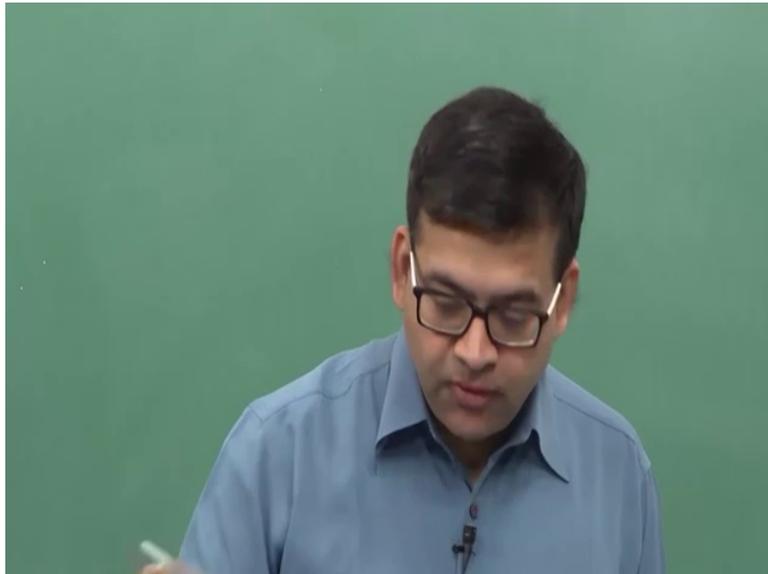


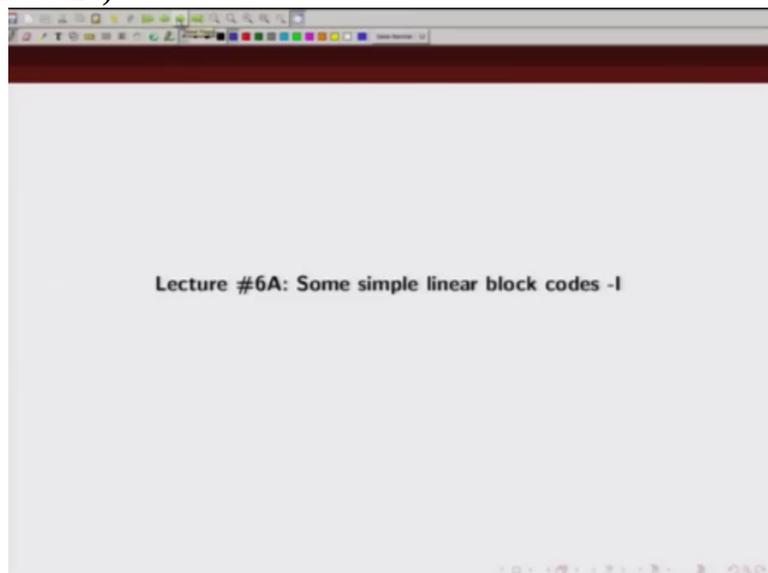
An Introduction to Coding Theory
Professor Adrish Banerji
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Module 3
Lecture Number 11
Some Simple Linear Block Codes-I

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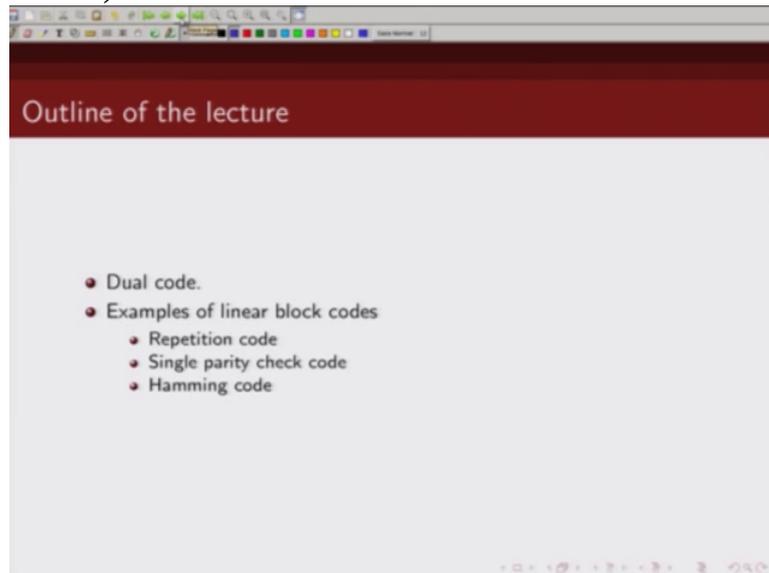
So today we will discuss about some very simple block codes.

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So this is the outline of today's talk. Before I discuss some examples

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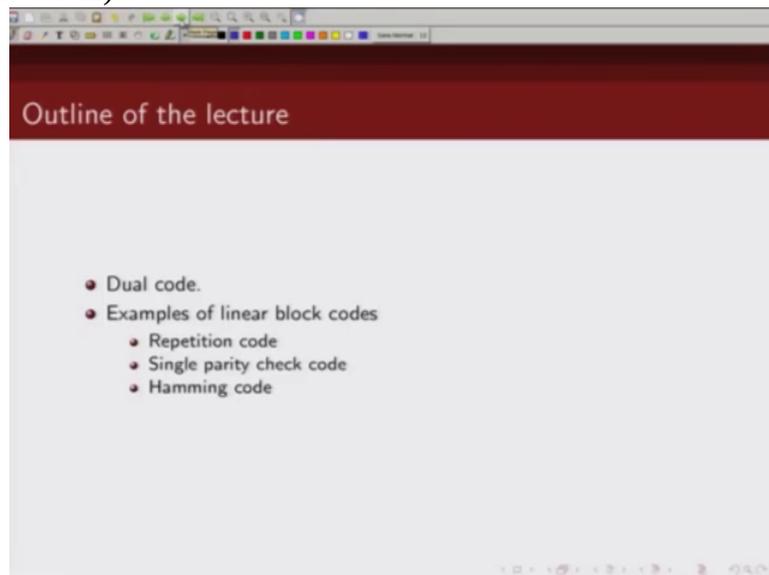
of linear block code I will first describe what do I mean by dual of a code. And then I will move on and describe some very simple linear block codes such as repetition code, single parity check code,

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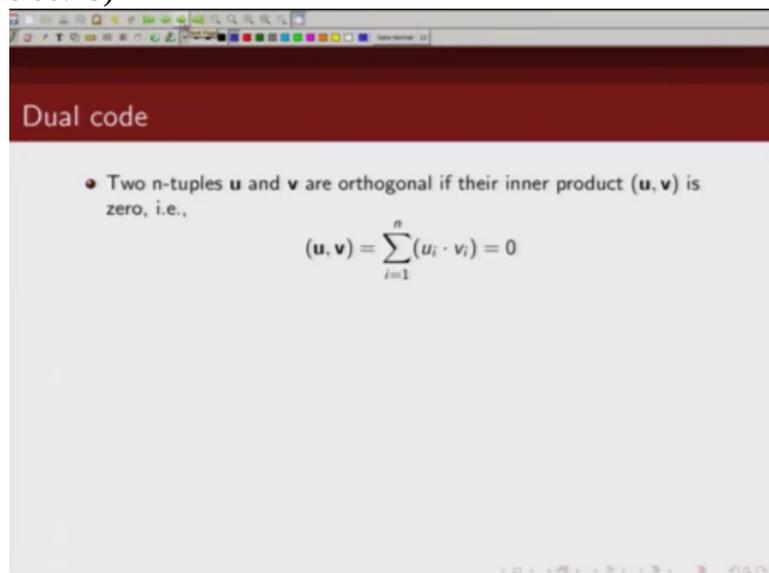


Hamming code.

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So before I discuss what is dual code, I would like to define what do I mean by two vectors \mathbf{u} and \mathbf{v} being orthogonal. So two vectors \mathbf{u} and \mathbf{v} are orthogonal if their inner product which is defined like this, so component wise dot product, if that inner product is zero, we call that these n tuples \mathbf{u} and \mathbf{v} as orthogonal.

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.

So, for a binary linear n k code, its dual has a parameter n and n minus k and it has the following properties. So a dual code has, is defined such that its set of codewords v are orthogonal to the set of codewords of the original code C . So if v is the codeword which belongs to the dual of the code c then v would be orthogonal to codewords u which belong to the original code C . We can show that,

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.

for a linear block code C the dual code is also a linear code.

So let's

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$

take \mathbf{x} and \mathbf{y} to codewords which belongs to this dual code of C which we are denoting by C_d , sometimes the people use this notation also for the dual. Now if \mathbf{x} and

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$ C^\perp

\mathbf{y} belongs to the dual code then we know that any codewords belonging to the dual code, they are orthogonal to the codewords in the original code C . So if \mathbf{u} belongs to C and \mathbf{x} belongs

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$

to dual code C_d of C then

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$

$\mathbf{x} \cdot \mathbf{u}$ will be zero because the codeword \mathbf{x} is orthogonal to codeword \mathbf{u} . Similarly codeword \mathbf{y} which belongs to the dual code and codeword \mathbf{u} which belongs to original code C , since they are orthogonal,

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,

$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$

- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$

their dot product will be zero. So we can write $\mathbf{x} \cdot \mathbf{u}$ is equal to $\mathbf{y} \cdot \mathbf{u}$ is equal to zero. This follows from the property that a code which belongs to the dual code is orthogonal to the codewords in the original code C . So therefore some binary

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,

$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$

- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$
- Thus,

$$(\lambda \mathbf{x} + \mu \mathbf{y}) \cdot \mathbf{u} = \lambda(\mathbf{x} \cdot \mathbf{u}) + \mu(\mathbf{y} \cdot \mathbf{u}) = 0$$

for every $\mathbf{u} \in C$

lambda and mu we can write this lambda plus mu times y dot product with u as lambda times x dot mu plus mu times lambda dot u. Now what is x dot u? Now x dot u is zero because x belongs to the

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$
- Thus,
$$(\lambda \mathbf{x} + \mu \mathbf{y}) \cdot \mathbf{u} = \lambda(\mathbf{x} \cdot \mathbf{u}) + \mu(\mathbf{y} \cdot \mathbf{u}) = 0$$
for every $\mathbf{u} \in C$

dual code and \mathbf{u} belongs to the original code C . So they are orthogonal. That's why $\mathbf{x} \cdot \mathbf{u}$ is zero. Similarly $\mathbf{y} \cdot \mathbf{u}$ is also zero. Hence we can write $\lambda \mathbf{x} + \mu \mathbf{y}$ dot product with the code \mathbf{u} is also zero. So we have shown basically this is also, this belongs to the dual

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$
- Thus,
$$(\lambda \mathbf{x} + \mu \mathbf{y}) \cdot \mathbf{u} = \lambda(\mathbf{x} \cdot \mathbf{u}) + \mu(\mathbf{y} \cdot \mathbf{u}) = 0$$
for every $\mathbf{u} \in C$

code. So we have shown that if our original code, linear block code is, n, k code is linear the dual code is also linear.

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$
- Thus,
$$(\lambda \mathbf{x} + \mu \mathbf{y}) \cdot \mathbf{u} = \lambda(\mathbf{x} \cdot \mathbf{u}) + \mu(\mathbf{y} \cdot \mathbf{u}) = 0$$
for every $\mathbf{u} \in C$
- This implies $\lambda \mathbf{x} + \mu \mathbf{y} \in C_d$

And why does this belong to the dual code? That is because this is orthogonal to the codeword which belongs to the, \mathbf{u} belongs to C and since this is orthogonal to

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$
- Thus,
$$(\lambda \mathbf{x} + \mu \mathbf{y}) \cdot \mathbf{u} = \lambda(\mathbf{x} \cdot \mathbf{u}) + \mu(\mathbf{y} \cdot \mathbf{u}) = 0$$
for every $\mathbf{u} \in C$
- This implies $\lambda \mathbf{x} + \mu \mathbf{y} \in C_d$

the codeword which belongs to C , so this must belong to the dual code.

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Dual code

- Two n -tuples \mathbf{u} and \mathbf{v} are orthogonal if their inner product (\mathbf{u}, \mathbf{v}) is zero, i.e.,
$$(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^n (u_i \cdot v_i) = 0$$
- For a binary linear (n, k) block code C , the $(n, n - k)$ dual code, C_d is defined as set of all codewords, \mathbf{v} that are orthogonal to all the codewords $\mathbf{u} \in C$.
- $(n, n - k)$ dual code, C_d is also a linear code.
- Proof: Let $\mathbf{x}, \mathbf{y} \in C_d$, then $\mathbf{x} \cdot \mathbf{u} = \mathbf{y} \cdot \mathbf{u} = 0$ for every $\mathbf{u} \in C$
- Thus,
$$(\lambda \mathbf{x} + \mu \mathbf{y}) \cdot \mathbf{u} = \lambda(\mathbf{x} \cdot \mathbf{u}) + \mu(\mathbf{y} \cdot \mathbf{u}) = 0$$
for every $\mathbf{u} \in C$ $\mathbf{x} \in C_d$ $\mathbf{y} \in C_d$
- This implies $\lambda \mathbf{x} + \mu \mathbf{y} \in C_d$

The next property which we are going to show

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Dual code

- Let C be a linear code with generator matrix \mathbf{G} . Then $\mathbf{x} \in C_d$ if and only if $\mathbf{x}\mathbf{G}^T = 0$

is, if we have a linear block code, we denote it by C , whose generator matrix is given by this capital G and if \mathbf{x} belongs to the dual code of this original code C . So the claim that we are making is if \mathbf{x} belongs to the dual code then this relation holds and if this relation holds \mathbf{x} belongs to dual code. That is what we mean by if and only if. So if \mathbf{x} belongs to the dual code then $\mathbf{x}\mathbf{G}^T$ should be zero and if $\mathbf{x}\mathbf{G}^T$ is zero \mathbf{x} should belong to dual code. So we are going to prove that if \mathbf{x} belongs to dual code then this relation holds and further we will show if this condition holds then \mathbf{x} will belong

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Dual code

- Let C be a linear code with generator matrix \mathbf{G} . Then $\mathbf{x} \in C_d$ if and only if $\mathbf{x}\mathbf{G}^T = 0$

to the dual code.

So let us

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Dual code

- Let C be a linear code with generator matrix \mathbf{G} . Then $\mathbf{x} \in C_d$ if and only if $\mathbf{x}\mathbf{G}^T = 0$
- Let \mathbf{G} be given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \dots \\ \mathbf{g}_{k-1} \end{bmatrix}$$

where $\{\mathbf{g}_0\}$ is some basis of \mathbf{G} .

write a generator matrix of a linear block code C . As you know this is a k cross n matrix and this \mathbf{g}_0 ,

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Dual code

- Let C be a linear code with generator matrix \mathbf{G} . Then $\mathbf{x} \in C_d$ if and only if $\mathbf{x}\mathbf{G}^T = 0$
- Let \mathbf{G} be given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \dots \\ \mathbf{g}_{k-1} \end{bmatrix} \quad k \times n$$

where $\{\mathbf{g}_i\}$ is some basis of \mathbf{G} .

$\mathbf{g}_0, \mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_{k-1}$ are these k generators each of length up to n these are length n so this \mathbf{G} is k cross n matrix and any codeword can be generated using these generators $\mathbf{g}_0, \mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_{k-1}$. Now

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Dual code

- Let C be a linear code with generator matrix \mathbf{G} . Then $\mathbf{x} \in C_d$ if and only if $\mathbf{x}\mathbf{G}^T = 0$
- Let \mathbf{G} be given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \dots \\ \mathbf{g}_{k-1} \end{bmatrix}$$

where $\{\mathbf{g}_i\}$ is some basis of \mathbf{G} .

- Also, $\mathbf{x}\mathbf{G}^T = (\mathbf{x} \cdot \mathbf{g}_0, \dots, \mathbf{x} \cdot \mathbf{g}_{k-1})$.

$\mathbf{x}\mathbf{G}^T$ is basically given by inner product of \mathbf{x} with \mathbf{g}_0 , \mathbf{x} with \mathbf{g}_1 , up to \mathbf{x} with \mathbf{g}_{k-1} . Now what happens if \mathbf{x} belongs to the dual code? If \mathbf{x} belongs to the dual code,

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then set of codewords which belongs to the dual code, they are orthogonal to the set of codewords which belongs to the original code C .

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Dual code

- Let C be a linear code with generator matrix G . Then $x \in C_d$ if and only if $xG^T = 0$
- Let G be given by

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \dots \\ g_{k-1} \end{bmatrix}$$

where $\{g_0\}$ is some basis of G .

- Also, $xG^T = (x \cdot g_0, \dots, x \cdot g_{k-1})$.

Now

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Dual code

- Let C be a linear code with generator matrix G . Then $x \in C_d$ if and only if $xG^T = 0$
- Let G be given by

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \dots \\ g_{k-1} \end{bmatrix}$$
 where $\{g_i\}$ is some basis of C .
- Also, $xG^T = (x \cdot g_0, \dots, x \cdot g_{k-1})$.
- If $x \in C_d$, then $x \cdot g_i = 0$ for every i , so $xG^T = 0$.

we know that if x belongs to the dual code then x , inner product of x with g_i should be zero. Why, because the original code is generated using these generator sequences g_0, g_1, \dots, g_{k-1} . Any linear combination of this g_0, g_1, \dots, g_{k-1} will give me my coded sequence. v is we can write $v_0 g_0 + \dots + v_{k-1} g_{k-1}$ right? So

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Dual code

- Let C be a linear code with generator matrix G . Then $x \in C_d$ if and only if $xG^T = 0$
- Let G be given by

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \dots \\ g_{k-1} \end{bmatrix} \quad v = u_0 g_0 + \dots + u_{k-1} g_{k-1}$$
 where $\{g_i\}$ is some basis of C .
- Also, $xG^T = (x \cdot g_0, \dots, x \cdot g_{k-1})$.
- If $x \in C_d$, then $x \cdot g_i = 0$ for every i , so $xG^T = 0$.

if x belongs to the dual code then inner product of x with g_i 's would be zero and hence xG^T will be zero. So what we have shown is, if x belongs to dual code, from the property that the codewords in the dual code and the codewords in the original codes, they are orthogonal to each other, from that property we get this condition that the inner product of x with g_i 's will be zero or in matrix form we can then write, because x , inner product of x with g_i 's is nothing but xG^T . So then xG^T would be zero.

