An Introduction to Coding Theory

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Module 02
Lecture Number 10
Problem Solving Session-II

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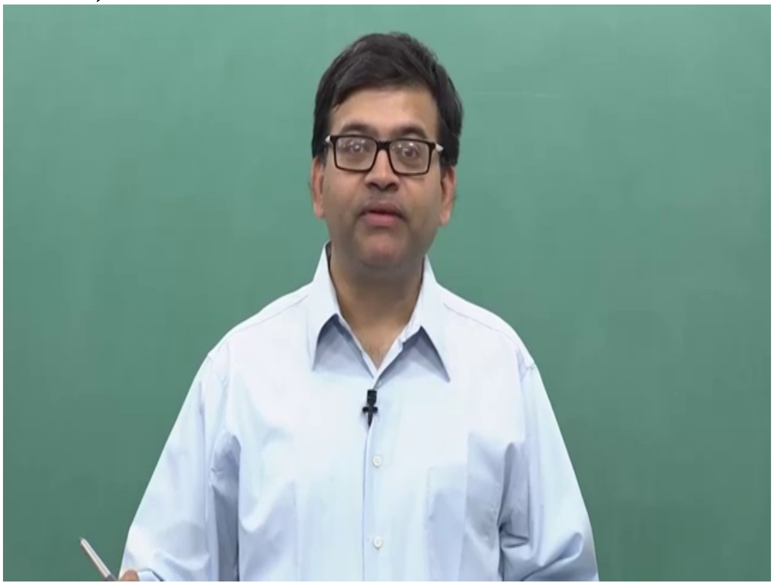

So, so far we have studied what are linear block codes, how do we describe linear block codes using generator matrix and parity check matrix, we talked about how we can use error correcting codes for error detection and error correction and we discussed the distance properties of linear block codes. Today we will spend some time solving some problems on linear block codes so today's session

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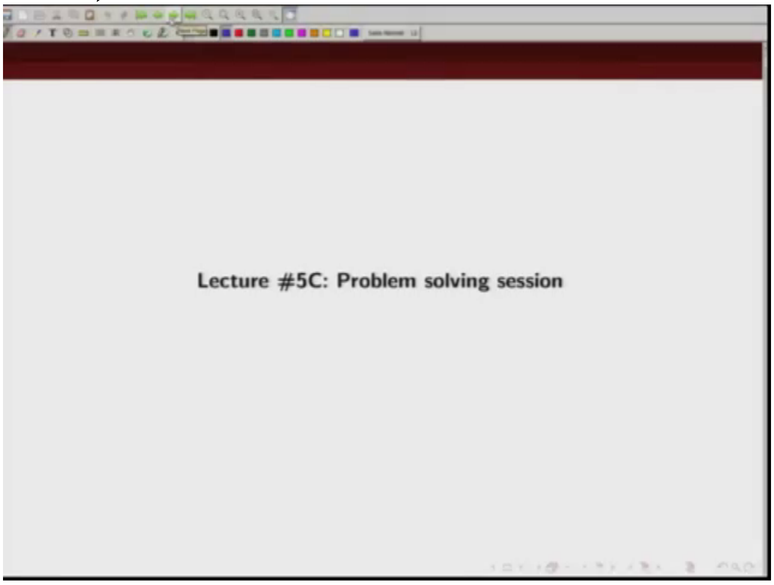

will be on problem solving.

So the first problem

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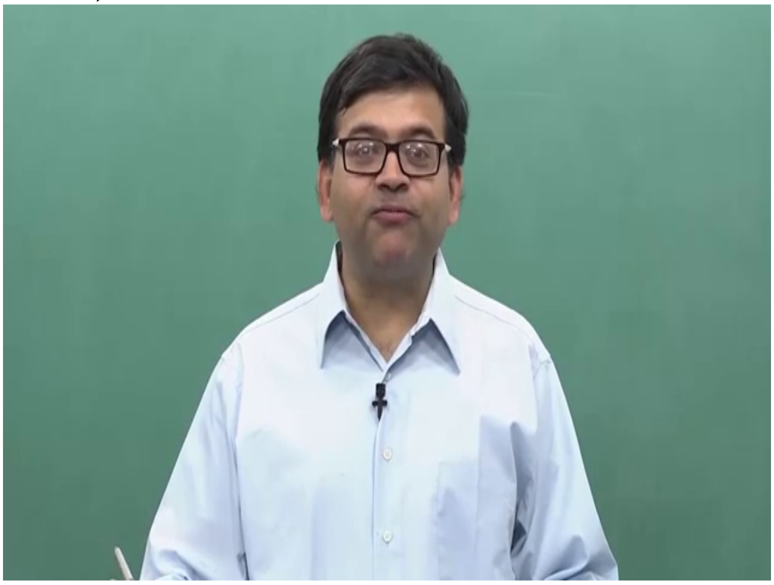

that we will look at is let C be a linear

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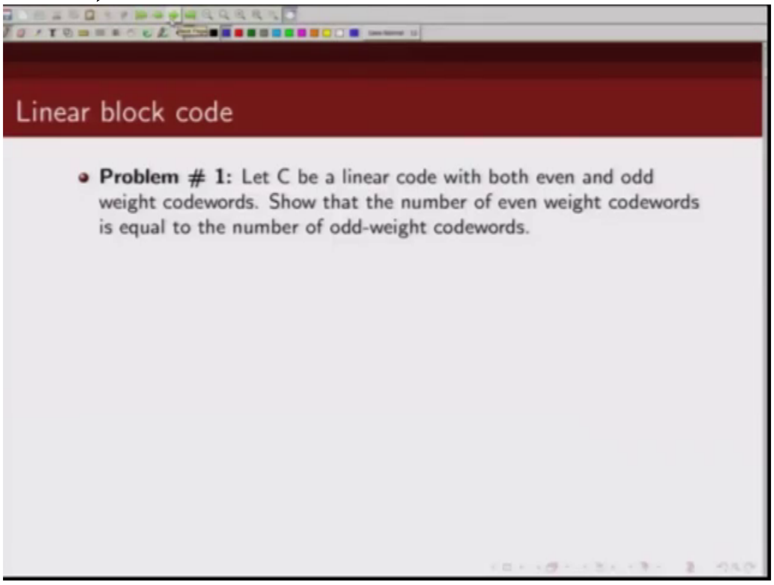

code with both even and odd weight codewords. Prove that the number of even bit codewords is equal to number of odd bit codewords.

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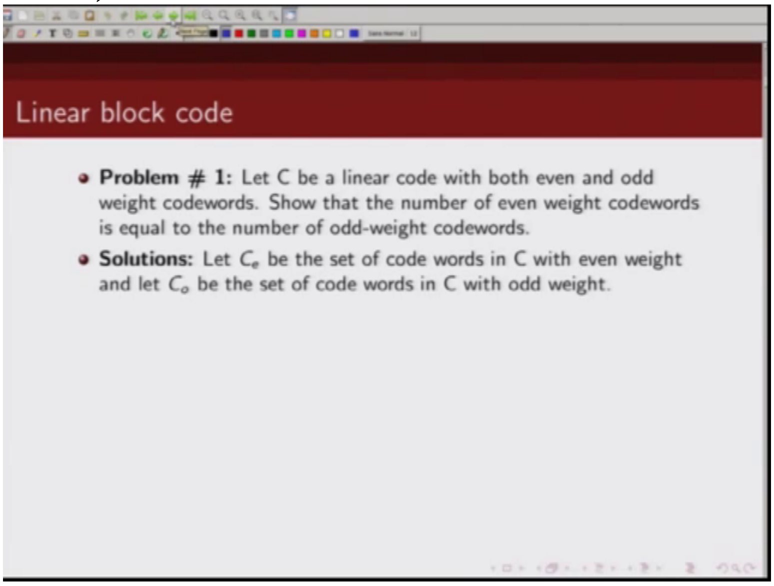

So let us denote the set of even codewords in C by C sub e and set of odd codewords in C by C sub o.

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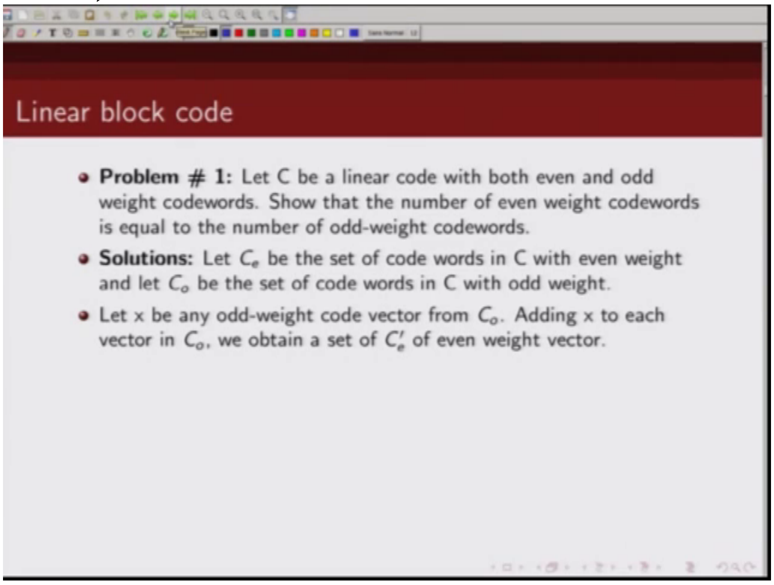

Now let us consider an odd weight codeword x which is taken from this set C o and let us add x to each of the codewords which are there in the set C o. So if we add a odd weight codeword to another odd weight codeword what we will get is a even weight codeword. For example let's say I add 1 1 1 0 0 0 and I add

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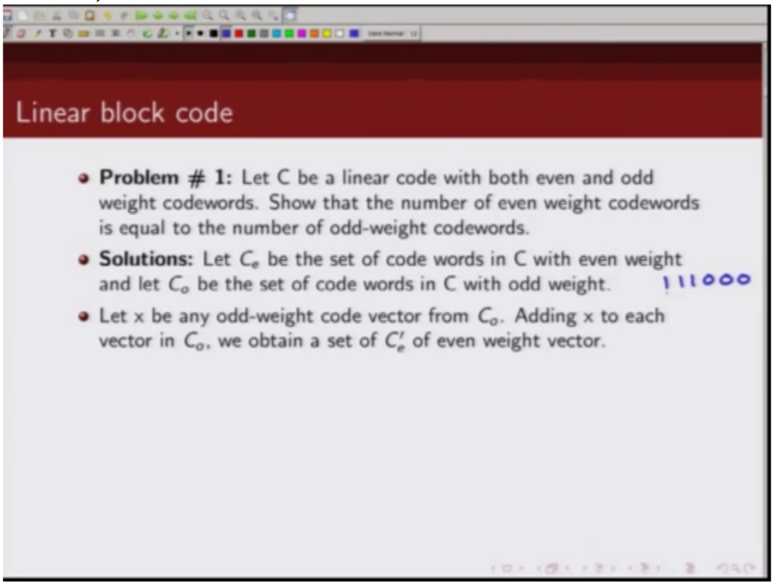

1 0 1 0 1 0 so

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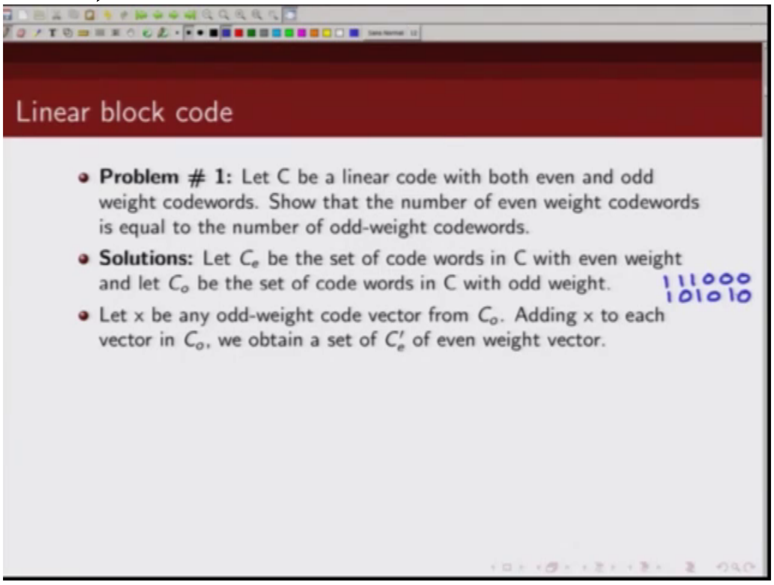

this first codeword this is odd weight codeword its weight is 3. Similarly this codeword also has weight 3. If I add both of them what do I get? I get 0 1 0 0 1 0 and this a

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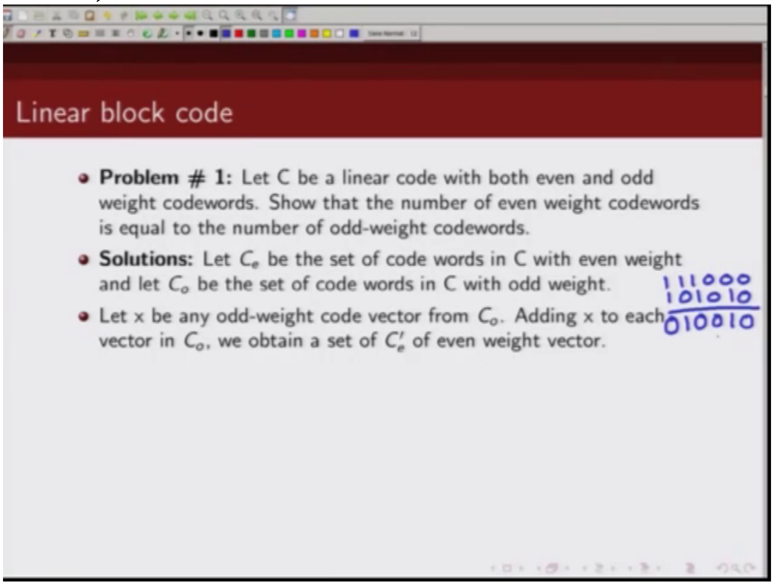

even weight codeword. So when I add x which is an odd weight code vector and I add x to each of the elements in this set C 0 what I get is a set of even codeword vectors and let us denote that set by

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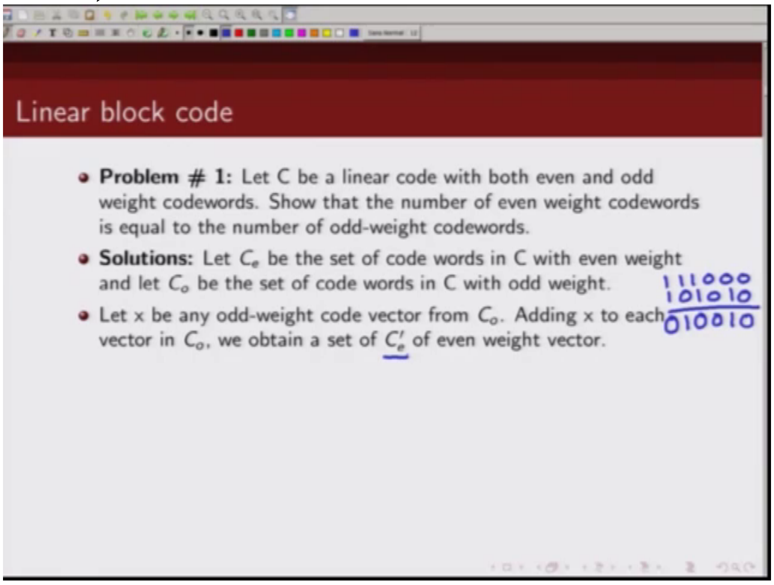

C e prime. Now

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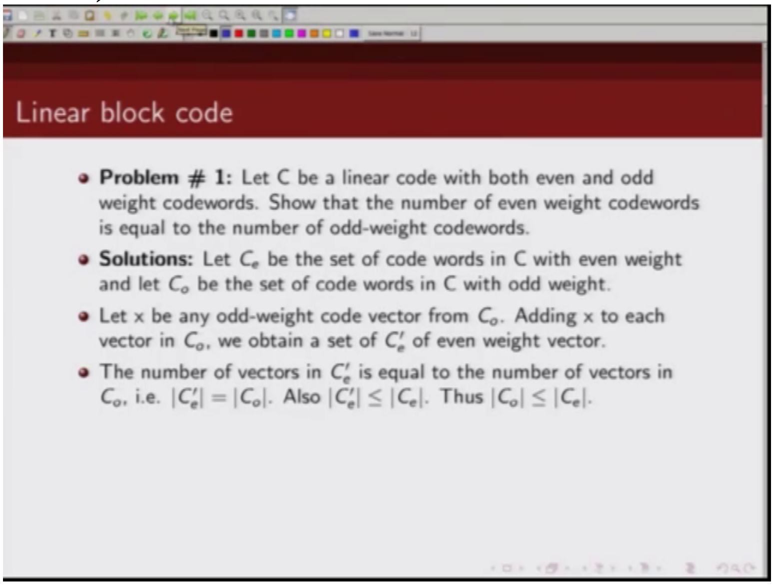

the number of code vectors in C e prime is going to be equal to number of vectors in C o. Why, because

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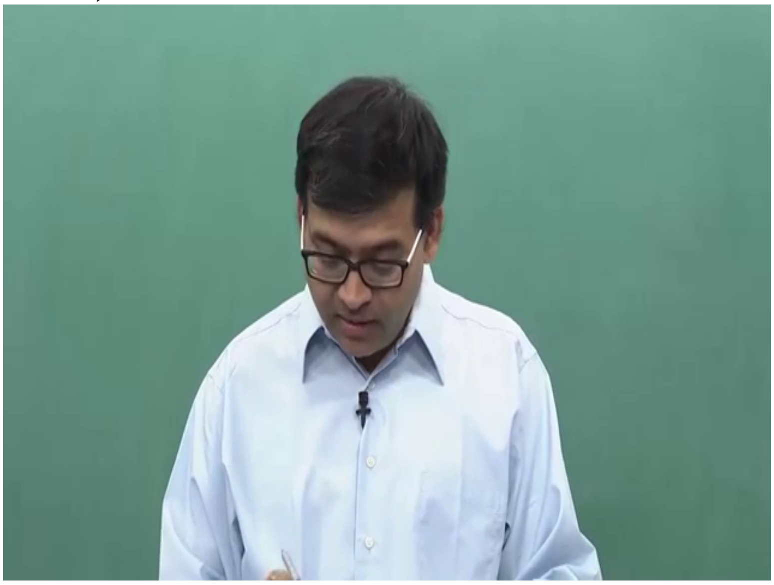

how did we get this c e prime? We added

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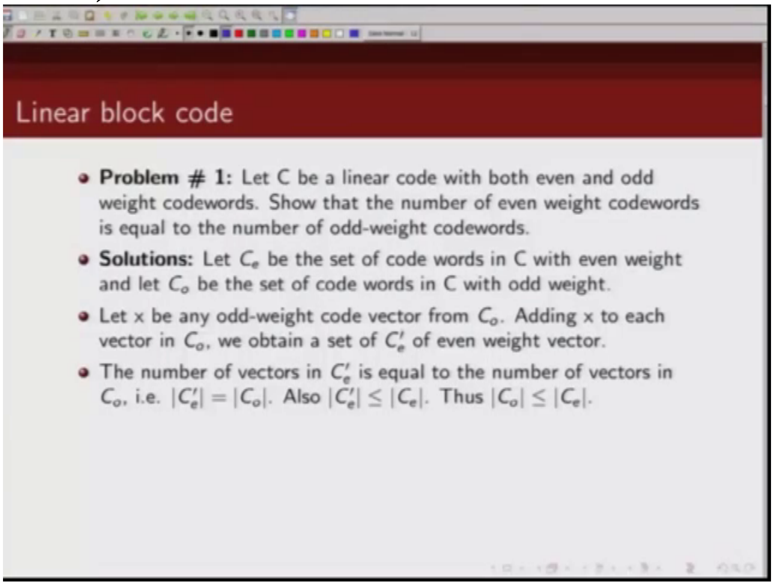

an odd vector x to the set C o. So number of vectors in this set is going to be equal to

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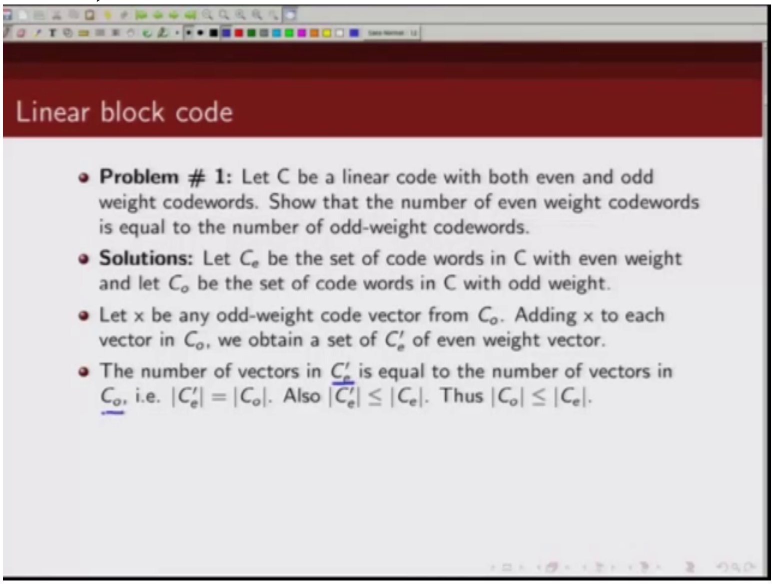

number of vectors in C o. Hence number of elements in

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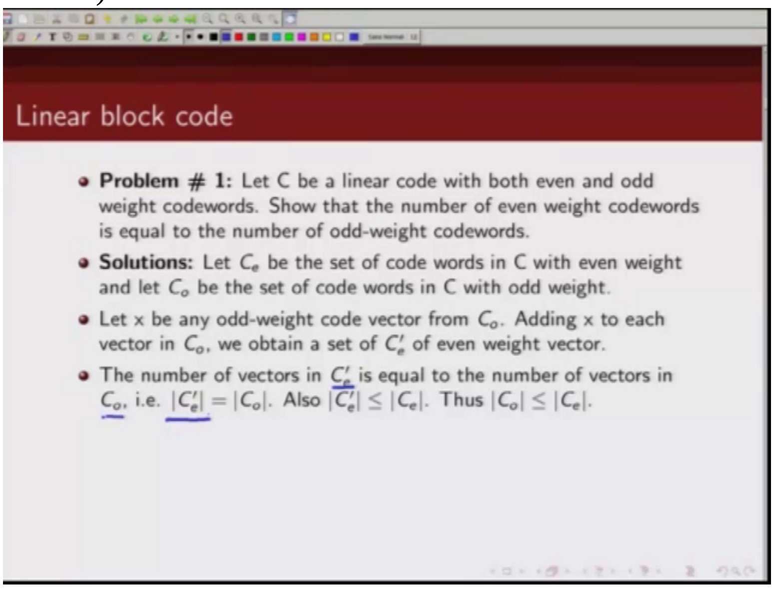

C e prime is going to be same as number of elements in

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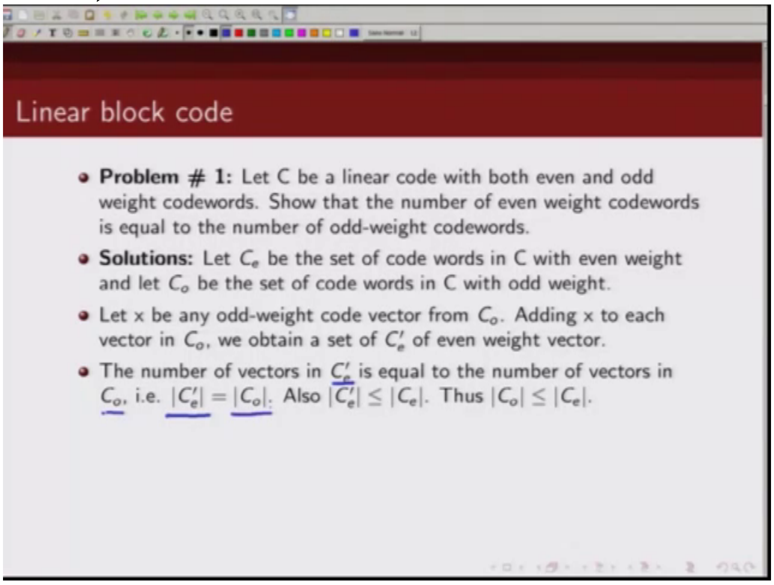

in C o and since we know that this set

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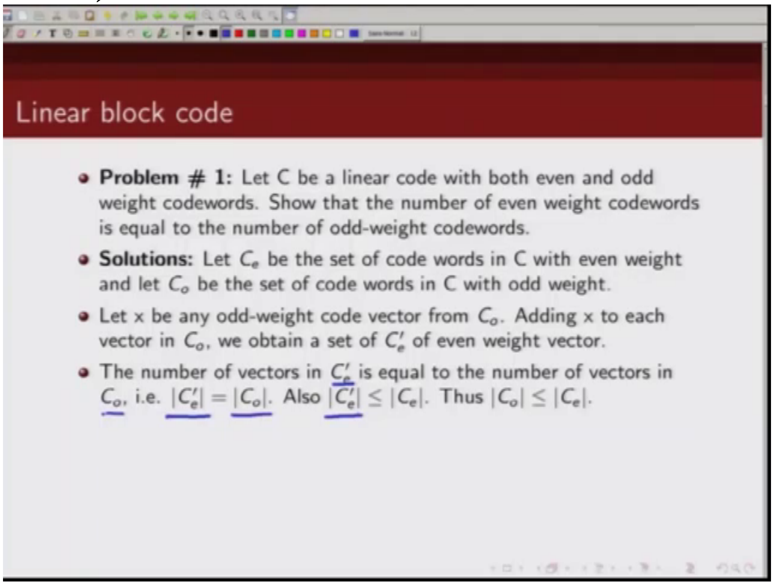

of even vectors C e hat is the subset of

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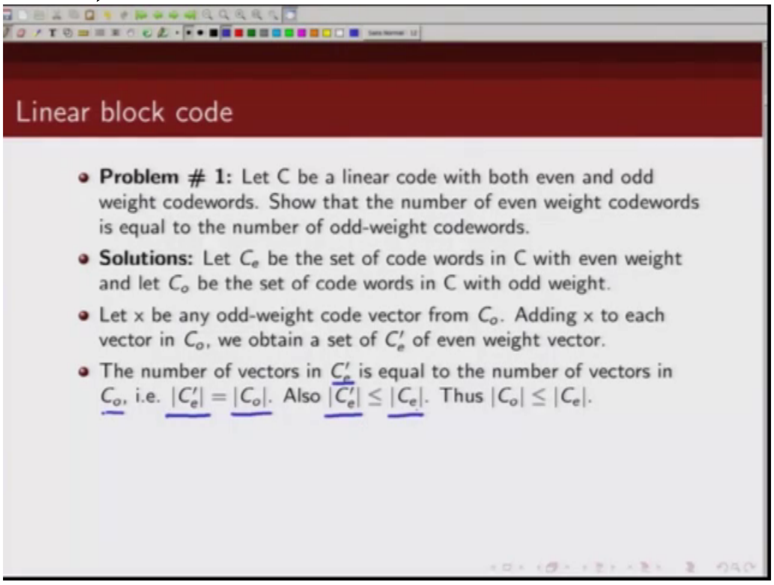

set of even vectors we can write from this that number of elements in the

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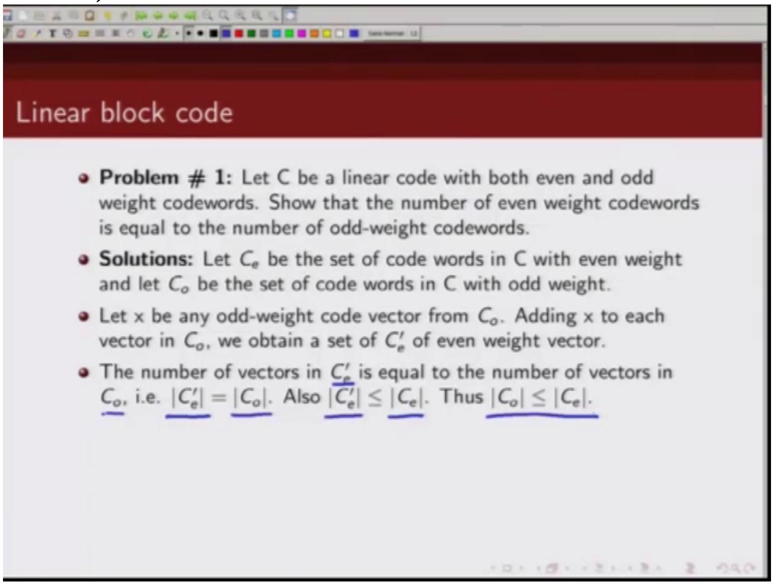

set of number of odd codewords is going to be a subset of number of even codewords. Next,

