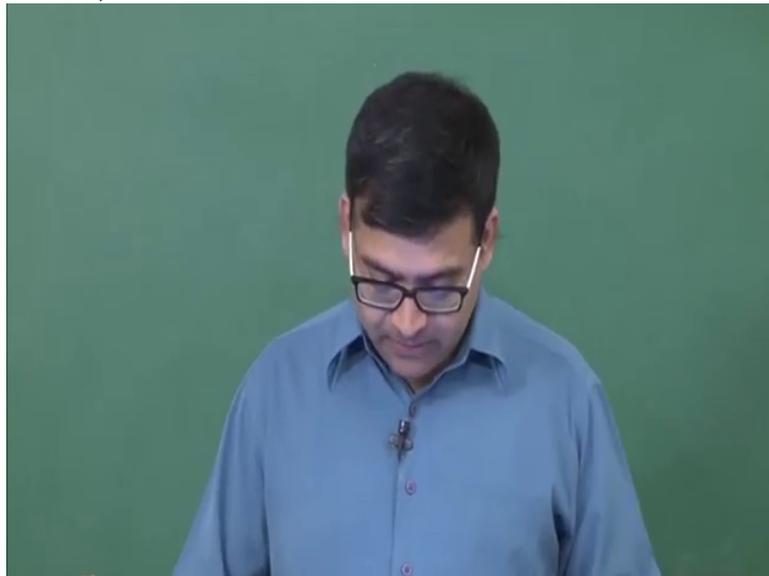
- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability $\frac{d}{7}$

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

$e = 4, 5, 6, 7$

errors happened, may be it will bring us closer to this incorrect path rather than all zero path.

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Now p

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d, a first event error happens with probability $\frac{d}{2}$

$d = 7$
 $e = 4, 5, 6, 7$

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \underline{d \text{ odd}} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

is the cross over probability of changing or flipping of the bits, so p times e will give us the probability that e number of bits have

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flipped and $1 - p$

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability $\frac{d}{2}$ if d is odd and $\frac{d-1}{2}$ if d is even.

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes on slide: $d=7$, $e=4,5,6,7$

raised to power $d - e$ will tell us the probability that $d - e$ bits have not flipped and $\binom{d}{e}$ is number of possible combinations of, in which we can have these error patterns, e error patterns among these set of d weight so this will give us the probability of first event error.

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$, $d \text{ odd}$, even

Again we will make a decision in favor of incorrect path if our received sequence is closer to

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incorrect path. And we know what is the maximum likelihood rule for binary symmetrical channel. It is we choose the codeword such that Hamming distance between the received codeword and this chosen codeword is minimized. So if there

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=0}^{d-1} \binom{d}{e} p^e (1-p)^{d-e} & d_{\text{odd}} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=d/2+1}^d \binom{d}{e} p^e (1-p)^{d-e} & \text{even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$, d_{odd}

are more than, there are four or more errors for the case when incorrect path has weight 7, then you are going to choose, basically go for this incorrect path. What about when d is even? When d is even, for all error patterns from $d/2 + 1$ to d ,

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you will decide in favor of incorrect path and whenever the error is

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First Error Event

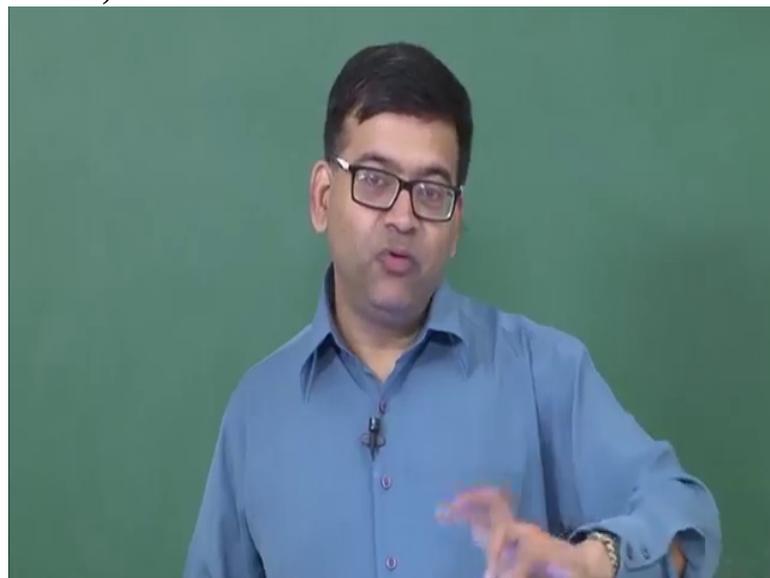
- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=d+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d_{\text{odd}} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=d/2+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d_{\text{even}} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$

d by 2, then there is a 50:50 chance because you can just

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then, then there is a tie in the matrix, so you can just flip and choose either of the path. And that's why for

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$

d by 2; I have written here it is half times

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$

this probability plus all error patterns of weight more than d by 2. So for example in this case we had one path of

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Introduction

- We will analyze the performance of maximum likelihood decoding for a convolutional code over binary symmetric channel.
- Without loss of generality, we assume that the all zero codeword $\mathbf{0}$ is transmitted from a $(3, 1, 2)$ nonsystematic feedforward encoder with

$$\mathbf{G}(D) = [1 + D \quad 1 + D^2 \quad 1 + D + D^2]$$
- The Input Output Weight Enumerating Function (IOWEF) of this encoder is given by

$$A(W, X, L) = \frac{X^7 W L^3}{1 - X W L (1 + X^2 L)}$$

$$= X^7 W L^3 + X^9 W^2 L^4 + X^{10} W^3 L^5 + X^{10} (W^2 L^5 + W^4 L^6) + \dots$$

The trellis diagram shows a path starting from the all-zero state, diverging at time 1, and remerging at time 3. The path is highlighted in red.

weight 8, right? So if our incorrect path was of weight 8, then

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d = 7$, $e = 4, 5, 6, 7$

all error patterns of 5, 6, 7, 8 they

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten annotations:
 For the first term: $d=7$, $e=4,5,6,7$
 For the second term: $d \text{ odd}$, $e=5,6,7,8$

would have caused decision in favor of this incorrect path

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and whenever there is

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$ for odd; $e=5,6,7,8$ for even.

error pattern of

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$ for odd; $e=4$ for the first term of the even case; $e=5,6,7,8$ for the second term of the even case.

weight 4, there is a 50:50 probability that I may choose

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an incorrect path or I may choose an all zero path because the Hamming distance is same from all zero path or this incorrect path. So that's why

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First Error Event

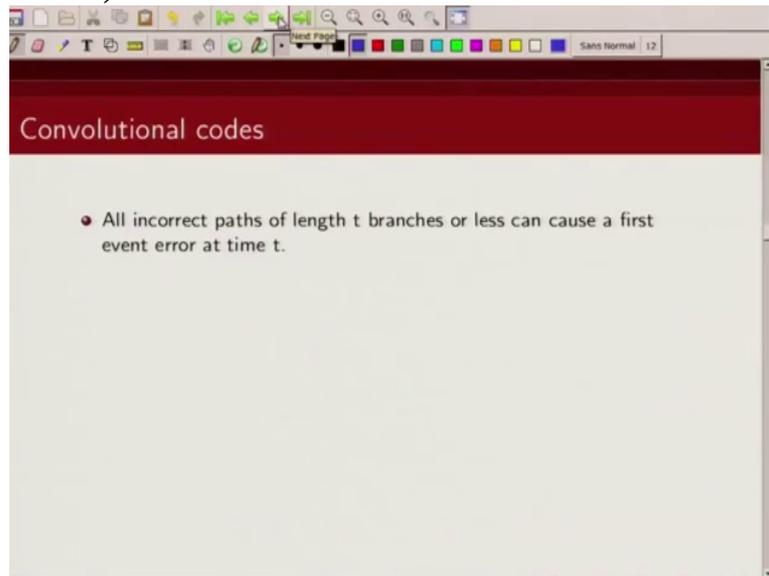
- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

$e = 4, 5, 6, 7$ $d = 7$
 $e = 4$ $e = 5, 6, 7, 8$

I have a half here. So this is a probability of first event error. Now

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if we are looking at time t then all incorrect path lengths of length t or

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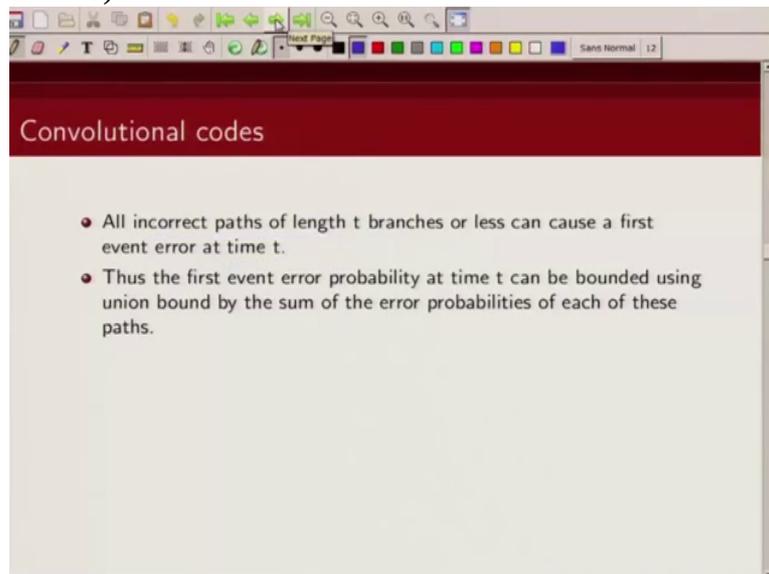


less can cause first event error.