Next we are going to show, if xG^T is zero then x must belong to the dual code.

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Dual code

- If $xG^T = 0$ then $x \cdot g_i = 0$ for every i . If $c \in C$, then $c = \sum_i \lambda_i g_i$ for some binary λ_i , so

$$x \cdot c = x \cdot \left(\sum_i \lambda_i g_i \right) = \sum_i \lambda_i (x \cdot g_i) = 0$$

and thus $x \in C_d$

Now if xG^T is zero, then this condition holds. That inner product of x with g_i is zero for i equal to zero to k minus 1. Now what is a codeword? Codeword is obtained by linear combinations of the, these generators, g_0, g_1, g_2, g_{k-1} . So if C belongs to the linear block code C then C is essentially generated by linear combinations of these generator sequences where these λ_i 's are zeroes and 1's because we are talking about binary code. Now note what do we want to show? We want

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Dual code

- Let C be a linear code with generator matrix G . Then $x \in C_d$ if and only if $xG^T = 0$
- Let G be given by

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \dots \\ g_{k-1} \end{bmatrix} \quad v = u_0 g_0 + \dots + u_{k-1} g_{k-1}$$

where $\{g_0\}$ is some basis of G .

- Also, $xG^T = (x \cdot g_0, \dots, x \cdot g_{k-1})$.
- If $x \in C_d$, then $x \cdot g_i = 0$ for every i , so $xG^T = 0$.

to show, if xG^T is zero then x must belong to the dual code. And when will x belongs to dual code? We have to show that a codeword which belongs

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to original linear block code C and set of codewords which belong to the dual code, they are orthogonal to each other. Or

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Dual code

- Let C be a linear code with generator matrix G . Then $x \in C_d$ if and only if $xG^T = 0$
- Let G be given by

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \dots \\ g_{k-1} \end{bmatrix} \quad v = u_0 g_0 + \dots + u_{k-1} g_{k-1}$$

where $\{g_i\}$ is some basis of G .

- Also, $xG^T = (x \cdot g_0, \dots, x \cdot g_{k-1})$.
- If $x \in C_d$, then $x \cdot g_i = 0$ for every i , so $xG^T = 0$.

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Dual code

- If $\mathbf{xG}^T = 0$ then $\mathbf{x} \cdot \mathbf{g}_i = 0$ for every i . If $\mathbf{c} \in C$, then $\mathbf{c} = \sum_i \lambda_i \mathbf{g}_i$ for some binary λ_i , so

$$\mathbf{x} \cdot \mathbf{c} = \mathbf{x} \cdot \left(\sum_i \lambda_i \mathbf{g}_i \right) = \sum_i \lambda_i (\mathbf{x} \cdot \mathbf{g}_i) = 0$$

and thus $\mathbf{x} \in C_d$

in other words, their dot product is zero. So let's take a dot product; this is \mathbf{x} which you want to show that it belongs to the dual code and \mathbf{c} is a codeword in the original code C . So what is $\mathbf{x} \cdot \mathbf{c}$? $\mathbf{x} \cdot \mathbf{c}$ can be written as $\mathbf{x} \cdot \sum_i \lambda_i \mathbf{g}_i$. This, I can write as summation over i , $\lambda_i \mathbf{x} \cdot \mathbf{g}_i$. And what do I know from this condition? That \mathbf{xG}^T is zero, I know from this condition that

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Dual code

- If $\mathbf{xG}^T = 0$ then $\mathbf{x} \cdot \mathbf{g}_i = 0$ for every i . If $\mathbf{c} \in C$, then $\mathbf{c} = \sum_i \lambda_i \mathbf{g}_i$ for some binary λ_i , so

$$\mathbf{x} \cdot \mathbf{c} = \mathbf{x} \cdot \left(\sum_i \lambda_i \mathbf{g}_i \right) = \sum_i \lambda_i (\mathbf{x} \cdot \mathbf{g}_i) = 0$$

and thus $\mathbf{x} \in C_d$

inner product of \mathbf{x} with \mathbf{g}_i is zero. That means this term is equal to zero. Then what I have shown? I have shown that $\mathbf{x} \cdot \mathbf{c}$ is equal to zero. That means \mathbf{x} and \mathbf{c} are orthogonal to each other. So if \mathbf{c} belongs to original code this capital C , then \mathbf{x} must belong to the dual code of this code C .

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Dual code

- If $\mathbf{xG}^T = 0$ then $\mathbf{x} \cdot \mathbf{g}_i = 0$ for every i . If $\mathbf{c} \in C$, then $\mathbf{c} = \sum_i \lambda_i \mathbf{g}_i$ for some binary λ_i , so

$$\mathbf{x} \cdot \mathbf{c} = \mathbf{x} \cdot \left(\sum_i \lambda_i \mathbf{g}_i \right) = \sum_i \lambda_i (\mathbf{x} \cdot \mathbf{g}_i) = 0$$

and thus $\mathbf{x} \in C_d$

So hence we have shown that \mathbf{x} belongs to the dual code of C . Now we

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Dual code

- If $\mathbf{xG}^T = 0$ then $\mathbf{x} \cdot \mathbf{g}_i = 0$ for every i . If $\mathbf{c} \in C$, then $\mathbf{c} = \sum_i \lambda_i \mathbf{g}_i$ for some binary λ_i , so

$$\mathbf{x} \cdot \mathbf{c} = \mathbf{x} \cdot \left(\sum_i \lambda_i \mathbf{g}_i \right) = \sum_i \lambda_i (\mathbf{x} \cdot \mathbf{g}_i) = 0$$

and thus $\mathbf{x} \in C_d$

- Thus the generator matrix \mathbf{G} of a linear (n, k) block code, is the parity check matrix \mathbf{H} of its dual code and vice-versa.

also know that if \mathbf{x} belongs to a particular code, let's say, dual code then we know the condition that, if, for any valid codeword we know that, let's say \mathbf{x} is a valid codeword then we know this property holds; that

(Refer Slide Time 11:49)

Dual code

- If $\mathbf{xG}^T = 0$ then $\mathbf{x} \cdot \mathbf{g}_i = 0$ for every i . If $\mathbf{c} \in C$, then $\mathbf{c} = \sum_i \lambda_i \mathbf{g}_i$ for some binary λ_i , so

$$\mathbf{x} \cdot \mathbf{c} = \mathbf{x} \cdot \left(\sum_i \lambda_i \mathbf{g}_i \right) = \sum_i \lambda_i (\mathbf{x} \cdot \mathbf{g}_i) = 0$$

and thus $\mathbf{x} \in C_d$ $\mathbf{xH}^T = 0$

- Thus the generator matrix \mathbf{G} of a linear (n, k) block code, is the parity check matrix \mathbf{H} of its dual code and vice-versa.

uh codeword parity check matrix transform is basically equal to zero. So if we compare this form with this, you can immediately guess that the generator matrix for the dual code is given by the parity check matrix of the original linear blockcode C . And the parity check matrix of the dual code is given by this generator matrix of the original code C . So this is what I am saying here. The generator matrix G of a linear blockcode is the parity check matrix for dual code and vice versa.

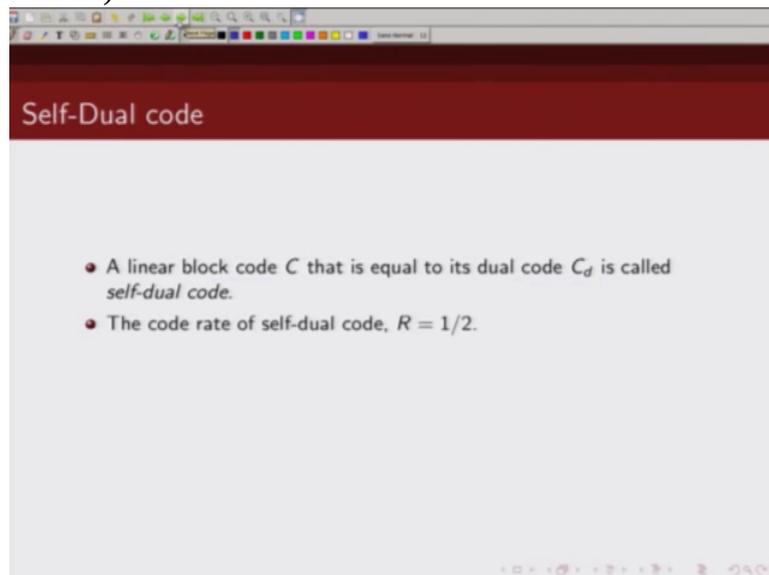
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Self-Dual code

- A linear block code C that is equal to its dual code C_d is called *self-dual code*.

Ok, now what is a self dual code? If a linear block code is same, is equal to its dual code then it is called a self dual code. So as you can make out

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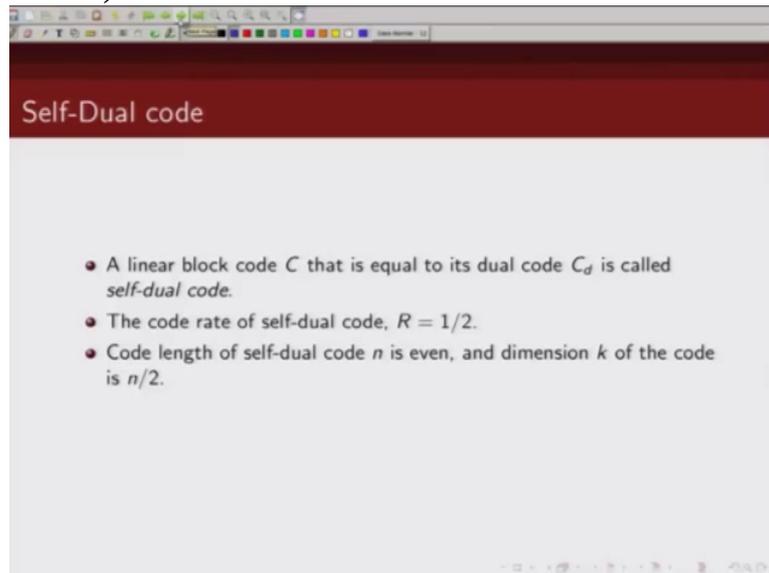
the rate for the self dual

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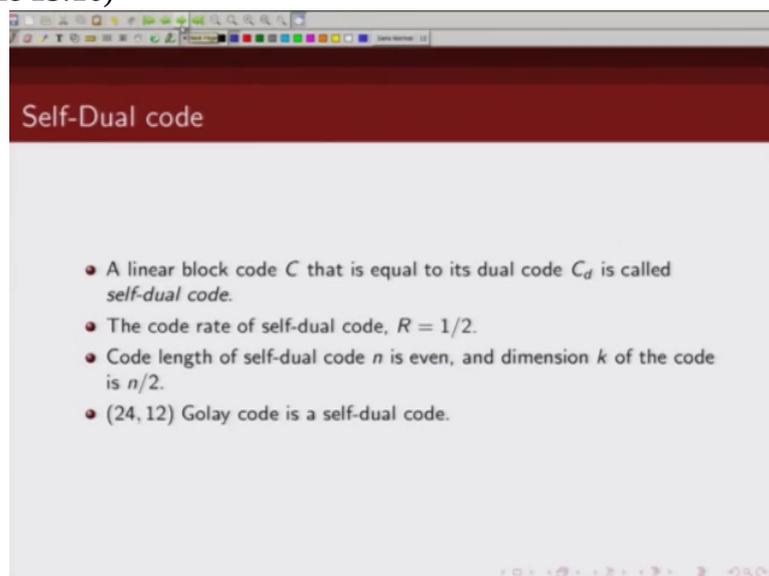
code should be half because k should be n by 2. So

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n is always even for a self dual code and the dimension of the code is always $n/2$. An example of self dual

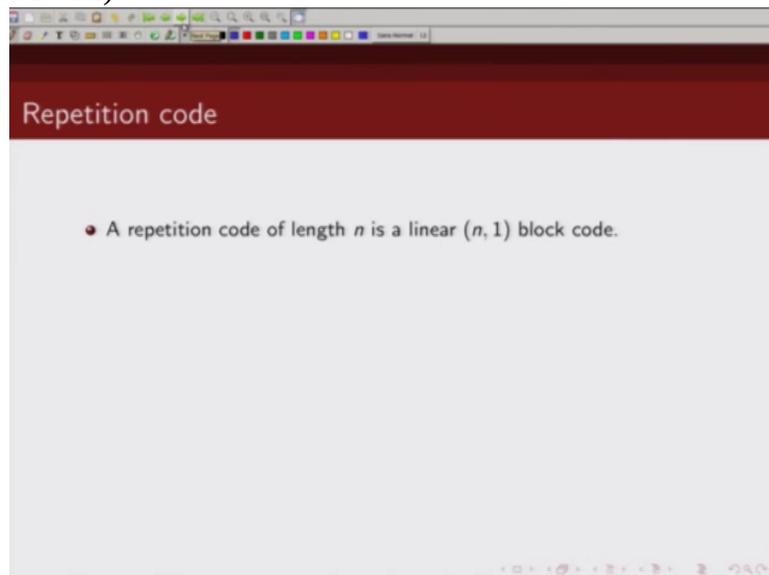
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is $24, 12$ Golay code.

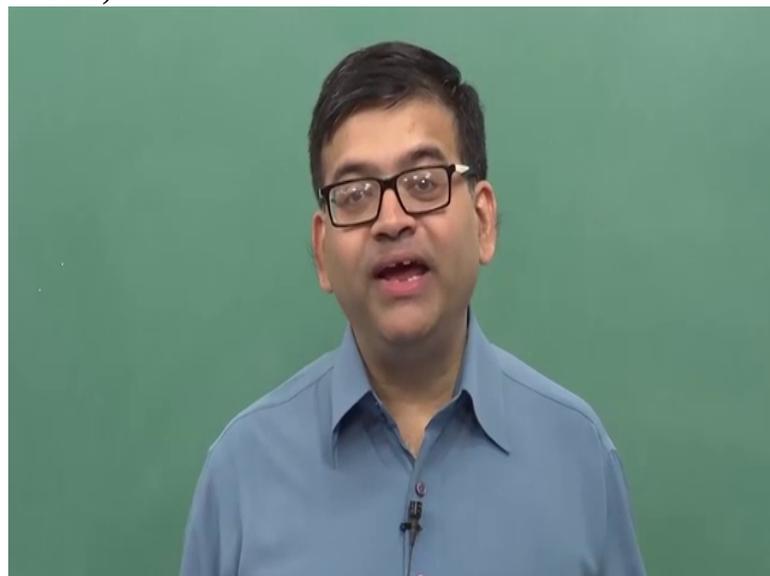
Ok now that we have

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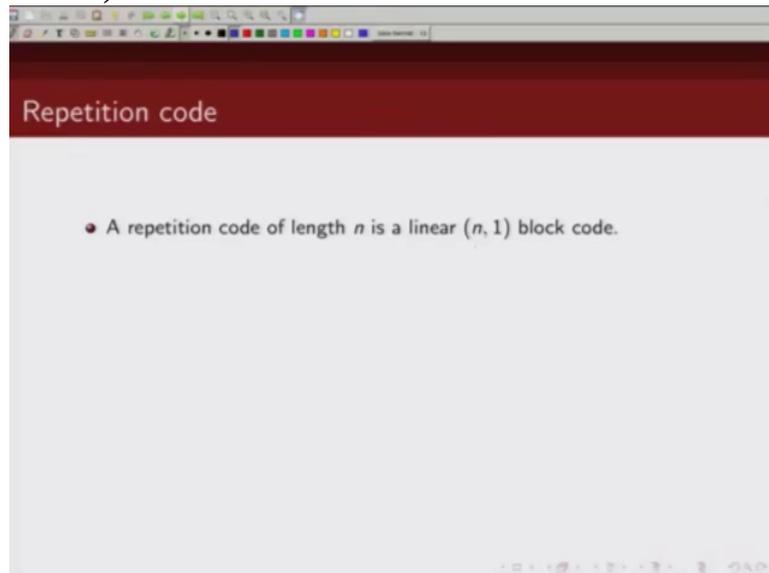
defined dual code let's now come to the other topic which is some examples of linear

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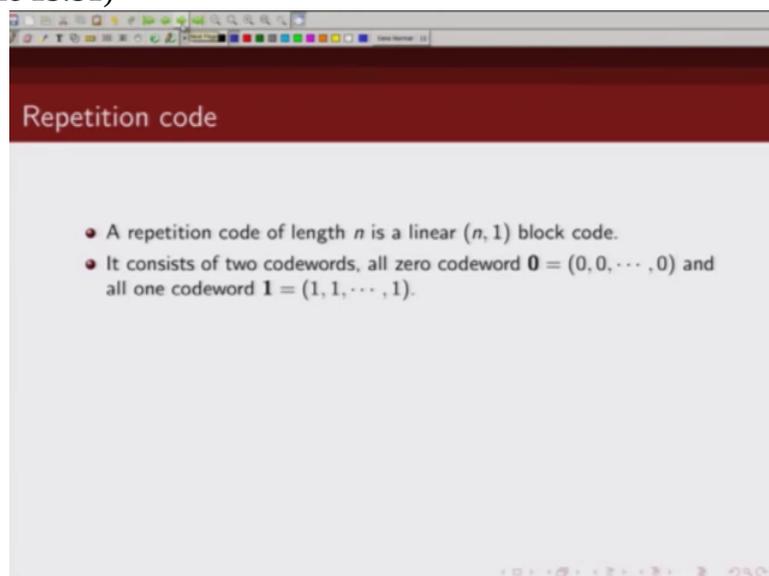
block code. So a very simple example is the repetition code. So for repetition

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code k is 1 and let's say we have a rate $1/n$ repetition code. So the codeword length is n , there is one input bit and n outputs. And how is

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repetition code generated? So we repeat the same information n times. So for a binary repetition code, it will have two codewords, 0 and 1. So if the information sequence, information bit is 0, the output will be all zeroes repeated n times. And if the input is 1, this will be, output will be all 1 repeated n times. So there are only two

(Refer Slide Time 14:24)

Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
n times

codewords in our repetition code. So we can then

(Refer Slide Time 14:31)

Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by

$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$

write down our generator matrix. How is generator matrix? If v is the coded sequence, u is the information sequence then generator matrix; they are related to generator matrix in this particular way.