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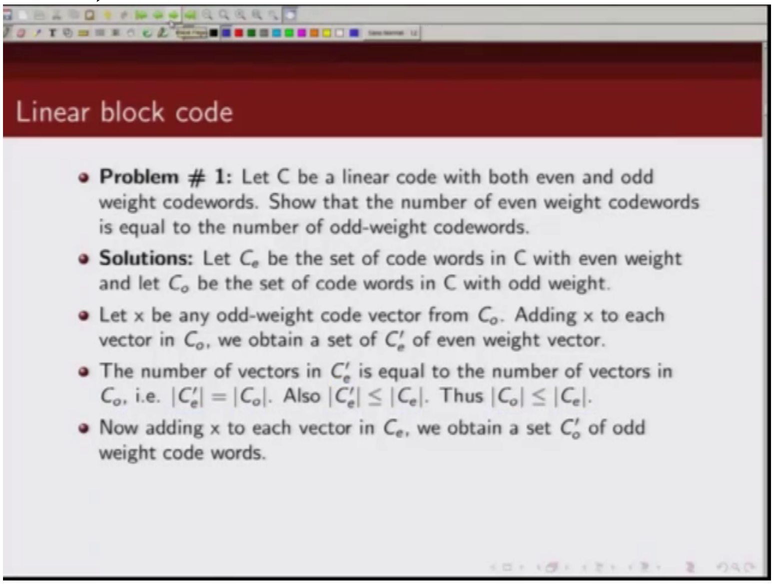

let us add the same odd weight codeword now to

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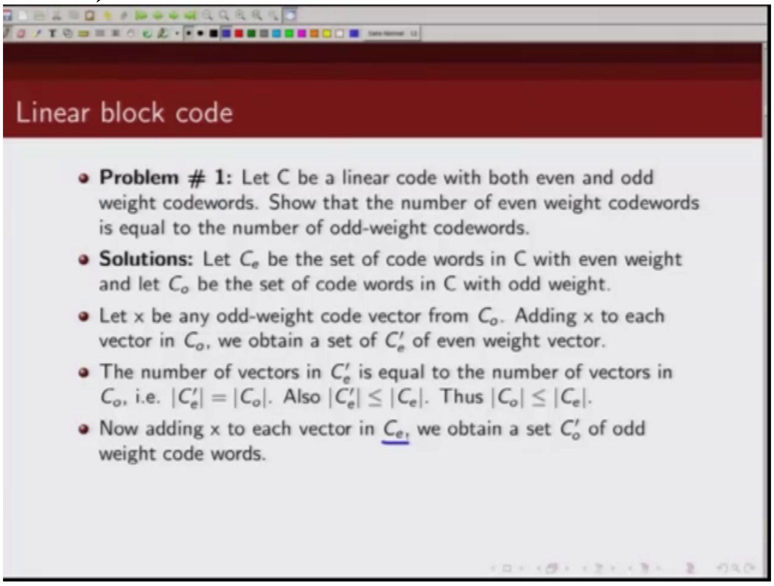

all the vectors in the set C e. So if we add an odd weight codeword to set of even weight codewords what we will get is a set of odd codewords. So the

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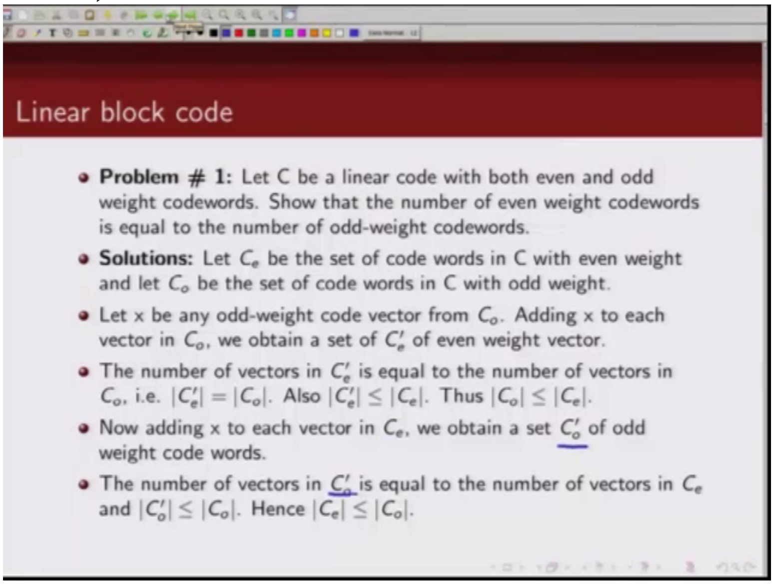

number of vectors in C o prime is going to be equal to number of vectors in, number of even vectors.

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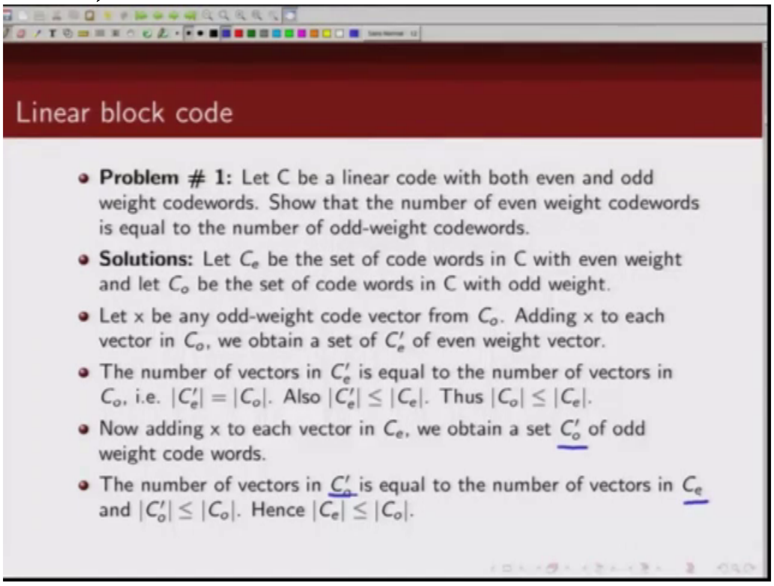

Why, because this set was generated by adding an odd vector x to the set of even codewords. So we can then write that C o prime is equal to this ok,

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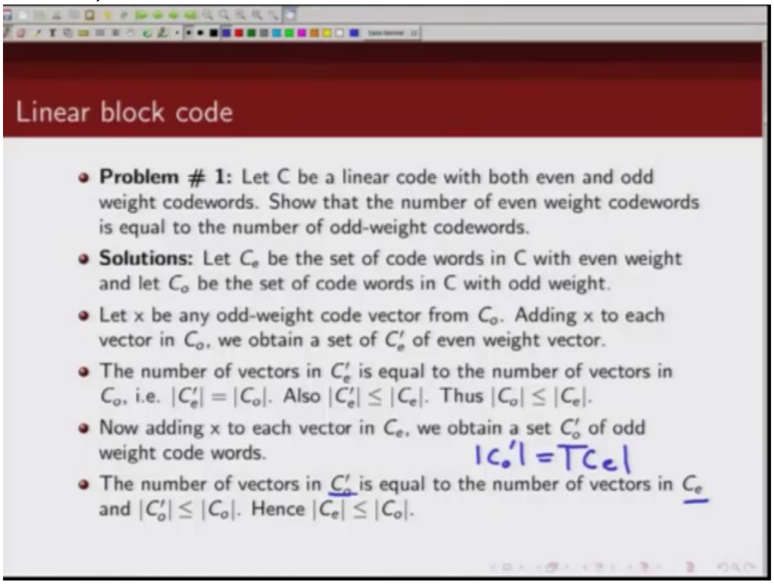

the set of codewords here is same as set of codewords here.

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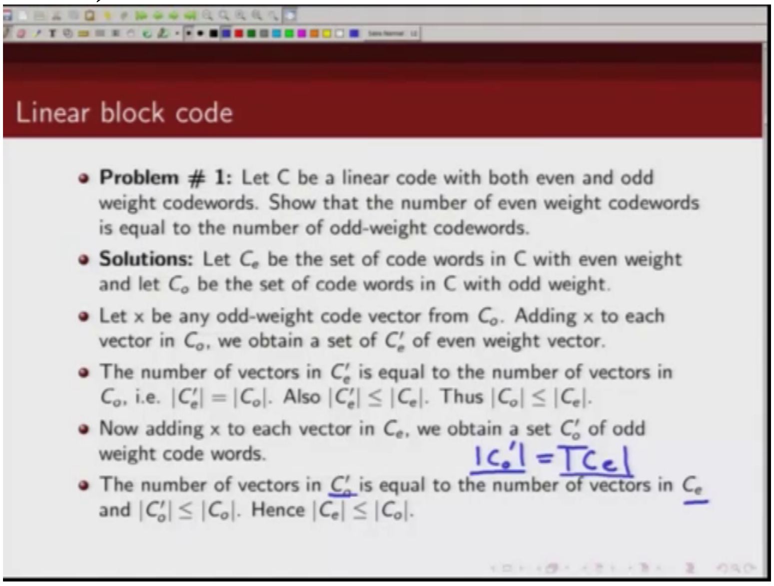

Now we know that C o dash is the subset of set of odd codewords. So then from this relation and this relation we can write that set of even codewords is the subset of set of number of elements in this is subset of number of is basically less than number of elements in this set. Now from this relation and this relation

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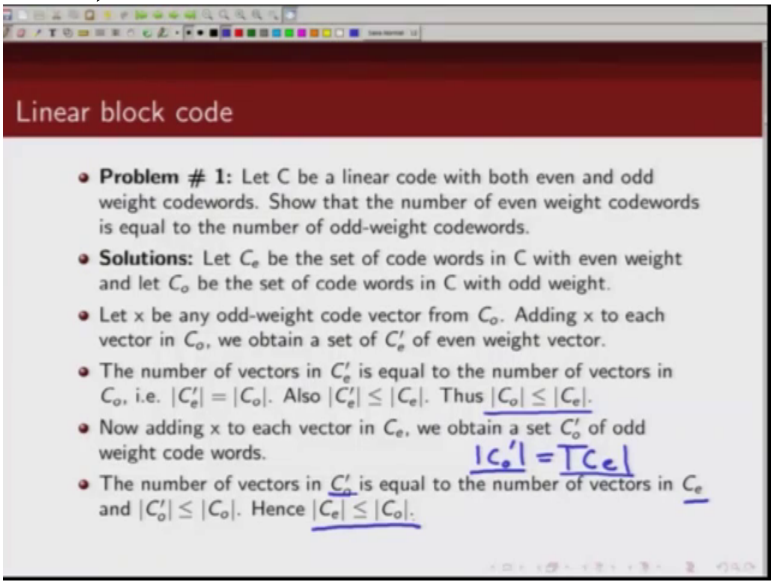

both of them can be true only if number of elements in C o is same as number of elements in C e. So this relation we call it 1 and let's call it 2. These

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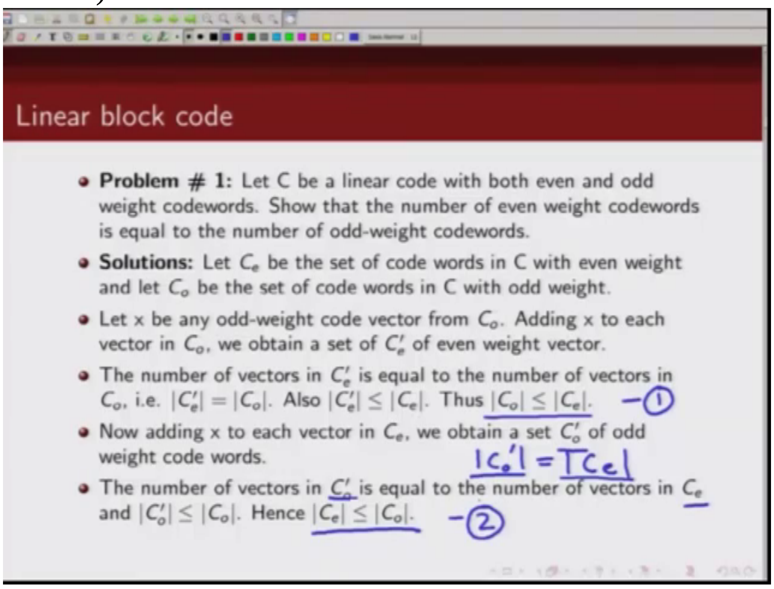

two relations are satisfied only if

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we have set of even codewords to be same as set of odd codewords.

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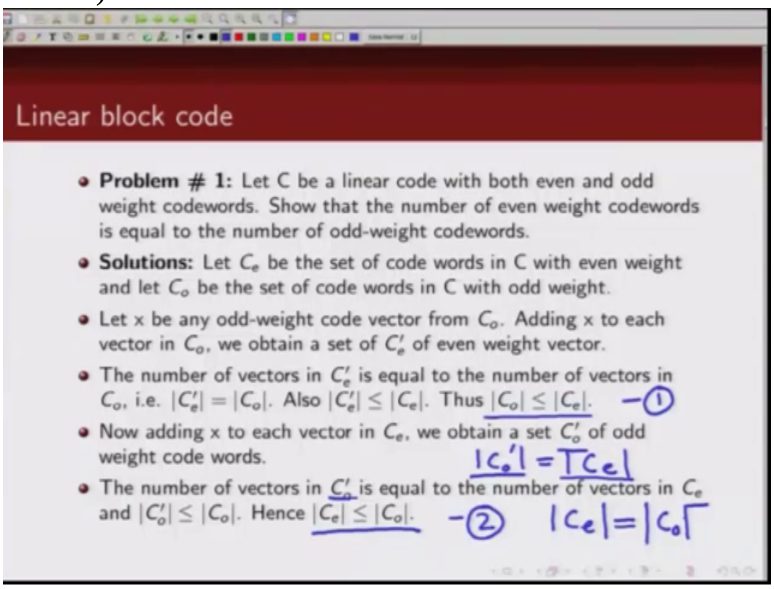

Hence we prove that in a linear code with both even and odd codewords the number of even weight codewords is same as number

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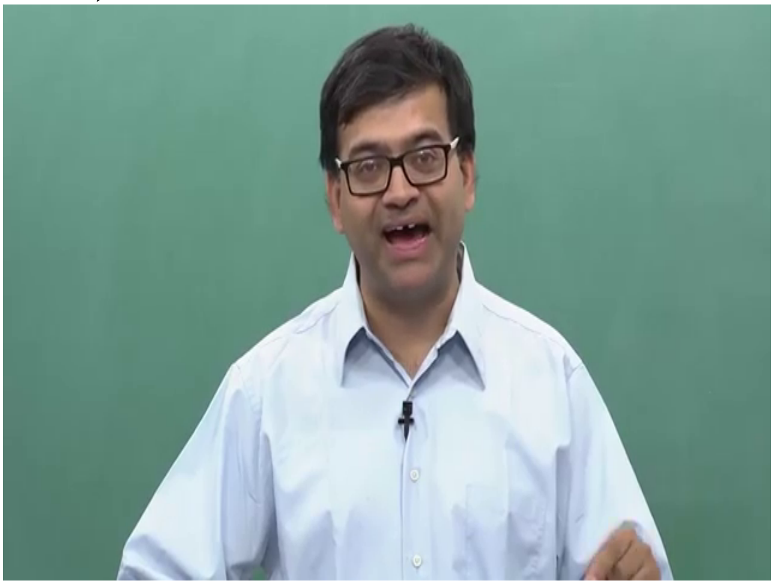

of odd weight codewords.

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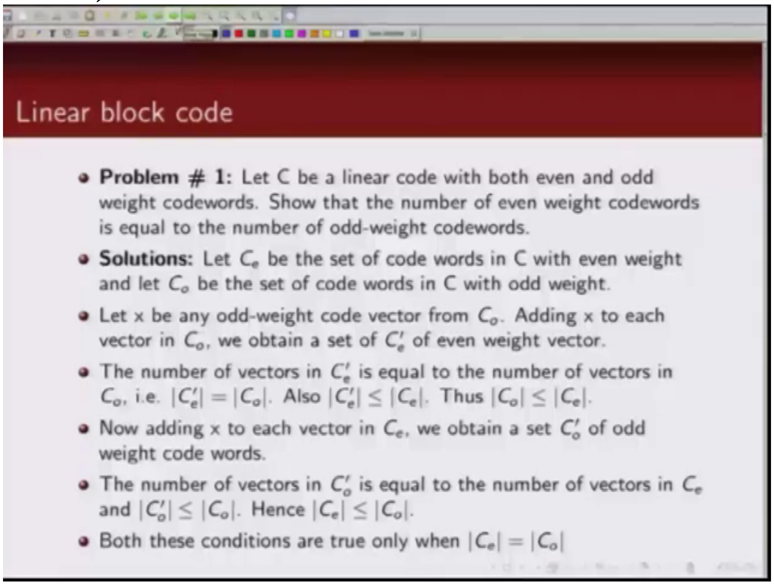

So I repeat, this condition and this condition

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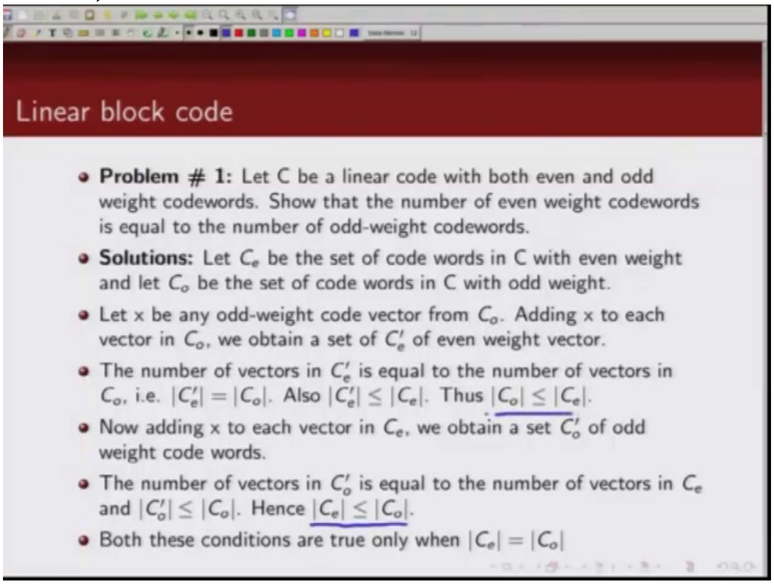

will be simultaneously satisfied only when

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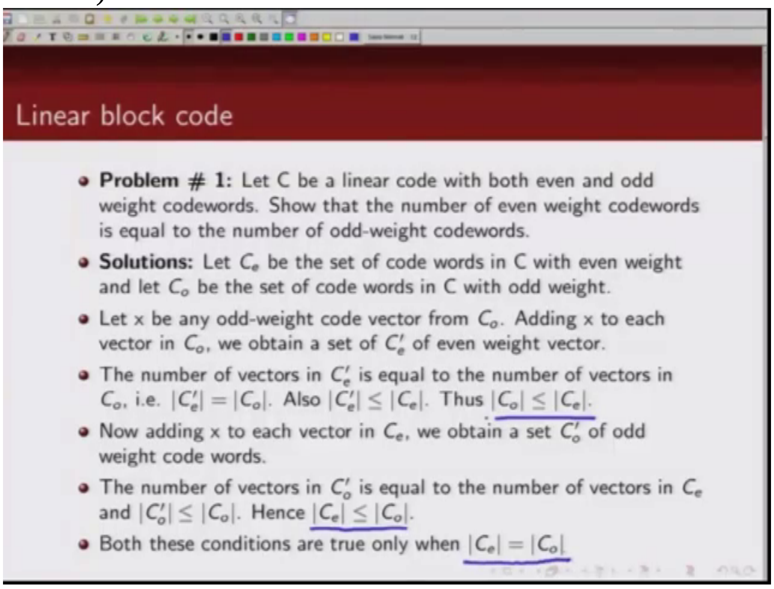

the set of number of even codewords is same as set of number of odd codewords. And this proves our result.