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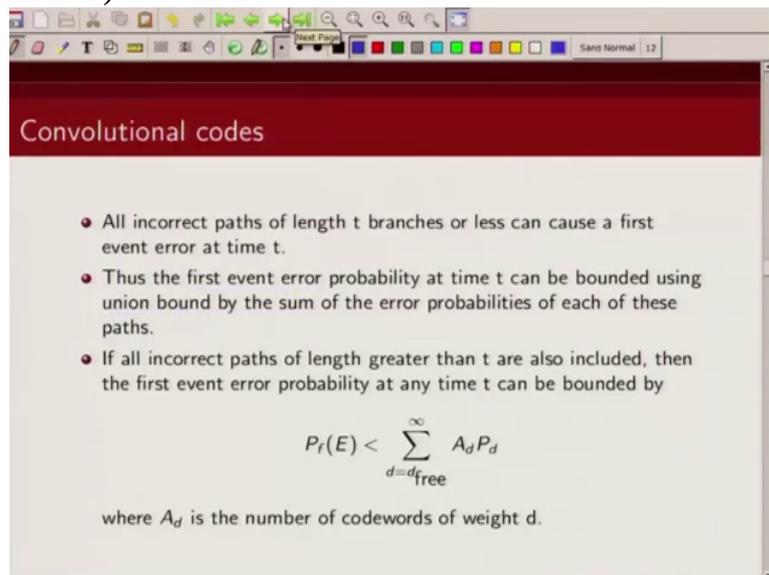


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So we would now use union bound to, essentially upper bound this probability of first event error probability. So what does union bound says? Probability of union of errors is basically upper bounded by sum of the probabilities of error. So we are going to upper bound this first event error probability by sum of error probabilities of all these incorrect paths. Now

(Refer Slide Time 15:01)



The image shows a screenshot of a presentation slide. The slide has a dark red header with the text "Convolutional codes" in white. Below the header, there is a list of three bullet points. The first bullet point states: "All incorrect paths of length t branches or less can cause a first event error at time t." The second bullet point states: "Thus the first event error probability at time t can be bounded using union bound by the sum of the error probabilities of each of these paths." The third bullet point states: "If all incorrect paths of length greater than t are also included, then the first event error probability at any time t can be bounded by" followed by a mathematical equation. The equation is
$$P_f(E) < \sum_{d=d_{free}}^{\infty} A_d P_d$$
 Below the equation, it says "where A_d is the number of codewords of weight d." The slide also shows a standard software toolbar at the top with various icons for editing and navigation.

Convolutional codes

- All incorrect paths of length t branches or less can cause a first event error at time t.
- Thus the first event error probability at time t can be bounded using union bound by the sum of the error probabilities of each of these paths.
- If all incorrect paths of length greater than t are also included, then the first event error probability at any time t can be bounded by

$$P_f(E) < \sum_{d=d_{free}}^{\infty} A_d P_d$$

where A_d is the number of codewords of weight d.

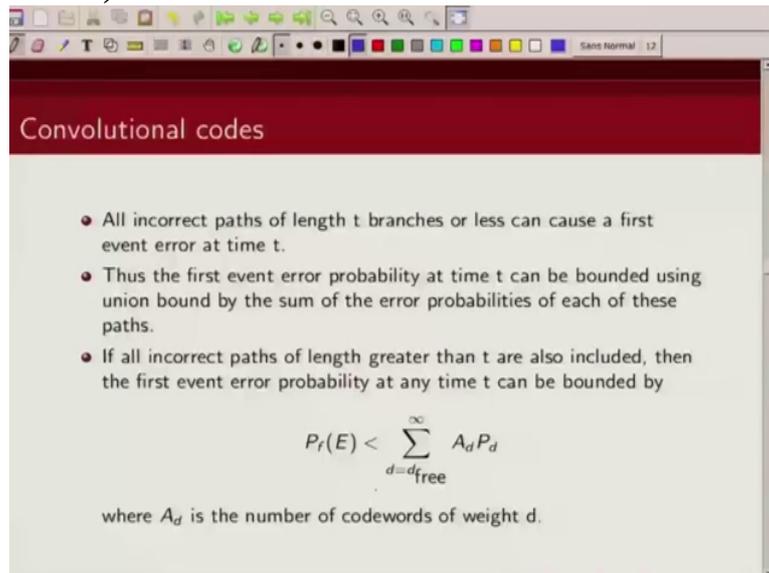
if we also allow

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incorrect path of any length even greater than t, then in that

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The slide is titled "Convolutional codes" and contains the following text:

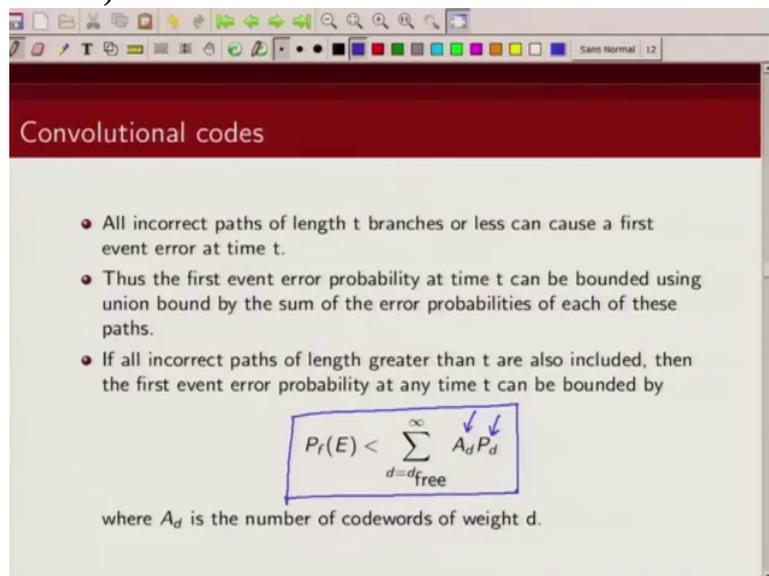
- All incorrect paths of length t branches or less can cause a first event error at time t .
- Thus the first event error probability at time t can be bounded using union bound by the sum of the error probabilities of each of these paths.
- If all incorrect paths of length greater than t are also included, then the first event error probability at any time t can be bounded by

$$P_f(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d P_d$$

where A_d is the number of codewords of weight d .

case the first event error probability can be upper bounded by this. So this is the probability of first event error of weight d and how many such weight d paths exist, that is given by this and you sum over your d which goes from free distance of convolutional code to infinity. So this will give you an upper bound

(Refer Slide Time 15:44)



The slide is titled "Convolutional codes" and contains the same text as the previous slide. The equation $P_f(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d P_d$ is highlighted with a blue box, and two blue arrows point to the A_d and P_d terms in the sum.

where A_d is the number of codewords of weight d .

on probability of first error event probability. Now can we make use of the weight enumerating function of the convolutional code to calculate this? And this is what we are going to show.

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First Error Event

- For odd d , we can write

$$\begin{aligned} P_d &= \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} \\ &< \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^{d/2} (1-p)^{d/2} \\ &= p^{d/2} (1-p)^{d/2} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} \\ &< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} \\ &= 2^d p^{d/2} (1-p)^{d/2} \end{aligned}$$

So let's first try to simplify the expression for probability of first event

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Convolutional codes

- All incorrect paths of length t branches or less can cause a first event error at time t .
- Thus the first event error probability at time t can be bounded using union bound by the sum of the error probabilities of each of these paths.
- If all incorrect paths of length greater than t are also included, then the first event error probability at any time t can be bounded by

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d P_d$$

where A_d is the number of codewords of weight d .

error. So

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$ for odd; $e=4$ for even; $e=5,6,7,8$ for even.

this was the expression for, this was the expression for first event error probability. So let us try to simplify this expression. So we will first do it for when d is odd and then we will do it for when d is even.

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First Error Event

- The incorrect path must have diverged from all zero state at some time and has now remerged at time t for the first time.
- So, it must be one of the path enumerated by the codeword weight enumerating function.
- Assuming that the incorrect path has weight d , a first event error happens with probability

$$P_d = \begin{cases} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ odd} \\ \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e} & d \text{ even} \end{cases}$$

Handwritten notes: $d=7$, $e=4,5,6,7$ for odd; $e=4$ for even; $e=5,6,7,8$ for even.

So when d is odd, the expression is given by this. Now note that this crossover probability is typically small. So and this again it is a number between 0 and 1, so if you raise it by a higher number you get smaller quantity. So the first upper bounding that we are doing is instead of raising it to e , we are raising it to d by 2 and again because it is a number between 0 and 1, see the actual expression would have been p raised to power d plus 1 by 2, then d plus 3 by 2 by 2, and like that, basically this number will go on decreasing because you are raising it to a

higher value. Now I can just fix, instead of varying it as fix, I keep it as fixed and I kept it as d by 2 and that's the first upper bound that we are getting. Next this term does not depend on

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First Error Event

- For odd d, we can write

$$P_d = \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{p/2} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e}$$

$$= 2^d p^{d/2} (1-p)^{p/2}$$

e. So I take this out. So what I am left with is this,

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First Error Event

- For odd d, we can write

$$P_d = \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{p/2} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e}$$

$$= 2^d p^{d/2} (1-p)^{p/2}$$

d choose e where e goes from d plus 1 by 2 to d. Now next upper bounding what I did was I replaced it by

(Refer Slide Time 18:06)

For odd d , we can write

$$P_d = \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e}$$

$$= 2^d p^{d/2} (1-p)^{d/2}$$

e equal to zero. So instead of e equal to d plus 1 by 2, I am adding more terms. I added from e equal to 0. So that's why this upper bound came and what is this,

(Refer Slide Time 18:23)

For odd d , we can write

$$P_d = \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e}$$

$$= 2^d p^{d/2} (1-p)^{d/2}$$

this is nothing but 2 raised to power d . So then for odd d I can write this first event error probability as upper bounded by

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First Error Event

- For odd d , we can write

$$P_d = \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e}$$

$$\leq p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e}$$

$$= 2^d p^{d/2} (1-p)^{d/2}$$

this quantity. Next

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First Error Event

- For even d , we can write

$$P_d = \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=\frac{d}{2}+1}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