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Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by

$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$

$v = uG$

So, since our output is the bit repeated n times, the generator matrix for repetition code will be all 1s. So this is 1 cross n . Now how do we decode

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Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by

$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$

- Decoding is based on majority decision of n coded bits.

a code which is encoded using repetition code? So what we do is we take a majority decision. So, let's say, for zero, we are sending n zeroes

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and for 1 we are sending n 1's. So at the receiver, when some of the bits get flipped or they are changed, what do we notice is that we look at block of n bits and we see what is the majority? Is it 0 or 1? If it is 0, if majority of the bits are 0 we decide in favor of 0. If it is a 1, we decide in favor of 1. And

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A slide titled "Repetition code" with a red header. The slide contains the following text:

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by
$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$
- Decoding is based on majority decision of n coded bits.
- Minimum distance of the code is n .

we can see the minimum distance of the code is n . Because there are only two codewords, all 0's codewords and all 1's codewords. So it can correct n minus 1 by 2, floor of that, it can correct so many errors,

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Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by

$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$

- Decoding is based on majority decision of n coded bits.
- Minimum distance of the code is n . $\left\lfloor \frac{n-1}{2} \right\rfloor$

it can correct so many errors and it can detect

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Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by

$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$

- Decoding is based on majority decision of n coded bits.
- Minimum distance of the code is n . $\left\lfloor \frac{n-1}{2} \right\rfloor$ correct

n minus 1 errors. Because any error pattern

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Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by
$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$
- Decoding is based on majority decision of n coded bits.
- Minimum distance of the code is n .

Handwritten note: $\left. \begin{matrix} \frac{n-1}{2} \\ n-1 \end{matrix} \right\} \begin{matrix} \text{correct} \\ \text{detect} \end{matrix}$

of weight less than n would not change it into any valid codeword. So all error patterns up to weight n minus 1 can be detected by this repetition code.

Let's look at

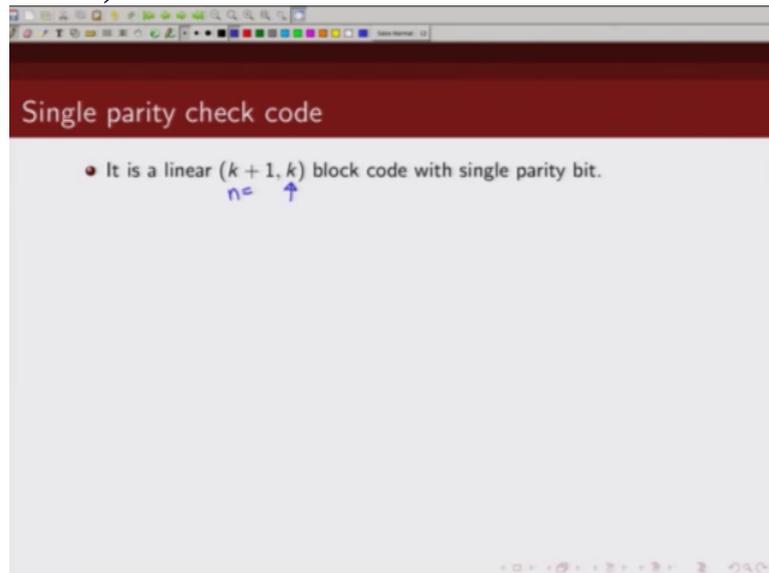
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Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.

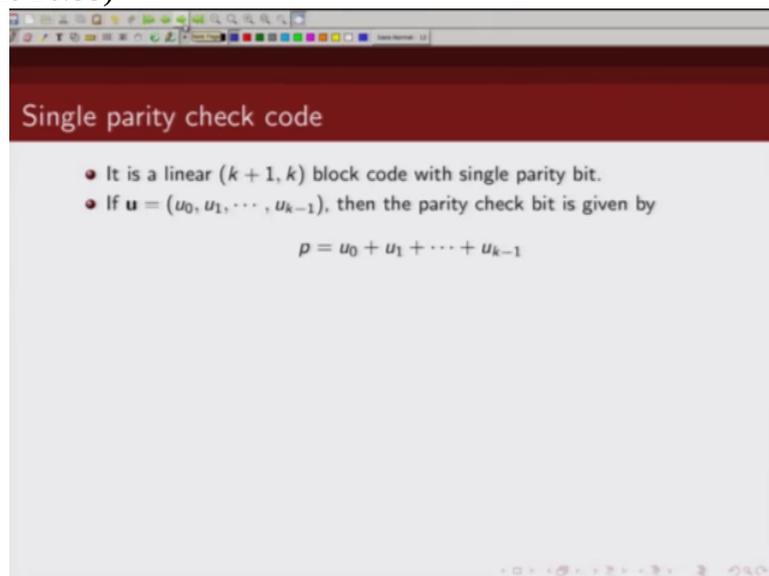
another example of linear block code. This is single parity check code. As the name suggests we are adding one single parity check bit. So the information sequence length is k and n here is given by k plus 1. And how do we generate this

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single parity check bit? This is as follows.

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So if your information sequence is given by this, so you have a k -bit information sequence. Let's call it $u_0, u_1, u_2, \dots, u_{k-1}$, then we generate this additional parity bit in this fashion. So p is equal to u_0 plus u_1 plus u_2 up to u_{k-1} . So in other words, if information sequence is even parity, that means some of them basically add up to zero, then p will be zero or else if the information sequence has odd parity, p will be 1. So as you can make out, this single parity check code will always have even weight

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codewords. So each codeword

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Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by
$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form
$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$

I can write in this particular fashion, so I have these k information bits and one parity bit.

(Refer Slide Time 17:58)

Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by
$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form
$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$

↑
parity bit

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Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by
$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form
$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$
- The generator matrix for the single parity check code in systematic form is given by
$$\mathbf{G} = \left[\begin{array}{c|cccc} 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & & & & \vdots & & \\ 1 & 0 & 0 & 0 & \dots & 1 \end{array} \right]$$

So I can write the same thing in the form of a generator matrix as we know our coded sequence can be written as input times

(Refer Slide Time 18:10)

Single parity check code

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$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form
$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$
- The generator matrix for the single parity check code in systematic form is given by
$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad \mathbf{v} = \mathbf{uG}$$

generator matrix. So you can see first bit is a parity bit which is sum of all u i's so first column would be all 1's and then this will be an identity matrix. Because the second bit is u 0, third bit is u 1, u 2 and so on. So this is a identity matrix.

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Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
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$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form
$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$
- The generator matrix for the single parity check code in systematic form is given by
$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad \mathbf{v} = \mathbf{uG}$$

Ok?

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Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

Now we can write down the parity check matrix for this also. As you can see this is,

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Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by

$$p = u_0 + u_1 + \dots + u_{k-1}$$

- Each codeword is of the form

$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$

- The generator matrix for the single parity check code in systematic form is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad \mathbf{v} = \mathbf{uG}$$

this generator matrix is in systematic form. So what would be

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Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by

$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form

$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$
- The generator matrix for the single parity check code in systematic form is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Handwritten notes: $\mathbf{v} = \mathbf{uG}$, $\mathbf{G} = [\mathbf{I} : \mathbf{P}]$

the corresponding, this is of the form like this. I have P I so parity check matrix will be I P transpose,

(Refer Slide Time 19:11)

Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by

$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form

$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$
- The generator matrix for the single parity check code in systematic form is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Handwritten notes: $\mathbf{v} = \mathbf{uG}$, $\mathbf{G} = [\mathbf{P} : \mathbf{I}]$, $\mathbf{H} = [\mathbf{I} : \mathbf{P}^T]$

right? So what I have here is

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Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

this is my P transpose and this is my I. This

(Refer Slide Time 19:27)

Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by

$$p = u_0 + u_1 + \dots + u_{k-1}$$

- Each codeword is of the form

$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$

- The generator matrix for the single parity check code in systematic form is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

$\mathbf{v} = \mathbf{uG}$
 $\mathbf{G} = [\mathbf{P} : \mathbf{I}]$
 $\mathbf{H} = [\mathbf{I} : \mathbf{P}^T]$

part is my, this part is my P and

(Refer Slide Time 19:34)

Single parity check code

- It is a linear $(k + 1, k)$ block code with single parity bit.
- If $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, then the parity check bit is given by
$$p = u_0 + u_1 + \dots + u_{k-1}$$
- Each codeword is of the form
$$\mathbf{v} = (p, u_0, u_1, \dots, u_{k-1})$$
- The generator matrix for the single parity check code in systematic form is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

$\mathbf{v} = \mathbf{uG}$
 $\mathbf{G} = [\mathbf{P} : \mathbf{I}]$
 $\mathbf{H} = [\mathbf{I} : \mathbf{P}^T]$

this part is my I. So parity check matrix would be I P transpose that's this.

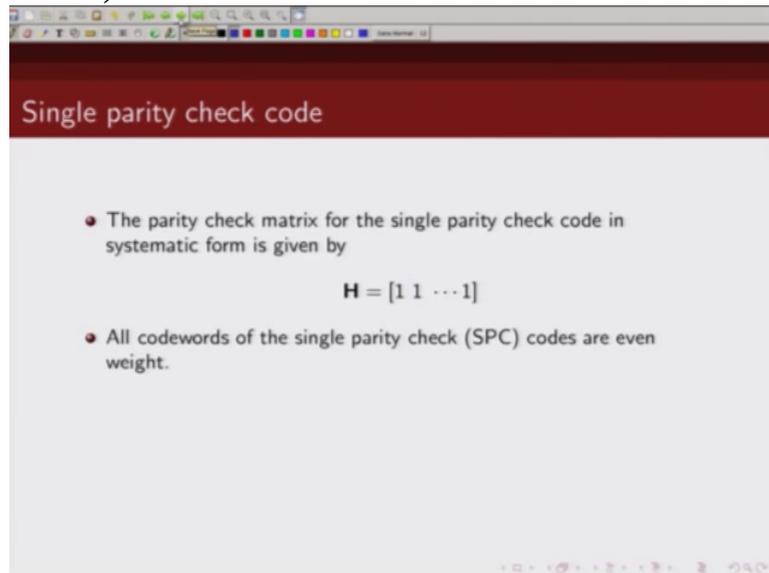
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Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1' 1 \dots 1]$$

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Single parity check code

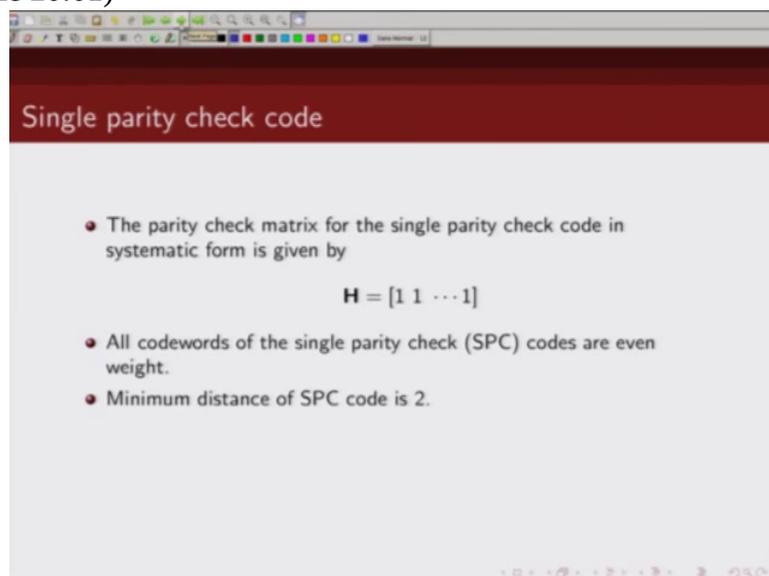
- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

- All codewords of the single parity check (SPC) codes are even weight.

As I mentioned, a very interesting property of single parity check code, all codewords of single parity check codes are of even weight. And

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Single parity check code

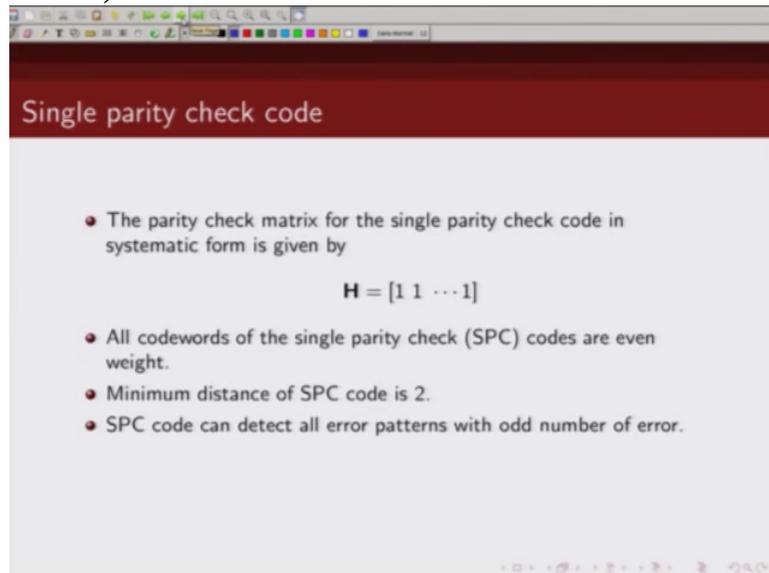
- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

- All codewords of the single parity check (SPC) codes are even weight.
- Minimum distance of SPC code is 2.

we can see the minimum distance of the code is 2. Simple, very simple way to check is look at the columns of the parity check matrix? What is the minimum number of columns that add up to zero? In this case 2. Any two columns will add up to zero. So the minimum distance of this code is 2.

(Refer Slide Time 20:24)



Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

- All codewords of the single parity check (SPC) codes are even weight.
- Minimum distance of SPC code is 2.
- SPC code can detect all error patterns with odd number of error.

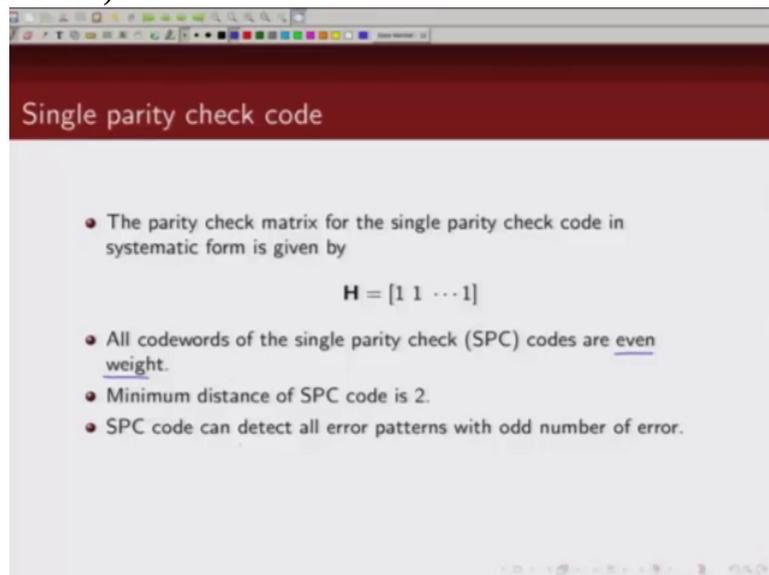
Now since all codewords have even weight any odd code, any odd error pattern can be determined by single parity check code, because any odd

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pattern, any odd weight pattern is not a valid codeword. So a single parity check

(Refer Slide Time 20:48)



Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

- All codewords of the single parity check (SPC) codes are even weight.
- Minimum distance of SPC code is 2.
- SPC code can detect all error patterns with odd number of error.

code can detect all error patterns which have even numbers of errors, odd numbers of errors, right? So as long as error pattern has odd weight,

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single parity check code can detect it. And it is not

(Refer Slide Time 21:08)

Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by

$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$

- All codewords of the single parity check (SPC) codes are even weight.
- Minimum distance of SPC code is 2.
- SPC code can detect all error patterns with odd number of error.
- The $(n, n - 1)$ SPC code and $(n, 1)$ repetition code are dual to each other.

very difficult to see that single parity check code and repetition codes are dual to each other. You can see basically uh, the parity check matrix of single parity check code is same as generator matrix of the repetition

(Refer Slide Time 21:29)

Repetition code

- A repetition code of length n is a linear $(n, 1)$ block code.
- It consists of two codewords, all zero codeword $\mathbf{0} = (0, 0, \dots, 0)$ and all one codeword $\mathbf{1} = (1, 1, \dots, 1)$.
- Codeword is obtained by repeating the information bit n times.
- Generator matrix is given by

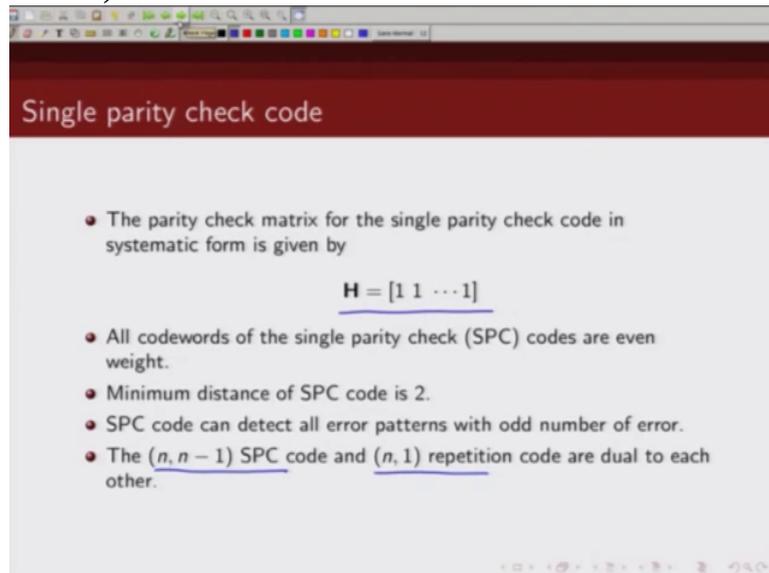
$$\mathbf{G} = [1 \ 1 \ \dots \ 1]$$

- Decoding is based on majority decision of n coded bits.
- Minimum distance of the code is n .