The next problem

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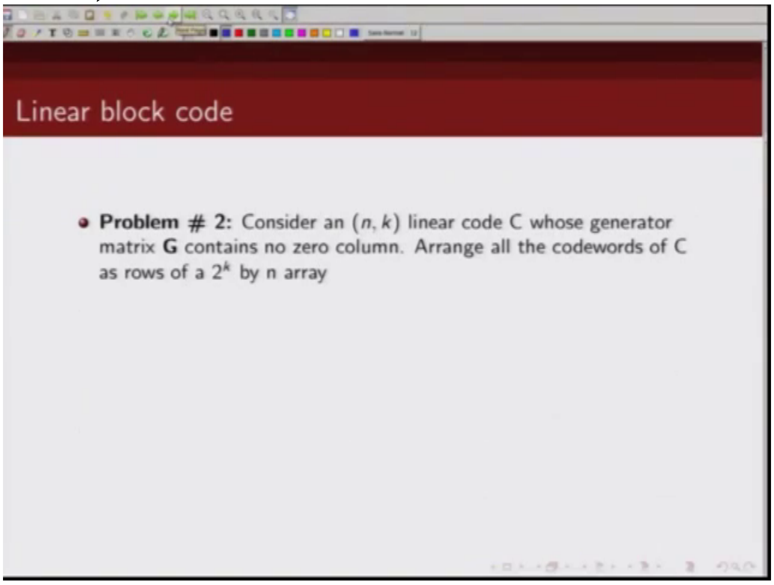

we will look out is as follows. Let us consider a linear n k code C whose generator matrix contains no zero column; now arrange all codewords of this linear code as rows of two raised to power k by n array. So what we are doing is we are arranging the 2 k codewords like this, in an array. So this array has dimensions 2 k cross n because total number of codewords

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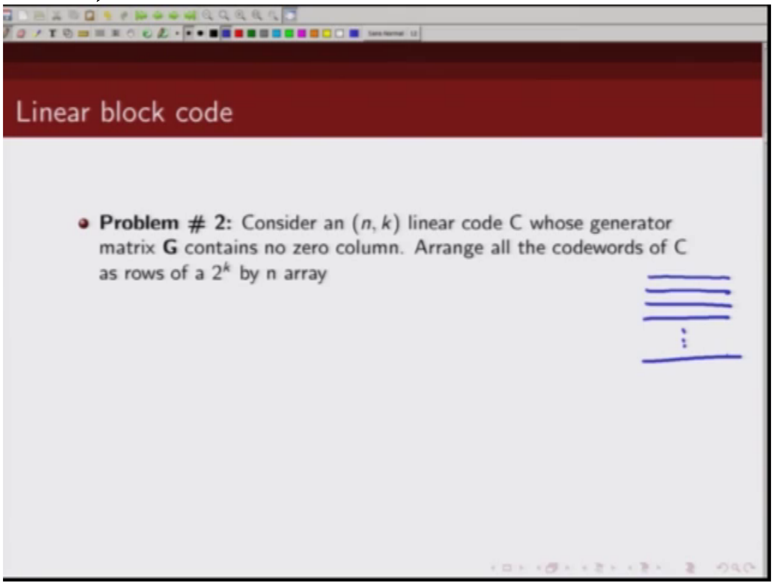

are 2 raised to power k for a n k binary code and they are all n-bit. The first result that we are

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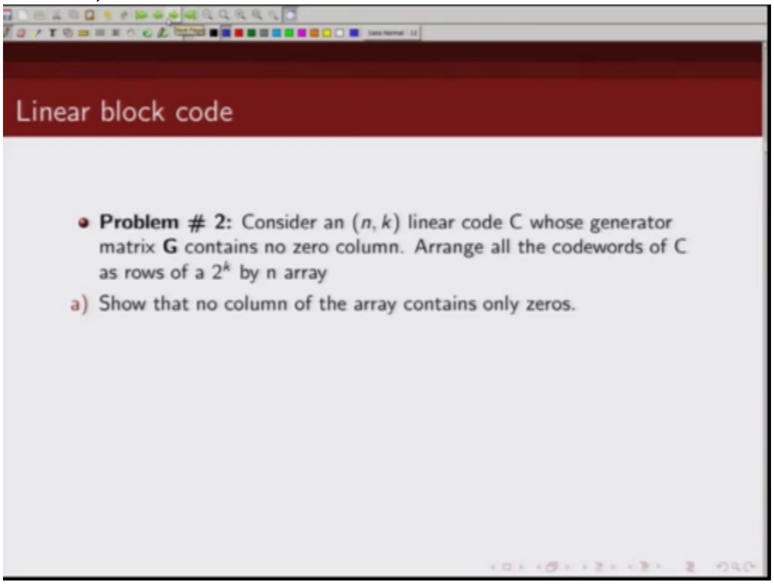

going to show is no columns of this array contain 0. Now please note that we have

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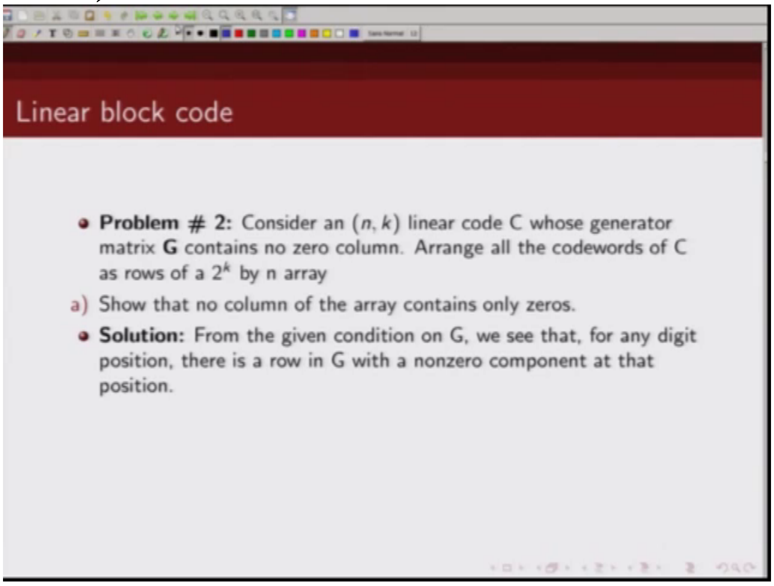

been given that this generator G does not contain any zero column ok so from the given condition on G we can see that for any position of any bit position there is a row in G which has non-zero component at that particular

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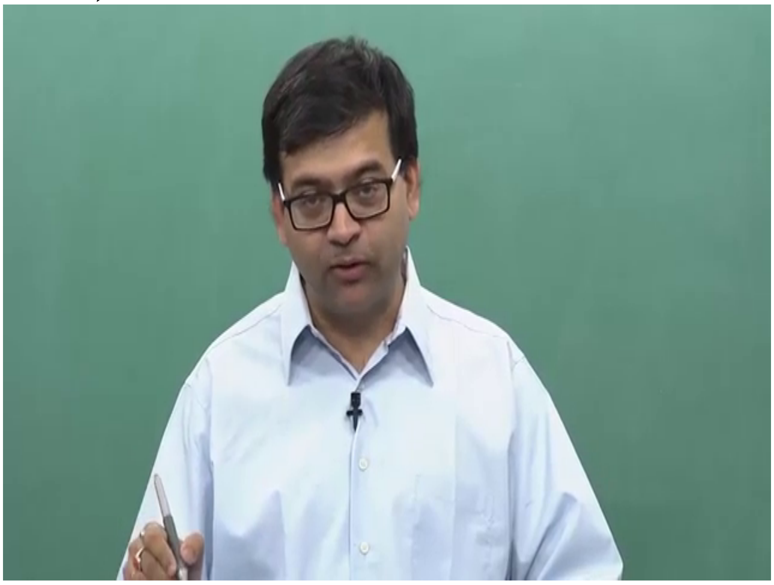

bit location and

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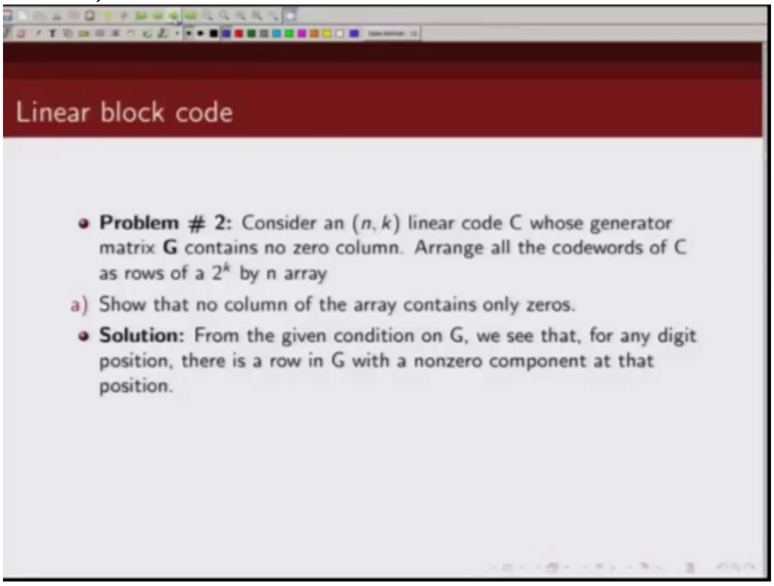

if this is

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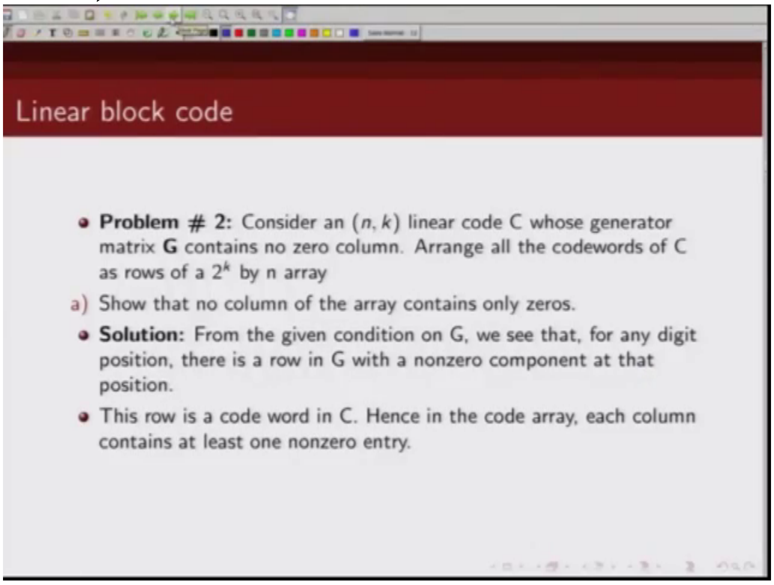

true what are the rows of, how do we generate the codewords? We generate the codewords by linear combination of these

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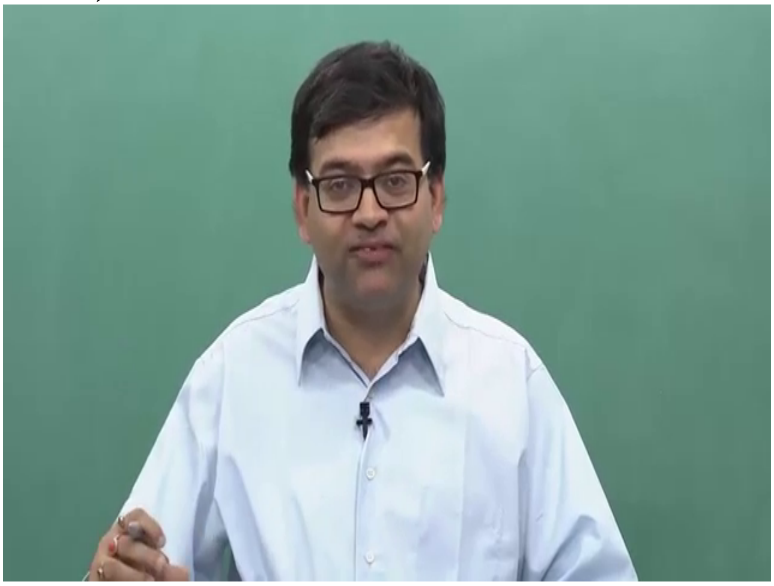

rows of the generator matrix. And since the generator matrix does not contain

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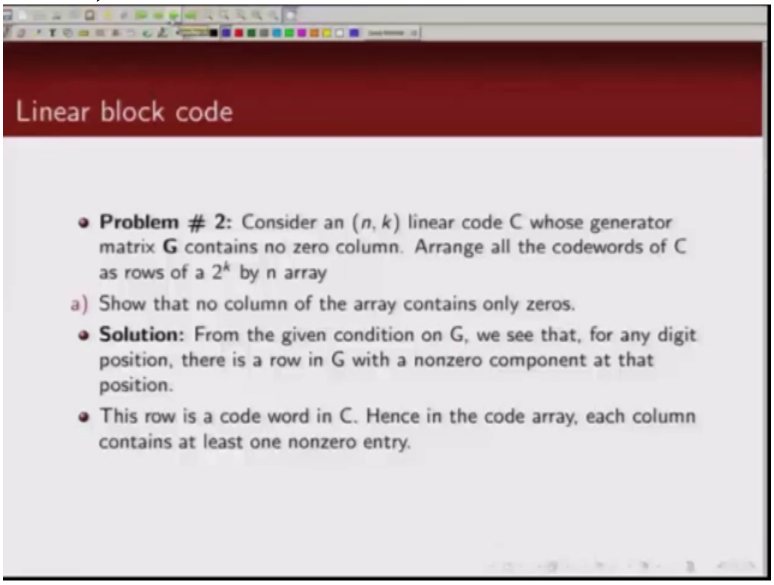

any zero column so each of these rows can be looked up as codeword in C so when we generate the codewords using this generator matrix in this code array, each column will have at least one

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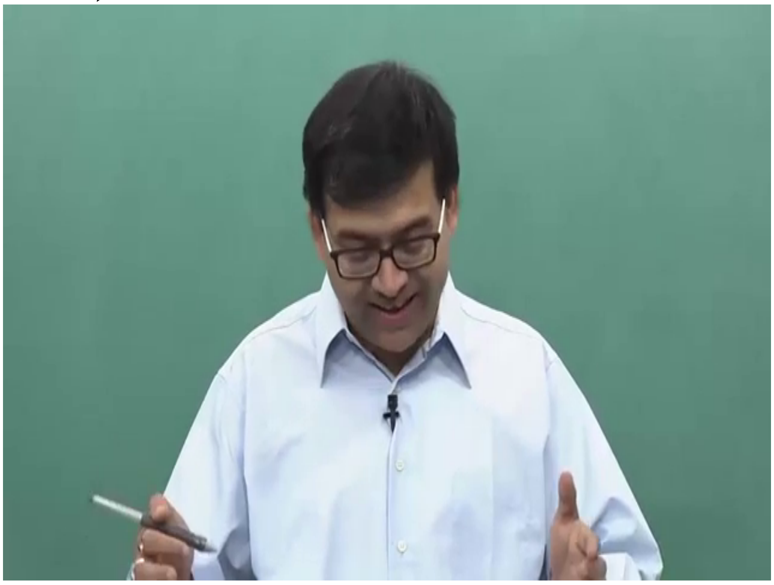

non zero entry. So this follows from the fact

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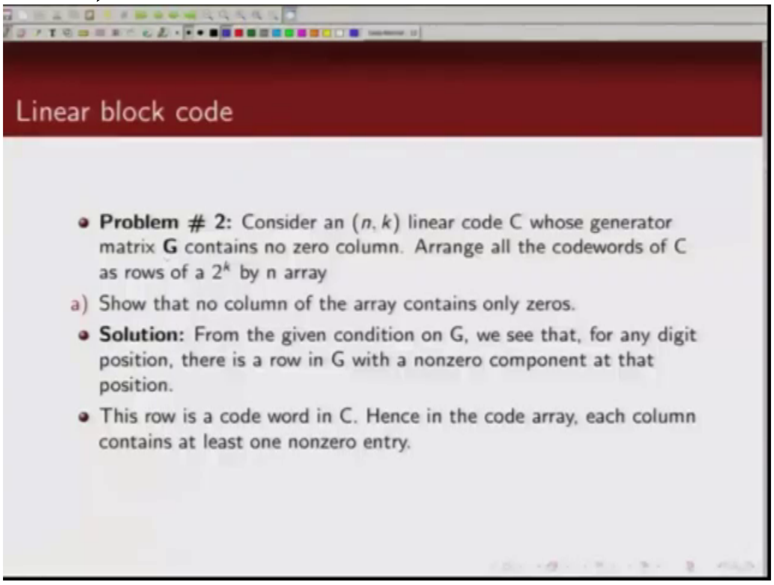

that our generator matrix G does contain any zero column

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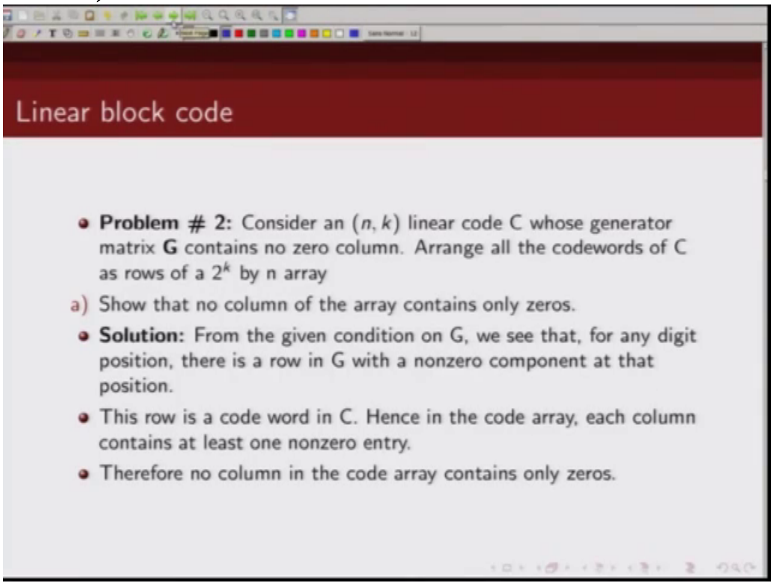

and hence no column in this code array will have zeroes.

The next result that we

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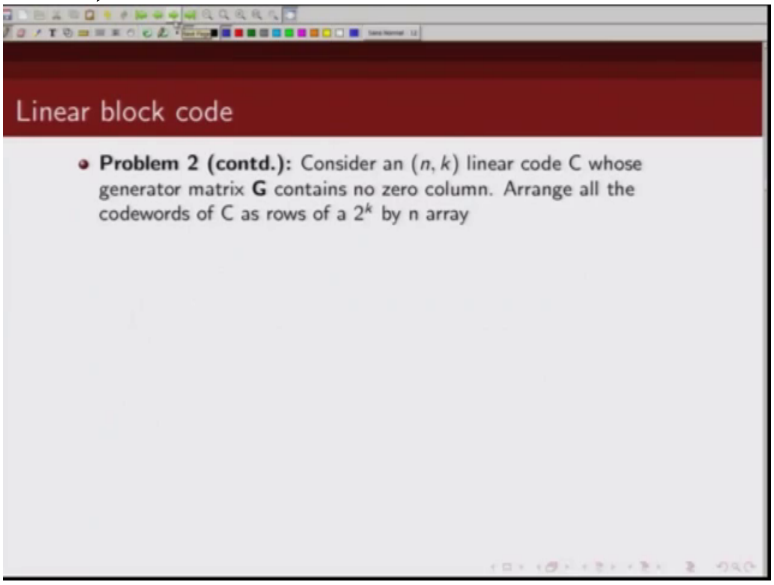

are going to show is in this array, in this 2 raised to power k cross n array each column consists of equal numbers of

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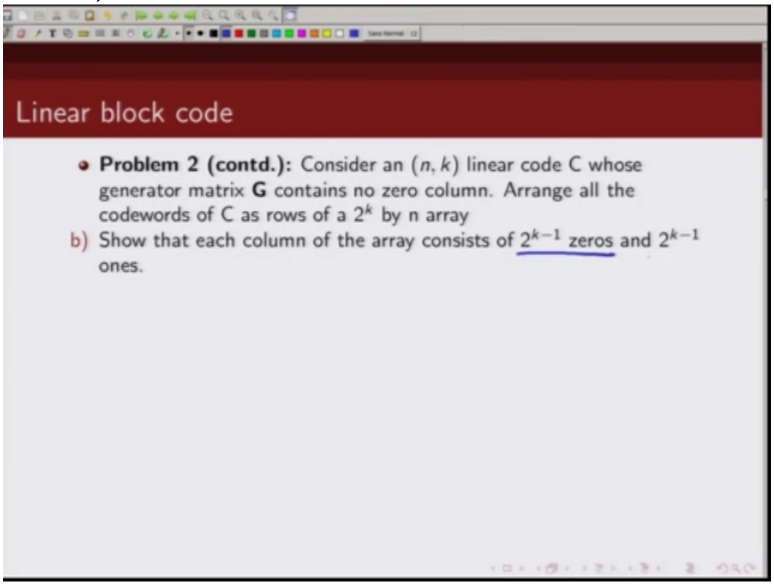

0s and 1s.

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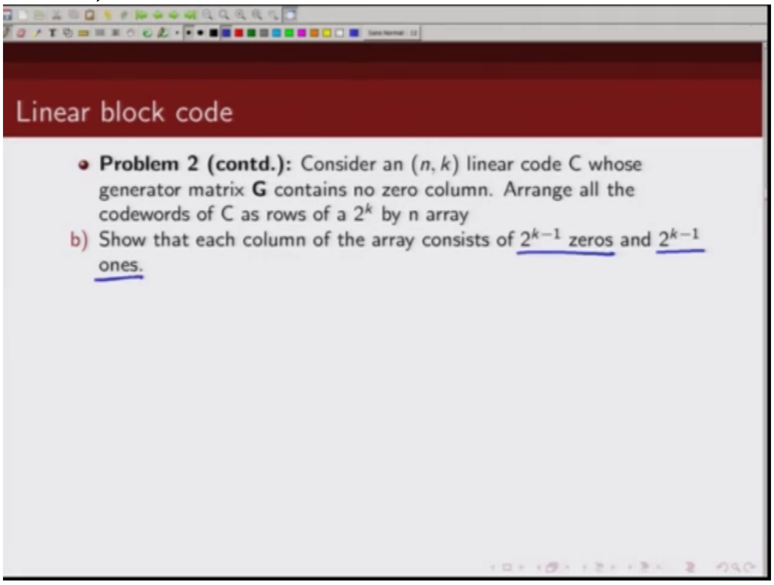

There are total 2 raised to power k minus 1 0s and 2 raised to power k minus 1 1s. So to prove

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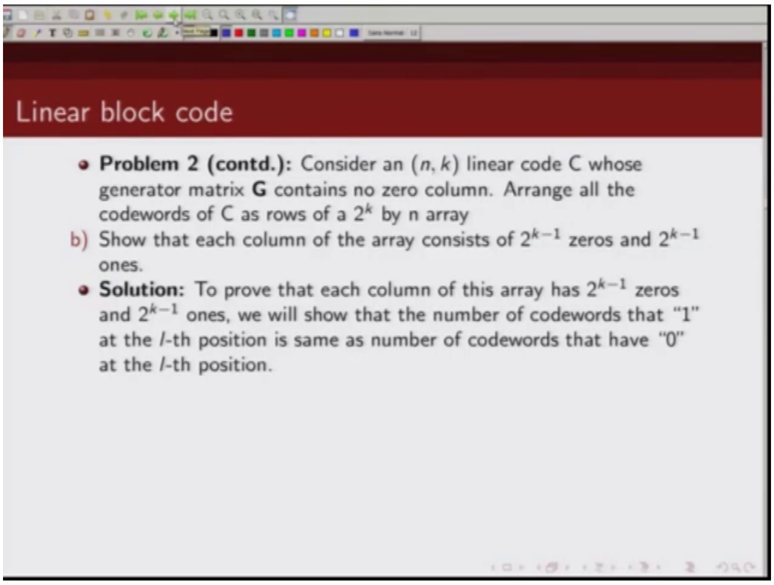

this what we will do is we will show that number of codewords that have 1 at lth location is same as number of codewords that have 0 at lth location. And in this way we will prove that this array has same number of 0s and 1s. So in this code array

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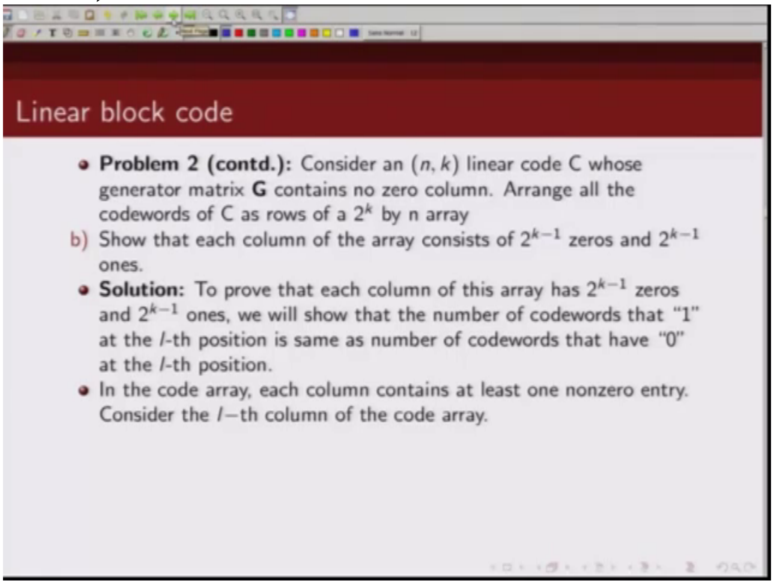

we know that each column will have at least one non-zero entry that we proved in the earlier result. So consider the lth column of this code array.

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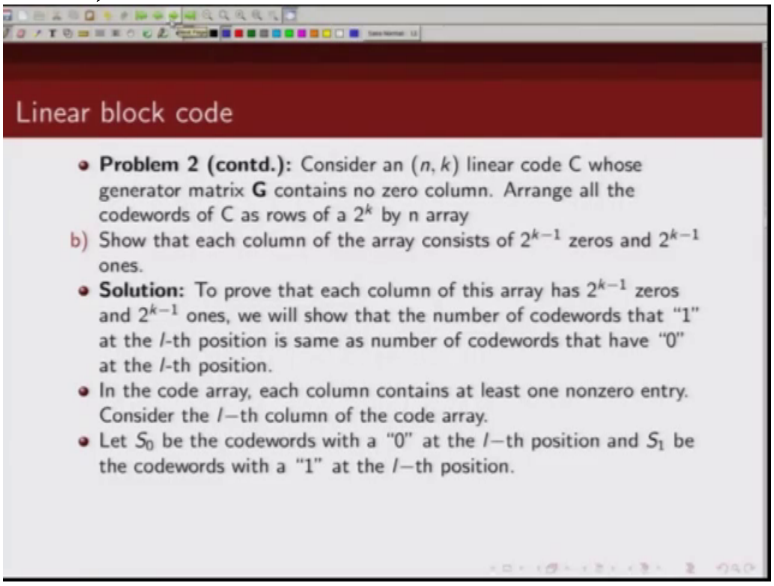

. Let us denote by S 0

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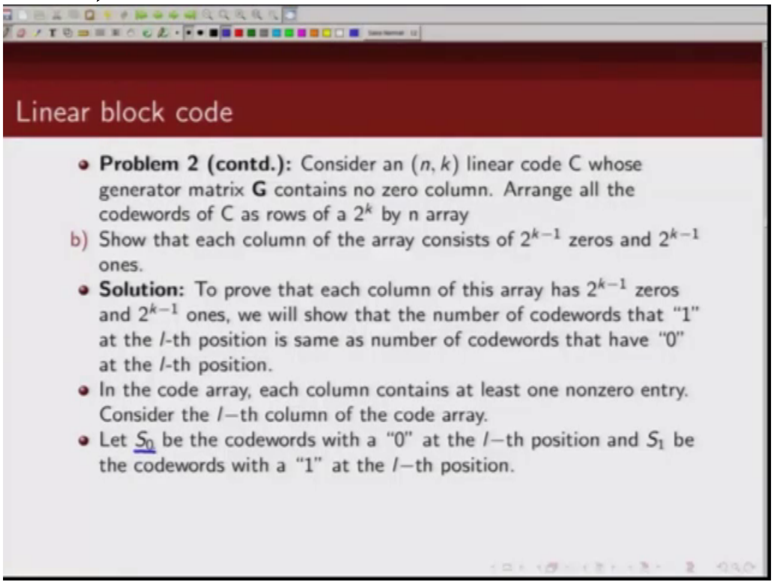

the set of codewords that has 0 at the lth location. And let us denote by S 1

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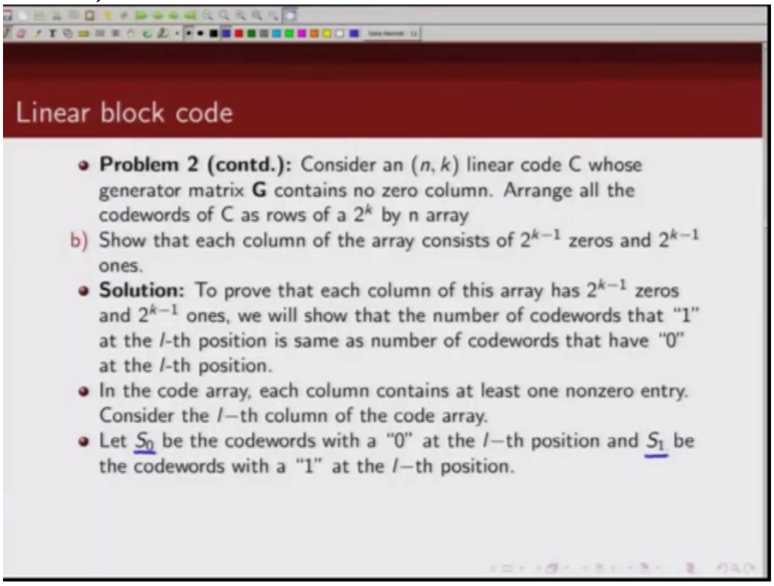

the set of codewords that have 1 at the lth location. Now we pick up a

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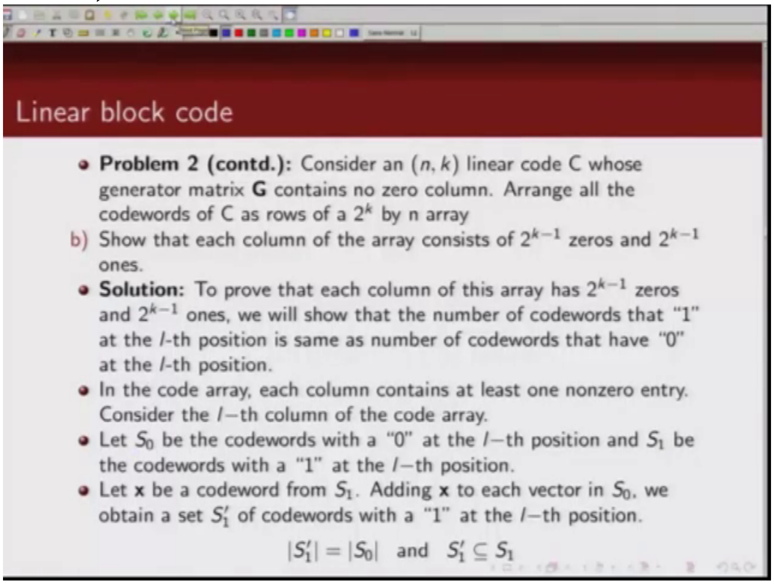

codeword x from the set S 1 that means x has 1 at lth location. Now if we add this codeword x to all the elements in the set S 0 what do we get?