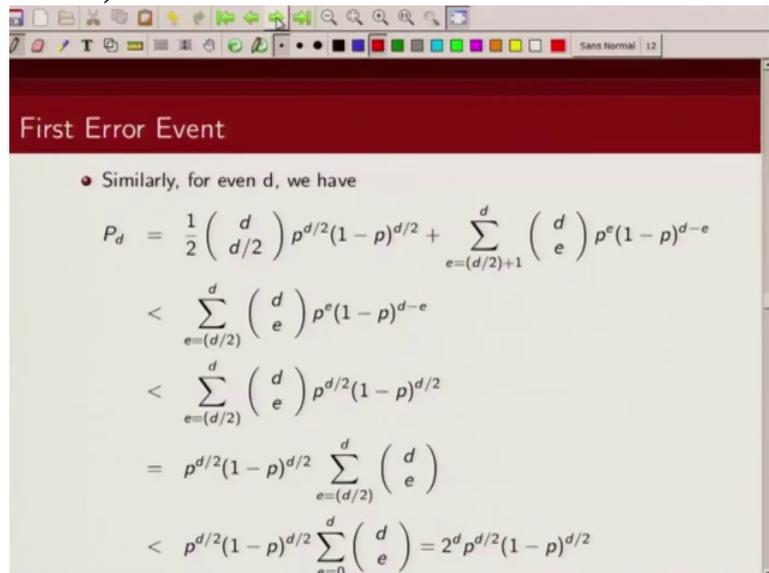
$$= p^{d/2} (1-p)^{d/2} \sum_{e=\frac{d}{2}+1}^d \binom{d}{e}$$

$$\leq p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e}$$

$$= 2^d p^{d/2} (1-p)^{d/2}$$

let's do the same thing for even d . So for an even d , the expression for first event error is given by this expression. So first thing that I do was I upper bound this by changing this exponent from e by $2 + 1$ to e by 2 . Note that in this expression when the error pattern is d by 2 , I considered a 50% probability of

(Refer Slide Time 19:15)



First Error Event

- Similarly, for even d , we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$
$$< \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$
$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$
$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$
$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

making an error. Now I just removed that and I added that here. I removed that

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50% thing here, so that's why I am getting an upper bound here. Next same as in previous expression I replace p raised to power e by p

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First Error Event

- Similarly, for even d, we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

raised to power d by 2. And again

(Refer Slide Time 19:33)

First Error Event

- Similarly, for even d, we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

because p is a number between 0 and 1, if you raise

(Refer Slide Time 19:37)

First Error Event

- Similarly, for even d, we have

$$\begin{aligned}
 P_d &= \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e} \\
 &\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e} \\
 &< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2} \\
 &= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e} \\
 &< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}
 \end{aligned}$$

it to a higher number it decreases. So I just

(Refer Slide Time 19:41)



kept it at $d/2$ which is a smallest value of e . Third thing which I do is again like for the odd case this term

(Refer Slide Time 19:51)

First Error Event

- Similarly, for even d, we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d-e}$$

$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

does not depend on e, so I bring it out. So I brought it out. And what I am left is with this,

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First Error Event

- Similarly, for even d, we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d-e}$$

$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

right and then I add terms from e 0 to up to d by 2 so I replace this

(Refer Slide Time 20:10)

First Error Event

- Similarly, for even d , we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$\leq p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

by this. So again I am upper bounding it, because I am adding additional terms here. So this quantity will be less than this quantity. And finally this is 2 raised to power d .

(Refer Slide Time 20:24)

First Error Event

- Similarly, for even d , we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$\leq p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

So I get my expression for

(Refer Slide Time 20:28)

First Error Event

• Similarly, for even d , we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

first event error probability for d being odd given by this expression. So p_d is upper bounded by this for when d is even and we showed even

(Refer Slide Time 20:45)

First Error Event

• Similarly, for even d , we have

$$P_d = \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2} + \sum_{e=(d/2)+1}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$\leq \sum_{e=(d/2)}^d \binom{d}{e} p^e (1-p)^{d-e}$$

$$< \sum_{e=(d/2)}^d \binom{d}{e} p^{d/2} (1-p)^{d/2}$$

$$= p^{d/2} (1-p)^{d/2} \sum_{e=(d/2)}^d \binom{d}{e}$$

$$< p^{d/2} (1-p)^{d/2} \sum_{e=0}^d \binom{d}{e} = 2^d p^{d/2} (1-p)^{d/2}$$

for

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Convolutional codes

- All incorrect paths of length t branches or less can cause a first event error at time t .
- Thus the first event error probability at time t can be bounded using union bound by the sum of the error probabilities of each of these paths.
- If all incorrect paths of length greater than t are also included, then the first event error probability at any time t can be bounded by

$$P_f(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d P_d$$

where A_d is the number of codewords of weight d .

when d is odd it is upper bounded by the same quantity. So then

(Refer Slide Time 20:55)

First Error Event

- For odd d , we can write

$$\begin{aligned}
 P_d &= \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^e (1-p)^{d-e} \\
 &< \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} p^{\frac{d}{2}} (1-p)^{\frac{d}{2}} \\
 &= p^{\frac{d}{2}} (1-p)^{\frac{d}{2}} \sum_{e=\frac{d+1}{2}}^d \binom{d}{e} \\
 &< p^{\frac{d}{2}} (1-p)^{\frac{d}{2}} \sum_{e=0}^d \binom{d}{e} \\
 &= 2^d p^{\frac{d}{2}} (1-p)^{\frac{d}{2}}
 \end{aligned}$$

what we can do is we can write that our first event error probability is upper bounded by this.

(Refer Slide Time 21:04)

Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$
$$= A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

We just showed for d equal to

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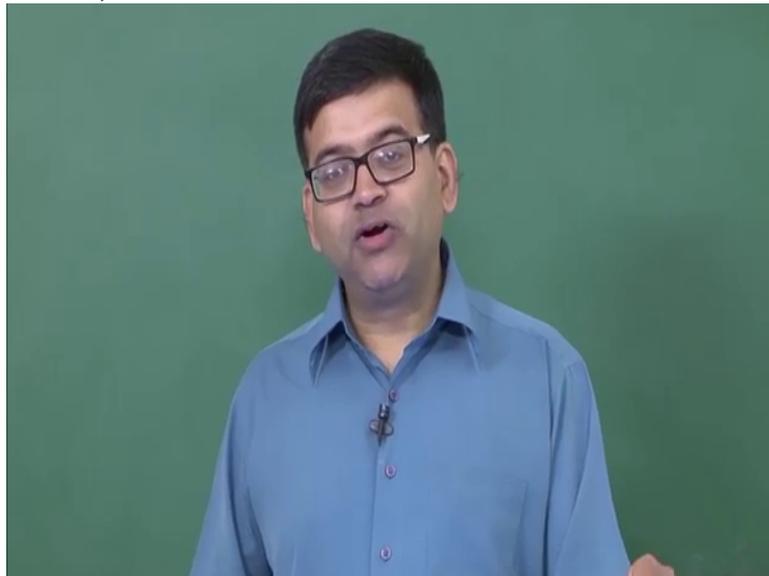
Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$
$$= A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

odd and d equal to even separately. So we can upper bound this

(Refer Slide Time 21:13)



probability like this. Now what we are going to do is we know our weight enumerating function so will give us the distribution, so if we have some weight enumerating function, it essentially will give us from d to d_{free} to infinity it will give us how many codewords of weight d are there and x raised to power d , Ok. So we are essentially making use of this weight enumerating

(Refer Slide Time 21:54)

A screenshot of a presentation slide. The title bar at the top is dark red with the text "Event error probability" in white. Below the title bar, the slide content is on a light gray background. It starts with a bullet point "Hence," followed by a mathematical equation enclosed in a red hand-drawn box. The equation is:
$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{p(1-p)}]^d$$
$$= A(X)_{X=2\sqrt{p(1-p)}}$$
The slide also shows a standard software toolbar at the top with various icons and a font selection menu showing "Sans Normal | 12".

function and we look at the form of our first event error probability, so looking at these two then we can write then that first event error probability can be computed from the weight enumerating function by replacing the weight, that x by this quantity. So this we obtain by comparing this equation with this equation, Ok. Now

(Refer Slide Time 22:32)

Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$

$$= A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

Handwritten note: $A(X) = \sum_{d=d_{\text{free}}}^{\infty} A_d X^d$

what sorts of error events can happen? So let's spend on that,

(Refer Slide Time 22:38)

Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$

$$= A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

- We have event error probability at time time t upper bounded by first event error probability, hence

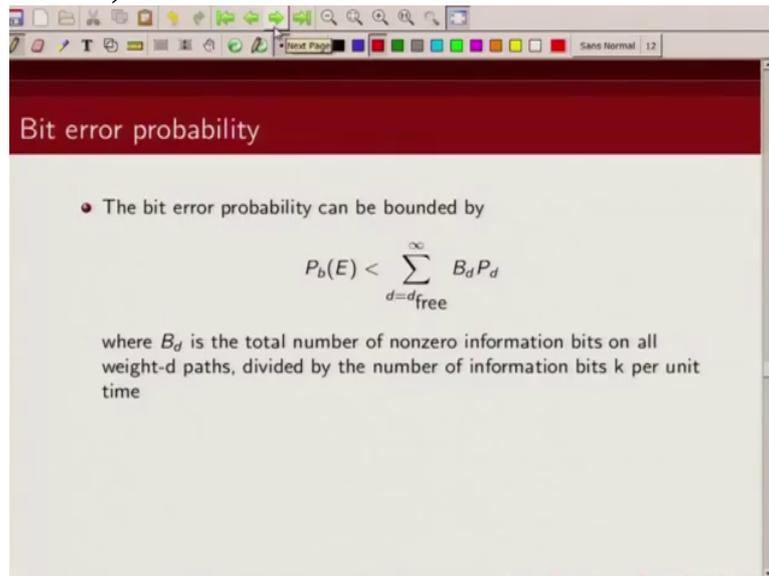
$$P(E) < A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

- For small ρ , the bound is dominated by the first time, thus event error probability can be approximated as

$$P(E) \approx A_{d_{\text{free}}} [2\sqrt{\rho(1-\rho)}]^{d_{\text{free}}}$$

what sorts of events can happen.