Handwritten notes: $\left[\frac{n-1}{2} \right]$ correct, $n-1$ detect

code. And similarly the parity check matrix of uh, repetition code is same as the generator matrix of the single parity check code and you can check the dimensions of the single parity check

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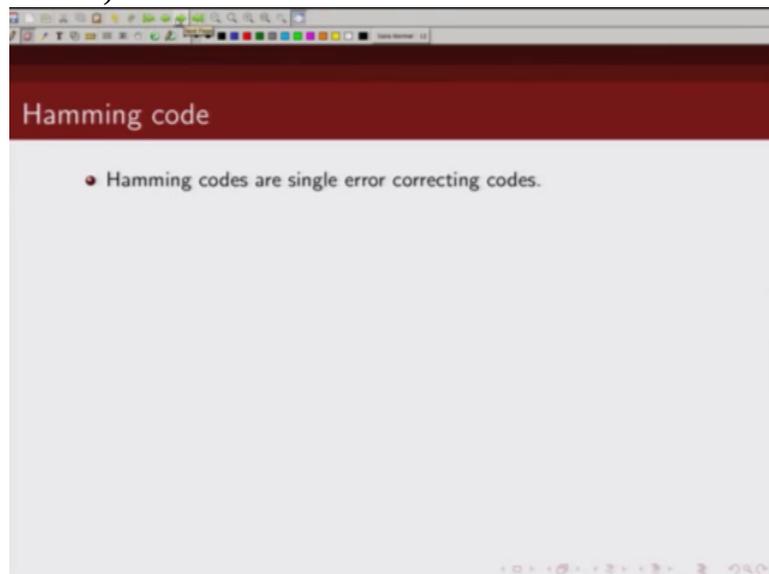
Single parity check code

- The parity check matrix for the single parity check code in systematic form is given by
$$\mathbf{H} = [1 \ 1 \ \dots \ 1]$$
- All codewords of the single parity check (SPC) codes are even weight.
- Minimum distance of SPC code is 2.
- SPC code can detect all error patterns with odd number of error.
- The $(n, n - 1)$ SPC code and $(n, 1)$ repetition code are dual to each other.

code is n minus 1 and this dimension is 1. So single parity check codes and repetition code are dual to each other.

Next we consider

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Hamming code

- Hamming codes are single error correcting codes.

Hamming codes. So Hamming codes are single error correcting codes. These are single error correcting codes and these are described by

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Hamming code

- Hamming codes are single error correcting codes.
- For any $m \geq 3$, there exist a Hamming code with following parameters

Code length:	$n = 2^m - 1$
Information bits:	$k = 2^m - m - 1$
Parity bits:	$n - k = m$
Error correcting capability:	$t = 1$
Minimum distance:	$d_{\min} = 3$

these parameters. So for any m greater than equal to 3, the codeword length is given by this. The number of parity bits is equal to m . So the number of information bits are n minus k , n minus m so this is 2 to power n minus m minus 1 . So these many number of information bits. It has minimum distance of 3 . So it can correct single error and it can detect 2 errors. So how do we describe Hamming code? We can describe

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Hamming code

- Hamming codes are single error correcting codes.
- For any $m \geq 3$, there exist a Hamming code with following parameters

Code length:	$n = 2^m - 1$
Information bits:	$k = 2^m - m - 1$
Parity bits:	$n - k = m$
Error correcting capability:	$t = 1$
Minimum distance:	$d_{\min} = 3$

- The parity check matrix in systematic form

$$\mathbf{H} = [\mathbf{I}_m : \mathbf{P}^T],$$

where the $2^m - m - 1$ columns of \mathbf{P}^T consists of all m -tuples of weight 2 or more.

the Hamming code by generator matrix or the parity check matrix. So a parity check matrix of a Hamming code consists of all non-zero m tuples. So if the number of parity bits is, as I said is, m , then codeword length is 2 raised to power m minus 1 and the entries in the parity check matrix are all non-zero m tuples. Now how many m -tuples do we have? We have total

2 raised to power m m-tuples and out of those, so these are like 0 0 0, 0 0 1, you can go on up to all 1s. And this is m times, m. Now if we

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Hamming code

- Hamming codes are single error correcting codes.
- For any $m \geq 3$, there exist a Hamming code with following parameters

Code length:	$n = 2^m - 1$
Information bits:	$k = 2^m - m - 1$
Parity bits:	$n - k = m$
Error correcting capability:	$t = 1$
Minimum distance:	$d_{\min} = 3$
- The parity check matrix in systematic form

$$\mathbf{H} = [\mathbf{I}_m : \mathbf{P}^T],$$

where the $2^m - m - 1$ columns of \mathbf{P}^T consists of all m-tuples of weight 2 or more.

Handwritten notes on the slide include a diagram of a matrix with m rows and $2^m - 1$ columns, and a bracket indicating the first m columns.

remove the all zero one, so total number of non-zero m-tuples is basically given by 2 raised to power m minus 1. So the

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Hamming code

- Hamming codes are single error correcting codes.
- For any $m \geq 3$, there exist a Hamming code with following parameters

Code length:	$n = 2^m - 1$
Information bits:	$k = 2^m - m - 1$
Parity bits:	$n - k = m$
Error correcting capability:	$t = 1$
Minimum distance:	$d_{\min} = 3$
- The parity check matrix in systematic form

$$\mathbf{H} = [\mathbf{I}_m : \mathbf{P}^T],$$

where the $2^m - m - 1$ columns of \mathbf{P}^T consists of all m-tuples of weight 2 or more.

Handwritten notes on the slide include a diagram of a matrix with m rows and $2^m - 1$ columns, and a bracket indicating the first m columns.

columns of the parity check matrix of Hamming code are nothing but non-zero m-tuples. So there are total $2^m - 1$ of them, $2^m - m - 1$ of them,

(Refer Slide Time 24:11)

Hamming code

- Hamming codes are single error correcting codes.
- For any $m \geq 3$, there exist a Hamming code with following parameters

Code length:	$n = 2^m - 1$
Information bits:	$k = 2^m - m - 1$
Parity bits:	$n - k = m$
Error correcting capability:	$t = 1$
Minimum distance:	$d_{\min} = 3$
- The parity check matrix in systematic form

$$\mathbf{H} = [\mathbf{I}_m : \mathbf{P}^T],$$
 where the $2^m - m - 1$ columns of \mathbf{P}^T consists of all m -tuples of weight 2 or more.

Handwritten diagram: A matrix structure $\mathbf{H} = [\mathbf{I}_m : \mathbf{P}^T]$ is shown. The identity matrix \mathbf{I}_m is represented by a vertical column of m boxes. The matrix \mathbf{P}^T is represented by a vertical column of $2^m - m - 1$ boxes. A bracket groups the m rows of \mathbf{P}^T .

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Hamming code

- For $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix \mathbf{H} ,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$
 and generator matrix \mathbf{G} ,

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Ok. And if you want to write a parity check matrix of a Hamming code in systematic form, then we can write it in this way. So parity check matrix will have one identity and then some matrix \mathbf{p} since identity matrix will consist of all patterns, all m -tuples of weight 1, \mathbf{p} \mathbf{t} should consist of all m -tuples of weight more than 1. So it consists of all m -tuples of weight 2, 3, 4 up to m .

So let us take an example. Let us consider m equal to 3. So if m equal to 3, we know m is 2 raised to the power m minus 1. So the length of the block code in this case will be 7.

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Hamming code

- For $m=3$, the Hamming code is of length $n = 2^3 - 1 = 7$,
 $k = 2^3 - 3 - 1 = 4$, that has parity check matrix **H**,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix **G**,

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes: $n = 2^m - 1 = 7$

Now how many number of parity bits? Number of parity bits is given by m. So this will be 3.

So then

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Hamming code

- For $m=3$, the Hamming code is of length $n = 2^3 - 1 = 7$,
 $k = 2^3 - 3 - 1 = 4$, that has parity check matrix **H**,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix **G**,

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes: $n = 2^m - 1 = 7$
 $n - k = m = 3$

what is the number of information bits? It is 7 minus 3 that is 4. So this is a

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Hamming code

- For $m=3$, the Hamming code is of length $n=2^3-1=7$, $k=2^3-3-1=4$, that has parity check matrix \mathbf{H} ,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix \mathbf{G} ,

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes: $n=2^m-1=7$, $n-k=m=3$, $k=7-3=4$.

7 4 code. Now as I said,

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Hamming code

- For $m=3$, the Hamming code is of length $n=2^3-1=7$, $k=2^3-3-1=4$, that has parity check matrix \mathbf{H} ,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix \mathbf{G} ,

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes: $n=2^m-1=7$, $n-k=m=3$, $k=7-3=4$, $(7,4)$.

the parity check matrix consists of all non-zero m -tuples. So m here is 3. So let's list all non-zero m -tuples. So we can write 0 0 1, 0 1 0, 0 1 1, 1 0 0, 1 0 1, 1 1 0, 1 1 1. So these are the seven non-zero m tuples,

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Hamming code

- For $m=3$, the Hamming code is of length $n=2^3-1=7$, $k=2^3-3-1=4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix G ,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes:

- $n=2^m-1=7$
- $n-k=m=3$
- $k=7-3=4$
- $(7,4)$
- Diagram: $\begin{matrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{matrix}$ with a bracket under the last four bits labeled "7-nonzero".

right and if you want to write your parity check matrix in a systematic form, what will you do? So this is your I , so I am writing these patterns which are like 1, 2 and 3, I am writing these patterns and then what I write here is the other m -tuples. So you can see this is m tuple of weight 1, these three are m -tuple of weight 1 and then this is m -tuple of weight 2, and this is m -tuple of weight 3. Now since this matrix is of the form I and P , I can

(Refer Slide Time 26:54)

Hamming code

- For $m=3$, the Hamming code is of length $n=2^3-1=7$, $k=2^3-3-1=4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix G ,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes:

- $n=2^m-1=7$
- $n-k=m=3$
- $k=7-3=4$
- $(7,4)$
- Diagram: $\begin{matrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{matrix}$ with a bracket under the last four bits labeled "7-nonzero".
- Arrows pointing to the first three columns of H labeled "I:P".

write the generator matrix. This will be P transpose I . So

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Hamming code

- For $m=3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix G ,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes on the slide include:

- $n = 2^m - 1 = 7$
- $n - k = m = 3$
- $k = 7 - 3 = 4$
- $(7, 4)$
- A diagram showing the 7-bit code structure: 0001111 , 0110011 , 1010101 , with a bracket under the last four bits labeled "7-paritybits".

we can write this, so P transpose, so 1 0 1 1 will come here as 1 0 1 1. 1 1 1 0 will come as 1 1 1 0. And 0 1 1 1 will come as 0 1 1 1. And then this is a identity matrix. So this is our generator matrix for this 7 4 Hamming code. The point to be noted here is, once you fix your m, all

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Hamming code

- For $m=3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix G ,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

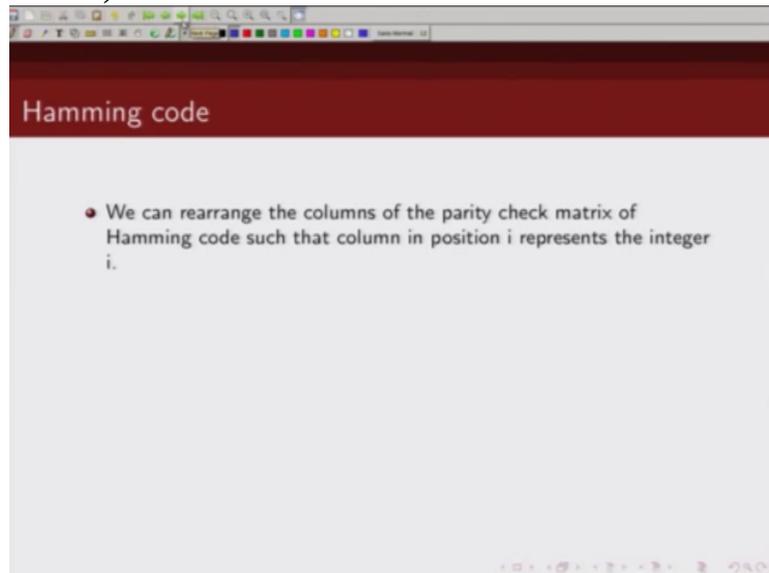
Handwritten notes on the slide include:

- $n = 2^m - 1 = 7$
- $n - k = m = 3$
- $k = 7 - 3 = 4$
- $(7, 4)$
- A diagram showing the 7-bit code structure: 0001111 , 0110011 , 1010101 , with a bracket under the last four bits labeled "7-paritybits".

other code parameters are fixed. n is fixed, right because n is 2 raised to power m minus 1. And number of parity bits is equal to m. For the Hamming code, once you fix m, the other parameters of the code are fixed.

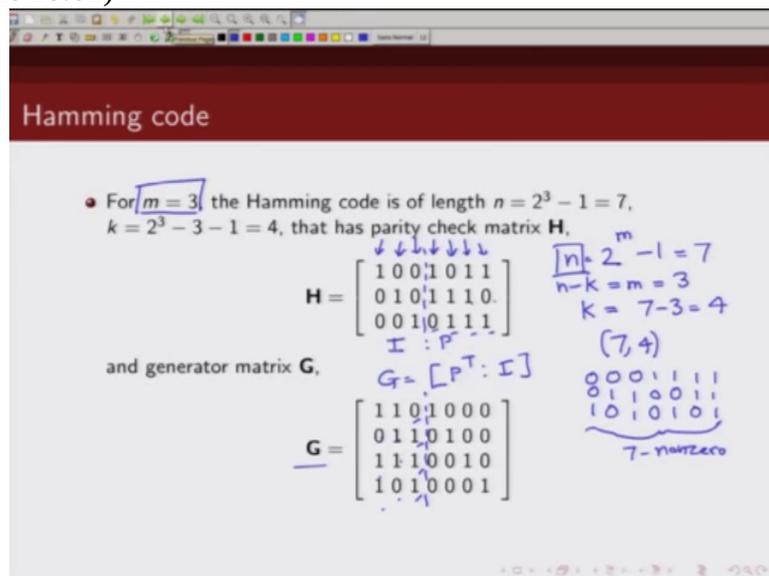
Now let us

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see how we can correct errors using Hamming code. As we said this is,

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this has minimum distance 3. You can see here. So let's take any three columns. Let's just take this column; column number 1, column number 2 and column number 4. You can see column number 1, 2 and 4, they will add up to 0 0 0, that is the minimum number of columns that will add up to zero,

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Hamming code

- For $m=3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix G ,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes on the slide include:

- $n = 2^m - 1 = 7$
- $n - k = m = 3$
- $k = 7 - 3 = 4$
- $(7, 4)$
- A diagram showing the 7-bit code structure with 4 data bits and 3 parity bits.
- A note: "7-nonzero"

so which means the minimum distance of this code is 3. If the minimum distance of this code is 3, what is the error correcting capability of this code? It is d minus 1 by 2, floor of that and this comes out to be 1. So Hamming code can correct single error.

So let's see how we can use

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Hamming code

- For $m=3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and generator matrix G ,

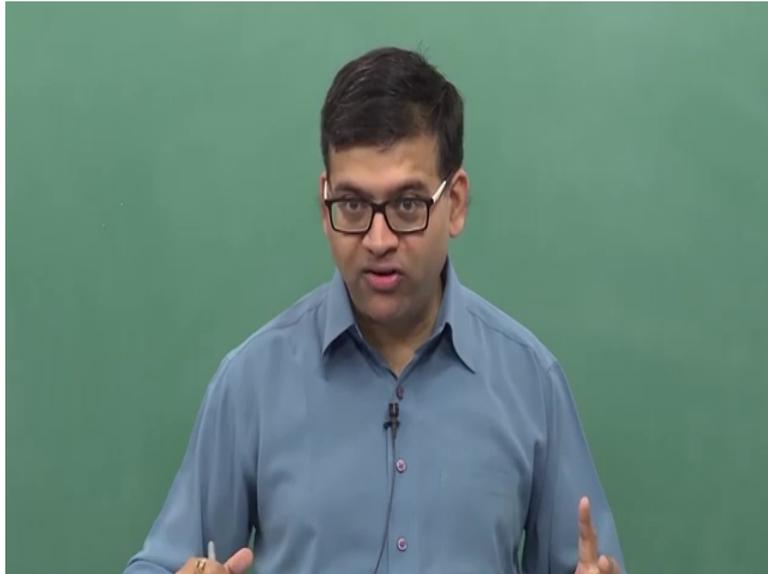
$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Handwritten notes on the slide include:

- $n = 2^m - 1 = 7$
- $n - k = m = 3$
- $k = 7 - 3 = 4$
- $(7, 4)$
- A diagram showing the 7-bit code structure with 4 data bits and 3 parity bits.
- A note: "7-nonzero"
- A calculation: $\left\lfloor \frac{d-1}{2} \right\rfloor = 1$

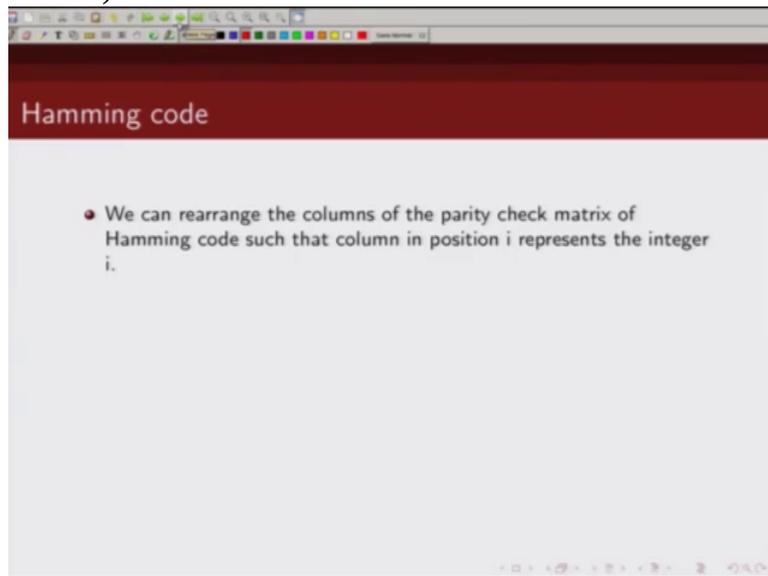
Hamming codes to

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correct single error. So first thing what we will

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do is, we will rearrange the columns of parity check matrix of Hamming code such that column in position i represent interior i .