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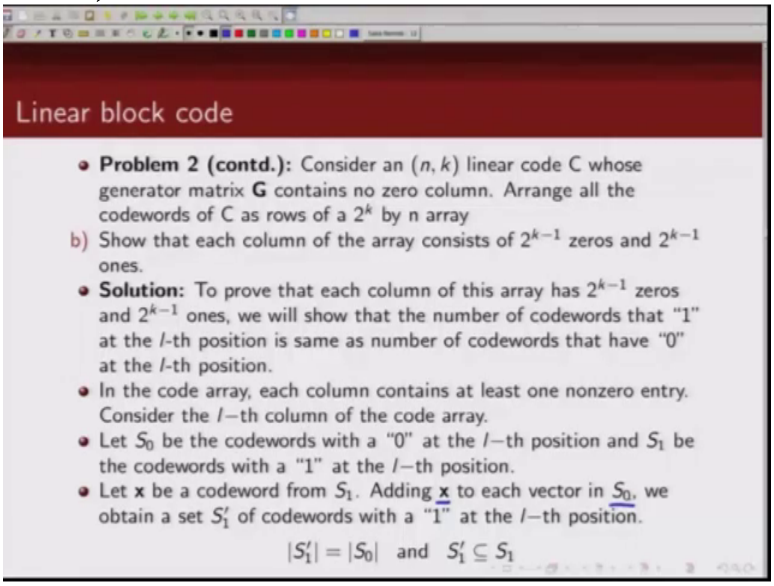

What we will get is a set containing 1 at lth location. Why, because S 0 is a set that has 0 at the lth location and x has, x is taken from the set S 1 so x has 1 at lth location. So if we add x to S, elements in S 0 what we will get is there will be a 1 at the lth bit location. So we denote this class

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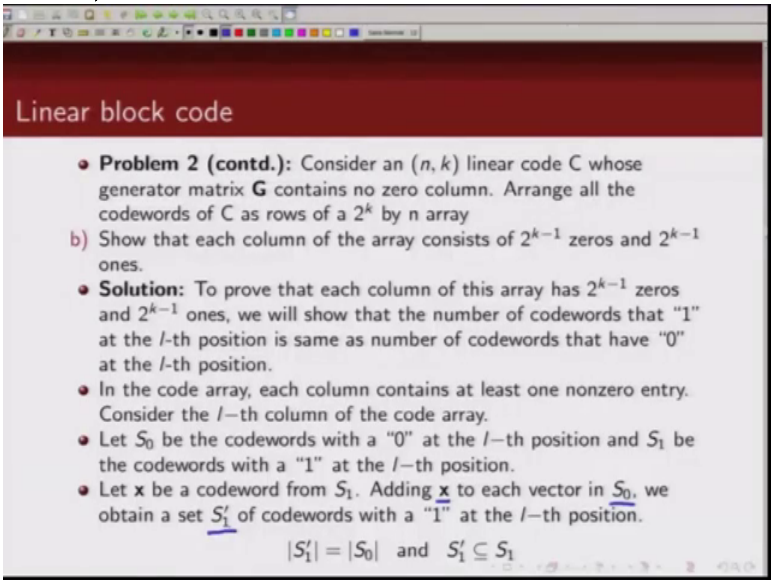

of codeword by S 1 prime and this S 1 prime will have 1 at the

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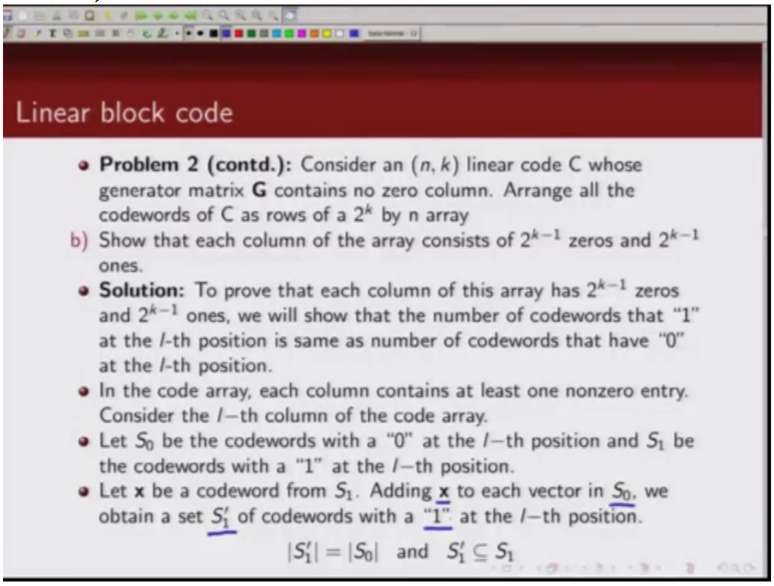

lth location and since this S 1 prime is generated by adding x to this set of vectors in S 0 so number of elements in S 0 is going to be

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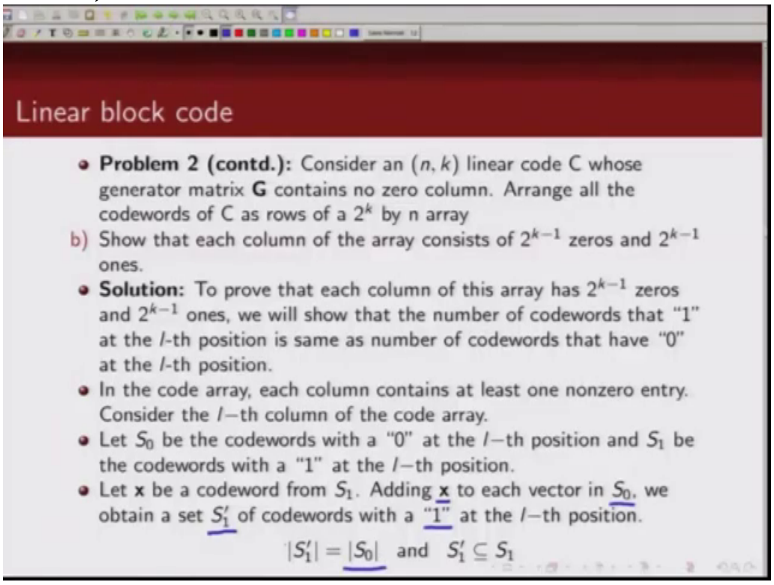

same as number of elements in S 1 prime and S 1 prime is a subset of S 1 which is a set of all codewords which have 1 at lth location.

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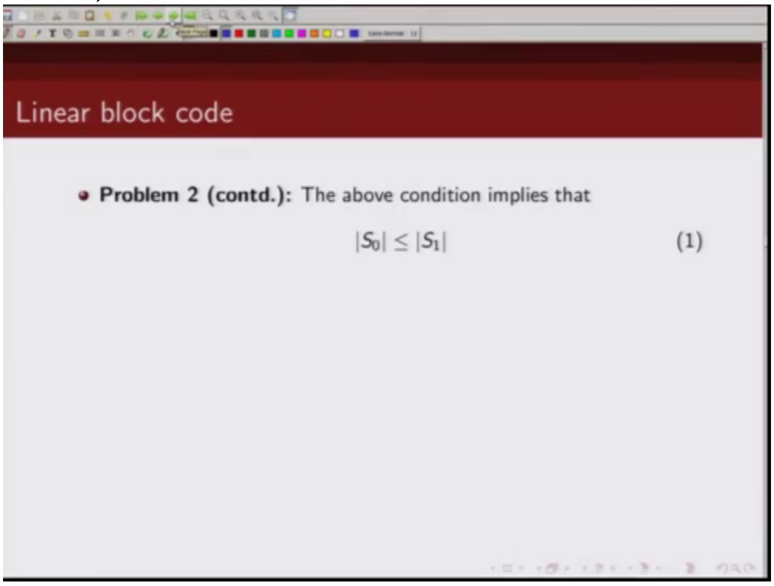

So from this we get this condition that set of codewords which has 0 at lth location is less than or equal to set of

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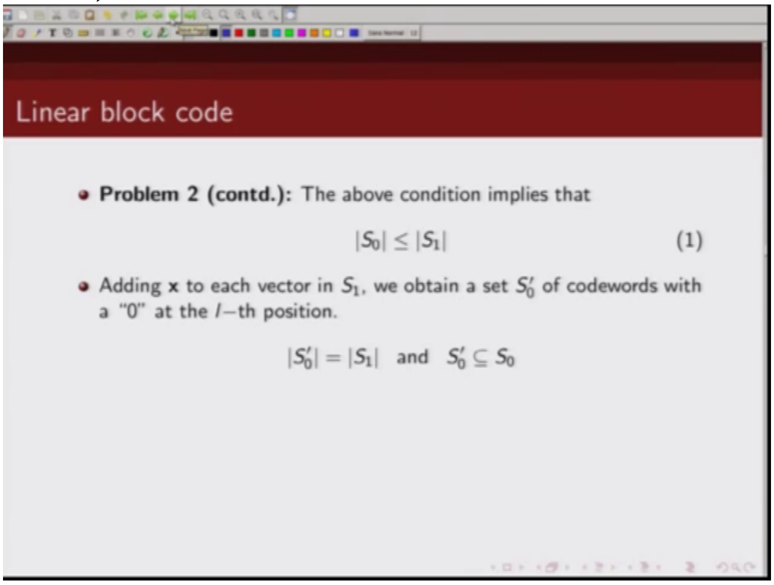

codewords which has 1 at lth location. Now add the same vector x which has 1 at lth location to all the elements in S 1. When we do that what we get is a new set of vectors which has 0 at lth location. We denote this set by S 0 prime. So S 0 prime is a set of codewords which are obtained by adding x to the set of vectors set of odd vectors which have 1 at lth location. So then we can write as set of vectors in S 0 prime is same as set of vectors in S 1 and since S 0 prime is a subset of S 0 what we can write then is,

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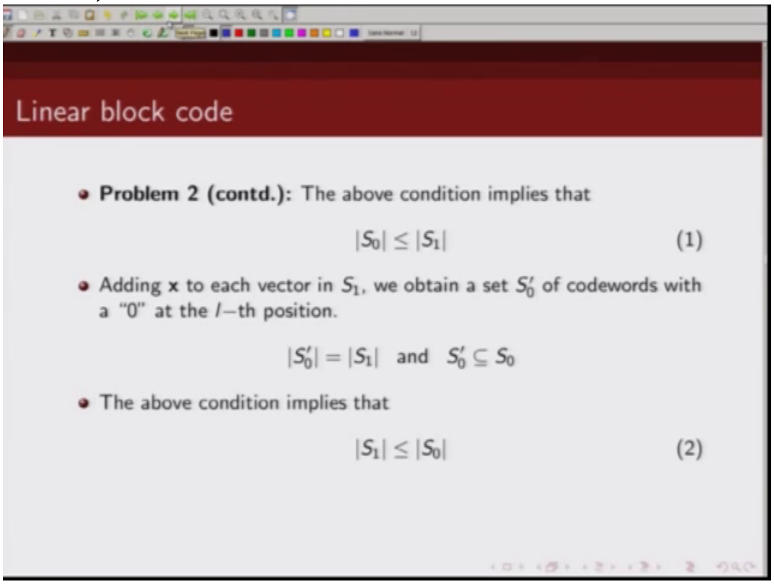

from this relation and this relation we

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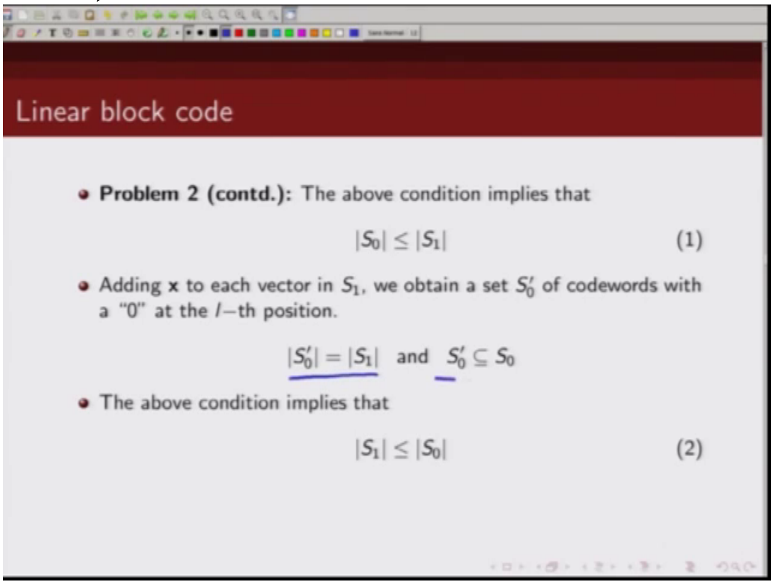

can write

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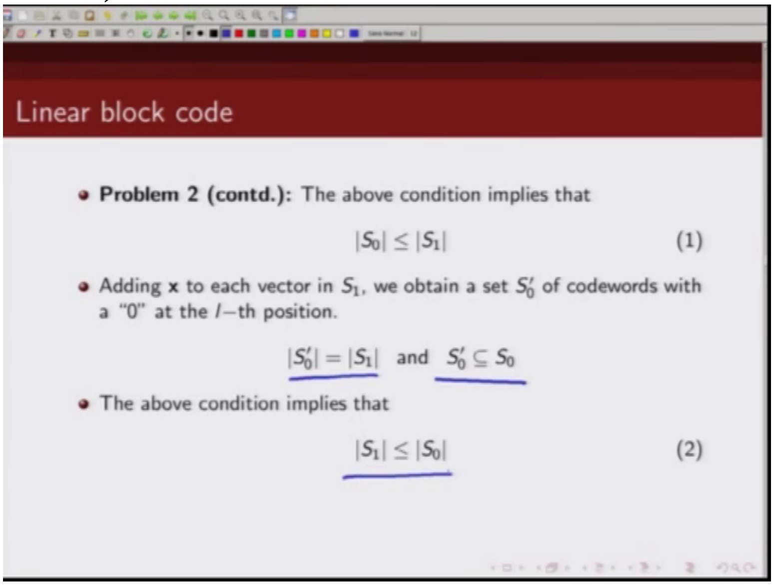

set of codewords which have 1 at lth location is less than set of codewords which has 0 at lth location. Now equation 1 and 2, they are going to be simultaneously

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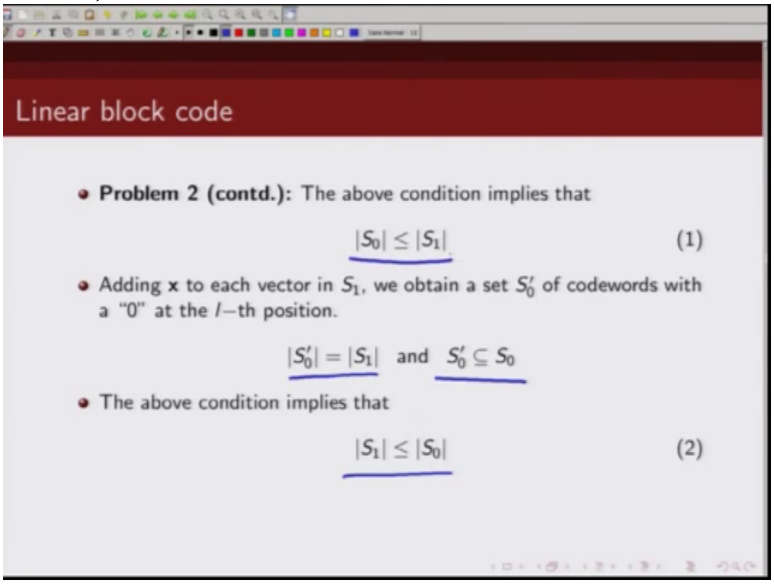

satisfied only when this is satisfied with equality. Then what it shows here is

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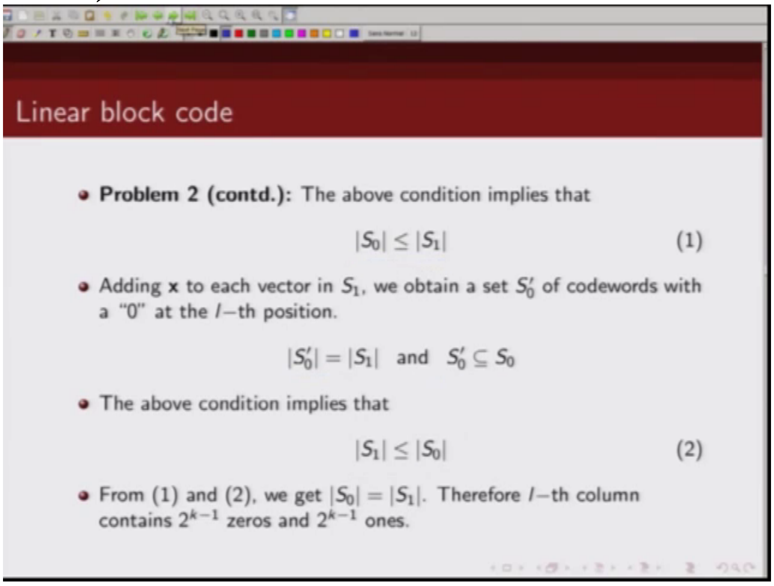

that at any lth location number of codewords which have 0 at lth location is same as number of codewords which have 1 at lth location. So basically each column will then have same number of 0s and

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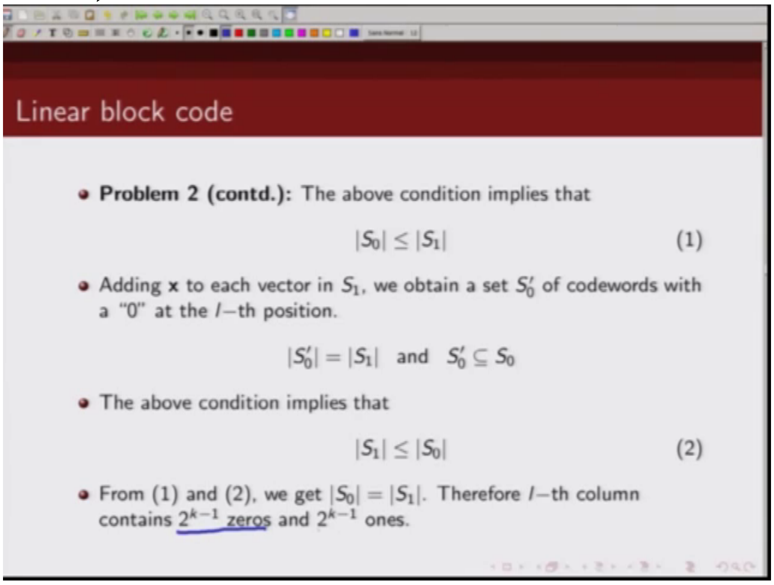

same number of 1s.

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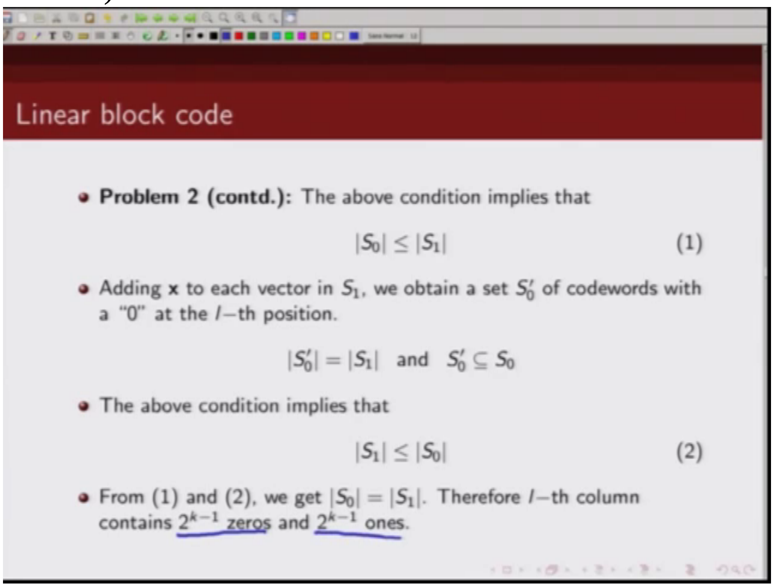

Now we

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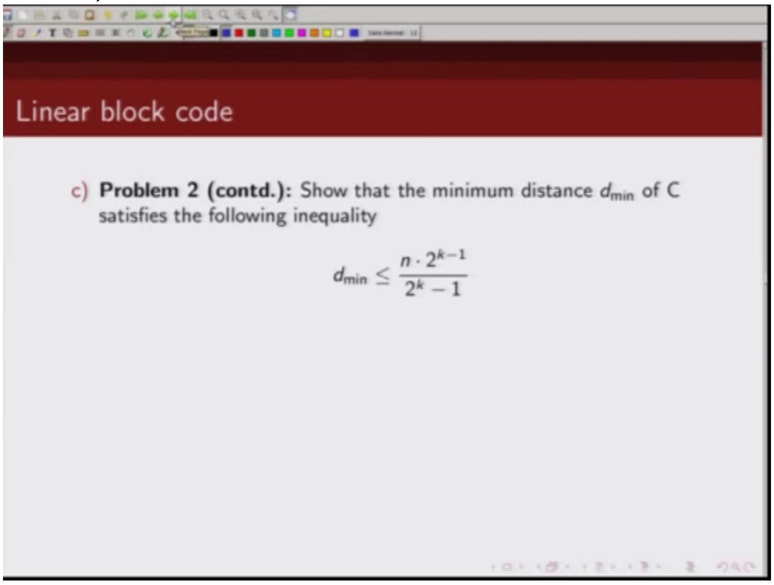

prove another result. We show that minimum distance of the code is upper bounded by this quantity and to prove this result we are just going to use the result we just proved in the previous section. So

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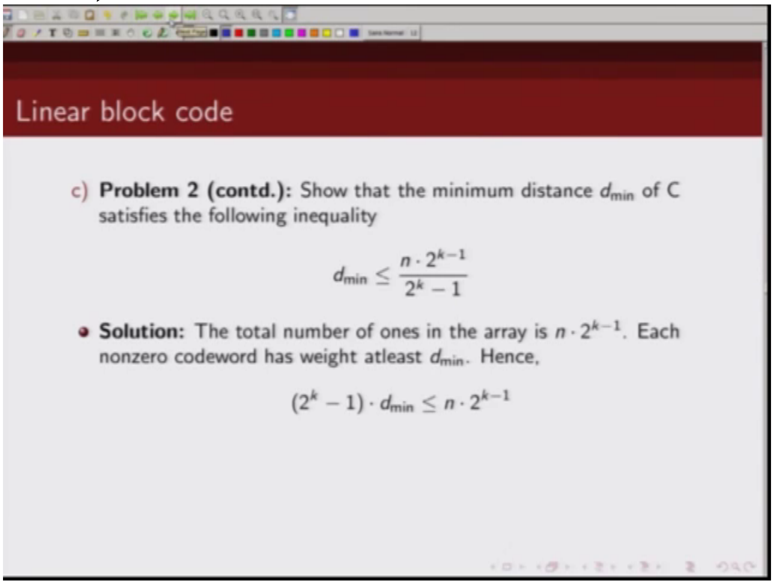

in the previous section what we did was we arranged these 2 k codewords in an array two k cross n array and we showed that each column of this array has 2 raised to power k minus 1 1s and 2 raised to k minus 1 0s. So in this whole array which has n columns total number of 1s is given by this,