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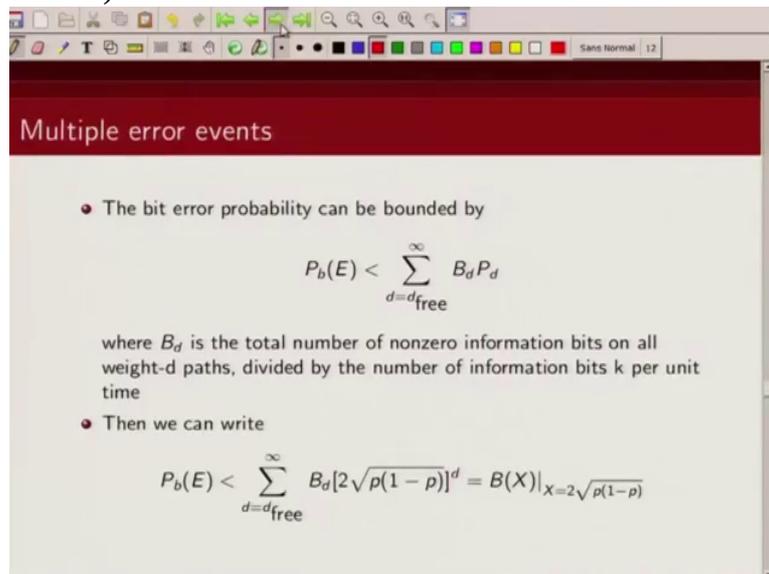
Bit error probability

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight- d paths, divided by the number of information bits k per unit time

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Multiple error events

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

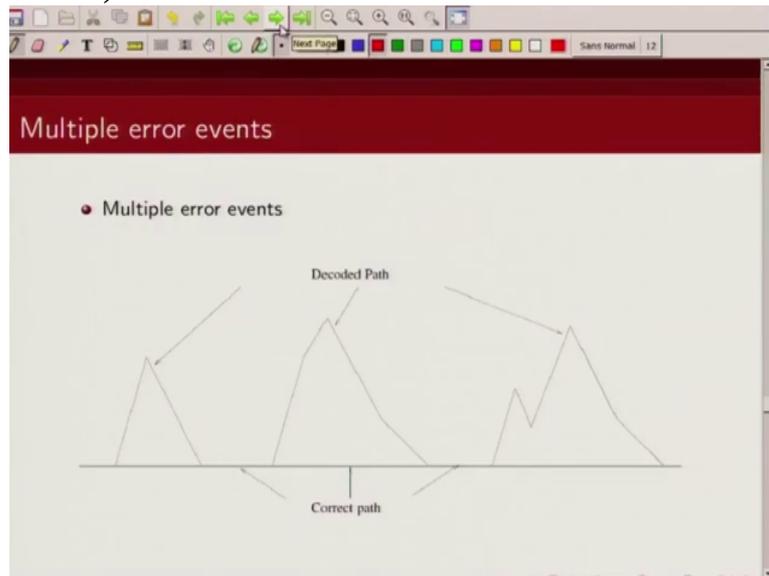
where B_d is the total number of nonzero information bits on all weight- d paths, divided by the number of information bits k per unit time

- Then we can write

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d [2\sqrt{\rho(1-\rho)}]^d = B(X)|_{X=2\sqrt{\rho(1-\rho)}}$$

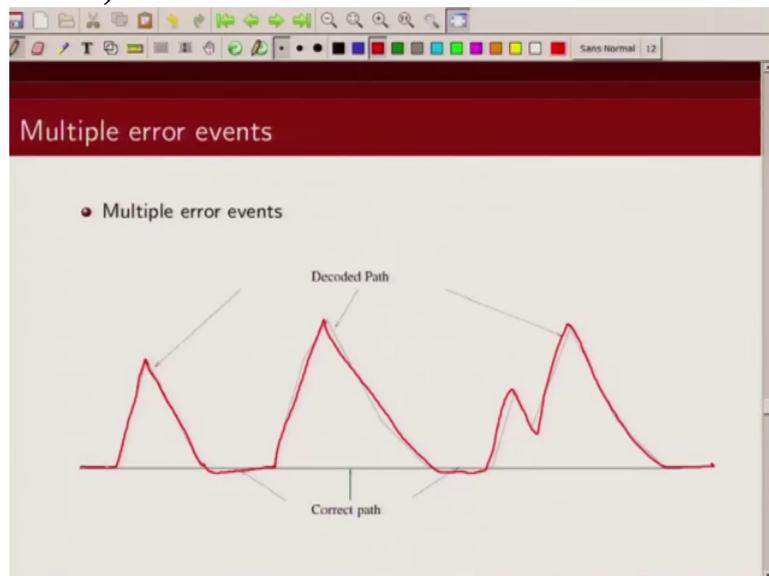
So

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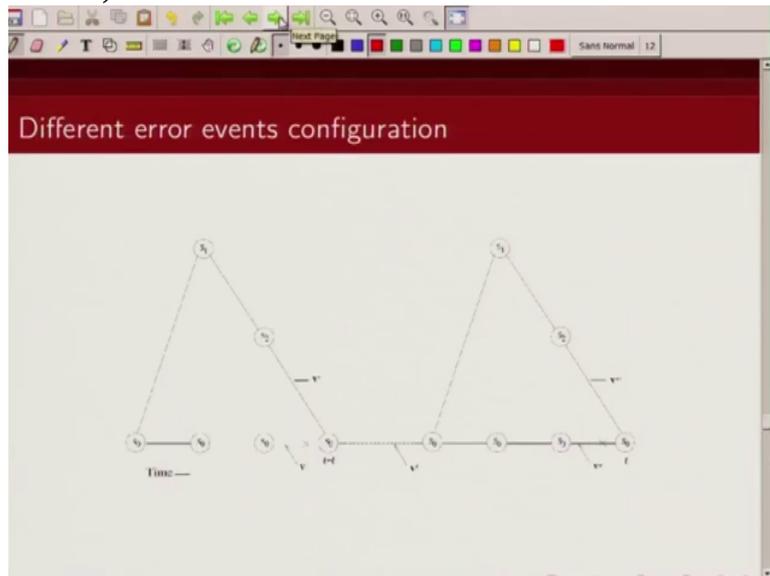
of course you can have a scenario where you have multiple error events, so you had, you diverged from all zero state, then you merged back, then you again diverged, you merged back, you diverged, you merged back, this can happen.

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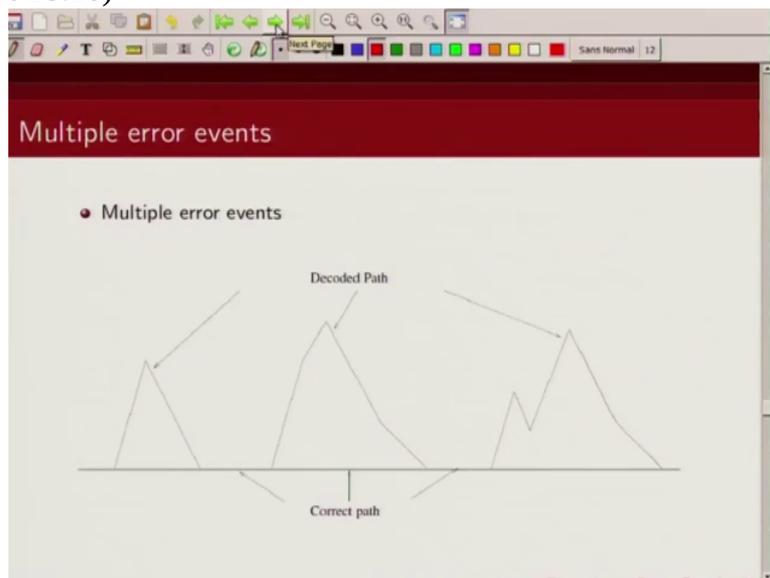
Now when will you, so if decoded path is this, that means the metric you are getting here is better than the metric value for the all zero state and that's why you are deciding in favor of this. Now there are multiple ways

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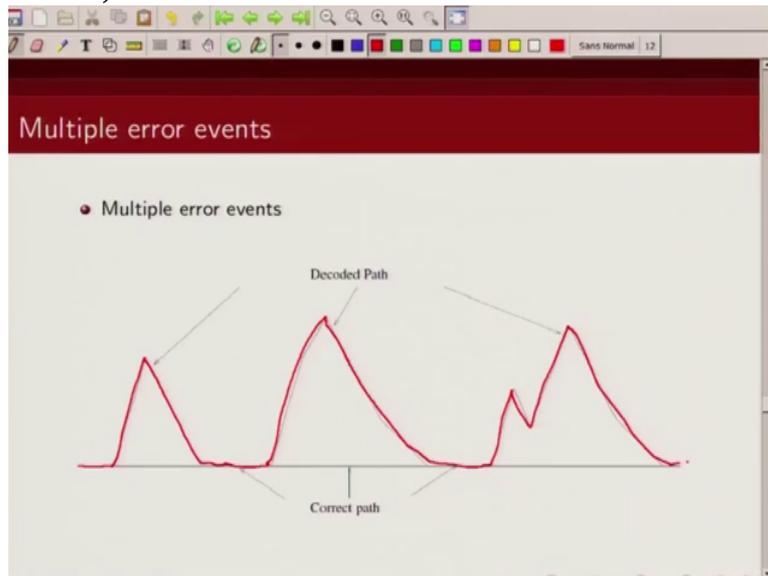
in which these errors can happen.