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Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

So as we said, the parity check matrix of Hamming code are all non-zero m -tuples. So we will arrange them in such a way such that the i th column represent i th bit. Now what do I mean by that? So let's go back to our same example m equal to 3, so in this case n is equal to 7 and k is equal to 4. Now note the way I have arranged this? So this is my, this is my M S B and this is my L S B, least significant bit and the

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Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} \leftarrow \text{LSB} \\ \leftarrow \text{MSB} \end{matrix}$$

most significant bit. So

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Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix \mathbf{H} .

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- Here the column $(x, y, z)^T$ represents the number $x(2^0) + y(2^1) + z(2^2)$.

each column represents a number in this particular range. So this is the least significant, this is least significant bit; this is the most significant bit. So this is 0 0 1, that's binary 1, so this is my column number 1. This is 0 1 0, that's 2. 0 1 1, that's 3. This is 4, 5, 6, 7. Note what I

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Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix \mathbf{H} .

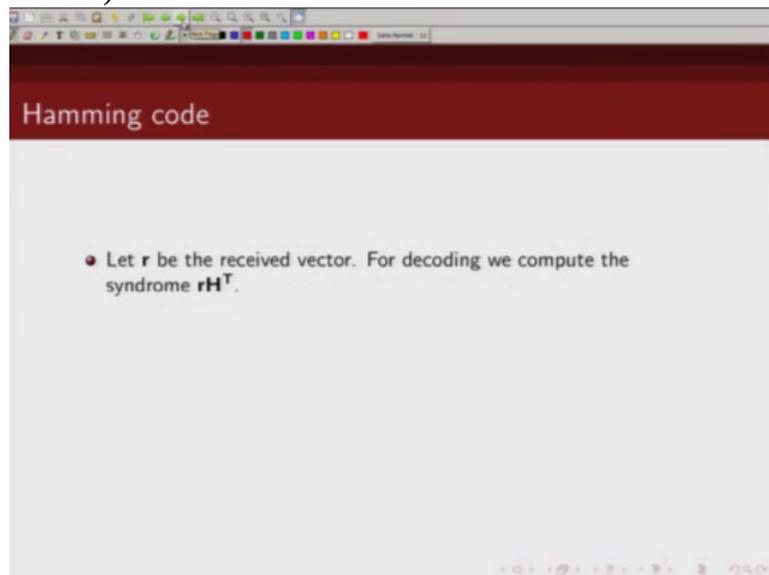
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1 2 3 4 5 6 7

- Here the column $(x, y, z)^T$ represents the number $x(2^0) + y(2^1) + z(2^2)$.

did was I rearranged a column of the parity check matrix of Hamming code such that now i th column represents i th, uh number basically, so column in i th position represents the integer i . Now if I do that then

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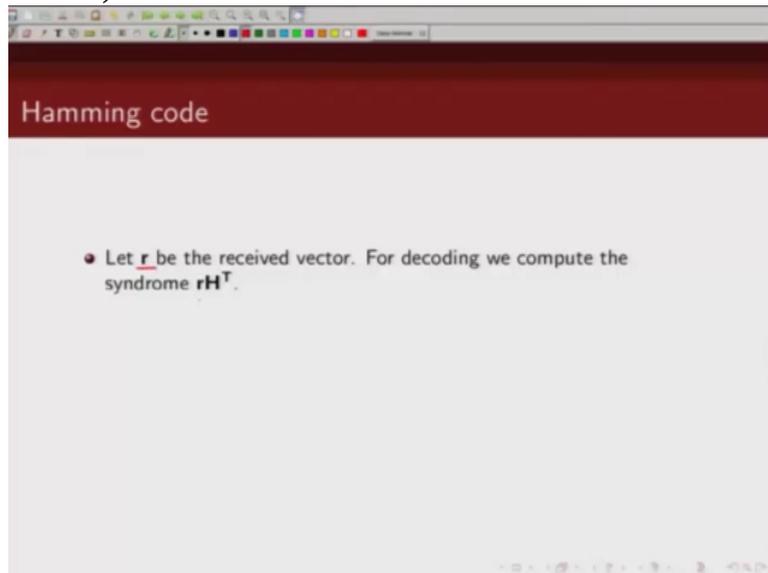
let's see how this helps us in locating the position of the error. So let's say \mathbf{r} is my received vector. This is my received n bit vector and I am interested in knowing whether \mathbf{r} is in error and if \mathbf{r} is in error, because it's a single error correcting code, let us say a single error has happened then I am interested in

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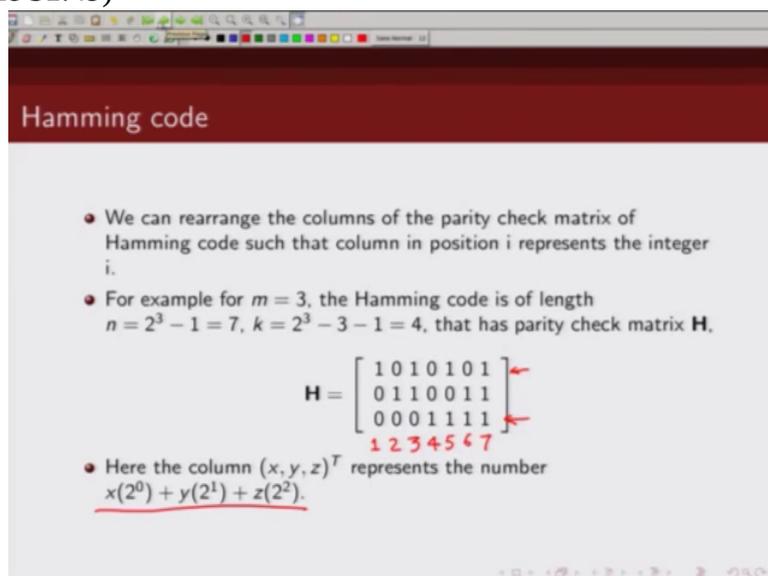
correcting this single error. So what do we do? The first thing to find out whether

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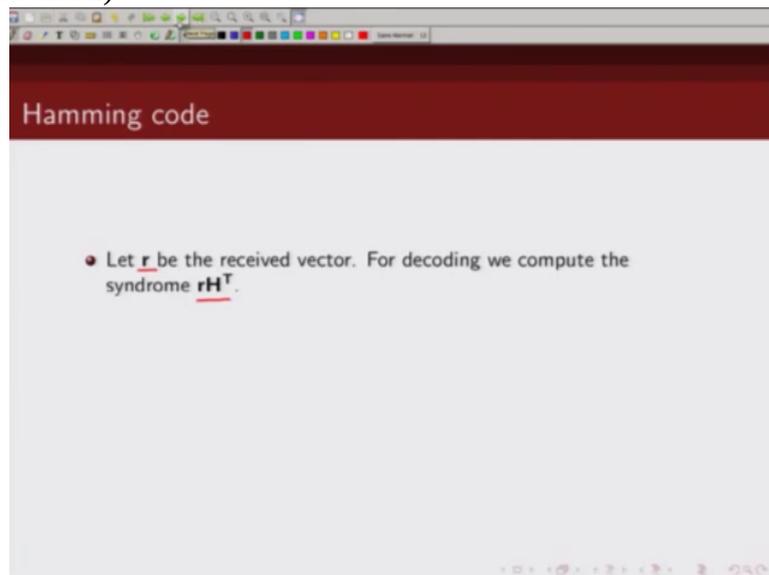
error has occurred or not, first thing that we do is we compute the syndrome. How do you compute the syndrome? We take \mathbf{rH}^T and what is our \mathbf{H} ? \mathbf{H} is

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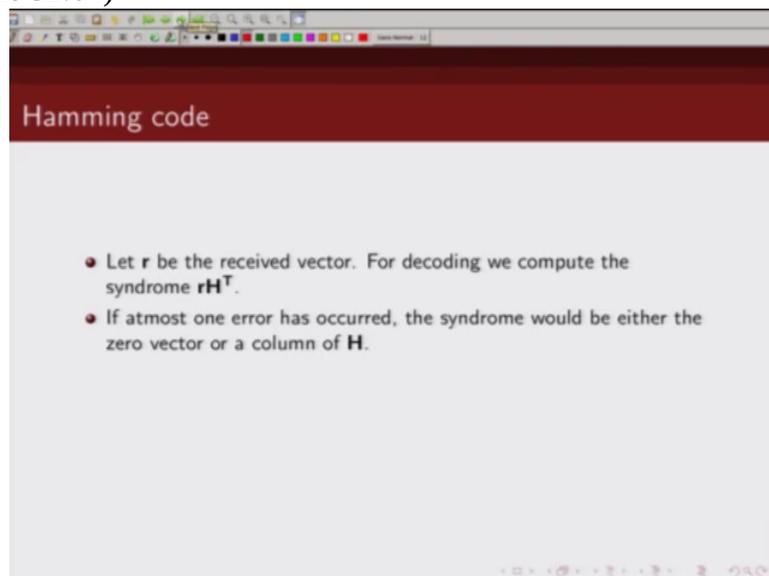
our matrix, this matrix which is arranged, where the columns are arranged in such a way such that column in i th position represents integer i .

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The slide is titled "Hamming code" in a dark red header. The main content area is white and contains a single bullet point: "Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T ." The text is in a black sans-serif font. At the bottom right of the slide, there are small navigation icons.

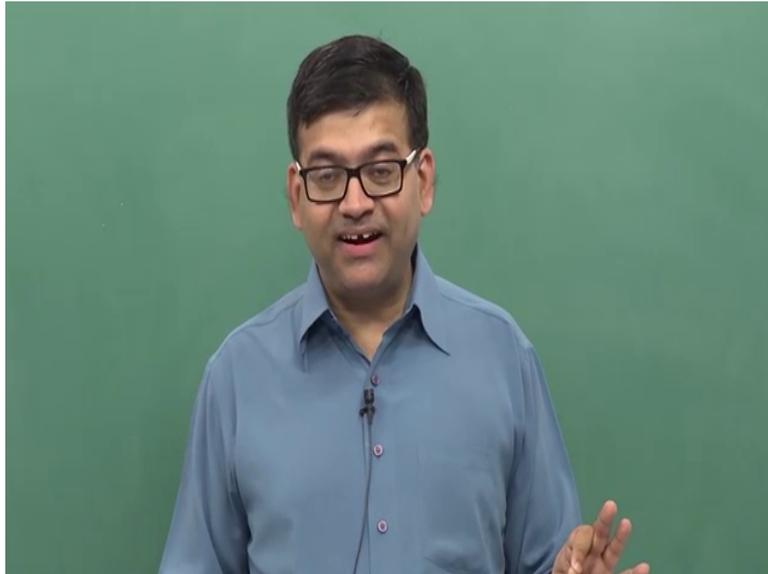
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The slide is titled "Hamming code" in a dark red header. The main content area is white and contains two bullet points: "Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T ." and "If at most one error has occurred, the syndrome would be either the zero vector or a column of \mathbf{H} ." The text is in a black sans-serif font. At the bottom right of the slide, there are small navigation icons.

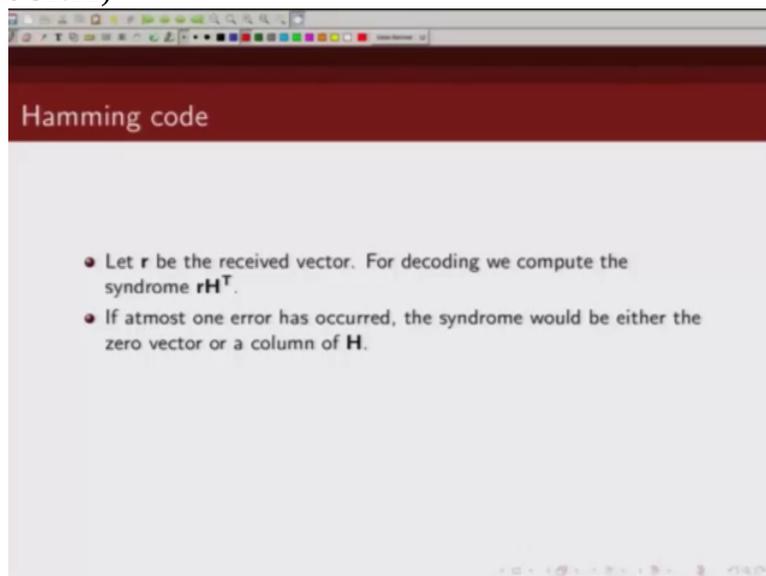
So, since it is single error correcting code,

(Refer Slide Time 32:06)



it can only correct at most one error. So we assume that say, at most

(Refer Slide Time 32:11)



one error has occurred, at most. Of course if there is no error, then syndrome would be an all-zero vector and if a single error has happened, the syndrome would have been a non-zero vector. Now what you will observe is, if a single error happens then that single error will be, whatever is the syndrome, from there we can find out which bit location is in error. So in case of Hamming code, if single error has happened and you have arranged your parity check matrix in such a way such that column in position i represent integer i then the syndrome would be either zero vector in case there is no error or otherwise in case of single error it would be

(Refer Slide Time 33:05)

Hamming code

- Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T .
- If atmost one error has occurred, the syndrome would be either the zero vector or a column of \mathbf{H} .
no error *single error*

column of this parity check matrix.

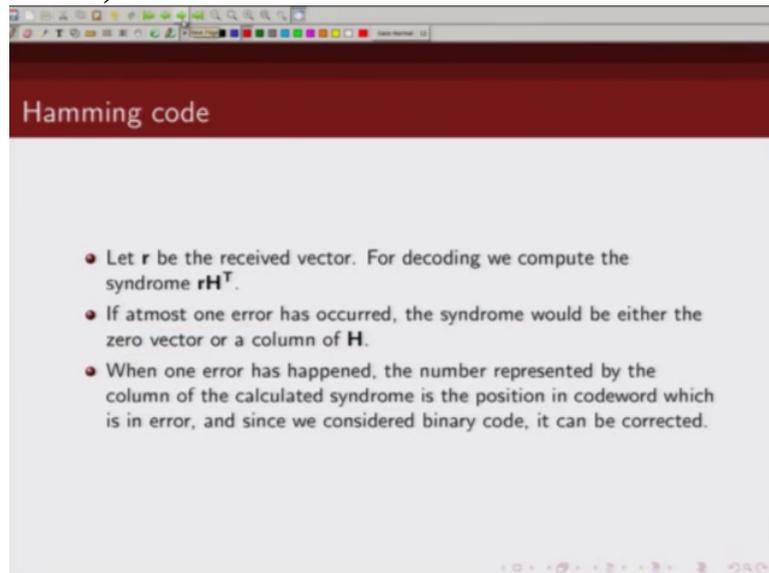
(Refer Slide Time 33:11)

Hamming code

- Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T .
- If atmost one error has occurred, the syndrome would be either the zero vector or a column of \mathbf{H} .
no error *single error*

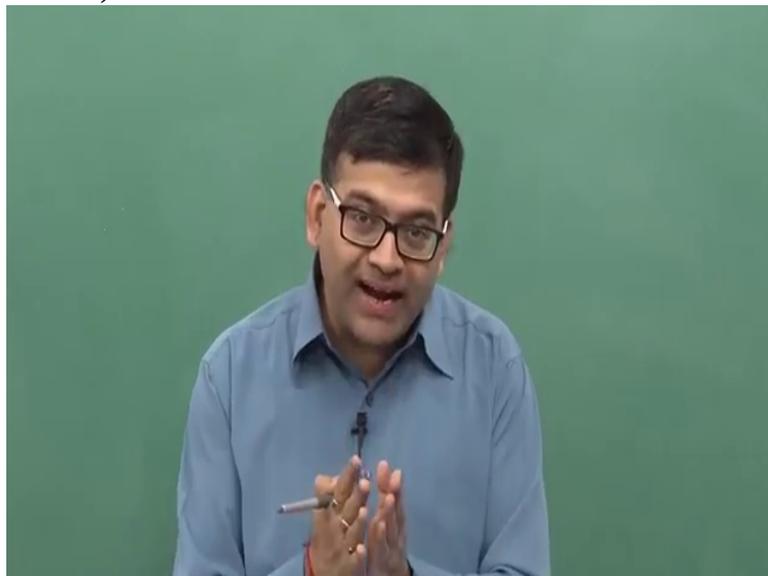
So

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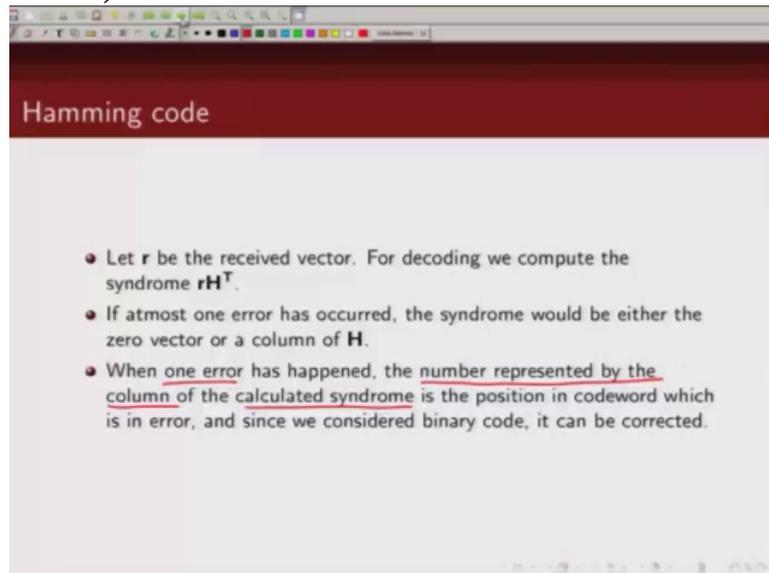
So when a single error happens, the number represented by the column of the calculated syndrome is the position in the codeword where the error has happened. So see, so nice, so by looking at the syndrome

(Refer Slide Time 33:34)



and comparing it with the column of the parity check matrix, our reordered parity check matrix, we can find out the location where the error has happened. And since for the binary code basically if we can locate the error, so the bit can either be 0 and 1, so whatever the bit is, we just flip that bit and get our

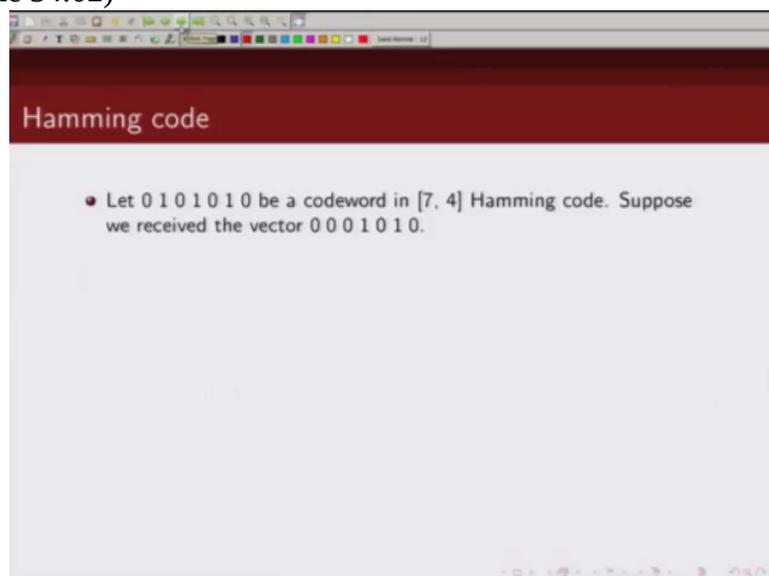
(Refer Slide Time 33:59)



corrected codeword.