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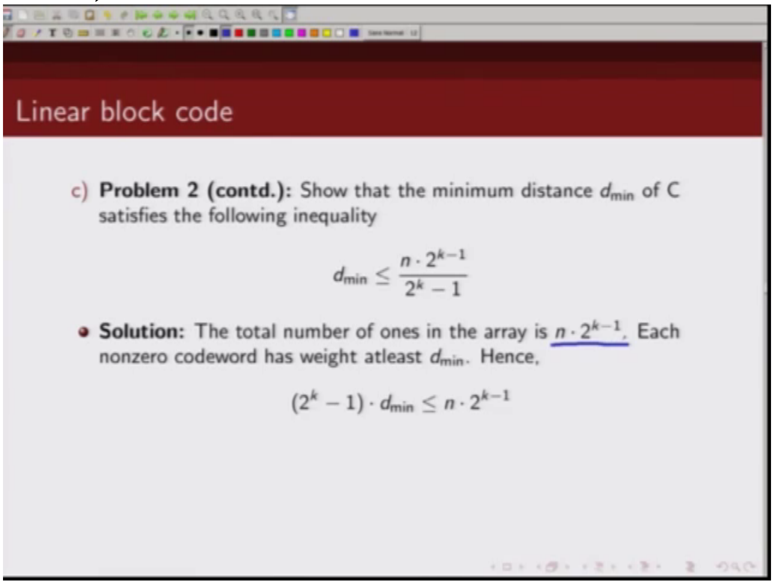

n times 2 raised to power k minus 1. Now since each non-zero codeword will have minimum distance at least d min and how many total codewords we have 2 raised to power k one of them is all zero codeword so how many non-zero codewords we have, that is given by 2 k minus 1

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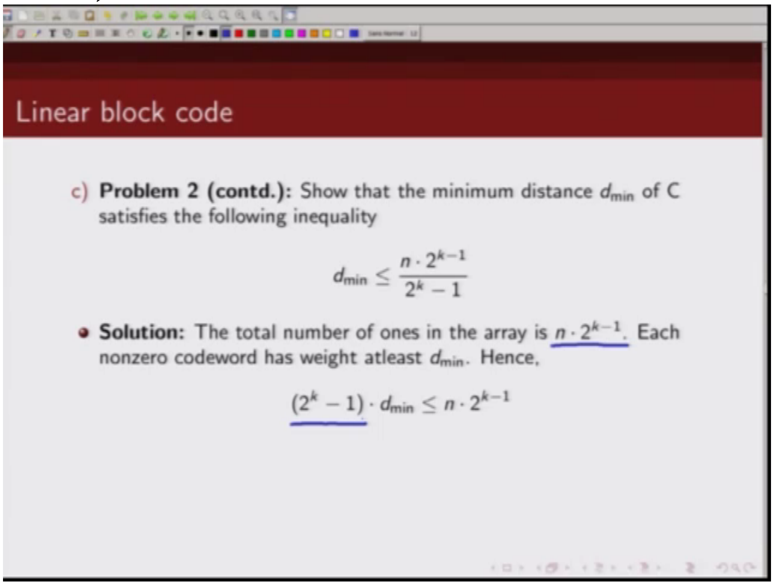

and each of these non-zero codewords have minimum distance at least d min so total number of non-zero codewords multiplied by d min must be

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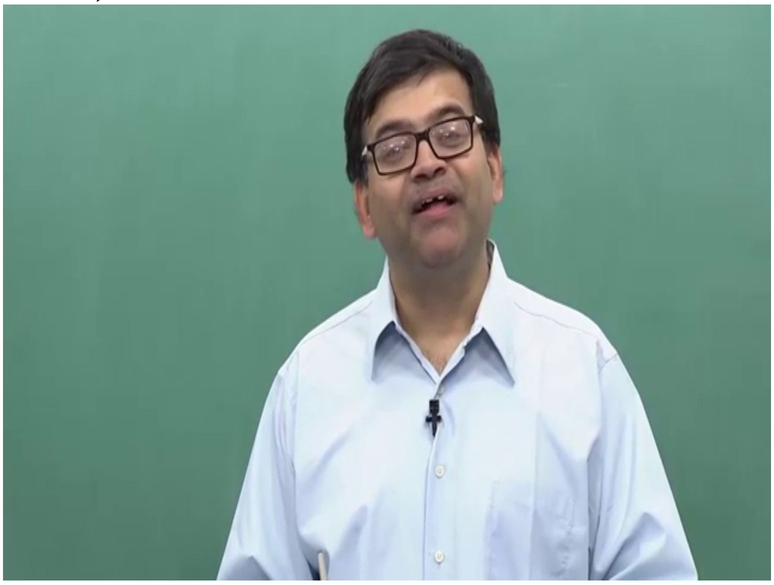

less than equal to total number of 1s

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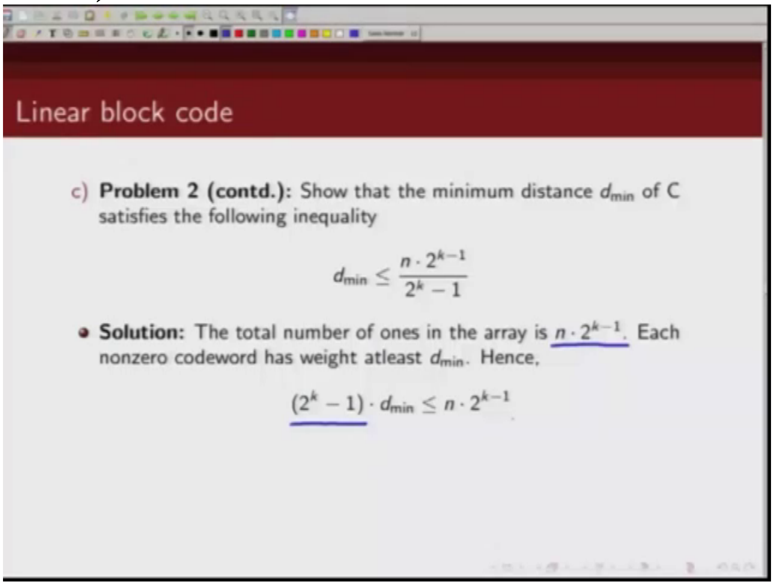

in this code array which is given by n times 2 k minus 1, 2 raised to power k minus 1. So from this relation then we can then write

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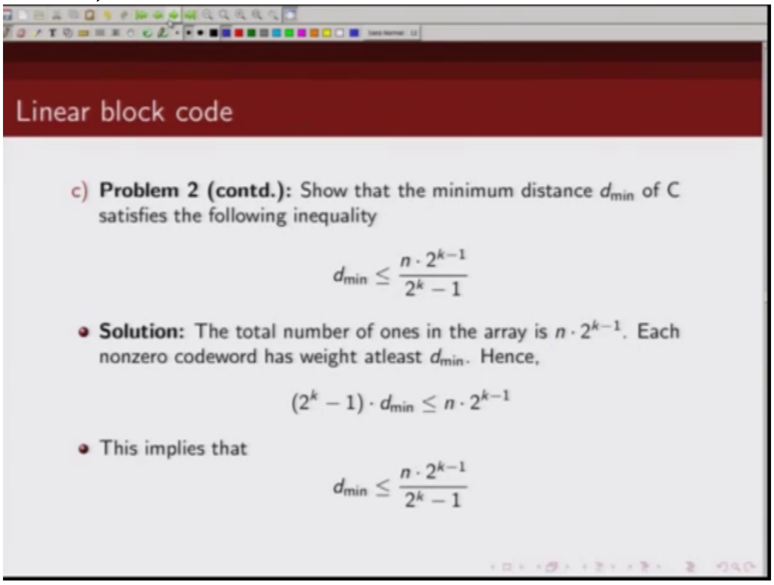

that minimum distance of a code is upper bounded by this relationship.

So

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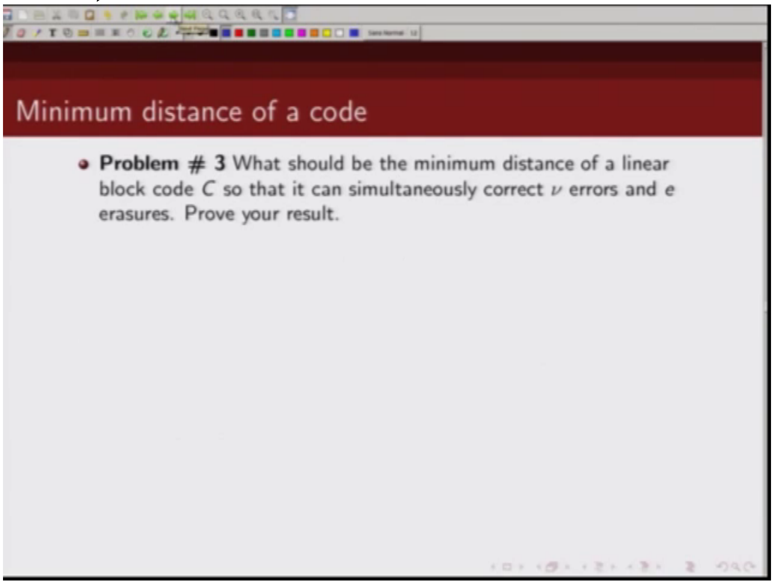

next problem that we will look at is what is a minimum distance of linear block C that can simultaneously correct mu errors and e erasures? Now just recall what do we mean by error correction and error

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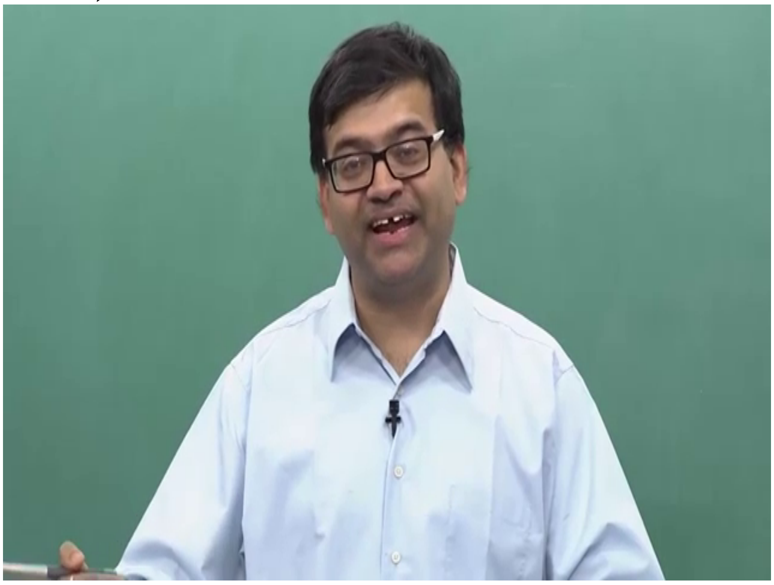

erasure correction. Basically, so erasure is basically, some of the bits are getting erased. So you send n bits. If e bits are getting erased what you are receiving is n minus e bits. And error correction you are familiar with, basically you want to correct errors that have happened in so many bit locations. So the question is

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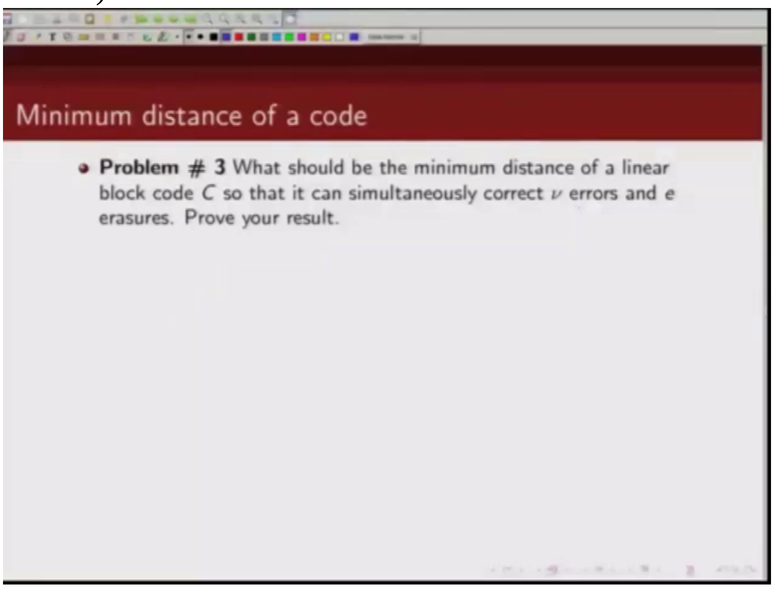

what should be the minimum distance of a linear block code that can simultaneously correct mu errors as well as e erasures?

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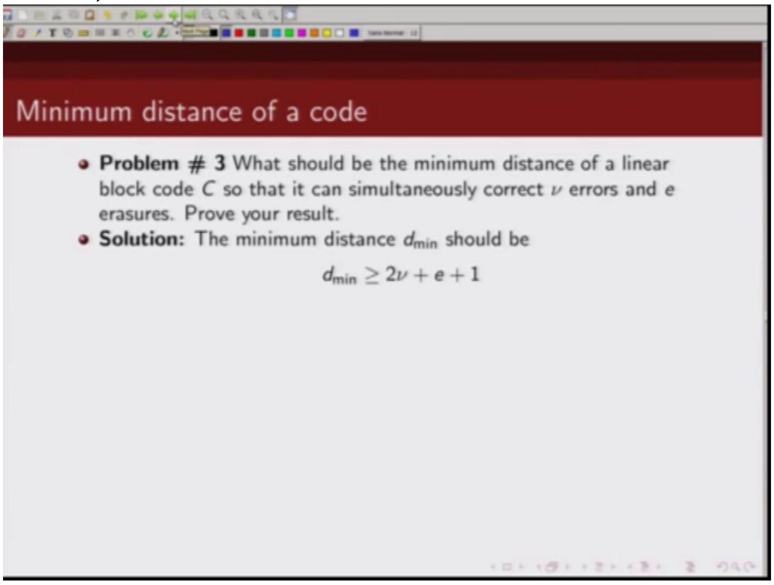

Now if the minimum distance of the code is at least two mu plus e plus 1 then it can simultaneously correct mu errors and e erasures. We are going to next prove this result.

So

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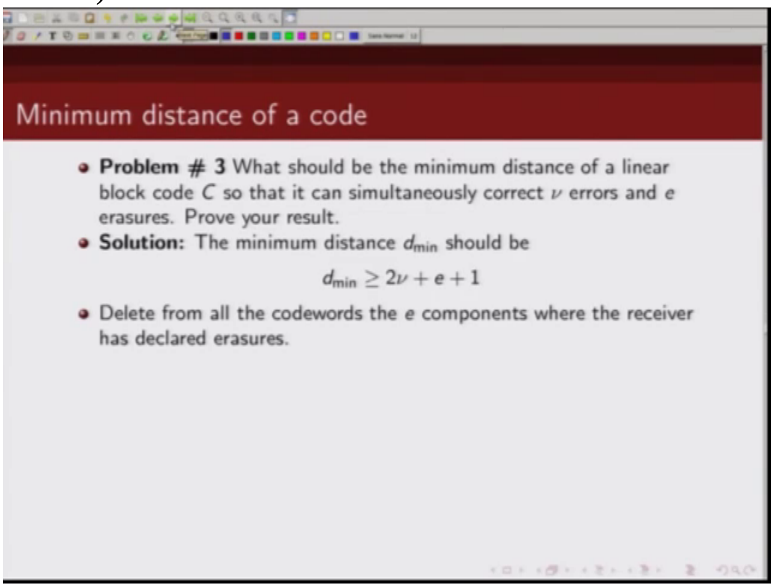

delete from all codewords e components which got erased. If we delete these e components what we are left is n minus e

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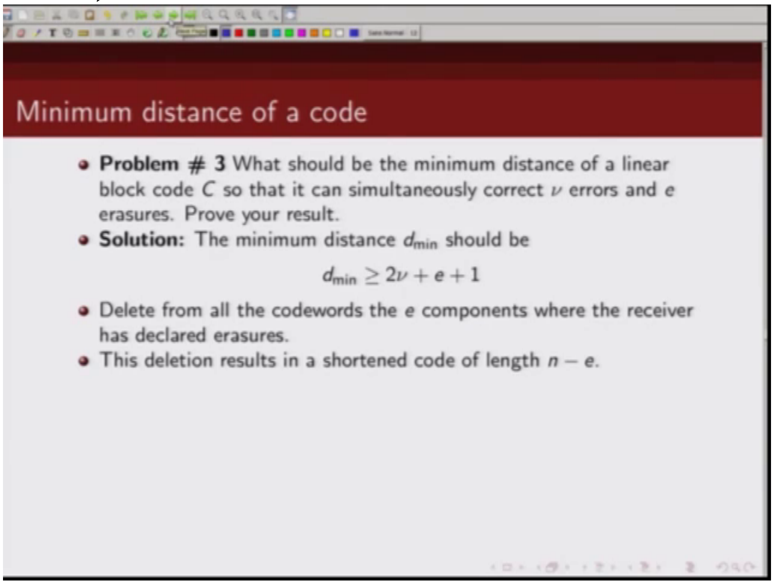

length shortened codeword. So this deletion of e component results in a shortened code of length n minus e.

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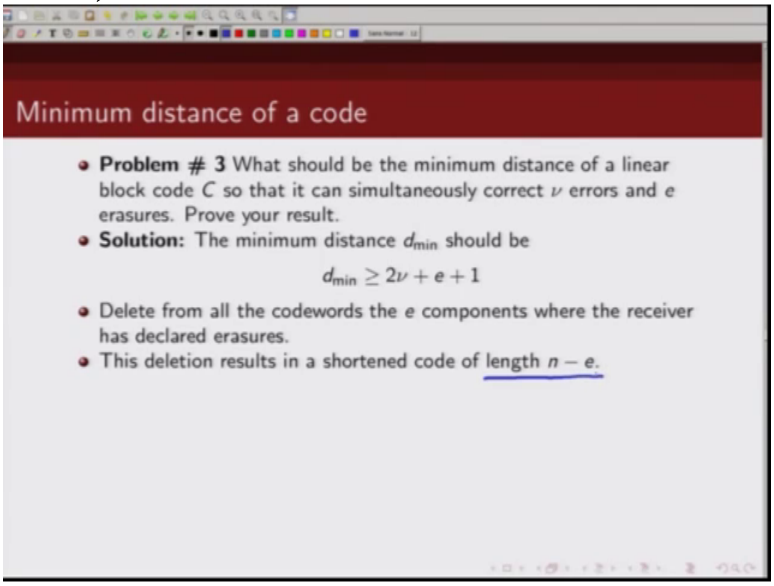

Now we

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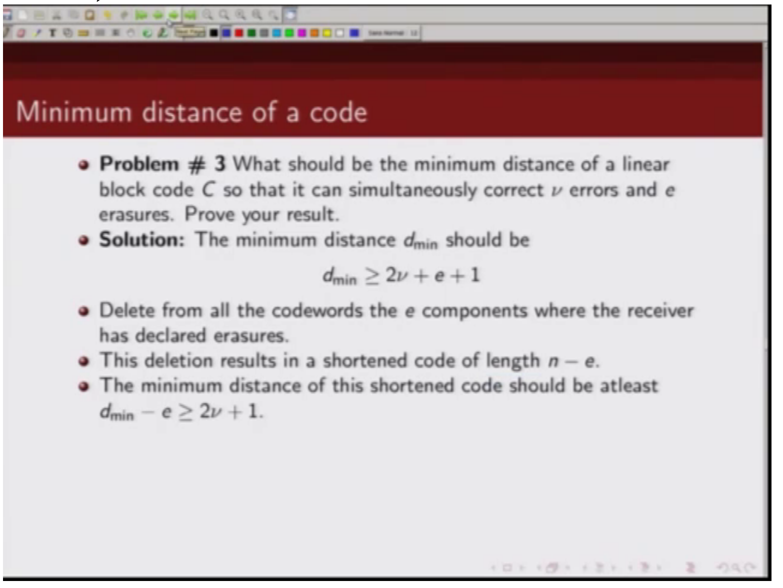

know that if we want to correct t errors, what should be minimum distance of the code? It should be at least

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2 t plus 1. So this code

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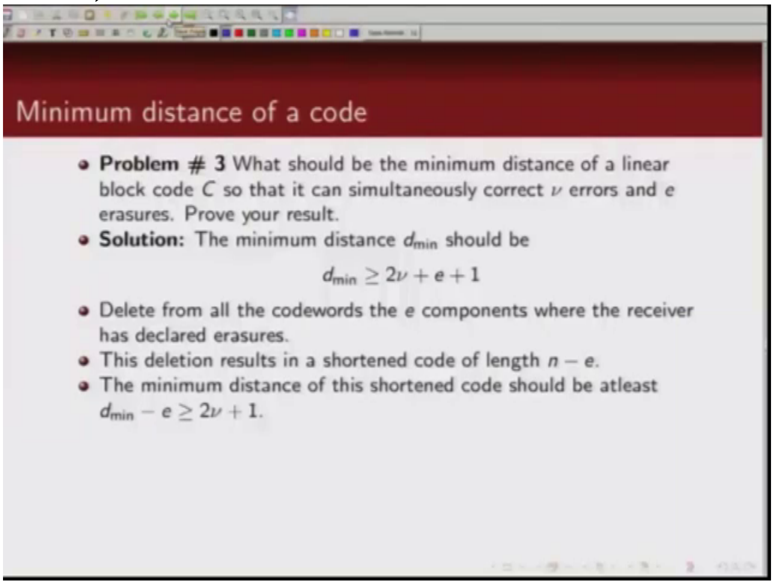

basically, if we want to correct mu errors the minimum distance of the code after these e erasures

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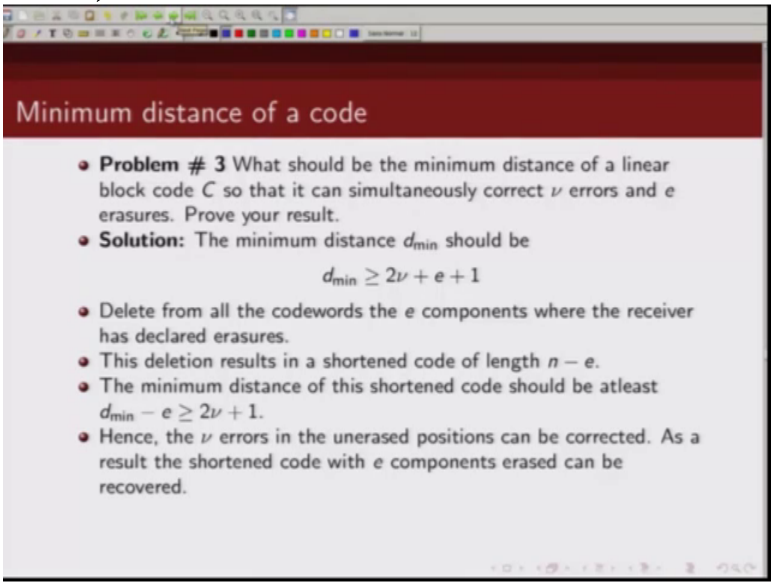

should be greater than equal to two mu plus 1. So if minimum distance of the code after these e erasures, the minimum distance is still larger than two mu plus 1, then this code can correct mu errors. So we want our minimum distance of the code to be at least 2 mu plus e plus 1. Now since these mu errors are in, in the unerased positions can be corrected if this condition holds so as a result basically we would be able to correct mu errors.