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So error can happen, error events can happen in multiple ways. So let's say there might be situation when you might have diverged from all zero state and then merged back into all zero state. And then after some time you again diverged from all zero state, stay away from all zero state and then you would merge back and then again you stay away from all zero state, diverge and then

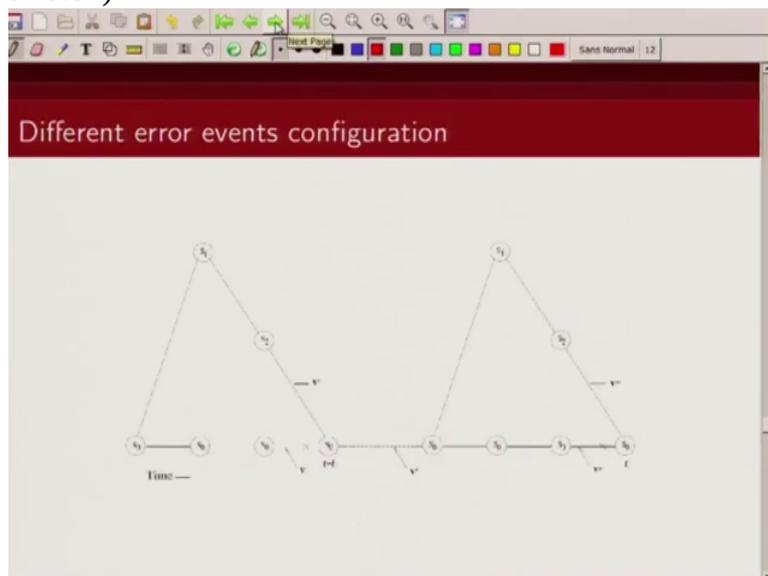
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then again merge back. So you can have multiple error events happening.

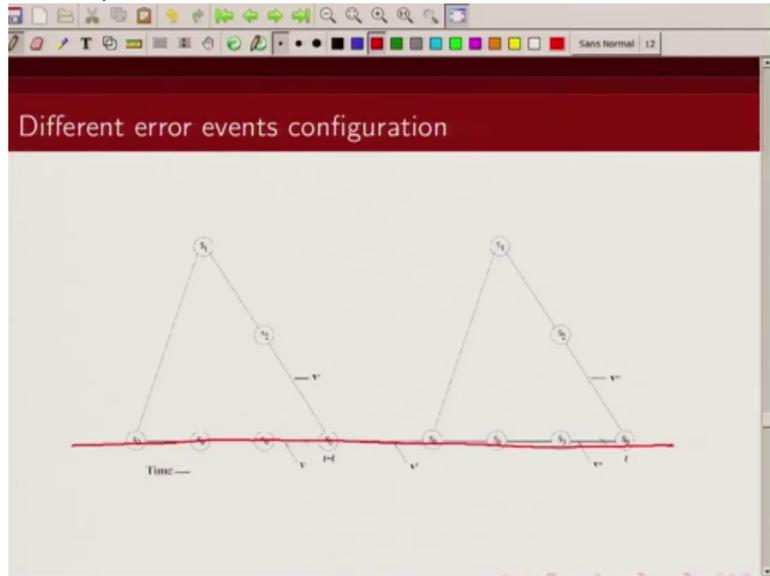
Now

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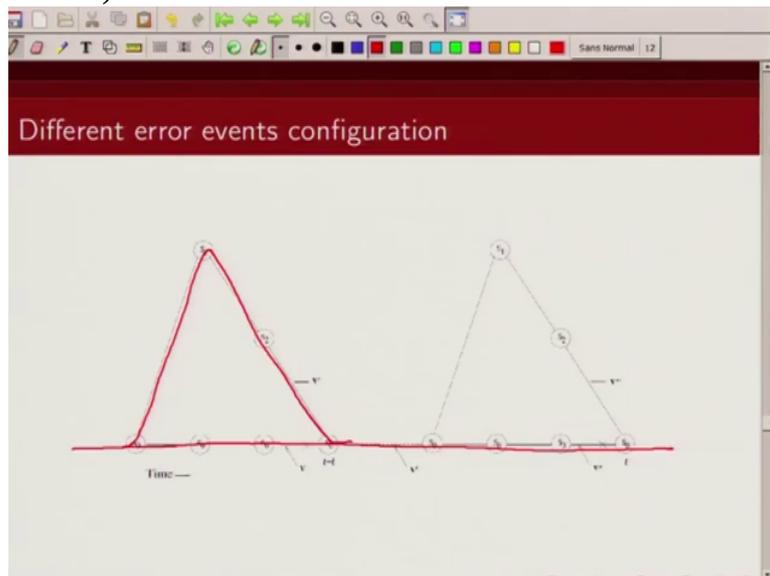
let's look at what are the various types of error configurations you can have. So for example you could have a scenario like this. So

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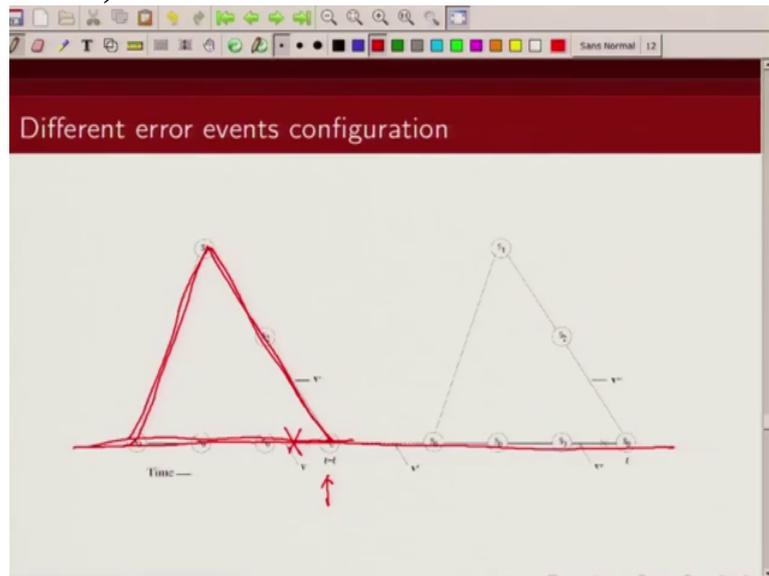
you diverge from all zero state and then you merge back

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and at this time instance let's say your metric corresponding to this path was better than metric corresponding to all zero state path. So you have discarded all zero

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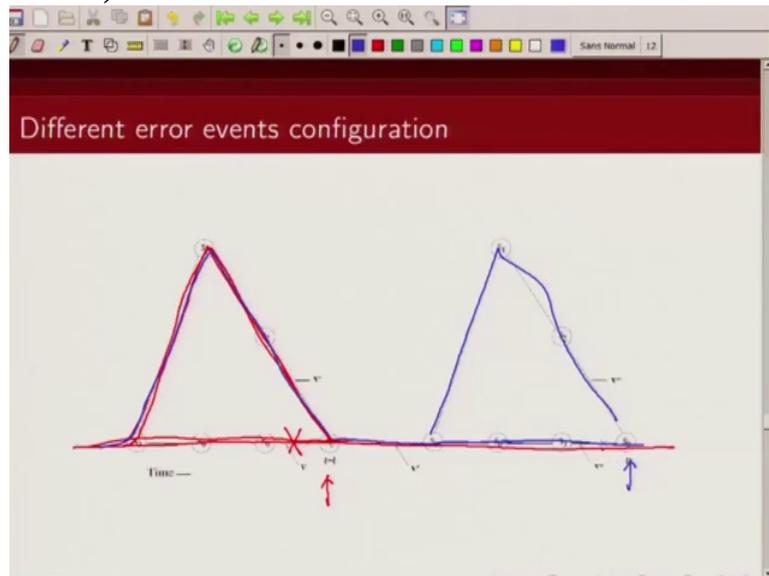
state path and

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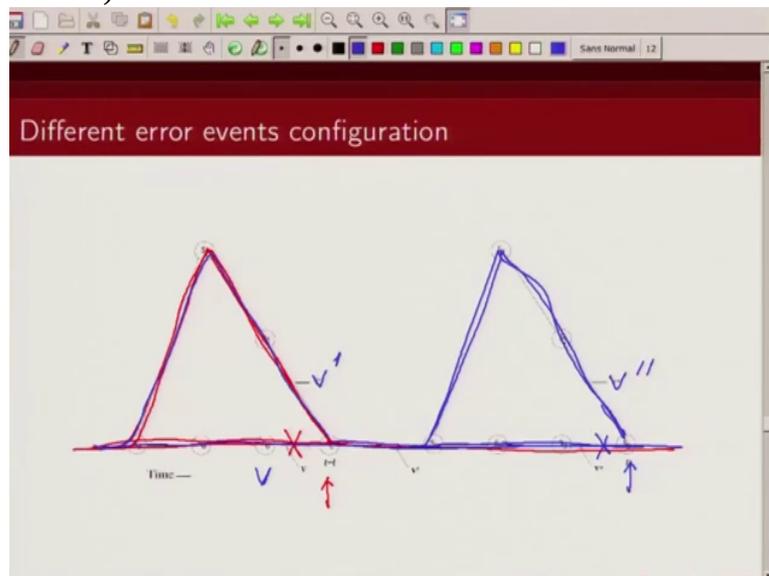
decided in favor of this path. Now further up when you came here you are here and there was one path which was going like this and when you came to this time instance t then again

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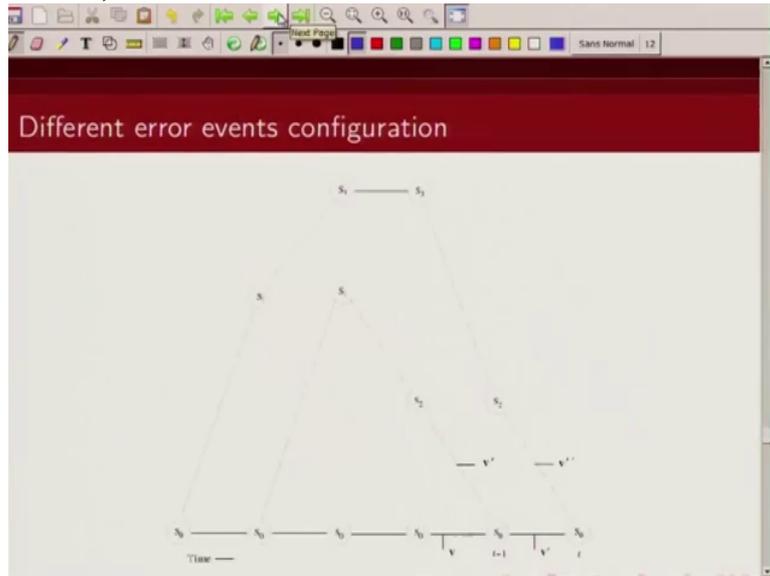
the metric corresponding to this was worse than the metric value corresponding to this path, so you decided to go for this particular codeword just calling this all zero state as vector v , I mean codeword v , this is codeword v dash and this is codeword v double dash.