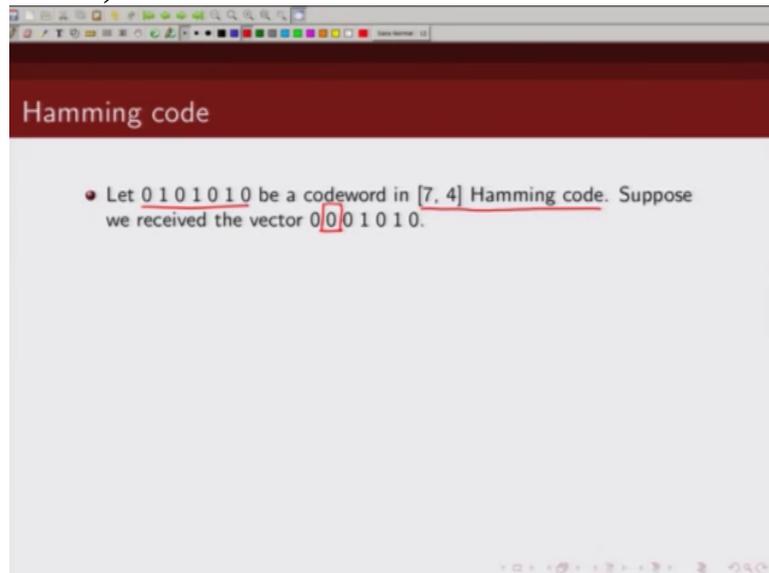
So let's take an example.

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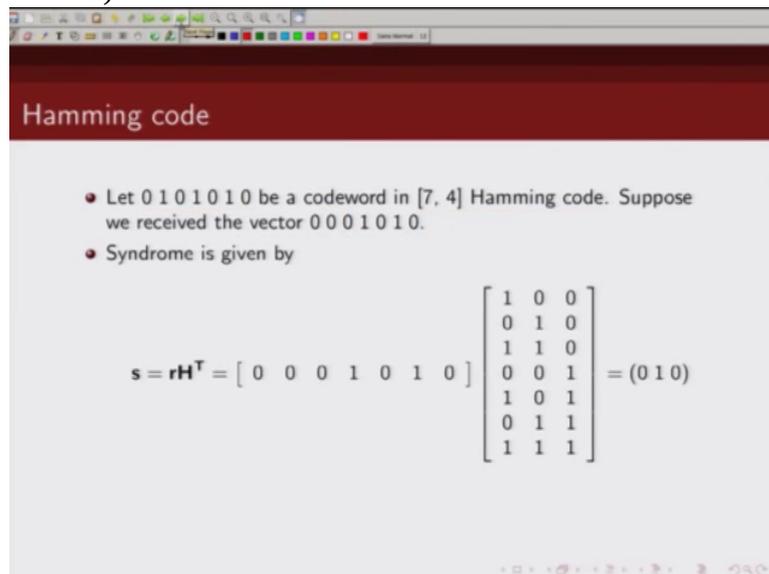
So let's say this is a codeword for this 7 4 Hamming code and let's assume that a single error has happened and this bit got changed from 1

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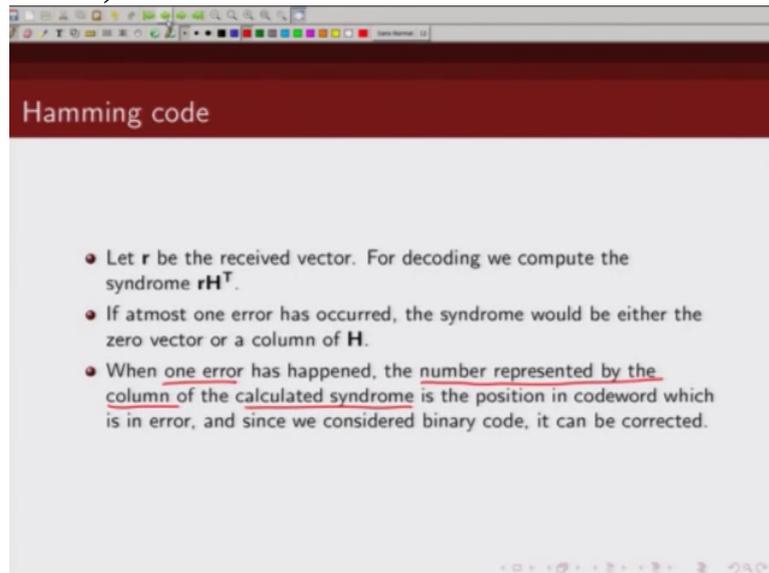
to 0. Now given this received sequence I am interested in finding out what is my estimated codeword. First of all I am interested in finding out whether an error has happened or not and if the error has happened I am interested in correcting that error. So

(Refer Slide Time 34:41)



as I said, the first thing I am going to do is I am going to compute the syndrome. So what is my r? This is my r. And what is my H? H matrix is basically my

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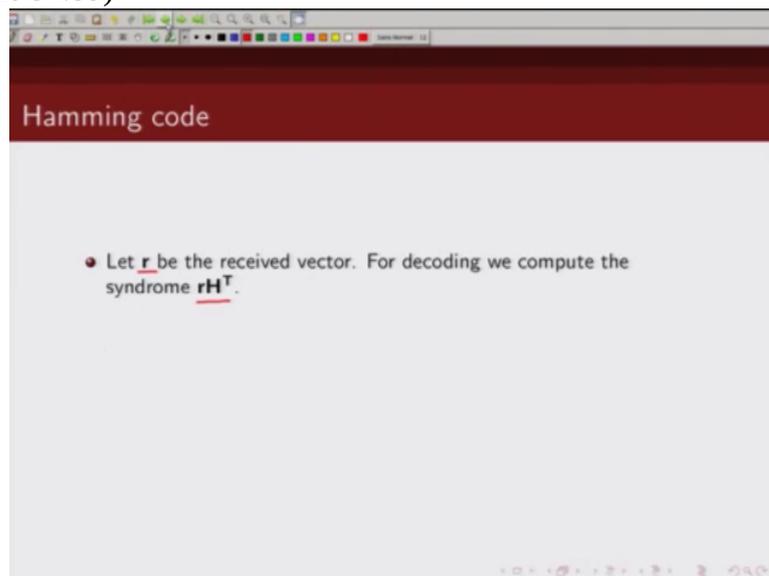


Hamming code

- Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T .
- If at most one error has occurred, the syndrome would be either the zero vector or a column of \mathbf{H} .
- When one error has happened, the number represented by the column of the calculated syndrome is the position in codeword which is in error, and since we considered binary code, it can be corrected.

parity check

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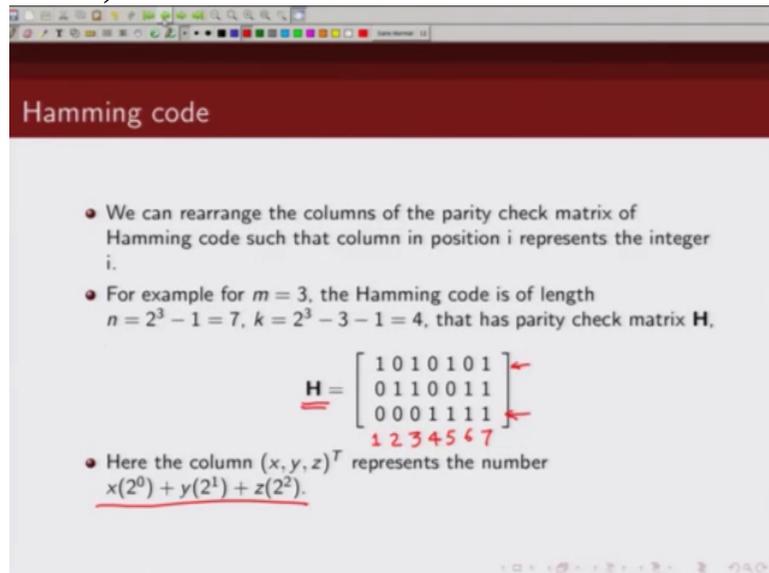


Hamming code

- Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T .

matrix

(Refer Slide Time 35:00)



Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix \mathbf{H} .

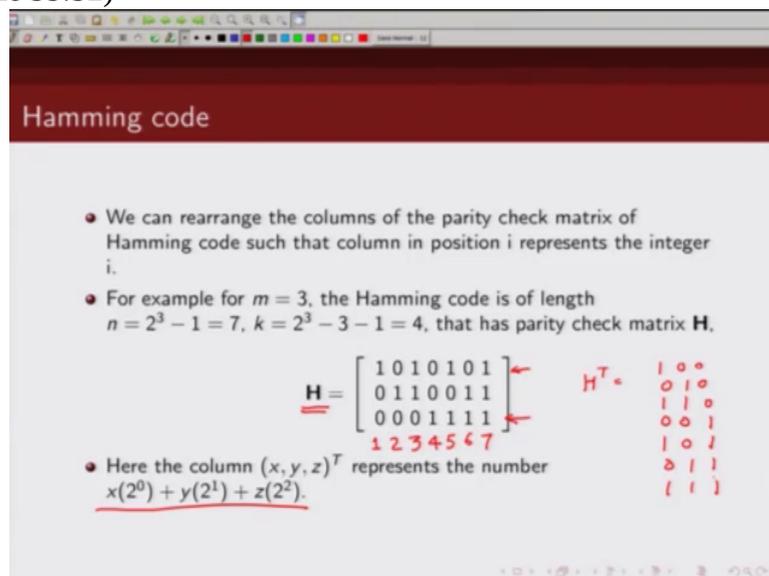
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1 2 3 4 5 6 7

- Here the column $(x, y, z)^T$ represents the number $x(2^0) + y(2^1) + z(2^2)$.

of the Hamming code where the columns are arranged in such a way that column in i th position represents i th integer. So if we take \mathbf{H} transpose we get 1 0 0, 0 1 0, 1 1 0, 0 0 1, 1 0 1, 0 1 1, 1 1 1, right and that's what I get here.

(Refer Slide Time 35:32)



Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix \mathbf{H} .

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1 2 3 4 5 6 7

$$\mathbf{H}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Here the column $(x, y, z)^T$ represents the number $x(2^0) + y(2^1) + z(2^2)$.

I get here.

(Refer Slide Time 35:33)

Hamming code

- Let \mathbf{r} be the received vector. For decoding we compute the syndrome \mathbf{rH}^T .
- If at most one error has occurred, the syndrome would be either the zero vector or a column of \mathbf{H} .
no error single error

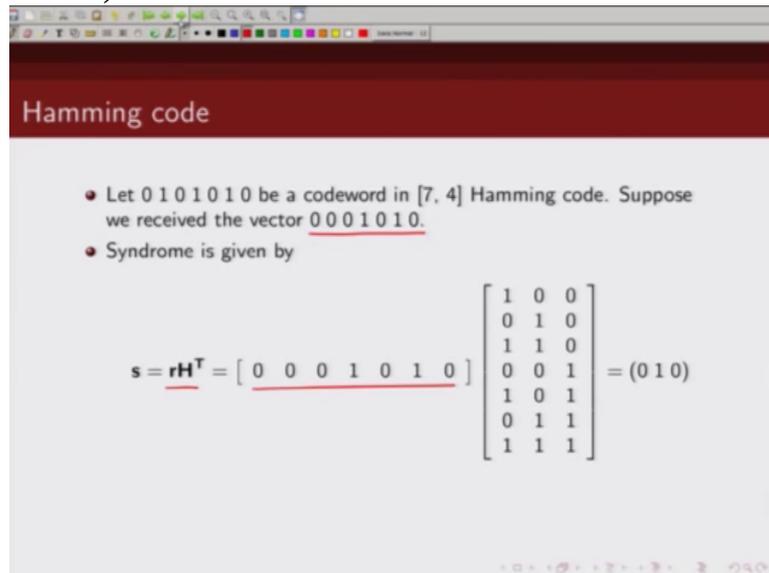
(Refer Slide Time 35:34)

Hamming code

- Let 0101010 be a codeword in [7, 4] Hamming code. Suppose we received the vector 0001010.

So this

(Refer Slide Time 35:35)



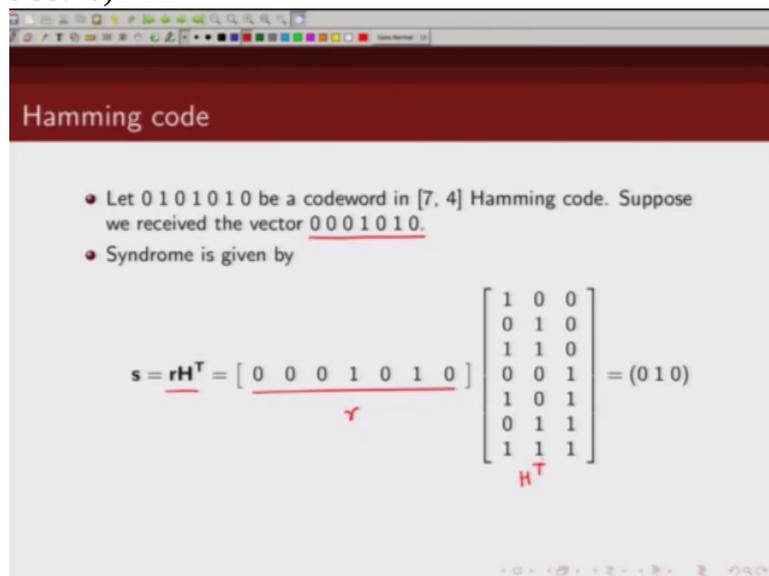
Hamming code

- Let 0101010 be a codeword in [7, 4] Hamming code. Suppose we received the vector 0001010.
- Syndrome is given by

$$s = rH^T = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = (010)$$

is my r, this is my H transpose, now H transpose,

(Refer Slide Time 35:40)



Hamming code

- Let 0101010 be a codeword in [7, 4] Hamming code. Suppose we received the vector 0001010.
- Syndrome is given by

$$s = rH^T = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = (010)$$

so this is 1, 2, 3, this row will participate this row, so my syndrome will be this plus this, this row plus this row. This will be 0 0 1 plus 0 1 1 which is 0 1 0.

(Refer Slide Time 36:02)

Hamming code

- Let 0101010 be a codeword in [7, 4] Hamming code. Suppose we received the vector 0001010.
- Syndrome is given by

$$s = rH^T = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = (010)$$

$001 + 011 = 010$
 $= 010$

H^T

So my syndrome here is 0 1 0. Now what do I do? I go back and check which column is 0 1 0. So if I go back to my parity check matrix I notice that

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Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H ,

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1 2 3 4 5 6 7

$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

- Here the column $(x, y, z)^T$ represents the number $x(2^0) + y(2^1) + z(2^2)$.

0 1 0 is this second column

(Refer Slide Time 36:24)

Hamming code

- We can rearrange the columns of the parity check matrix of Hamming code such that column in position i represents the integer i .
- For example for $m = 3$, the Hamming code is of length $n = 2^3 - 1 = 7$, $k = 2^3 - 3 - 1 = 4$, that has parity check matrix H .

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Here the column $(x, y, z)^T$ represents the number $x(2^0) + y(2^1) + z(2^2)$.