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Now remember we have to simultaneously

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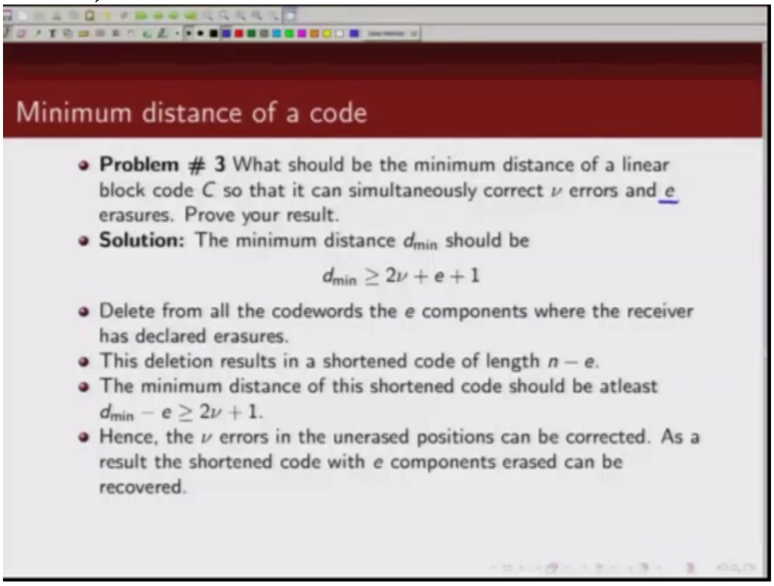

also be

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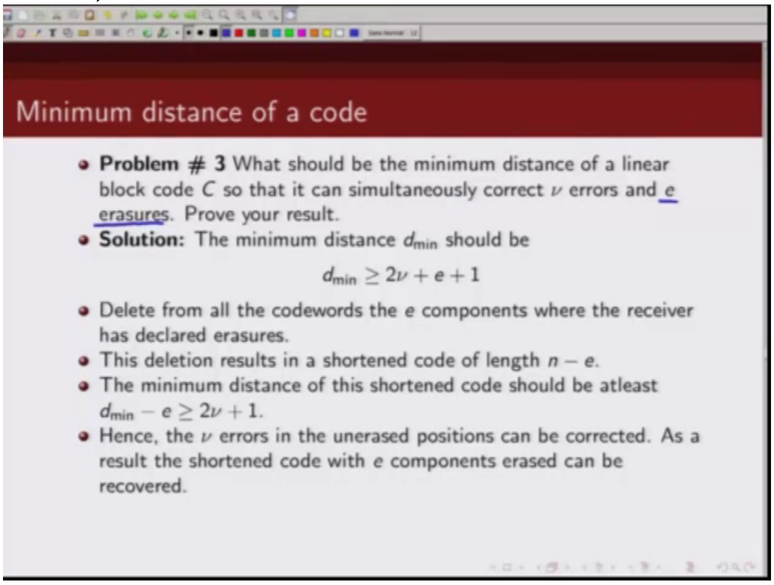

able to basically, it would be not only simultaneously correct mu errors but we have to correct e erasures also. Now what is the condition on minimum distance such that e erasures can also be corrected? The minimum distance of the code should be at least greater than number of erasures plus 1. So if the minimum distance of the code

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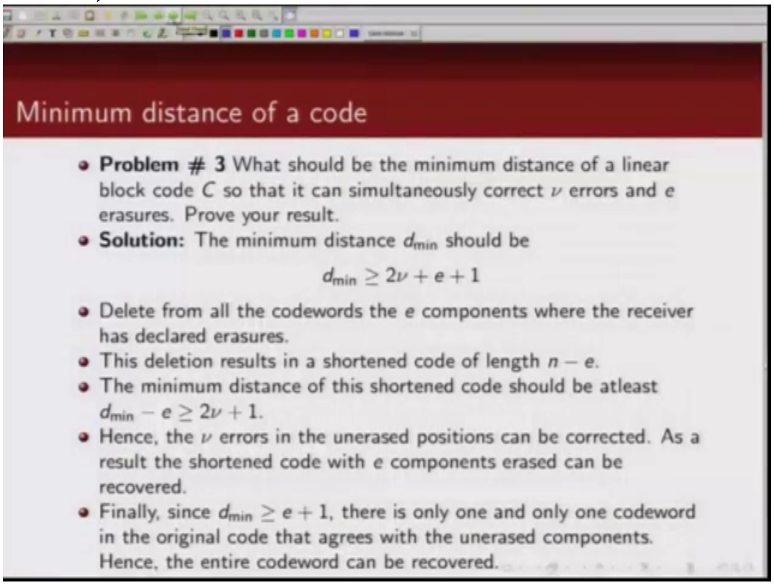

is greater than e plus 1

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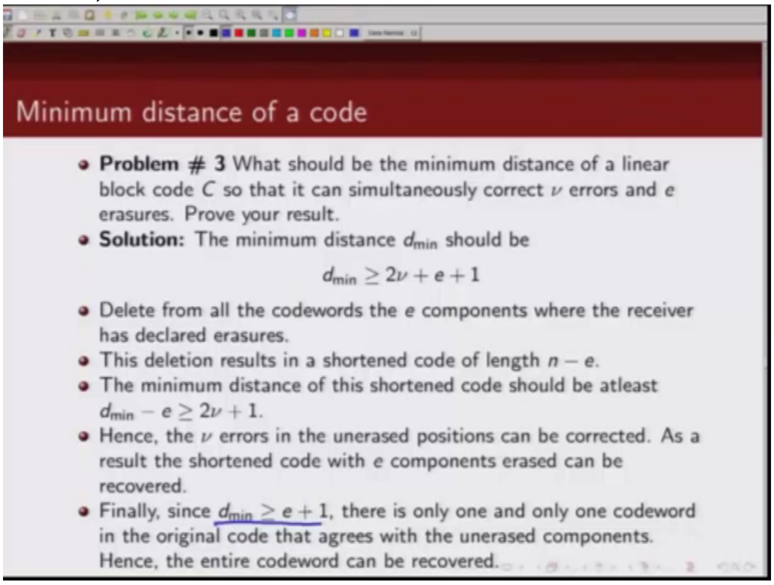

then there is only one codeword in the original code that maps to the shortened code. So as long as minimum distance of the code is greater than e plus 1, there is only one and one codeword in the original code that agrees with the unerased component. So there is as long as minimum distance of the code is greater than e plus 1, there is only one code that maps from erased shortened code to the original code. And since in this case the d minimum is already 2 mu plus e plus 1 which is greater than e plus 1, this code would be able to correct e erasures as well. So if we choose our minimum distance of the code to be greater than equal to two mu plus e plus 1, it would be able to correct mu errors as well as e erasures.

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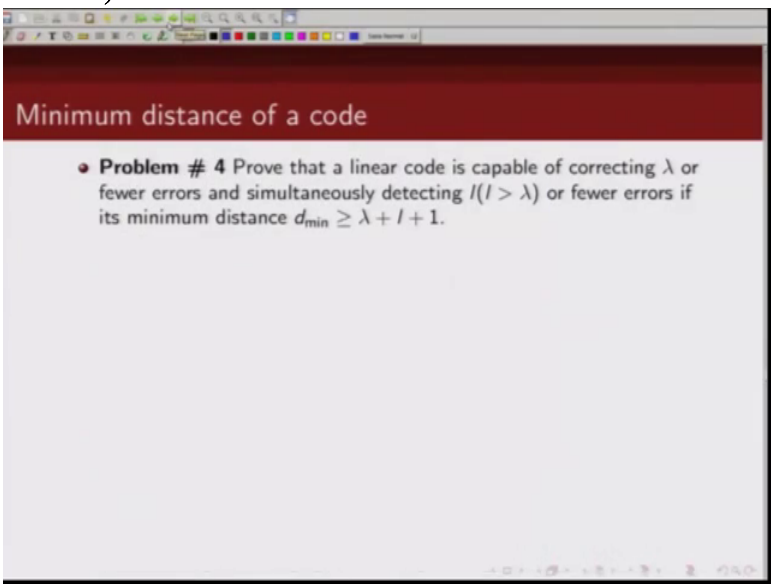

The next problem that we are going to solve is as follows. Prove that a linear block code is capable of correcting lambda or fewer errors and simultaneously detecting l where l is greater than lambda or fewer errors if the minimum distance of the code is at least lambda plus l plus 1. Please pay attention to the word simultaneously.

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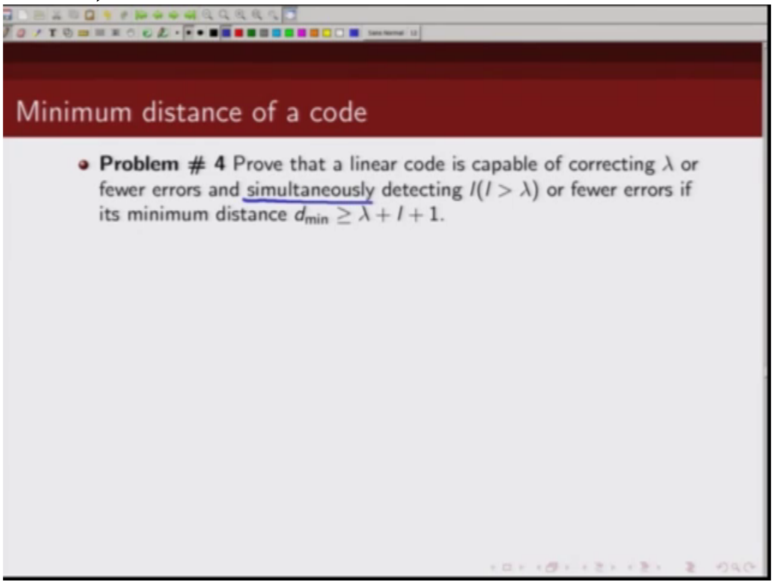

So we want not only to correct mu errors, along with that we should be able

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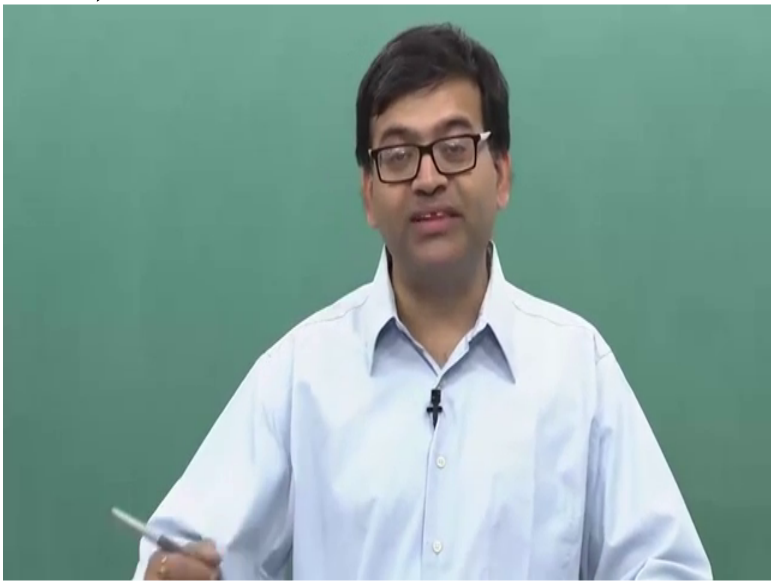

to detect l errors as well. That's what we mean by

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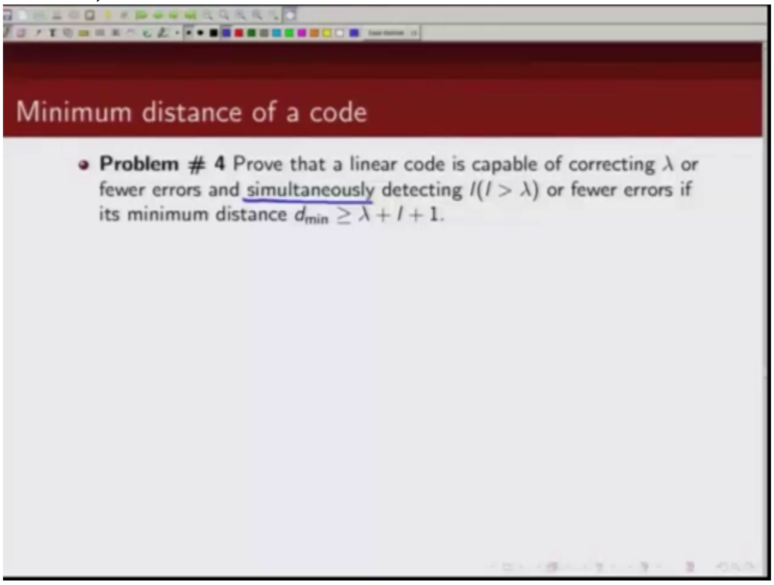

simultaneous error detection and correction. So let's prove this result. Now note

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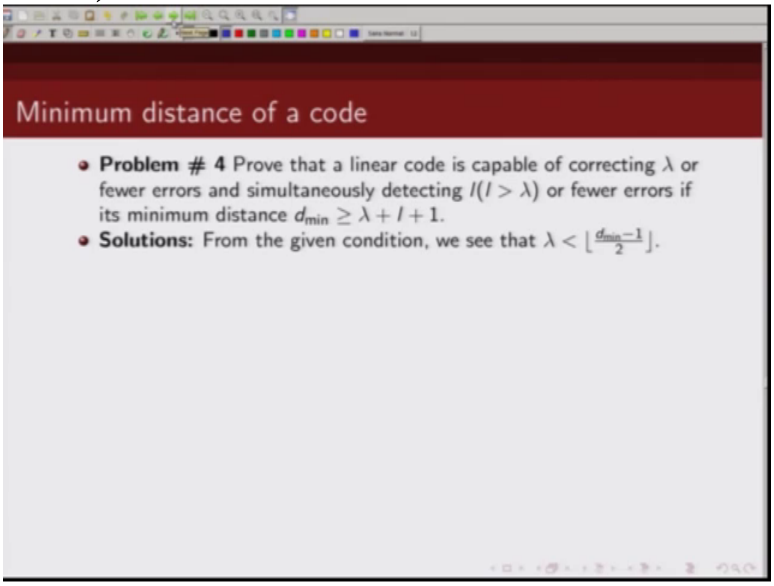

lambda is less than l. So if minimum distance is lambda plus l plus 1, this is basically greater than 2 lambda plus 1.

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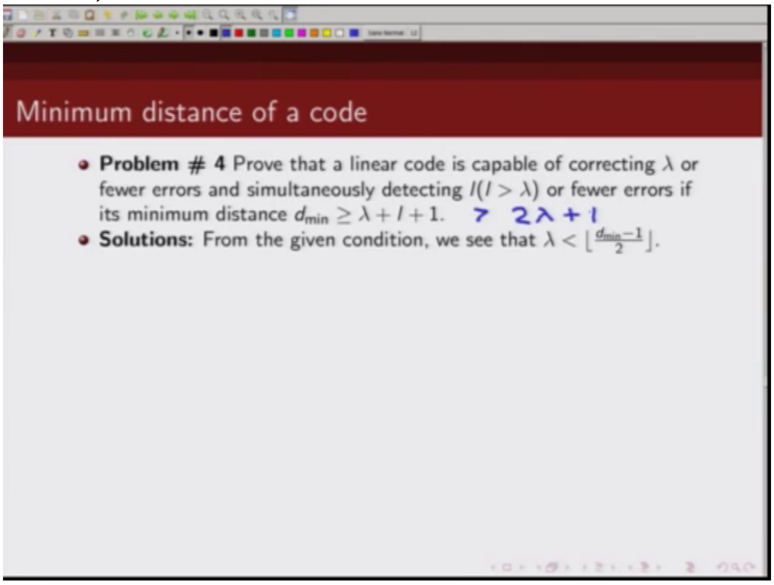

And if the minimum distance is greater than 2 lambda plus 1, it would be able to correct lambda errors. So from this given condition that d min is at least lambda plus l plus 1 where l is greater than lambda we know

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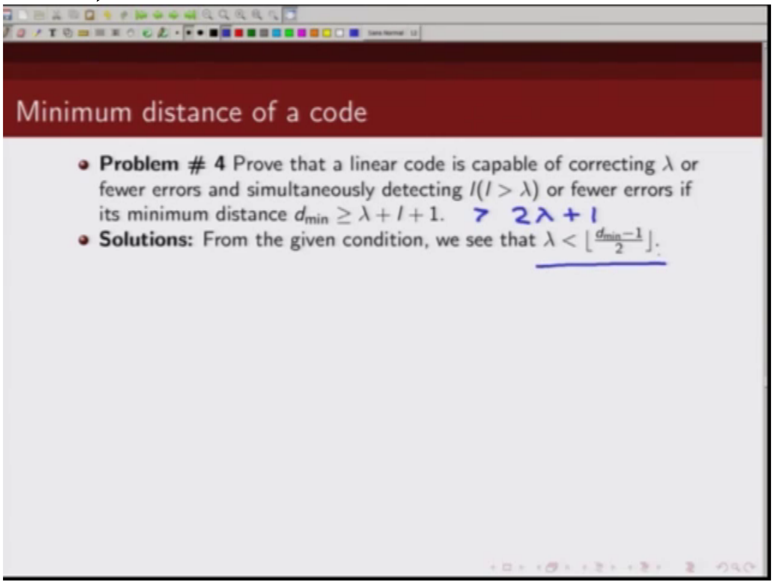

that the minimum distance is greater than 2 lambda plus 1.

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So it should be able to correct lambda errors.

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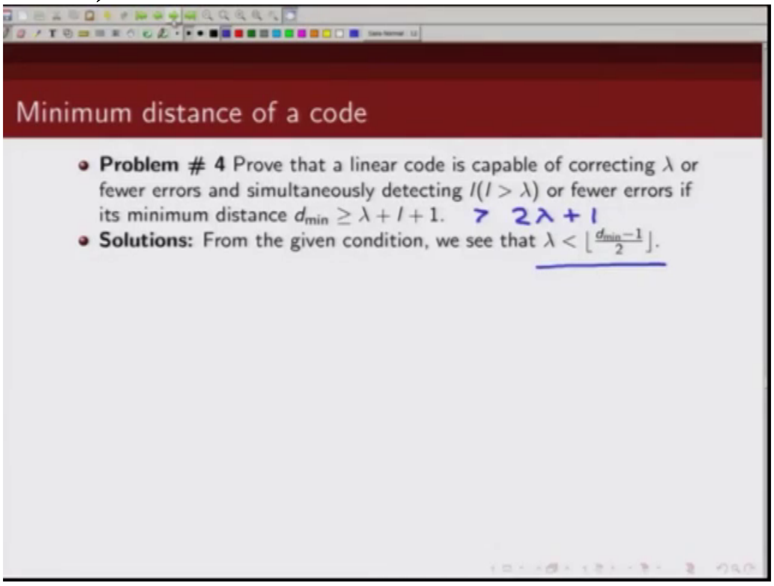

Now note we want to,

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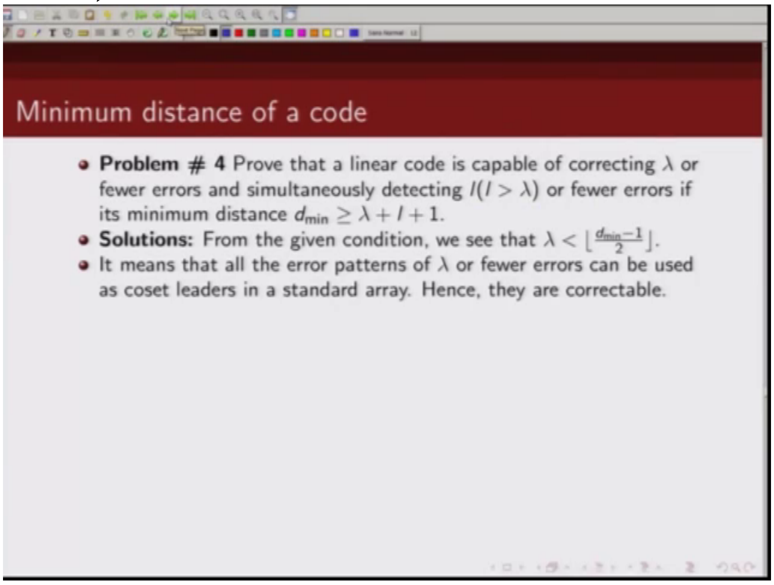

in addition to correcting lambda or fewer errors, we also want to simultaneously detect l errors. Now if we want to simultaneously detect those l or fewer errors we have to ensure that

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those error patterns of weight l or less are not in the same coset as the error patterns that we are

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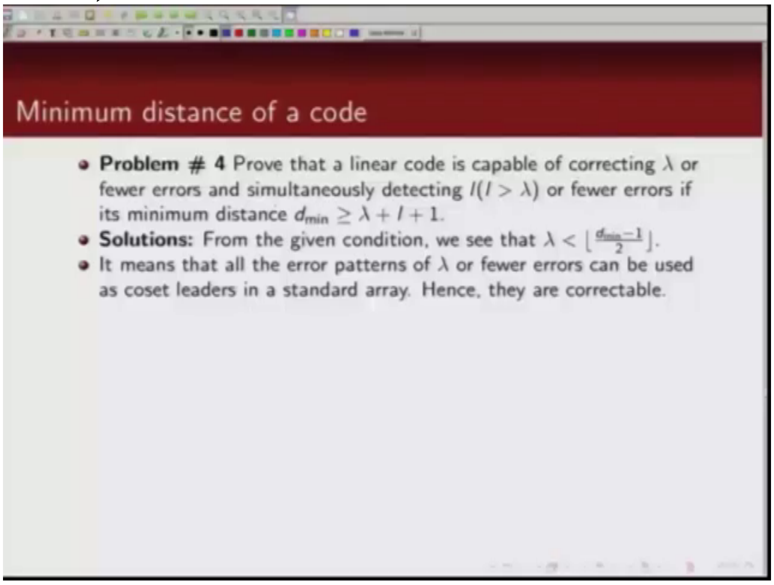

trying to correct. Now since lambda errors can be corrected we can put all error patterns of lambda or fewer errors as coset leader in our standard array and they can be correctable.

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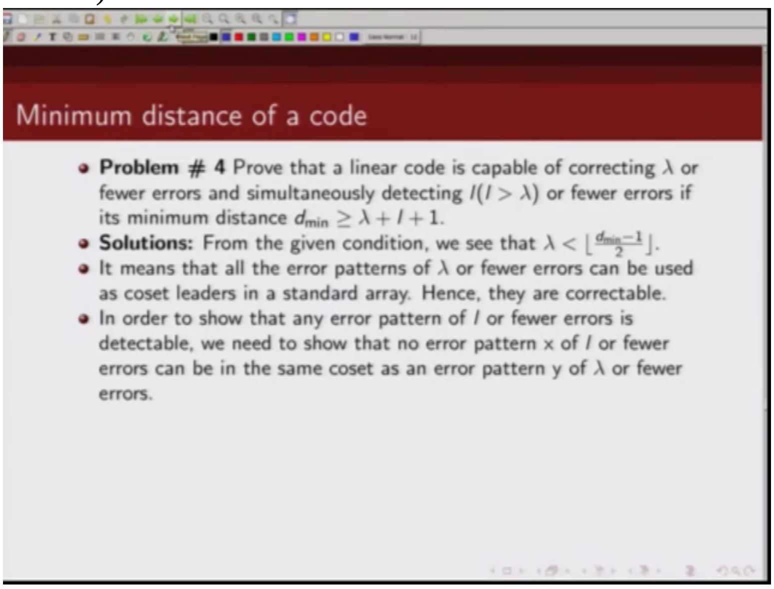

Next, to simultaneously detect l errors we have to show that none of these error patterns of weight l or less

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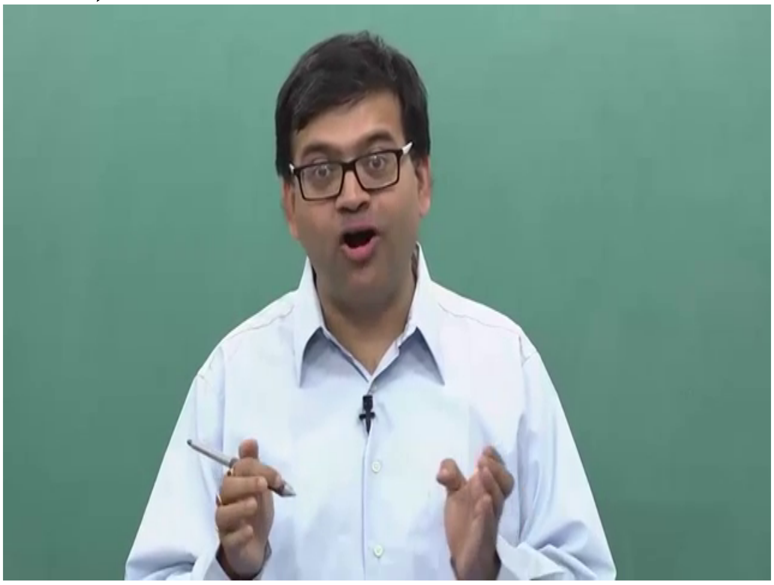

are in the same coset as these error

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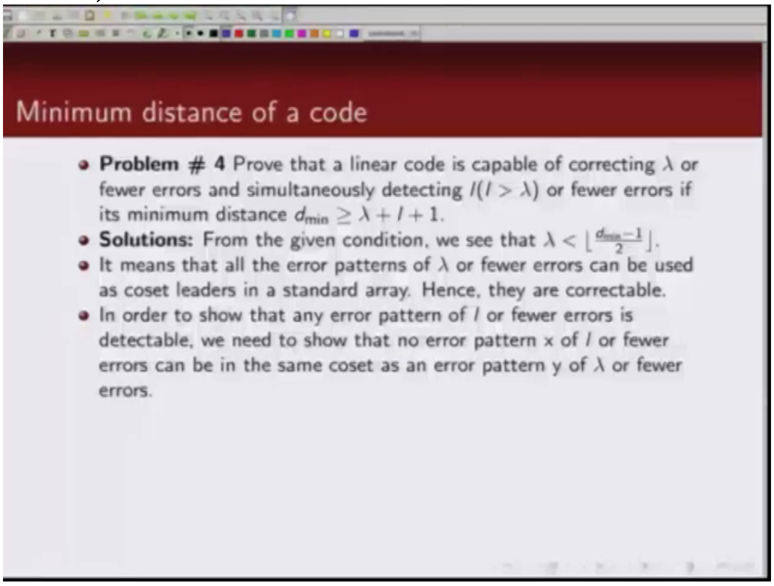

patterns of lambda or less error. So we need to show that no error pattern x of length l or fewer are in the same coset as error pattern y of lambda or fewer errors. If they are in the same coset, because we are using those coset leaders for error correction, we would not be able to detect those error patterns. So it is important that those error patterns of weight l or less,

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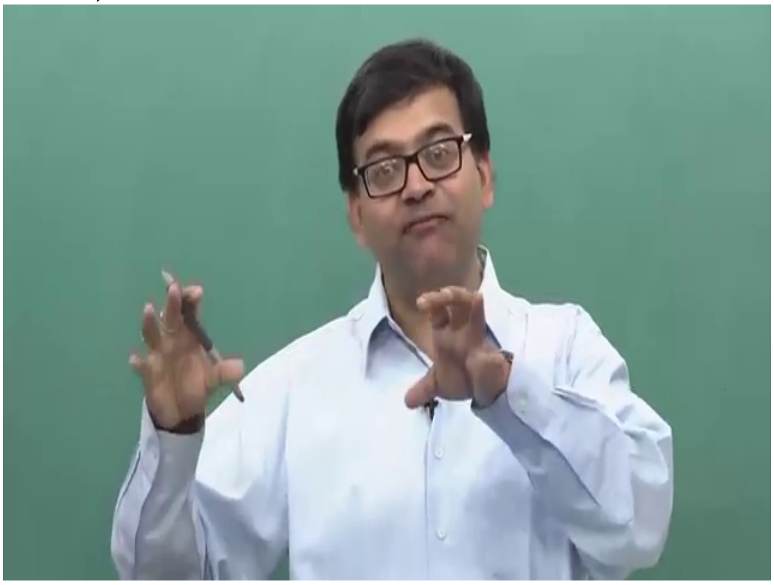

if we want to detect them, they should not be in the same coset as the correctable error patterns.