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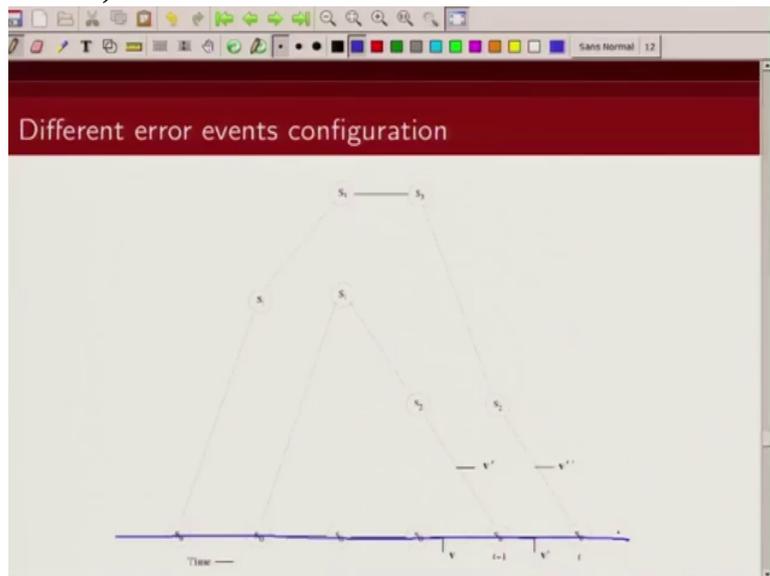
So the point which I am trying to make is whenever you have these kinds of multiple error events you can always upper bound your event probability by error probability of the first event error. So when you decide in favor of this particular codeword; that means this has better metric corresponding to our v , right. So we can upper bound the probability of event error by first event error probability and

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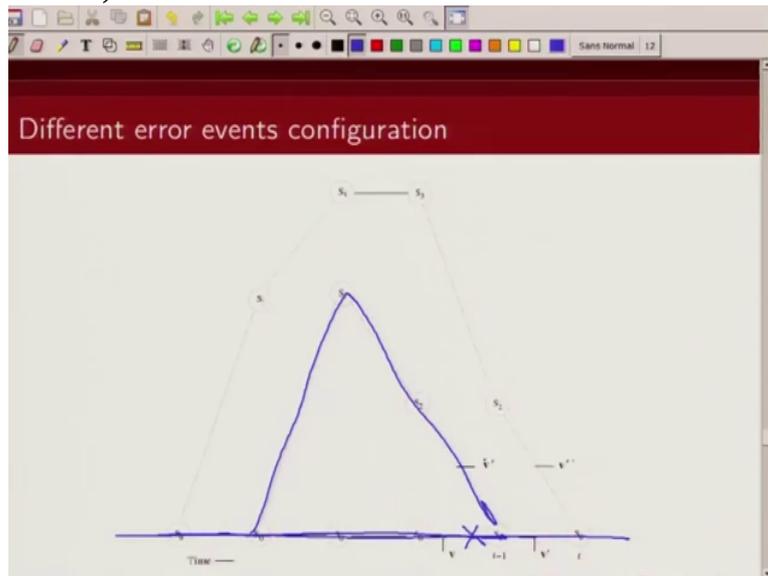
this could have multiple configuration. For example this is your all zero state. So you are here,

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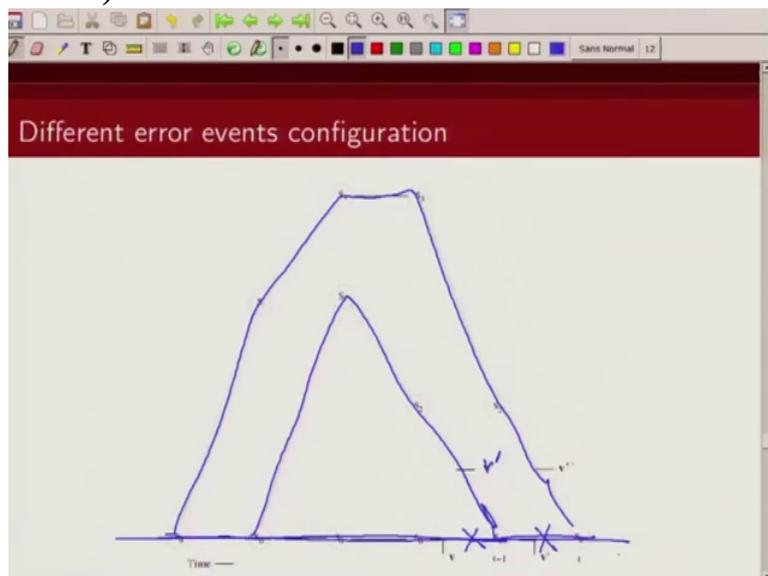
all zero state. When you came to this point you noticed that this metric was better than this metric so you discarded this and you decided in favor of this particular codeword which is your

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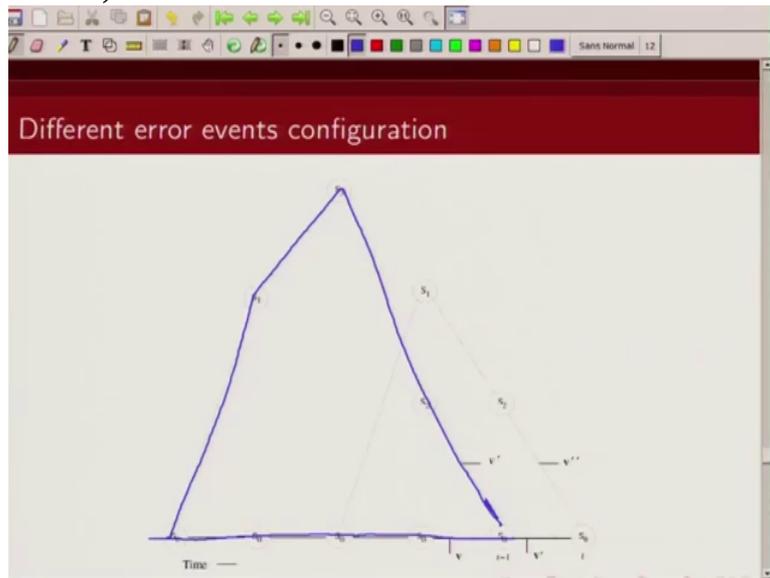
codeword v dash but when you came here again this metric was better than this, so you discarded this and you decided in favor of this. So this is another way

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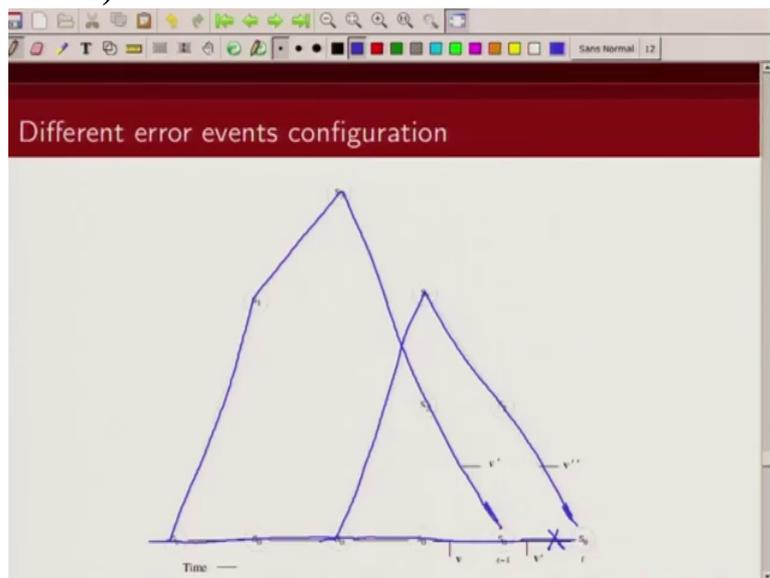
multiple error events can happen. Or you could have a situation like this. You are at all zero state. When you are at this particular instance you notice this has a better metric. So you decide in favor of this incorrect path

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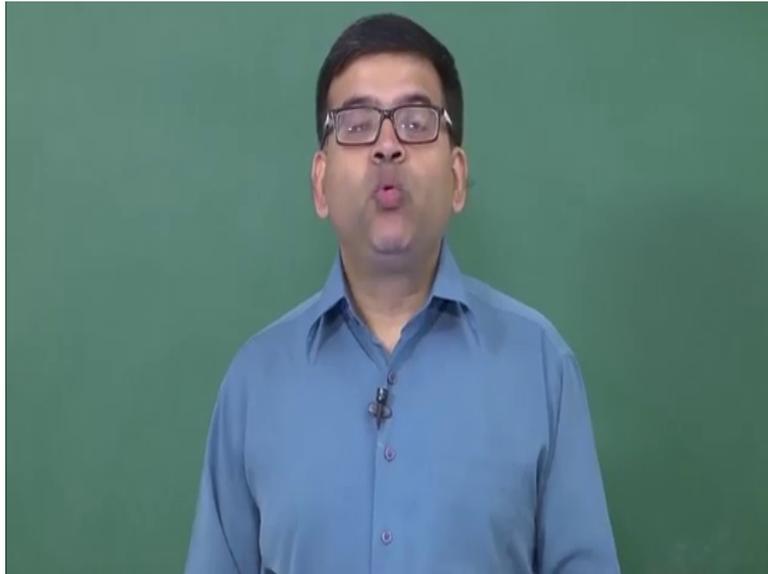
but then when you reached here, you noticed this path has a better metric so you discarded this and then you decided in favor of this. Now for each