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

of my parity check matrix, right? So what does it indicate then? It indicates that second bit of my codeword is in error. So then I can go ahead and correct this error. So this second bit,

(Refer Slide Time 36:44)

Hamming code

- Let 0101010 be a codeword in $[7, 4]$ Hamming code. Suppose we received the vector 0001010 .
- Syndrome is given by

$$s = rH^T = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = (010)$$

- The number represented by syndrome is 2, hence the error is in second bit position. Hence estimated codeword is 0101010 .

from the syndrome I can find out that this syndrome is nothing but the column in the second location. So my second bit is in error. And since we are talking about binary code so then this 0 should have been 1. So my estimated codeword is then 0101010 . And you can see that's what I had transmitted. This is the same

(Refer Slide Time 37:15)

Hamming code

- Let 0101010 be a codeword in $[7, 4]$ Hamming code. Suppose we received the vector $0\hat{0}1010$.
- Syndrome is given by $\hat{v} = 0101010$

$$s = rH^T = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \underline{(010)}$$

- The number represented by syndrome is 2, hence the error is in second bit position. Hence estimated codeword is 0101010 .

codeword that I have transmitted. You can yourself verify, change some other bit location, introduce some other single error and you will see, from the syndrome you can find out the location of your error. So it has a very simple

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Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$
Information bits:	$k = 2^m - m - l - 1$
Parity bits:	$n - k = m$
Minimum distance:	$d_{\min} \geq 3$

decoding algorithm. Now as I said before, in case of Hamming code, once I fix my m ,

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my code parameters are fixed. So for example, the examples that we were dealing with, we had

(Refer Slide Time 37:52)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$
Information bits:	$k = 2^m - m - l - 1$
Parity bits:	$n - k = m$
Minimum distance:	$d_{\min} \geq 3$

taken m equal to 3. So once

(Refer Slide Time 37:55)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length: $n = 2^m - l - 1$

Information bits: $k = 2^m - m - l - 1$

Parity bits: $n - k = m$

Minimum distance: $d_{\min} \geq 3$

$m = 3$

the moment I fixed m equal to 3, my n is fixed which is 2 raised to power 3 minus 1 that is 7 so k is

(Refer Slide Time 38:03)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length: $n = 2^m - l - 1$

Information bits: $k = 2^m - m - l - 1$

Parity bits: $n - k = m$

Minimum distance: $d_{\min} \geq 3$

$m = 3$
 $n = 2^3 - 1 = 7$

7 minus 3 that's 4. So this becomes a 7 4 code, right? What

(Refer Slide Time 38:10)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$	$m = 3$ $n = 2^3 - 1 = 7$
Information bits:	$k = 2^m - m - l - 1$	$k = 4$
Parity bits:	$n - k = m$	
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$

if I am interested in designing, let's say 8 4 code or

(Refer Slide Time 38:16)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$	$m = 3$ $n = 2^3 - 1 = 7$
Information bits:	$k = 2^m - m - l - 1$	$k = 4$
Parity bits:	$n - k = m$	
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$ $(8, 4)$

let's say I am interested in 7 3 codes, 6 3 codes. So there are various techniques

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Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

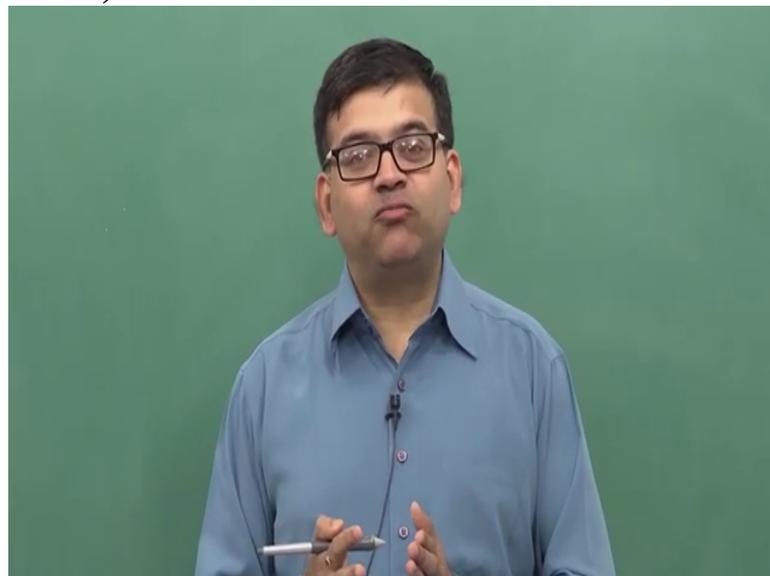
Code length:	$n = 2^m - l - 1$	$m = 3$
Information bits:	$k = 2^m - m - l - 1$	$n = 2^3 - 1 = 7$
Parity bits:	$n - k = m$	$k = 4$
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$

Handwritten examples in red:

- $(8, 4)$
- $(7, 3)$
- $(6, 3)$

available to tweak the parameters of the code, we are going to talk about some of them, we are not exhaustively covering all possible ways of lengthening or shortening a code, we just give you just few examples to illustrate this idea of changing the parameters of the code. So the first thing we are considering is a shortened Hamming code. So how do you get a shortened Hamming code? You delete l columns from the parity check matrix of your Hamming code. Now if I delete l columns, that means my codeword length is decreased by l .

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So my new codeword length is then

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Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

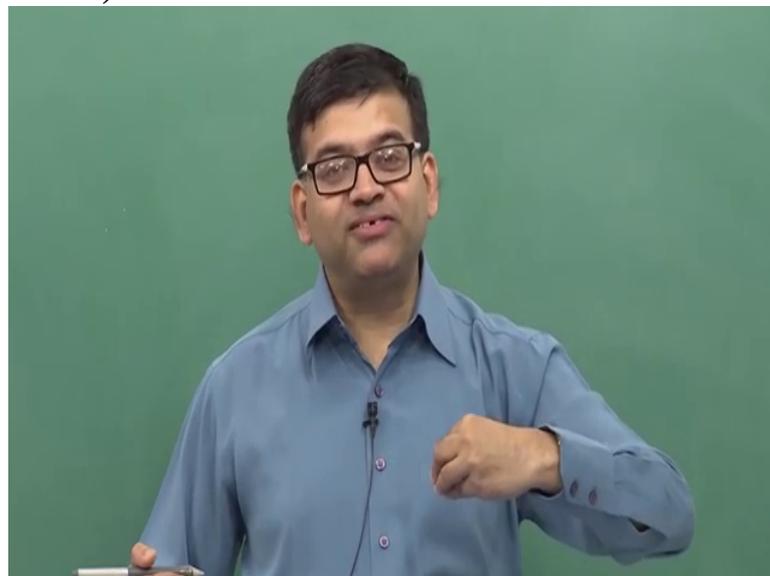
Code length:	$n = 2^m - l - 1$	$m = 3$
Information bits:	$k = 2^m - m - l - 1$	$n = 2^3 - 1 = 7$
Parity bits:	$n - k = m$	$k = 4$
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$

Handwritten notes on the right side of the slide:

- $(8, 4)$
- $(7, 3)$
- $(6, 3)$

$2^m - 1 - l$. So this is my codeword length. Now number of rows are still the same. That's m so number of parity bits are still same. But since I deleted l coded bits so number of information bits also got decreased by l . Now minimum distance of Hamming code is 3. If I am deleting some columns, then minimum distance cannot

(Refer Slide Time 39:46)



be less than what

(Refer Slide Time 39:48)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$	$m = 3$
Information bits:	$k = 2^m - m - l - 1$	$n = 2^3 - 1 = 7$
Parity bits:	$n - k = m$	$k = 4$
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$
		$(8, 4)$
		$(7, 3)$
		$(6, 3)$

was the original minimum distance of the code. Why, because you can see if I have a parity check matrix here and let's say I have these columns, let's call it a 1, a 2... a n. Now if I am removing some of the columns I can carefully

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Shortened Hamming code

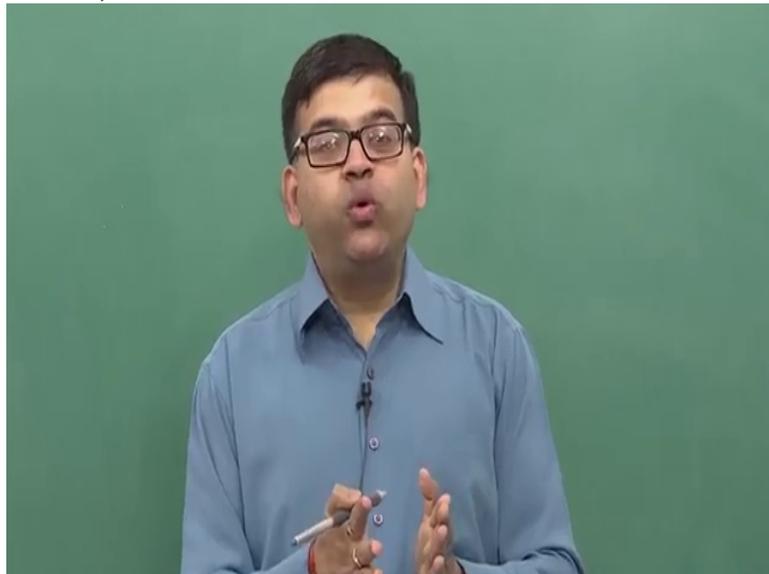
- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$	$m = 3$
Information bits:	$k = 2^m - m - l - 1$	$n = 2^3 - 1 = 7$
Parity bits:	$n - k = m$	$k = 4$
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$
		$(8, 4)$
		$(7, 3)$
		$(6, 3)$

$H = [a_1, a_2, \textcircled{a_3}, \textcircled{a_4}, \dots, a_n]$

remove those columns which

(Refer Slide Time 40:14)



are causing the sum of, let's say d columns to be linearly dependent. If I remove some of those columns I can possibly increase my minimum distance. And that's why I wrote

(Refer Slide Time 40:30)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$	$m = 3$
Information bits:	$k = 2^m - m - l - 1$	$n = 2^3 - 1 = 7$
Parity bits:	$n - k = m$	$k = 4$
Minimum distance:	$d_{\min} \geq 3$	$(7, 4)$

$H = [a_1, a_2, \otimes, \otimes, a_n]$

$(8, 4)$
 $(7, 3)$
 $(6, 3)$

by whenever you delete columns on the parity check matrix, the minimum distance is at least what was there in the original code. This is one example of a 7 4 Hamming code. So if I delete, let's say I delete

(Refer Slide Time 40:48)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length: $n = 2^m - l - 1$
Information bits: $k = 2^m - m - l - 1$
Parity bits: $n - k = m$
Minimum distance: $d_{\min} \geq 3$

- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Shortened (6,3) Hamming code has a parity check matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

this, if I delete

(Refer Slide Time 40:52)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length: $n = 2^m - l - 1$
Information bits: $k = 2^m - m - l - 1$
Parity bits: $n - k = m$
Minimum distance: $d_{\min} \geq 3$

- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Shortened (6,3) Hamming code has a parity check matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

this column, what do I get? I get a shortened 6 3 Hamming code. So when I delete this my n becomes 6 from 7 and my information bits also reduce from 4 to 3. This code also has minimum distance 3, you can see 3 columns will add up to zero.

(Refer Slide Time 41:16)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$
Information bits:	$k = 2^m - m - l - 1$
Parity bits:	$n - k = m$
Minimum distance:	$d_{\min} \geq 3$
- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$
- Shortened (6,3) Hamming code has a parity check matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad d = 3$$

Let's say this column, this column, and this column add up to zero so minimum distance is still 3.

(Refer Slide Time 41:24)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length:	$n = 2^m - l - 1$
Information bits:	$k = 2^m - m - l - 1$
Parity bits:	$n - k = m$
Minimum distance:	$d_{\min} \geq 3$
- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$
- Shortened (6,3) Hamming code has a parity check matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad d = 3$$

Or this one. So whenever you want shortened code, make sure you remove

(Refer Slide Time 41:31)



those columns in a judicious way so that potentially you can increase the minimum distance. In this particular example, if I delete one column I cannot increase the Hamming distance but if I delete more columns from this parity check matrix I can potentially increase the minimum distance of the code.

(Refer Slide Time 41:55)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters
 - Code length: $n = 2^m - l - 1$
 - Information bits: $k = 2^m - m - l - 1$
 - Parity bits: $n - k = m$
 - Minimum distance: $d_{min} \geq 3$
- H matrix of (7,4) Hamming code given by
$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$
- Shortened (6,3) Hamming code has a parity check matrix
$$H_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad d = 3$$

(Refer Slide Time 41:56)

Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \begin{pmatrix} H \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix}$$

Another class of code is what is called expurgated codes. So what is expurgated Hamming code? So H is the parity check matrix of Hamming code, then I define a new parity check matrix which has all 1's at the last row. Now note that the parity check matrix

(Refer Slide Time 42:22)

Shortened Hamming code

- If we delete any l columns from the parity check matrix of a Hamming code, we get shortened Hamming code with following parameters

Code length: $n = 2^m - l - 1$
 Information bits: $k = 2^m - m - l - 1$
 Parity bits: $n - k = m$
 Minimum distance: $d_{\min} \geq 3$

- H matrix of $(7,4)$ Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Shortened $(6,3)$ Hamming code has a parity check matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad d = 3$$

of Hamming code does not have all 1's. So when I add an all 1 row,

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Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix}$$

essentially the rank of the matrix will be one more than the rank of the original matrix H because

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Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix}$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.

any linear combination of last row with the earlier parity check matrix H would not be dependent. Hence

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Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix}$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

the rank of this new matrix \mathbf{H}_1 is $n - k + 1$. And what is the number of columns of this matrix? That is n . So rank of this matrix is $n - k + 1$. Then what is the dimension of the null space of this? Or what is the dimension

(Refer Slide Time 43:14)

Expurgated Hamming code

- The dimension of its null space C_1 is:

$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$

of this code sequence here? So that is basically, this is n minus dimension of \mathbf{H} matrix so that becomes $k - 1$. So the new code that we derive by adding

(Refer Slide Time 43:31)

Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ 1 \cdots 1 \end{pmatrix} \quad n$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

an all 1 row in the parity check matrix of the Hamming code, we get a new

(Refer Slide Time 43:37)

Expurgated Hamming code

- The dimension of its null space C_1 is:

$$\dim(C_1) = \underset{\uparrow}{(n)} - \underset{\uparrow}{(n - k + 1)} = \underline{k - 1}$$

code which is a $n - k + 1$

(Refer Slide Time 43:40)

Expurgated Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.

code and this is known as expurgated Hamming code. Now one of

(Refer Slide Time 43:48)

Expurgated Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.
- Now, since the last row of \mathbf{H}_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{v}\mathbf{H}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.

the interesting property of this code is this code contains all even weight codewords. It is not very difficult to prove. If

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Expurgated Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.

v is a valid codeword

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Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)
$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix} \quad n$$
- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

then we know $v \mathbf{H}^T$ should be

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Expurgated Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix} \quad n \quad v\mathbf{H}_1^T = 0$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

zero. And last row of \mathbf{H}_1 has all 1's. So when we take transpose,

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Expurgated Hamming code

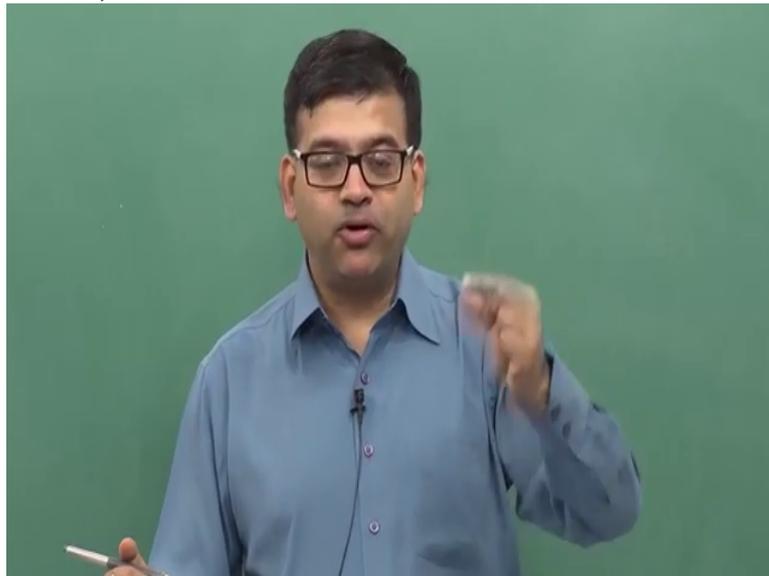
- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ 1 \dots 1 \end{pmatrix} \quad n \quad v\mathbf{H}_1^T = 0$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

the first column will be all 1's. And when we do $v\mathbf{H}_1^T$ what we will get is, let's say the components of v are

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v_0, v_1, v_2 so v_{n-1} so what you will get is basically sum of these

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Expurgated Hamming code

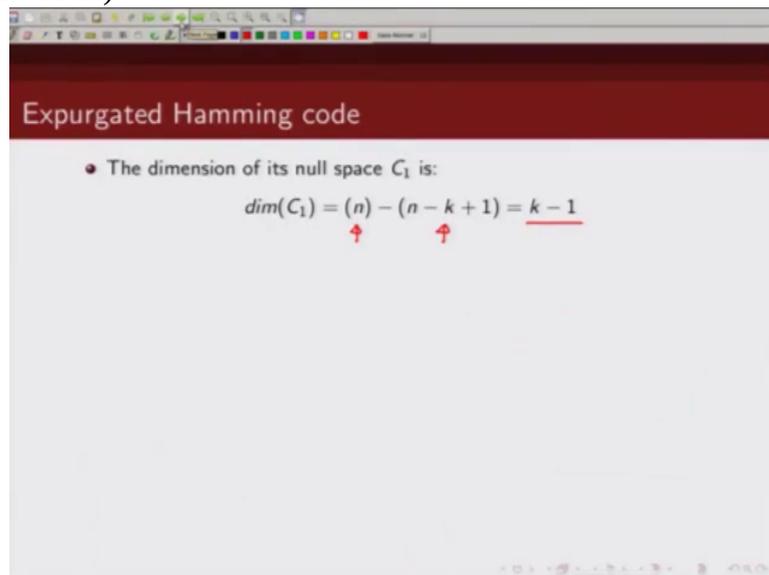
- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H} \\ \dots\dots\dots \\ \boxed{1 \dots 1} \end{pmatrix} \quad n \quad v\mathbf{H}_1^T = 0$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

component of codewords,

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Expurgated Hamming code

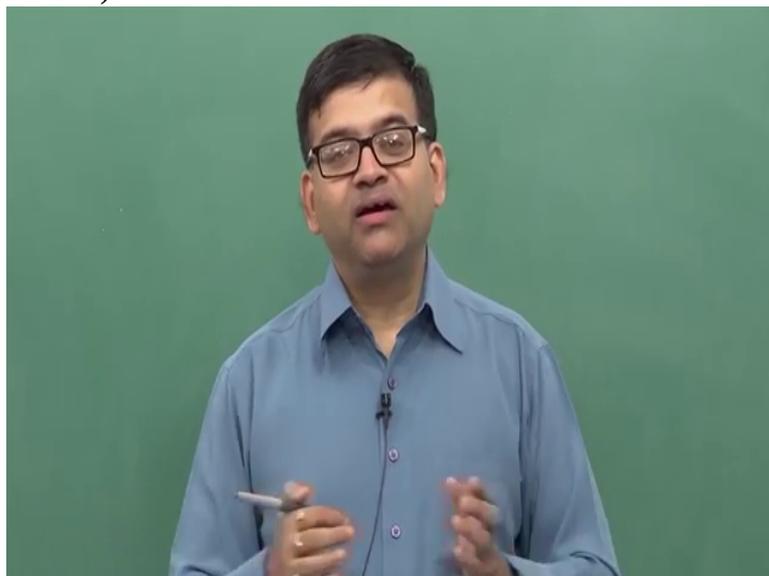
- The dimension of its null space C_1 is:

$$\dim(C_1) = (n) - (n - k + 1) = \underline{k - 1}$$

The formula shows the dimension of the null space C_1 is calculated as $(n) - (n - k + 1) = k - 1$. Red arrows point to the (n) and $(n - k + 1)$ terms, and a red underline is under $k - 1$.

they should add up to zero and this will happen only when

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v has even weight.