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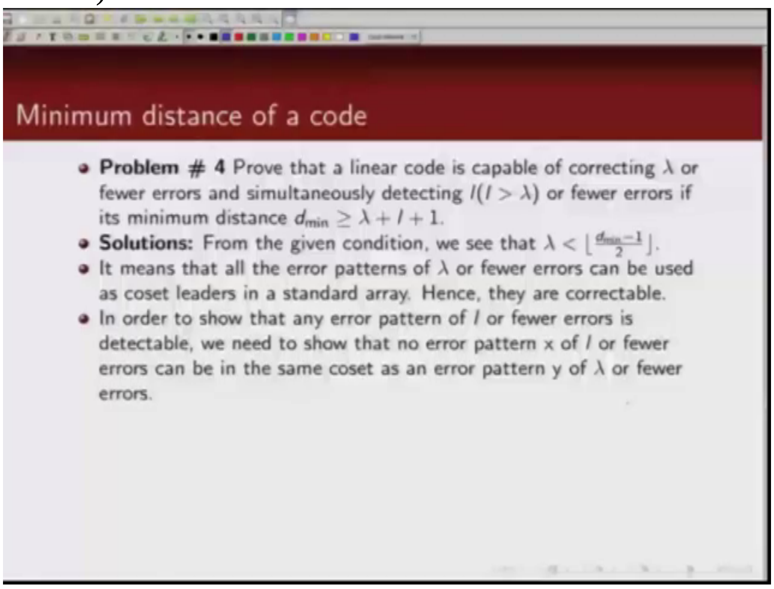

So

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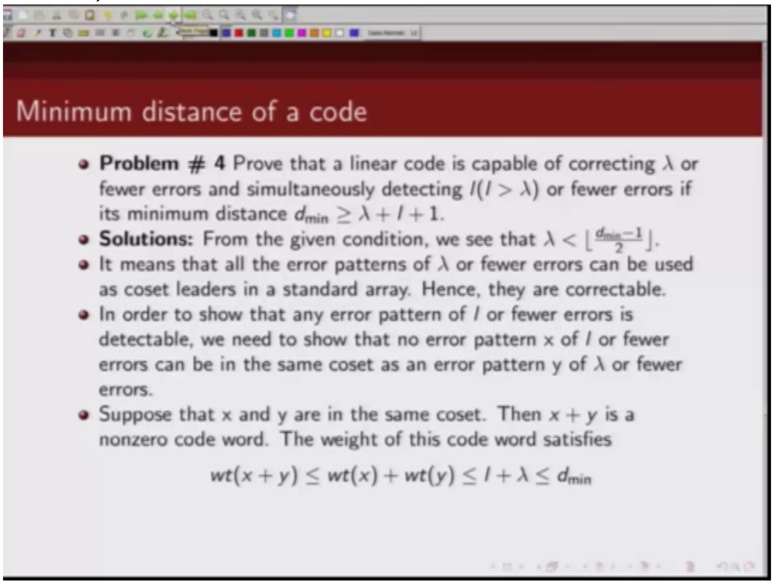

So we are now going to use method of contradiction to show that it is not possible to have these error pattern x of

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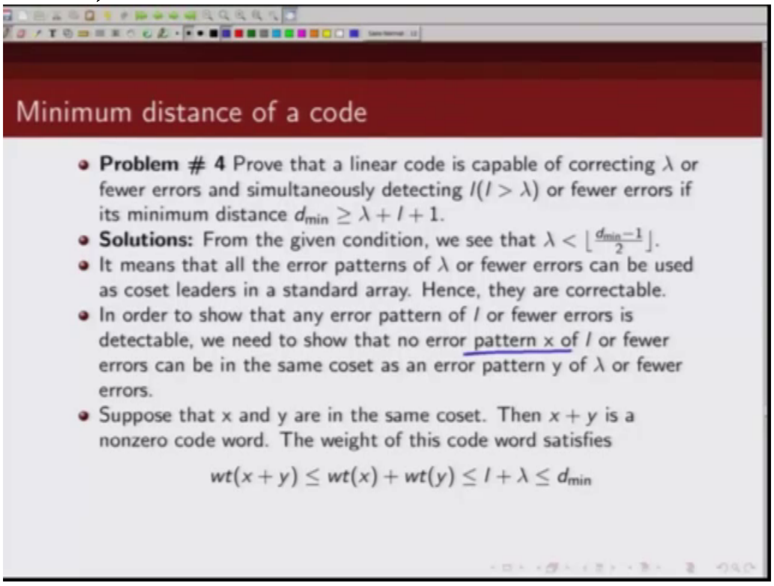

l or fewer errors in the same coset as these error patterns y

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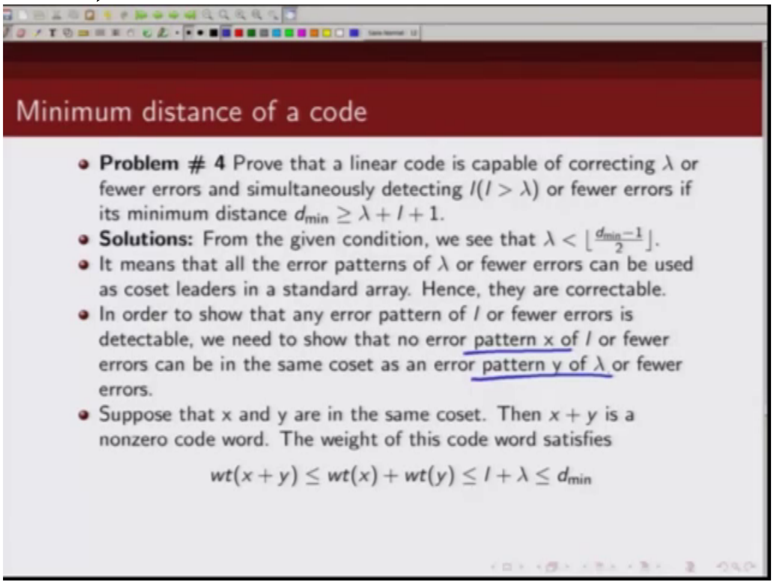

of lambda or fewer errors which we are trying to correct. So how does this method of contradiction work? We will first assume that they are in the same coset and then we will show that this is not possible. Hence our assumption that they are in the same coset is wrong.

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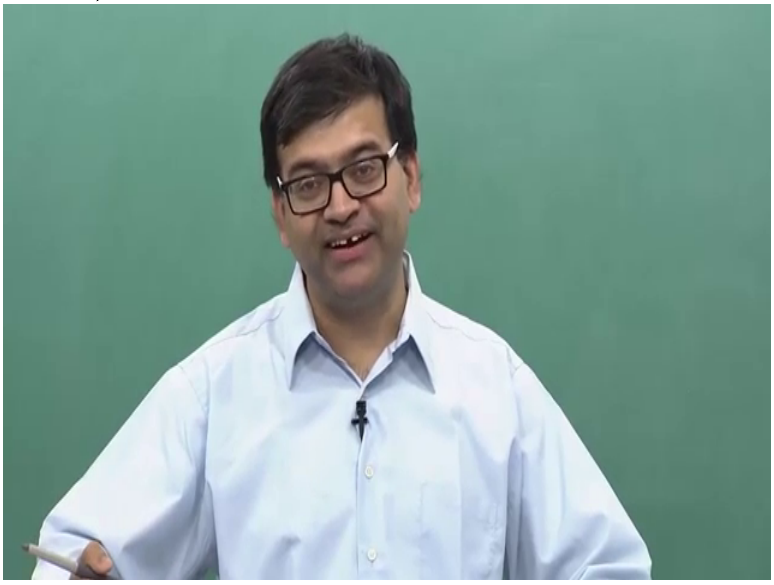

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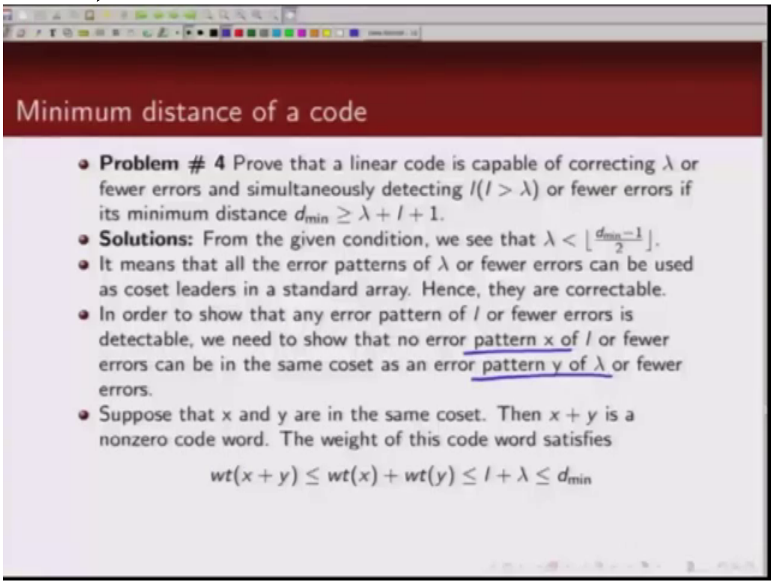

So we start our proof by saying these error pattern x of weight l or less and error pattern y of weight lambda or less, they are in the same coset. Now if x and y are in the same coset we know from our

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standard array that x plus y

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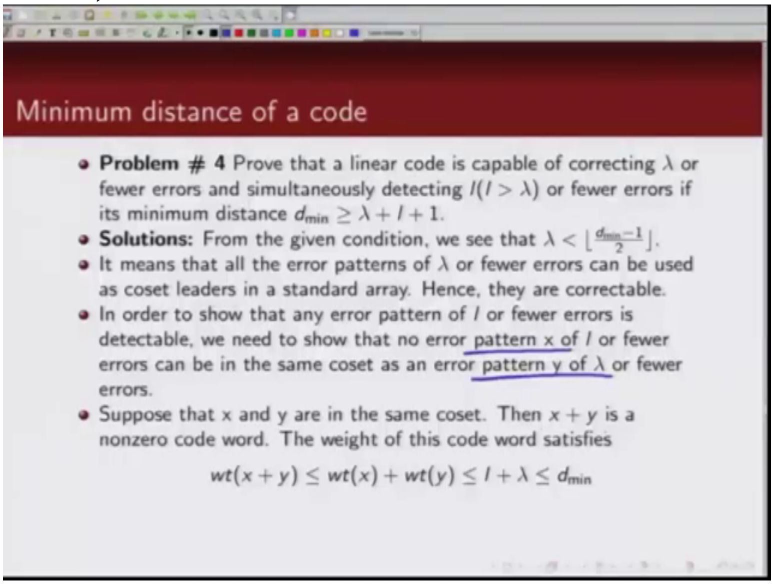

should be a non-zero codeword. If you recall the entries of standard array we have in the first column an all zero codeword and then we had other codeword v 2, v 3 ... and then what we had was error pattern

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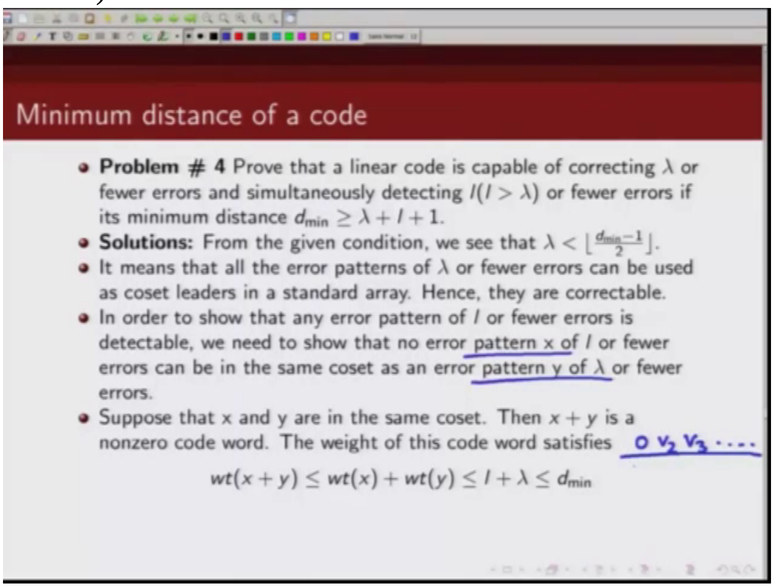

e 2 and then we had basically this was e 2 plus v 2 like that we had and if you add any two elements of a coset or a row

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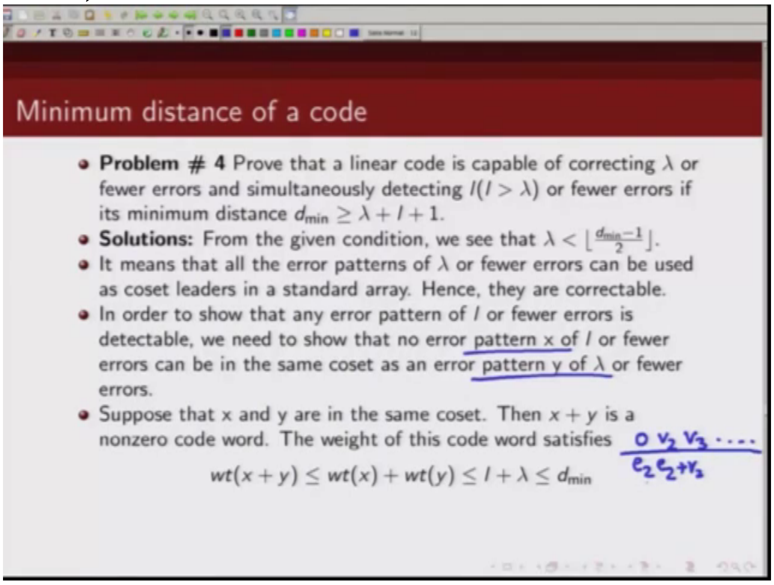

what you will notice is sum of them is a valid codeword. So if x and y are in the same coset, x plus y must be a non-zero codeword. Now let's look at what is the weight of x plus y. So weight of x plus y is less than equal to weight of x plus weight of y because it is possible that there are some common elements between x and y,

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that's why the weight of x plus y

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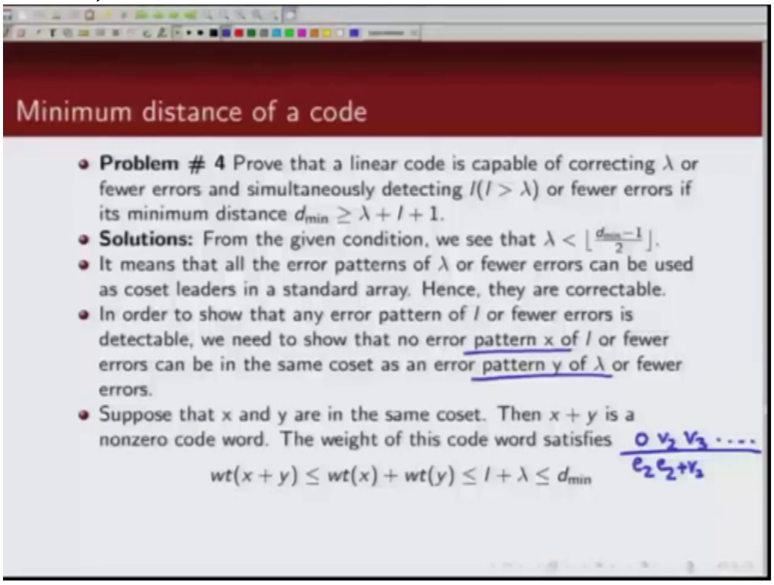

is less than equal to weight of x plus weight of y. And what is weight of x? x are the error pattern of weight l or less. So the maximum weight of x is l. Similarly maximum weight of y is lambda. So weight of x plus y is then less than equal to lambda plus l. And what is the minimum distance? Minimum distance code is at least lambda plus l plus 1. So weight of x plus y is then less than d min. So what we have shown is the weight of x plus y, x plus y should have been a codeword, is a codeword if they are in the same coset. If x and y are in the same coset, x plus y is a valid non-zero codeword but what we have shown here is weight of x plus y is less than d min so x plus y is a valid codeword. Its minimum weight

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should be at least d min. So

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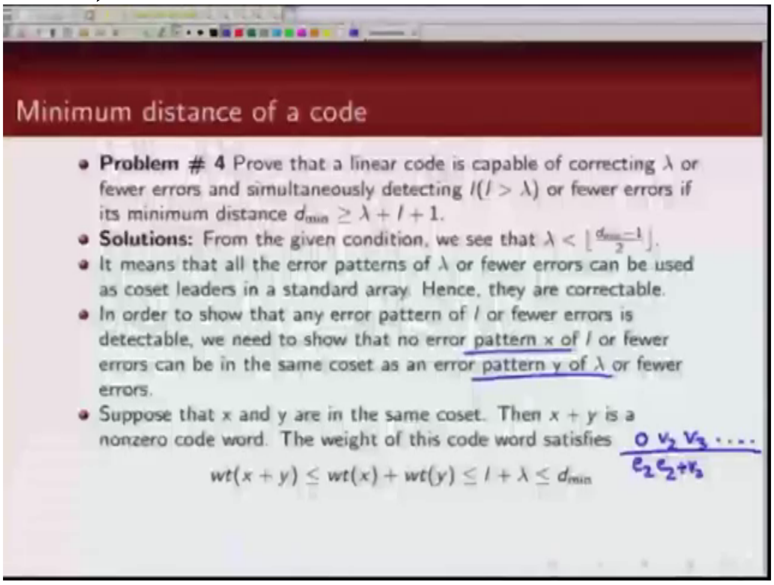

from here, basically what we get is it is not possible to have x plus y in the same coset. Because if they were in the same coset x plus y would have been a valid codeword and its weight of x plus y should have been more than d min but here in this case it is coming out to be less than d min. Hence our assumption that x and y are in the same coset

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is wrong. Now if

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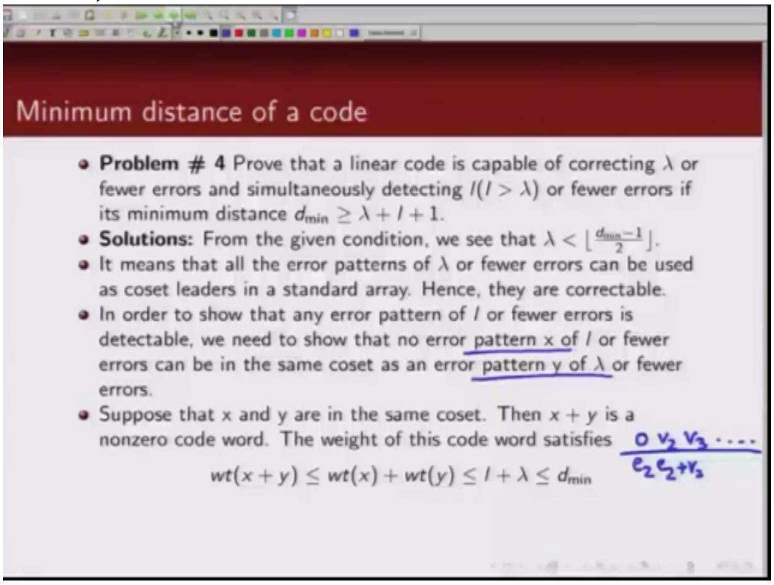

x and y are not in the same coset

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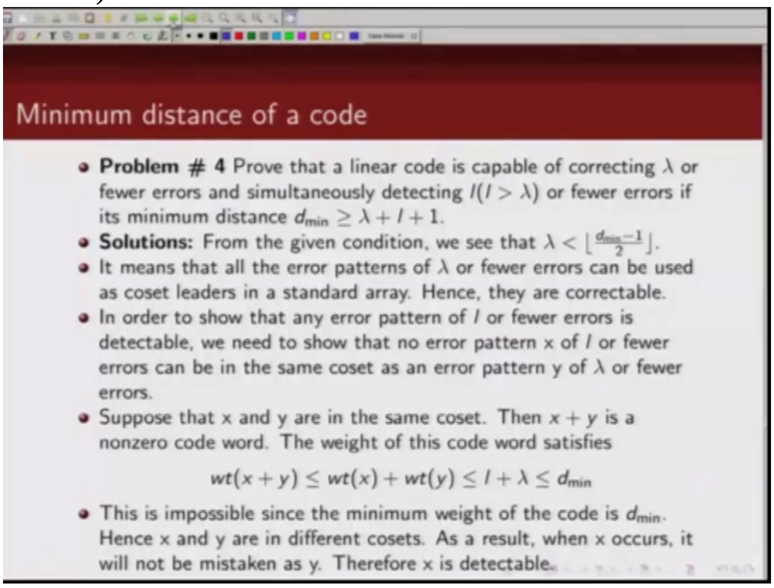

then we can always put those error patterns of

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y and x in different cosets and hence we can simultaneously

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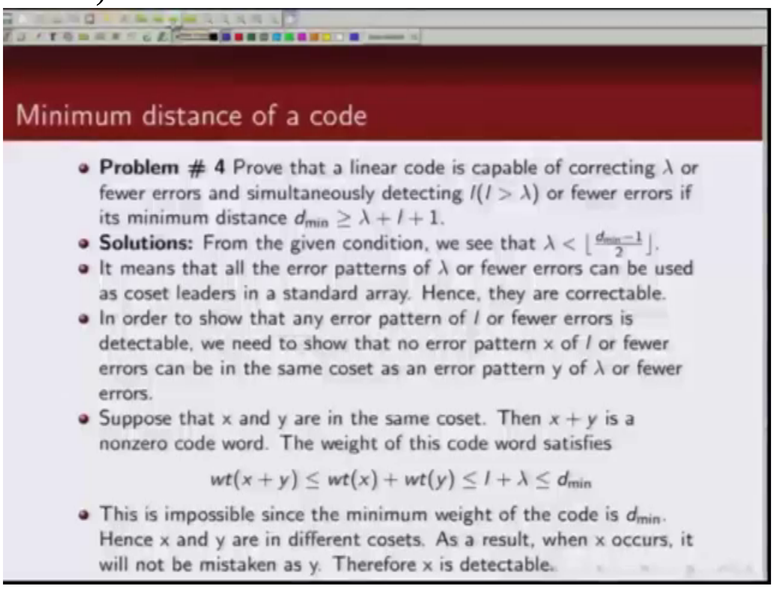

detect and correct errors. So we can simultaneously correct lambda errors while detecting also l errors, Ok. So again to recap, basically we prove this result by showing that if you want to simultaneously correct and detect errors

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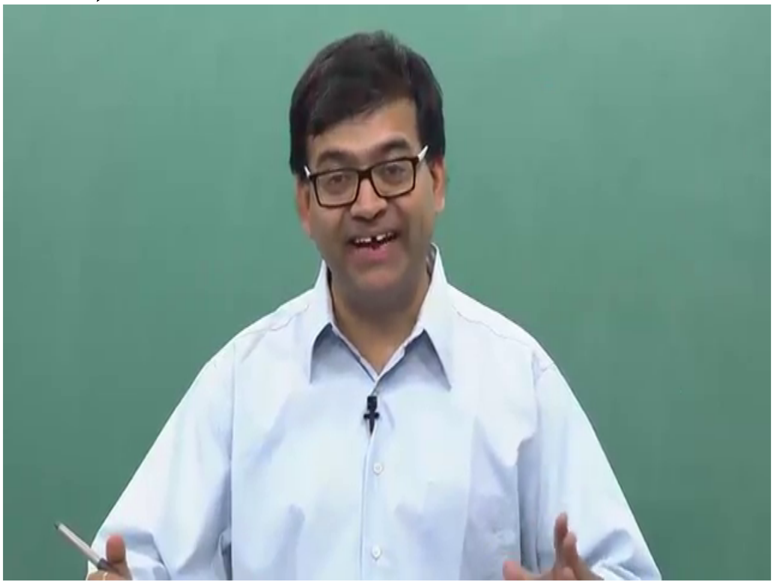

those error patterns should be in the different cosets and hence we can

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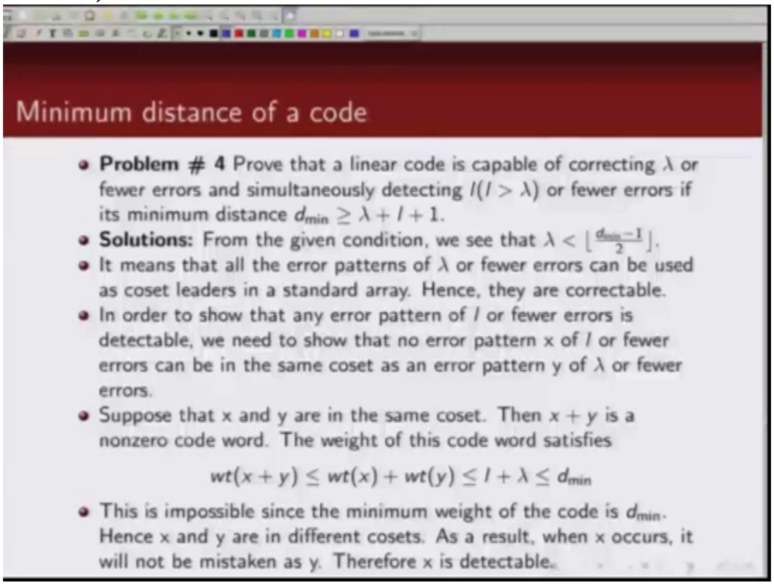

simultaneously correct and detect those error patterns.

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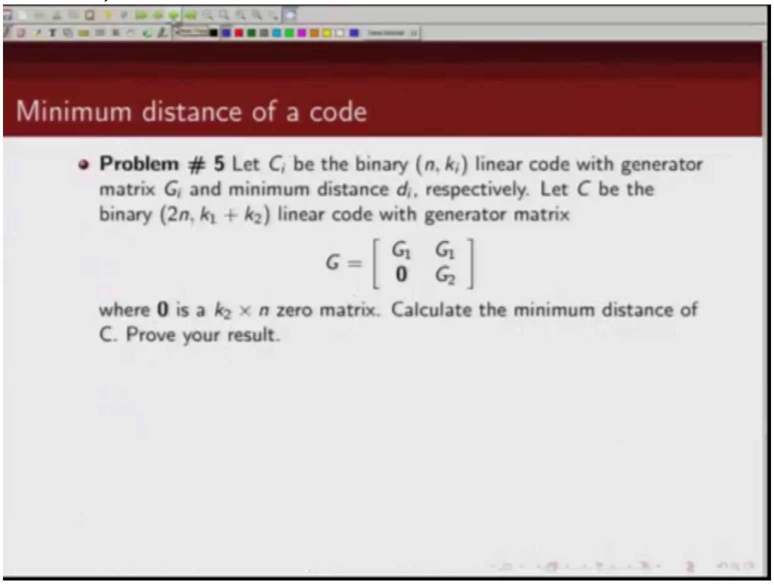

The next problem that we are going to solve is as follows. Let C i be a binary linear code with code parameter given by n, k i with generator matrix G i and minimum distance d i. And let us consider a new code C, a new binary code linear code of length 2 n and message bit length k 1 plus k 2 whose generator matrix is given by this expression. Now what is the minimum distance of this new code C?