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of these cases

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we can upper bound the event probability by the first

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A screenshot of a presentation slide. The slide has a dark red header with the title "Event error probability" in white. Below the header, there is a list of bullet points and mathematical equations. The first bullet point says "Hence," followed by an equation:
$$P_t(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d = A(X)_{X=2\sqrt{\rho(1-\rho)}}$$
 The second bullet point says "We have event error probability at time time t upper bounded by first event error probability, hence" followed by the equation:
$$P(E) < A(X)_{X=2\sqrt{\rho(1-\rho)}}$$
 The third bullet point says "For small p, the bound is dominated by the first time, thus event error probability can be approximated as" followed by the equation:
$$P(E) \approx A_{d_{\text{free}}} [2\sqrt{\rho(1-\rho)}]^{d_{\text{free}}}$$

event error probability which we have already computed. So this event error probability can be upper bounded by the first event error probability so we can write a probability of error

(Refer Slide Time 27:06)

Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$

$$= A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

- We have event error probability at time time t upper bounded by first event error probability, hence

$$P(E) < A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

- For small ρ , the bound is dominated by the first time, thus event error probability can be approximated as

$$P(E) \approx A_{d_{\text{free}}} [2\sqrt{\rho(1-\rho)}]^{d_{\text{free}}}$$

in this particular fashion. Now note here you had terms corresponding to various these, so you had terms corresponding to d_{free} , $d_{\text{free}} + 1$, $d_{\text{free}} + 2$, so since the value of ρ is typically very small, the most dominating term in this error probability expression is your first term which is the d_{free} term. So you can also approximate your probability of event error by how many such events are there which have

(Refer Slide Time 27:51)

Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$

$$= A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

- We have event error probability at time time t upper bounded by first event error probability, hence

$$P(E) < A(X)_{X=2\sqrt{\rho(1-\rho)}}$$

- For small ρ , the bound is dominated by the first time, thus event error probability can be approximated as

$$P(E) \approx A_{d_{\text{free}}} [2\sqrt{\rho(1-\rho)}]^{d_{\text{free}}}$$

paths which have weight d_{free} and this raised to power d_{free} . So case when ρ is very small, this bound is dominated by the first term which is the term corresponding to d_{free} . And in that case we can write down an approximate expression for

(Refer Slide Time 28:16)

The slide is titled "Event error probability" and contains the following content:

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{\rho(1-\rho)}]^d$$
$$= A(X)|_{X=2\sqrt{\rho(1-\rho)}}$$

- We have event error probability at time t upper bounded by first event error probability, hence

$$P(E) < A(X)|_{X=2\sqrt{\rho(1-\rho)}}$$

- For small ρ , the bound is dominated by the first time, thus event error probability can be approximated as

$$P(E) \approx A_{d_{\text{free}}} [2\sqrt{\rho(1-\rho)}]^{d_{\text{free}}}$$

Handwritten annotations in blue ink include a box around the first equation, a box around the second equation, and the label d_{free} next to the exponent in the third equation.

probability of error like this.

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The slide is titled "Bit error probability" and contains the following content:

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight- d paths, divided by the number of information bits k per unit time

Now can we modify this expression

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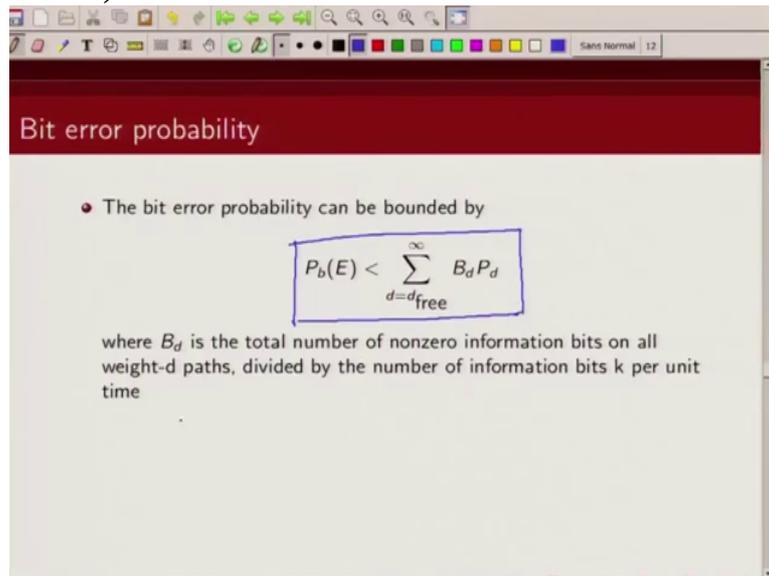
to calculate bit error rate probability? The answer is yes. So you know what is the weight corresponding to an incorrect path. Now if you can find out what is the information weight corresponding to these incorrect path and then

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A screenshot of a presentation slide. The slide has a red header with the title "Bit error probability". Below the header, there is a bullet point that says "The bit error probability can be bounded by". Underneath this, there is a mathematical equation:
$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$
 Below the equation, there is a text explanation: "where B_d is the total number of nonzero information bits on all weight-d paths, divided by the number of information bits k per unit time". The slide is shown within a software window with a toolbar at the top.

we divide it by total number of information bits, we can find out what is the bit error rate probability. So bit error rate probability can be computed from this first event error probability. This is given by this expression where

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The slide is titled "Bit error probability" and contains the following text:

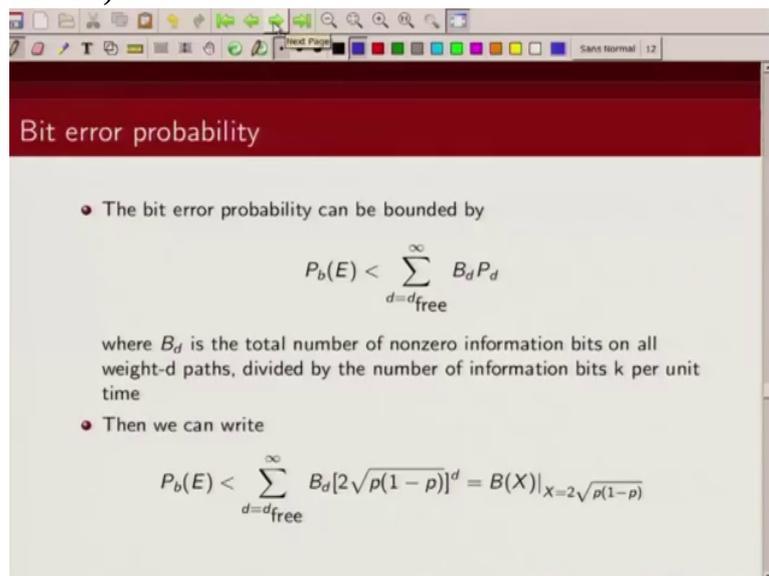
- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight-d paths, divided by the number of information bits k per unit time

B_d is the total number of non zero information bits on all these weight d incorrect paths divided by k information bits per unit time.

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The slide is titled "Bit error probability" and contains the following text:

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight-d paths, divided by the number of information bits k per unit time

- Then we can write

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d [2\sqrt{\rho(1-\rho)}]^d = B(X)|_{X=2\sqrt{\rho(1-\rho)}}$$

So if we plug that value of bit error rate probability we can see that,

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The slide is titled "Bit error probability" and contains the following content:

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight- d paths, divided by the number of information bits k per unit time

- Then we can write

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d [2\sqrt{\rho(1-\rho)}]^d = B(X)|_{X=2\sqrt{\rho(1-\rho)}}$$

we can similarly write the expression for bit error rate probability; it is the bit, this is bit weight enumerating function and in a minute we will talk about

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The slide is titled "Bit error probability" and contains the following content:

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight- d paths, divided by the number of information bits k per unit time

- Then we can write

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d [2\sqrt{\rho(1-\rho)}]^d = B(X)|_{X=2\sqrt{\rho(1-\rho)}}$$

Bit WEF

how to

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generate bit weight enumerating function from

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A screenshot of a presentation slide. The title is "Bit error probability" in white text on a dark red background. Below the title, there is a bullet point: "The bit error probability can be bounded by". This is followed by the mathematical inequality $P_b(E) < \sum_{d=d_{free}}^{\infty} B_d P_d$. Below this, a text block explains that B_d is the total number of nonzero information bits on all weight-d paths, divided by the number of information bits k per unit time. Another bullet point says "Then we can write". Below this, the inequality is repeated: $P_b(E) < \sum_{d=d_{free}}^{\infty} B_d [2\sqrt{p(1-p)}]^d = B(X)|_{X=2\sqrt{p(1-p)}}$. The entire equation is enclosed in a hand-drawn blue box. Above the box, the text "Bit WEF" is written in blue. The slide is shown within a software window with a standard toolbar at the top.

input output weight enumerating function or from weight enumerating function. We will talk about this, so

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Bit error probability

- The bit error probability can be bounded by

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d P_d$$

where B_d is the total number of nonzero information bits on all weight- d paths, divided by the number of information bits k per unit time

- Then we can write

$$P_b(E) < \sum_{d=d_{\text{free}}}^{\infty} B_d [2\sqrt{p(1-p)}]^d = B(X) \Big|_{X=2\sqrt{p(1-p)}}$$

IOWEF
Bit WEF
WEF

this can be computed from bit weight enumerating function by substituting x equal to this quantity, Ok.