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Expurgated Hamming code

- The dimension of its null space C_1 is:

$$\dim(C_1) = (n) - (n - k + 1) = \underline{k - 1}$$

The slide shows the equation $\dim(C_1) = (n) - (n - k + 1) = k - 1$. Red arrows point to the (n) and $(n - k + 1)$ terms, and a red underline is under $k - 1$.

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Expurgated Hamming code

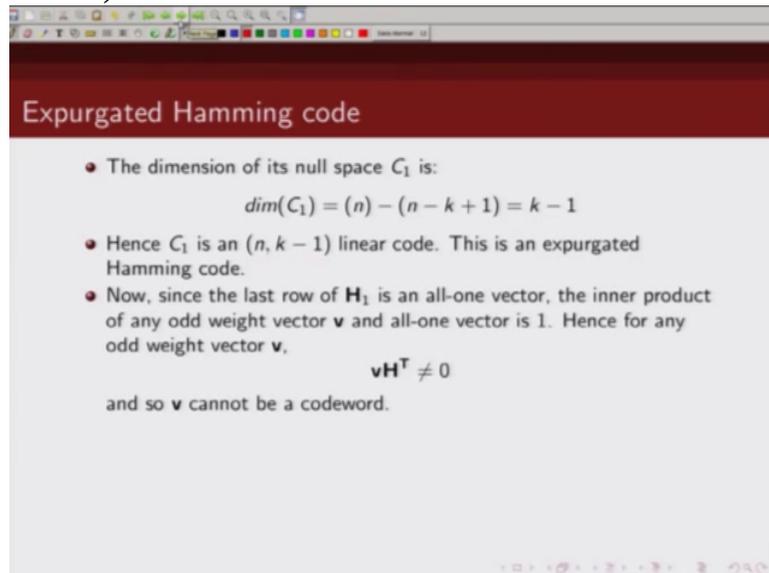
- The dimension of its null space C_1 is:

$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$

- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.

So this new code that we generated, expurgated code is basically has all even weight codewords.

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The slide is titled "Expurgated Hamming code" in a dark red header. It contains the following text:

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.
- Now, since the last row of H_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.

So if you have an odd weight vector then \mathbf{vH}^T cannot be zero because of all these, all 1's

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in the parity check matrix

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Expurgated Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.
- Now, since the last row of H_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}_1^T \neq 0$$

and so \mathbf{v} cannot be a codeword.

of this new matrix H_1 , expurgated code parity check matrix, Ok. So \mathbf{vH}_1^T cannot be zero if \mathbf{v} has odd weight. So you can see that

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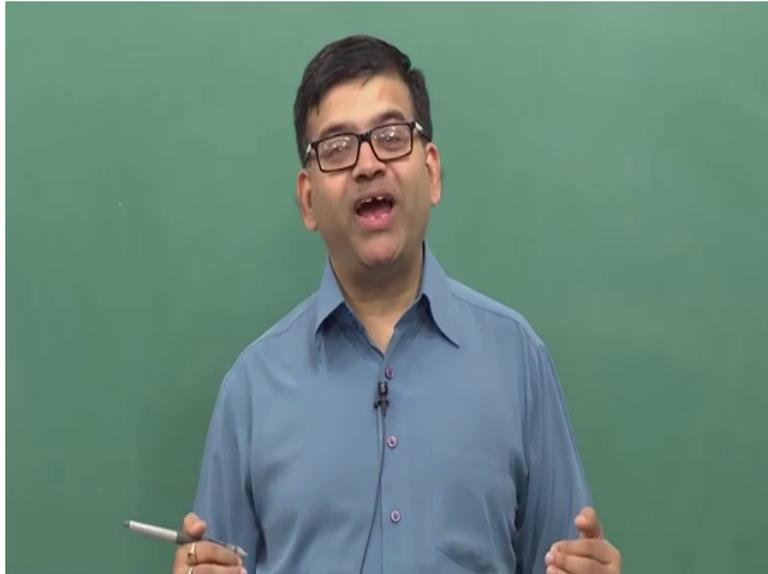
Expurgated Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.
- Now, since the last row of H_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}_1^T \neq 0$$

and so \mathbf{v} cannot be a codeword.
- Thus, this expurgated Hamming code only has even weight codewords (all odd weight codewords are expurgated).

this is one way of

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getting rid of odd codewords. So by adding an additional all 1 rows in the parity check matrix of the original code, we actually got rid of all odd weight codewords. So the minimum distance of this expurgated Hamming code would be then 4. Why? Original code has minimum distance 3, but now we got rid of all odd weight codewords. So the minimum distance of this new code, expurgated Hamming code would be 4.

(Refer Slide Time 46:11)

Expurgated Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n) - (n - k + 1) = k - 1$$
- Hence C_1 is an $(n, k - 1)$ linear code. This is an expurgated Hamming code.
- Now, since the last row of H_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.
- Thus, this expurgated Hamming code only has even weight codewords (all odd weight codewords are expurgated).
- The submatrix formed by the original Hamming code insures that all nonzero codewords must have a weight of atleast three.
- The expurgated parity check matrix defines a code with minimum distance four.

So minimum distance is 4.

So this is

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Expurgated Hamming code: Example

- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

an example, this is the parity check matrix of the original Hamming code and if we add

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Expurgated Hamming code: Example

- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Distance-4 expurgated Hamming code has a parity check matrix H_1 given by

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

all 1 rows in the parity check matrix, we get

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Expurgated Hamming code: Example

- H matrix of (7,4) Hamming code given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Distance-4 expurgated Hamming code has a parity check matrix H_1 given by

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ \boxed{1 & 1 & 1 & 1 & 1 & 1 & 1} \end{bmatrix}$$

parity check matrix of expurgated code. Finally

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \dots & \dots \\ \mathbf{1 \dots 1} & \mathbf{1} \end{array} \right)$$

I will conclude this lecture with another class which is basically extension; it is called extension of the code. So this is an example of extended Hamming code. So how do I generate an extended Hamming code? So note, so this is the parity check matrix of the original Hamming code. I add an zero, I add an additional column which is zero here, so these are all zero here and then I add 1 row which is

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \mathbf{1 \cdots 1} & \mathbf{1} \end{array} \right)$$

all 1. Now it is not very difficult to see that the rank of the matrix will be if , the rank of the H is n minus k , the rank of this matrix

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \mathbf{1 \cdots 1} & \mathbf{1} \end{array} \right) \quad H_{n-k}$$

H_1 will be n minus k plus 1

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Extended Hamming code

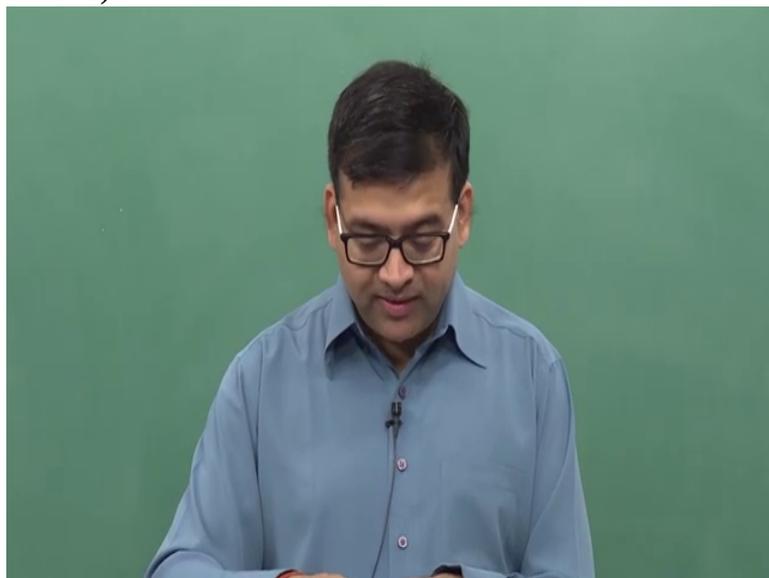
- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \mathbf{1 \cdots 1} & \mathbf{1} \end{array} \right)$$

H_{n-k}
 $H_1 = n-k+1$

because the parity check matrix

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of the Hamming code did not have an all 1 row and then

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \mathbf{1} \cdots \mathbf{1} & \mathbf{1} \end{array} \right)$$

H_{n-k}
 $H_1 = n-k+1$

these are all zeroes and this is 1. So any linear combination of this row with these rows basically would not, would not change the, would not decrease

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \cdots & \cdots \\ \mathbf{1} \cdots \mathbf{1} & \mathbf{1} \end{array} \right)$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of H_1 will never yield a zero vector.

the Hamming distance actually. So this will have, rank of this matrix would be one more than the rank of the original parity check matrix H . So what

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \left(\begin{array}{c|c} \mathbf{H} & \mathbf{0} \\ \hline \dots & \dots \\ 1 \dots 1 & 1 \end{array} \right)$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

we are saying then is, all rows of this parity check matrix H_1 are linearly independent. As I said that is because original H matrix did not have all 1s here and this is 1 here, this is 0 here so if we take linear combination

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \left(\begin{array}{c|c} \mathbf{H} & \mathbf{0} \\ \hline \dots & \dots \\ \boxed{1 \dots 1} & \boxed{1} \end{array} \right)$$

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

of this with any of the rows of the original parity check matrix they would not be linearly dependent. So all rows are linearly independent, hence the rank of this matrix is n minus k plus 1. And what is the number of columns? The original matrix had n columns. And we added one column, one more column.

(Refer Slide Time 48:36)

Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

$$\mathbf{H}_1 = \left(\begin{array}{c|c} \mathbf{H} & \mathbf{0} \\ \hline \dots & \dots \\ \boxed{1 \dots 1} & \boxed{1} \end{array} \right)$$

$\xrightarrow{n \text{ columns}}$ \downarrow

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

So number of columns here is $n + 1$. So this will define the code of length

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix \mathbf{H} . Let us define a new code C_1 with parity check matrix \mathbf{H}_1 . (all one vector as the last row.)

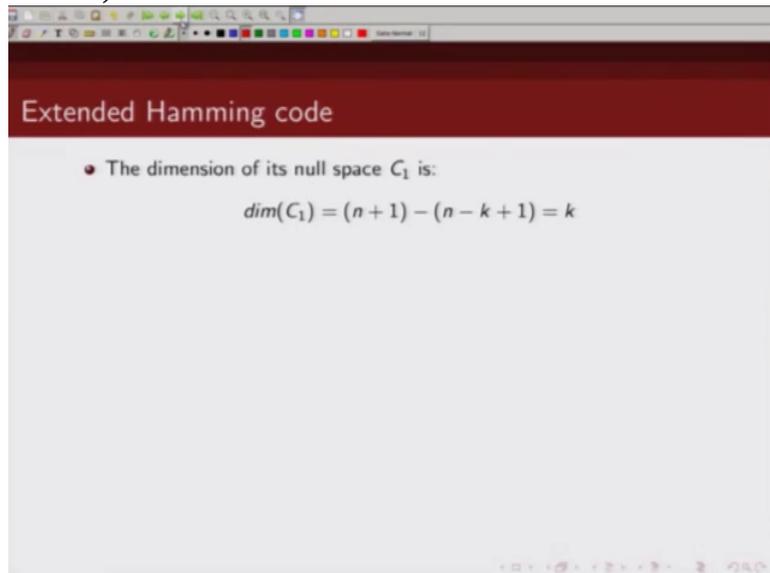
$$\mathbf{H}_1 = \left(\begin{array}{c|c} \mathbf{H} & \mathbf{0} \\ \hline \dots & \dots \\ \boxed{1 \dots 1} & \boxed{1} \end{array} \right) \quad n+1$$

$\xrightarrow{n \text{ columns}}$ \downarrow

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of \mathbf{H}_1 will never yield a zero vector.
- Thus all the rows of \mathbf{H}_1 are linearly independent. Hence the row space of \mathbf{H}_1 has dimension $(n - k + 1)$.

$n + 1$. So then we can find out

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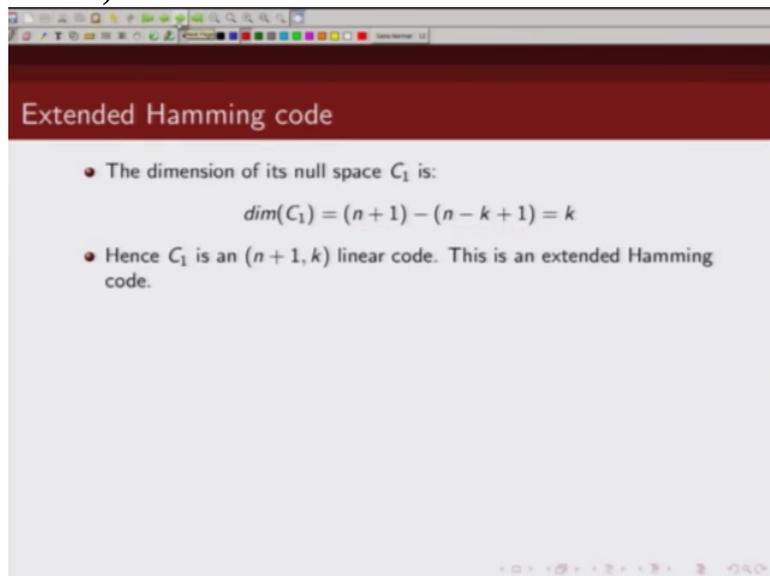


Extended Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$

what is the dimension of null space or so, this is number of coded bits, this is the rank of the new parity check matrix so number of coded, so the dimension of null space is k. So So this will then generate an

(Refer Slide Time 49:04)



Extended Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$
- Hence C_1 is an $(n + 1, k)$ linear code. This is an extended Hamming code.

n plus 1 k linear block code. And this is known as extension code or extended Hamming code. If this H is matrix,

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Extended Hamming code

- Let C be a (n, k) Hamming code with parity check matrix H . Let us define a new code C_1 with parity check matrix H_1 . (all one vector as the last row.)

$$H_1 = \left(\begin{array}{c|c} H & \mathbf{0} \\ \hline \dots & \dots \\ \mathbf{1 \dots 1} & \mathbf{1} \end{array} \right)$$

Handwritten annotations: "n columns" above H, "↓ 1" above the 0 vector, "n+1" to the right of the matrix.

- Since the parity check matrix of Hamming code doesn't have an all one vector in any of the rows, any linear combination including the last row of H_1 will never yield a zero vector.
- Thus all the rows of H_1 are linearly independent. Hence the row space of H_1 has dimension $(n - k + 1)$.

parity check matrix with Hamming code, the code described by H_1 which is given by this would be extended Hamming code. And again

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Extended Hamming code

- The dimension of its null space C_1 is:

$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$

- Hence C_1 is an $(n + 1, k)$ linear code. This is an extended Hamming code.

we can see that the extended Hamming code will have only even codewords. That is because

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the last row of the parity check contains all 1's. So \mathbf{vH}^T cannot be zero if \mathbf{v}

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Extended Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$
- Hence C_1 is an $(n + 1, k)$ linear code. This is an extended Hamming code.
- Now, since the last row of \mathbf{H}_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.

has odd weight. So the

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Extended Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$
- Hence C_1 is an $(n + 1, k)$ linear code. This is an extended Hamming code.
- Now, since the last row of \mathbf{H}_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{v}\mathbf{H}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.
- Thus, this extended Hamming code only has even weight codewords.

minimum distance of extended Hamming code is 4. Now please

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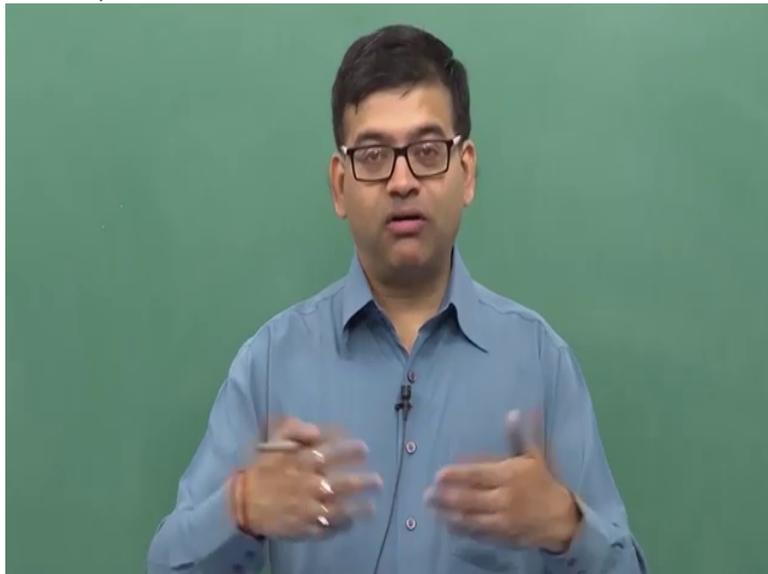
Extended Hamming code

- The dimension of its null space C_1 is:
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- Now, since the last row of \mathbf{H}_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{v}\mathbf{H}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.
- Thus, this extended Hamming code only has even weight codewords.
- The submatrix formed by the original Hamming code insures that all nonzero codewords must have a weight of atleast three.

note that some of the techniques what we mentioned here, extension, shortening, expurgated, that's valid for

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any other linear block codes too

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Extended Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$
- Hence C_1 is an $(n + 1, k)$ linear code. This is an extended Hamming code.
- Now, since the last row of H_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.
- Thus, this extended Hamming code only has even weight codewords.

and these are some of the ways in which we can change

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Extended Hamming code

- The dimension of its null space C_1 is:
$$\dim(C_1) = (n + 1) - (n - k + 1) = k$$
- Hence C_1 is an $(n + 1, k)$ linear code. This is an extended Hamming code.
- Now, since the last row of H_1 is an all-one vector, the inner product of any odd weight vector \mathbf{v} and all-one vector is 1. Hence for any odd weight vector \mathbf{v} ,
$$\mathbf{vH}^T \neq 0$$

and so \mathbf{v} cannot be a codeword.

the code parameters. So with this we conclude this

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lecture, thank you.