Let's just take an example to illustrate this. So let us

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Example: Computation of Event error probability

- For the (3, 1, 2) encoder calculate the event error probability for crossover probability of $p = 10^{-2}$ for binary symmetric channel.

consider the same encoder that we have considered. It's a 3 1 2 convolutional encoder, it's a feed forward encoder and what is given to us is the crossover probability is point 0.1. You are asked to compute what is the event error probability. Now we know the expression of event error probability, the bound on event error probability that's given by this

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Event error probability

- Hence,

$$P_r(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d [2\sqrt{p(1-p)}]^d$$

$$= A(X)_{X=2\sqrt{p(1-p)}}$$
- We have event error probability at time t upper bounded by first event error probability, hence

$$P(E) < A(X)_{X=2\sqrt{p(1-p)}}$$
- For small p, the bound is dominated by the first time, thus event error probability can be approximated as

$$P(E) \approx A_{d_{\text{free}}} [2\sqrt{p(1-p)}]^{d_{\text{free}}}$$

expression. And in this particular example p is very small. p is point 0 1. So we can approximate, we can get the approximate expression of event error probability from this. Now what is the free distance of this convolutional code? Again if you go back, the minimum weight sequence was 7. So d free for this particular example was 7.

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Example: Computation of Event error probability

- For the (3, 1, 2) encoder calculate the event error probability for crossover probability of $p = 10^{-2}$ for binary symmetric channel.
- $d_{\text{free}} = 7$ and $A_{d_{\text{free}}} = 1$, then we have

$$P(E) \approx 2^7 p^{7/2} = 1.28 \times 10^{-5}$$

So free distance was 7. And there was only one such sequence of weight 7. So we plugged those values of d free and a d free in the expression for probability of event error and we get probability of event error to be roughly 1 point 2

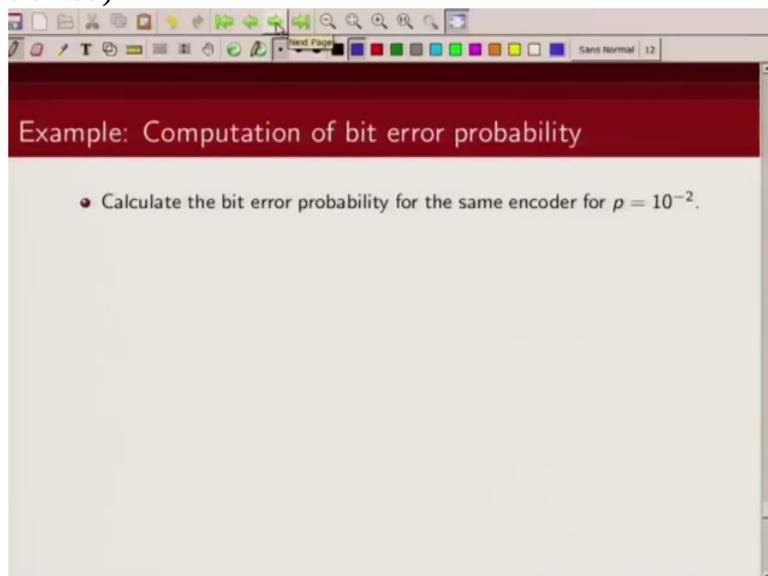
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2 times into 10 to power minus 5.

Now how do we

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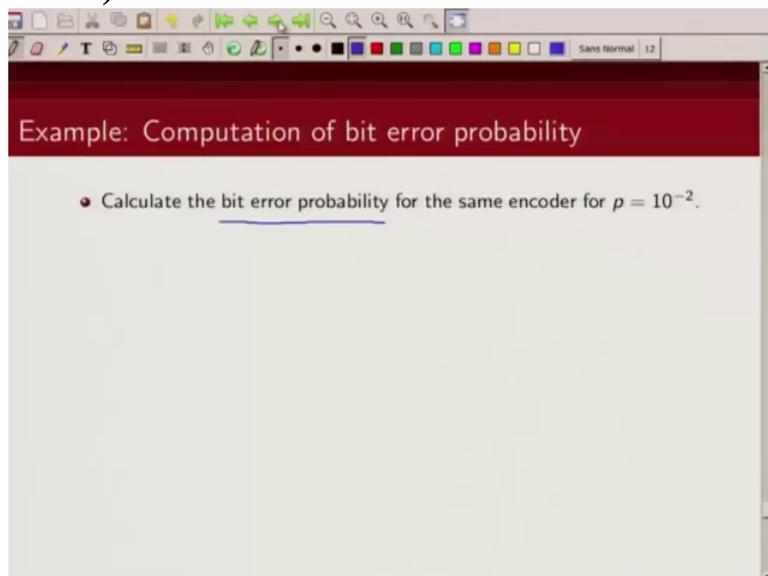
compute the bit error probability? So first we have to generate bit weight

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enumerating function. We know how to compute weight enumerating function. It has been explained in lecture 2 c.

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So assuming

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Example: Computation of bit error probability

- Calculate the bit error probability for the same encoder for $p = 10^{-2}$.
- The bit weight enumerating function is given by

$$\begin{aligned}
 B(X) &= (1/k) \frac{\partial A(W, X)}{\partial W} \Big|_{W=1} \\
 &= \frac{\partial [X^7 W / (1 - XW - X^3 W)]}{\partial W} \Big|_{W=1} \\
 &= \frac{X^7}{(1 - 2X + X^2 - 2X^3 + 2X^4 + X^6)}
 \end{aligned}$$

you have the weight enumerating function which tells you and you have the input output weight enumerating function which will tell you what information bit causes or what is the corresponding output weight so you could compute bit weight enumerating function by partial derivative of your input output weight enumerating function with respect to w and putting w to be equal to be 1 and 1 by k times that. If you do that you will get the expression for bit weight enumerating function. So for the example that we have considered we already had the expression for input output weight enumerating function. So if we plug that in we get the expression for bit weight enumerating

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Example: Computation of bit error probability

- Calculate the bit error probability for the same encoder for $p = 10^{-2}$.
- The bit weight enumerating function is given by

$$\begin{aligned}
 B(X) &= (1/k) \frac{\partial A(W, X)}{\partial W} \Big|_{W=1} \\
 &= \frac{\partial [X^7 W / (1 - XW - X^3 W)]}{\partial W} \Big|_{W=1} \\
 &= \frac{X^7}{(1 - 2X + X^2 - 2X^3 + 2X^4 + X^6)}
 \end{aligned}$$

function like this and then we can divide this by this. So we will get x 7 plus some some terms like that.

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Example: Computation of bit error probability

- Calculate the bit error probability for the same encoder for $p = 10^{-2}$.
- The bit weight enumerating function is given by

$$\begin{aligned}
 B(X) &= (1/k) \frac{\partial A(W, X)}{\partial W} \Big|_{W=1} \\
 &= \frac{\partial [X^7 W / (1 - XW - X^3 W)]}{\partial W} \Big|_{W=1} \\
 &= \frac{X^7}{(1 - 2X + X^2 - 2X^3 + 2X^4 + X^6)} \\
 &= X^7 + \dots + \dots
 \end{aligned}$$

So in this case also, what is d_{free} ? d_{free} is 7 and $B_{d_{\text{free}}}$ is also 1.

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Example: Computation of bit error probability

- Calculate the bit error probability for the same encoder for $p = 10^{-2}$.
- The bit weight enumerating function is given by

$$\begin{aligned}
 B(X) &= (1/k) \frac{\partial A(W, X)}{\partial W} \Big|_{W=1} \\
 &= \frac{\partial [X^7 W / (1 - XW - X^3 W)]}{\partial W} \Big|_{W=1} \\
 &= \frac{X^7}{(1 - 2X + X^2 - 2X^3 + 2X^4 + X^6)}
 \end{aligned}$$

- $d_{\text{free}} = 7$ and $B_{d_{\text{free}}} = 1$, then we have

$$P(E) \approx 2^7 p^{7/2} = 1.28 \times 10^{-5}$$

So this quantity is 1, d_{free} is 7 so we can approximate the expression because the first term corresponding to the free distance will be the dominating term. So we can write the probability of bit error

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Example: Computation of bit error probability

- Calculate the bit error probability for the same encoder for $p = 10^{-2}$.
- The bit weight enumerating function is given by

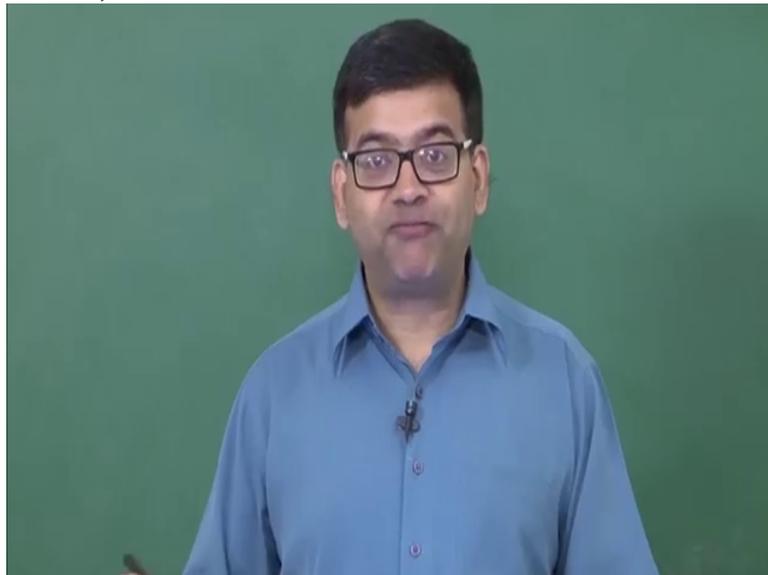
$$\begin{aligned} B(X) &= (1/k) \frac{\partial A(W, X)}{\partial W} \Big|_{W=1} \\ &= \frac{\partial [X^7 W / (1 - XW - X^3 W)]}{\partial W} \Big|_{W=1} \\ &= \frac{X^7}{(1 - 2X + X^2 - 2X^3 + 2X^4 + X^6)} \end{aligned}$$

- $d_{\text{free}} = 7$ and $B_{d_{\text{free}}} = 1$, then we have

$$\underline{P_b(E) \approx 2^7 p^{7/2} = 1.28 \times 10^{-5}}$$

to be this, Ok. So with this I will conclude this discussion on

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performance analysis of convolutional code over binary symmetric channel. Now similar analysis can be done for other channels as well, thank you