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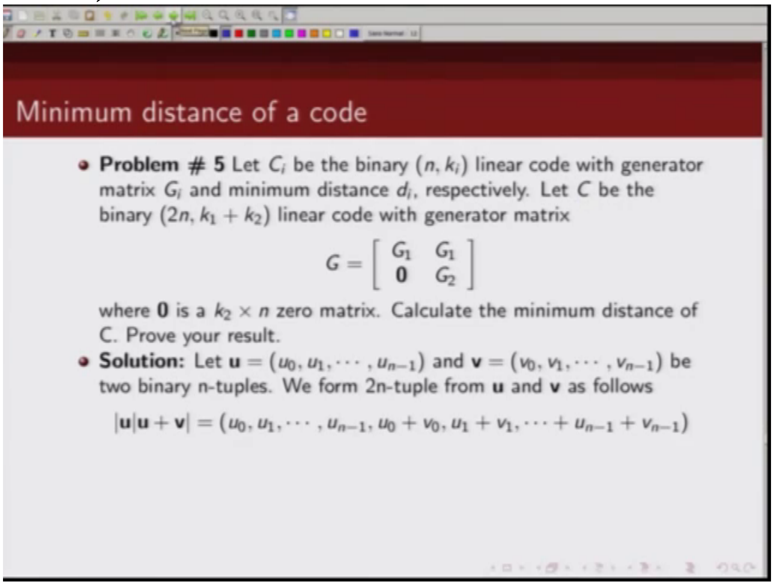

So to find out the minimum distance, so let's consider let u and v are two binary n-tuples. And we form a 2 n-tuples as follows. If we look at this codeword v how is this codeword generated? So it's one n-bit codeword, another n-bit codeword. First n-bit codeword is generated by using u times v 1 and second one you get basically u times G 1 plus v times G 2.

So essentially the way you are generating this codeword, the first part contains n-bit codeword u and the second part is n-bit codeword which is

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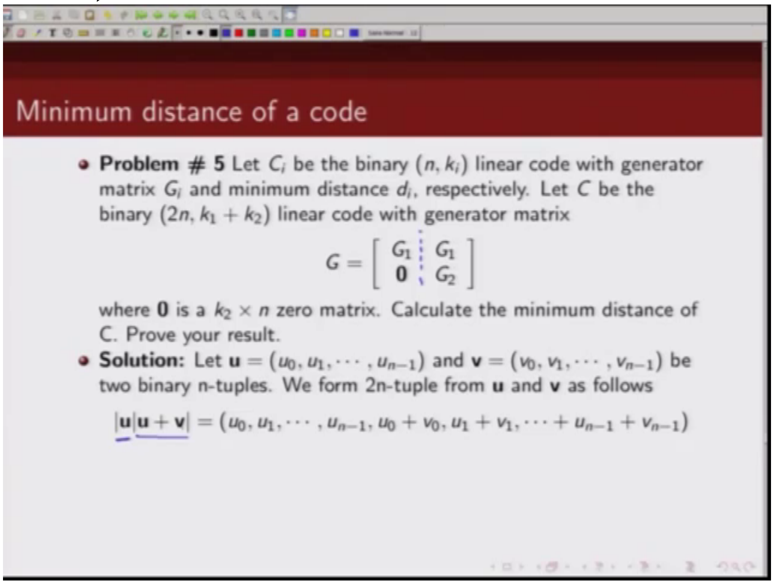

u plus v. So this 2 n length codeword will be of the form like this

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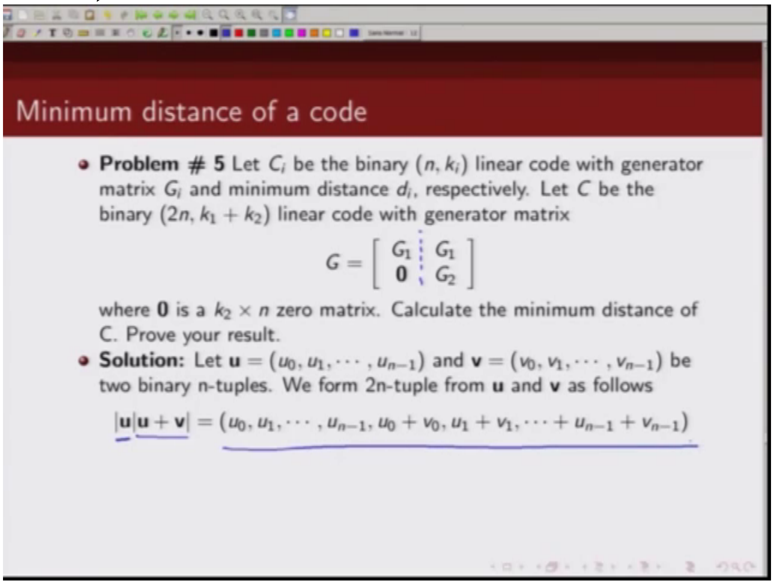

where the first 10 bits are u 0, u 1, u 2, u n minus 1 and then next n bits are of the form u 0 plus v 0, u 1 plus v 1 and like that, so

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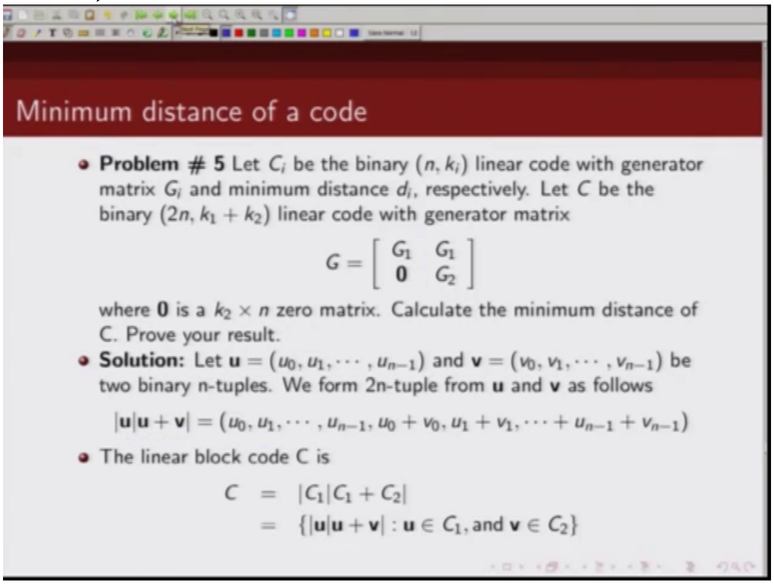

as I said our linear block code, see the new code of length 2 n can be written of this form where you have a code u of length n which belongs to C 1 and then the second part which is n bit part is u plus v where u belongs to C 1 and v belongs to C 2.

Now we will

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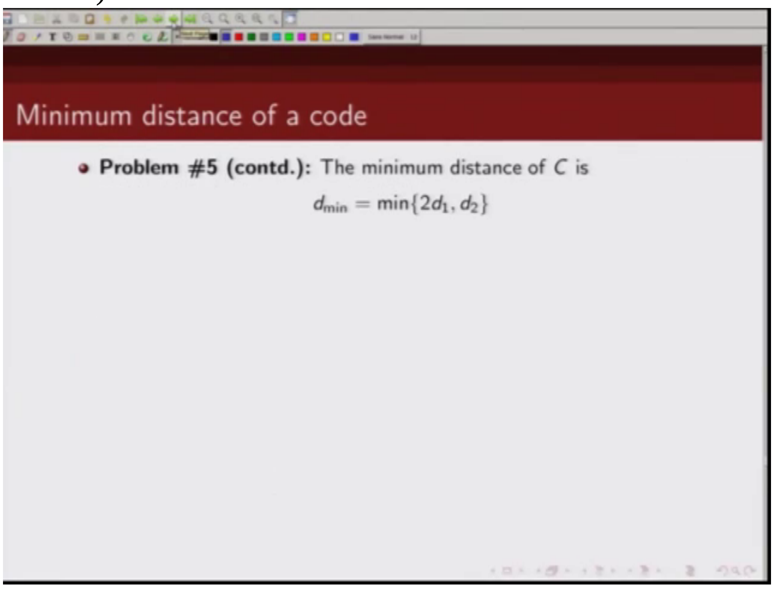

show that minimum distance of the code is minimum of two d 1 or d 2 where d 2 is the minimum distance of the code n k 2 and d 1 is the minimum distance of the code n k 1. So

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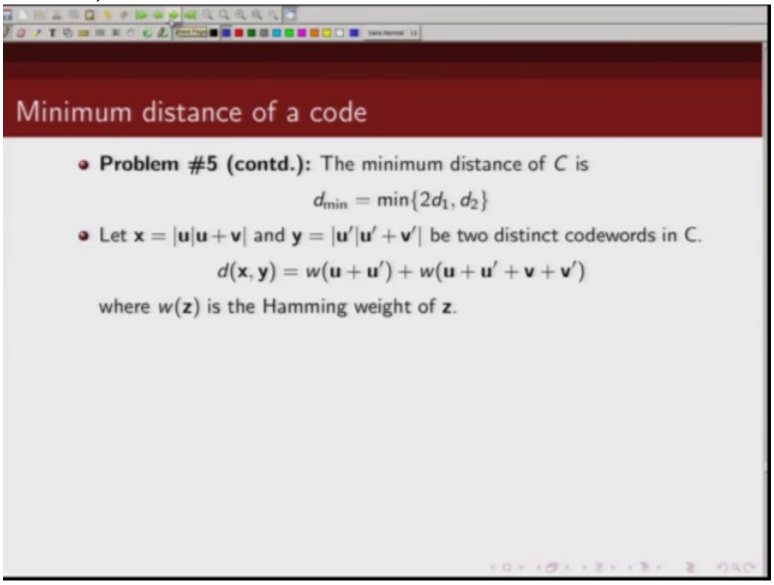

let us consider two distinct codewords x and y. So x we denote as concatenation of u and u plus v and this is u prime, u prime plus v prime. Let x and y be two distinct codewords in C. Now what is the Hamming distance

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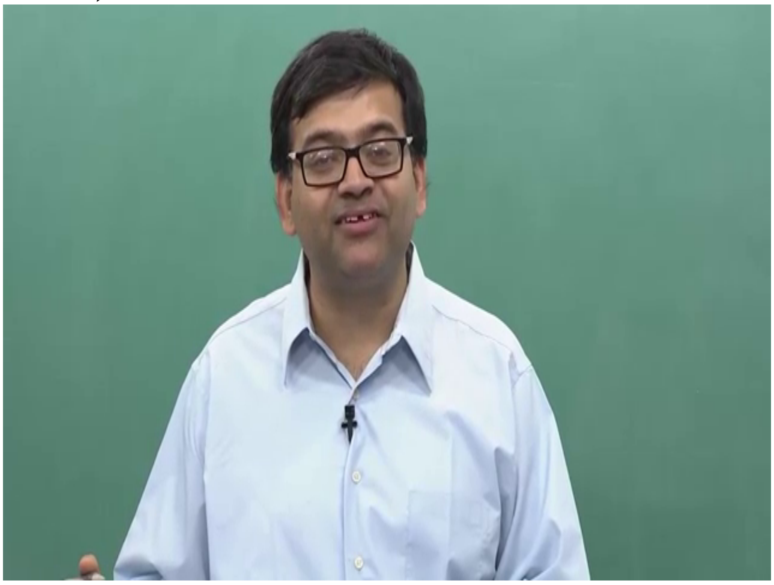

between x and y? We can write down the Hamming distance between x and y as

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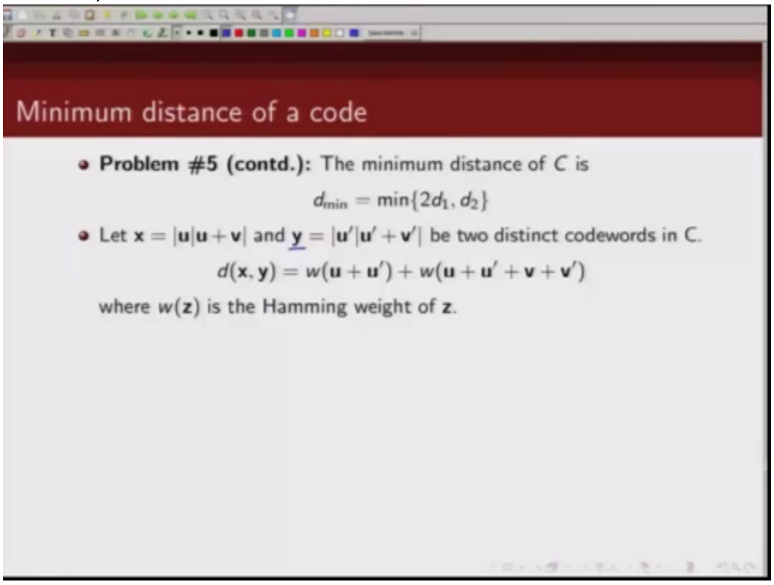

the Hamming weight between u plus u prime plus Hamming weight between u plus v plus v hat plus u hat. So the Hamming distance between x and y can be written as Hamming weight of u plus u hat plus Hamming weight of u plus u hat plus v plus v hat.

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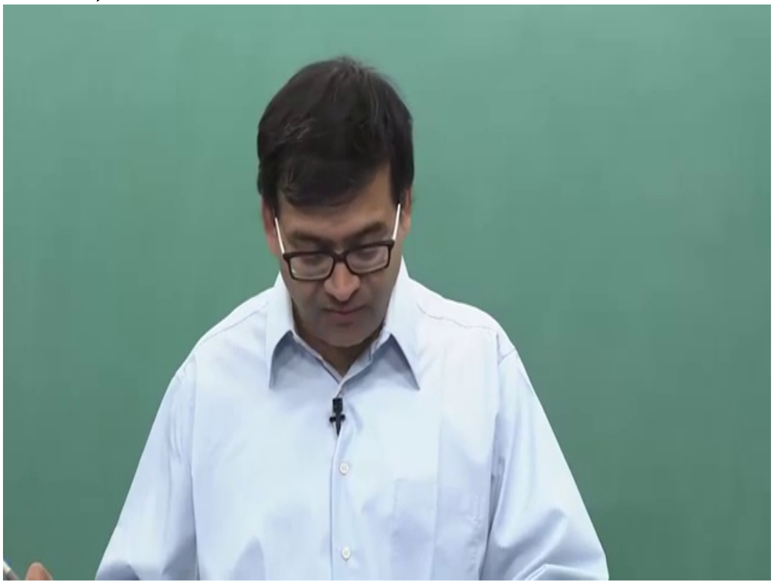

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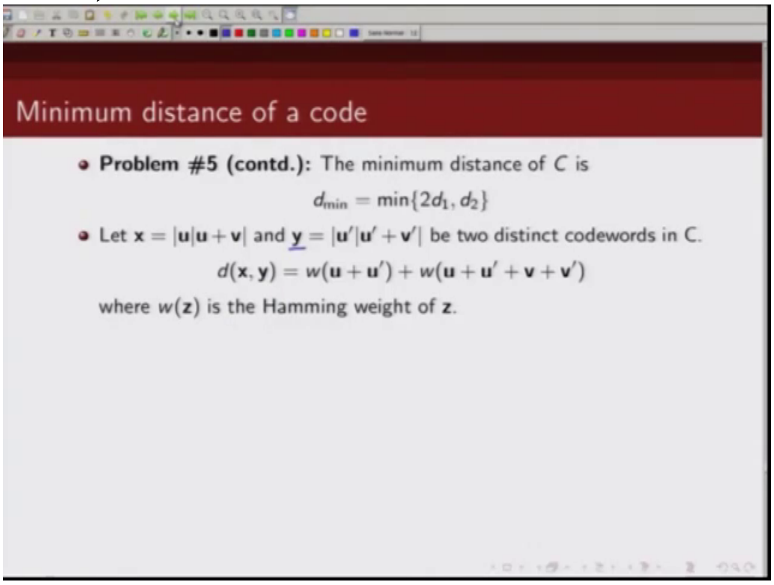

Now note

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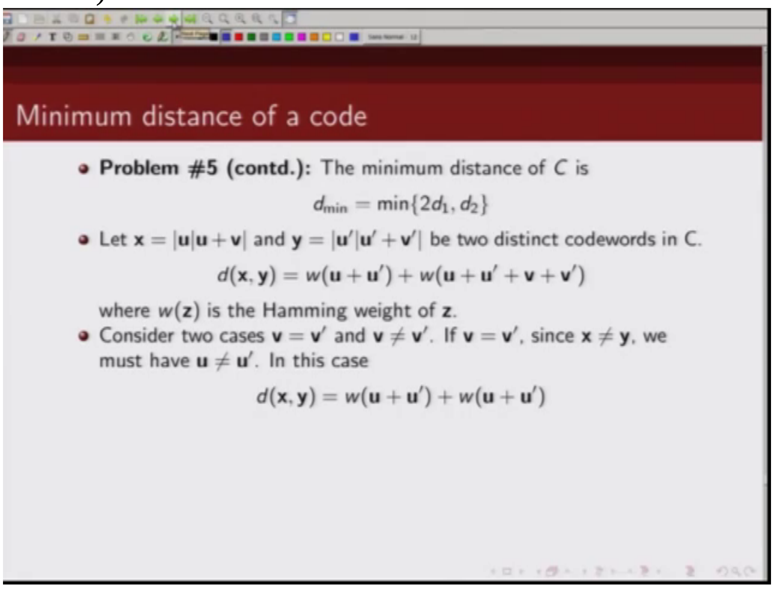

x and y are distinct codewords. So let us consider two scenarios. In first case we will consider v is same as v prime. In second case we will consider v is not same as v prime. So if we consider v as same as v prime, since x and y are distinct codewords what we will have is u is not same as

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u prime. So in this case the Hamming

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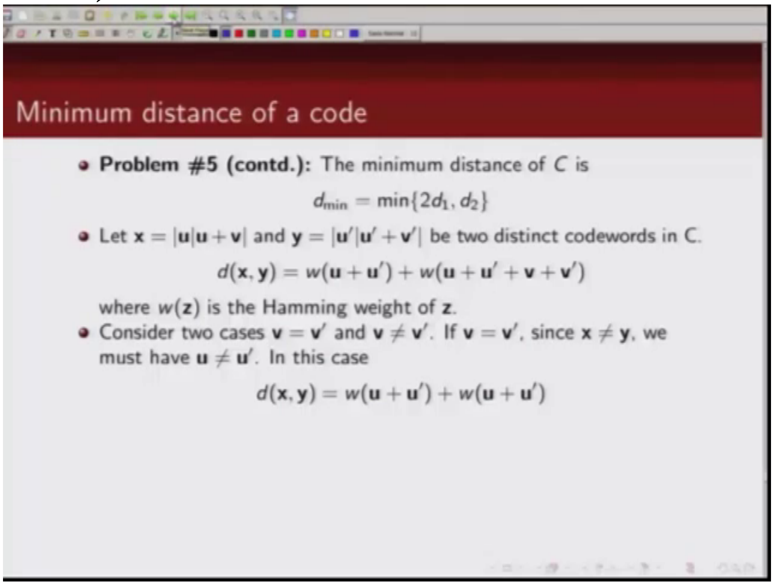

distance between x and y will be given by Hamming weight of u plus u prime plus, now since v and v prime are same this will be zero so this will be same as Hamming weight of u plus u prime. So then in this case when v is same as v prime we can write the Hamming distance between x and y as Hamming weight of u plus u prime plus Hamming weight of u plus u prime. And since what is u plus u prime?

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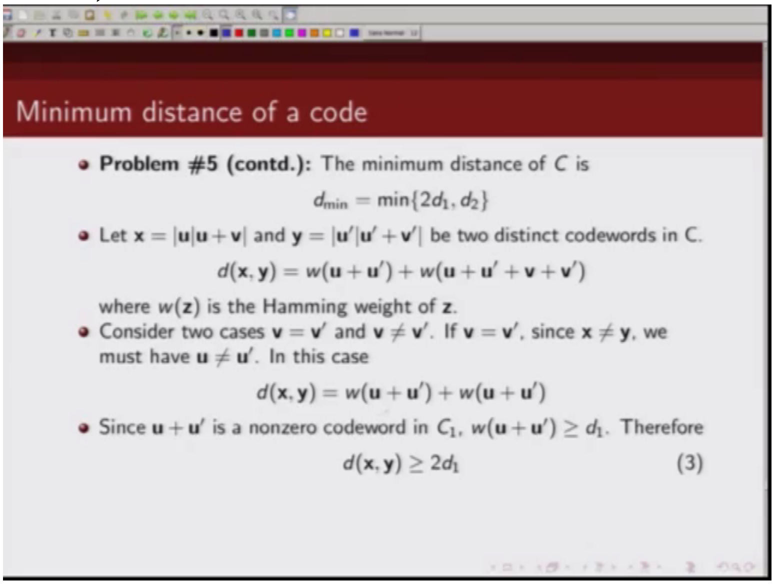

u and u prime are two codewords belonging to

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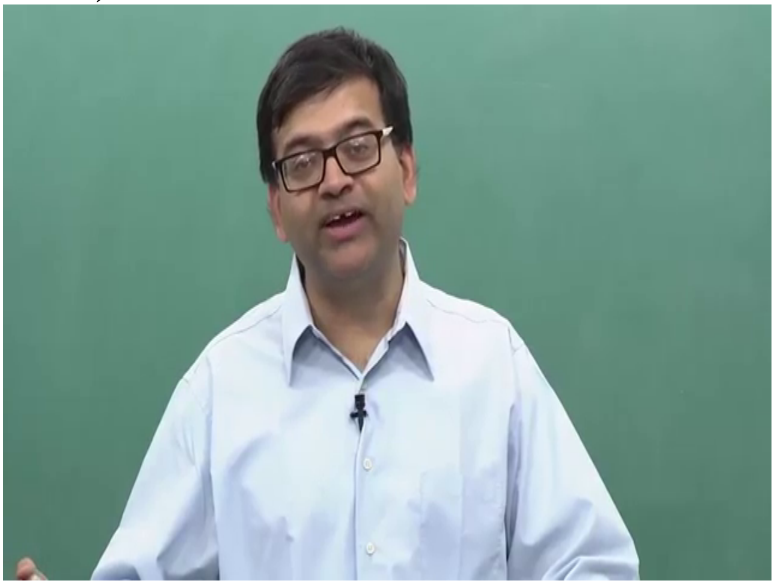

C 1. So sum of two codewords for a linear block code is another valid codeword. So u plus u prime is going to be

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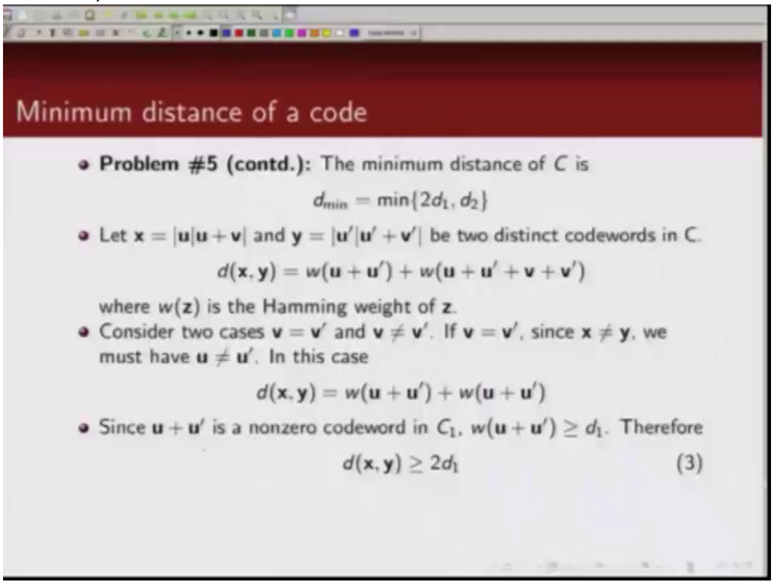

another valid codeword. So then, so what would be the minimum distance of u plus u prime? It would be at least the minimum distance of the code C 1 which is d 1.

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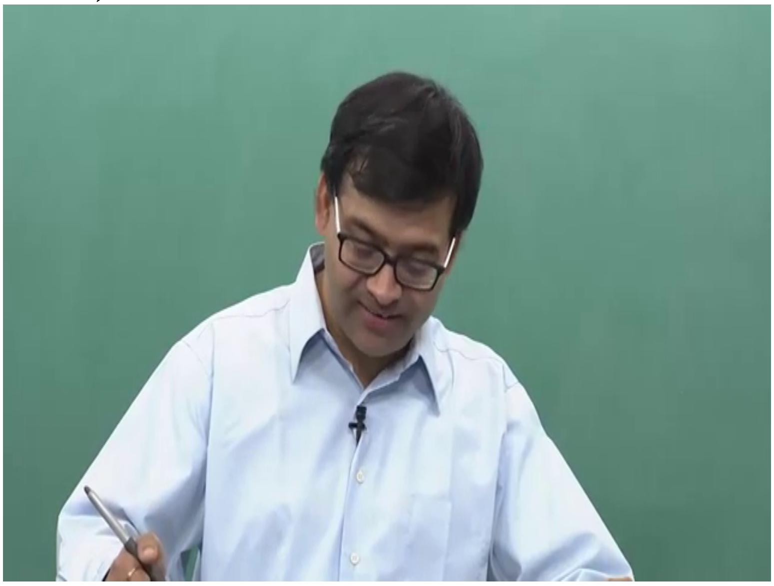

So then Hamming distance between x and y would be

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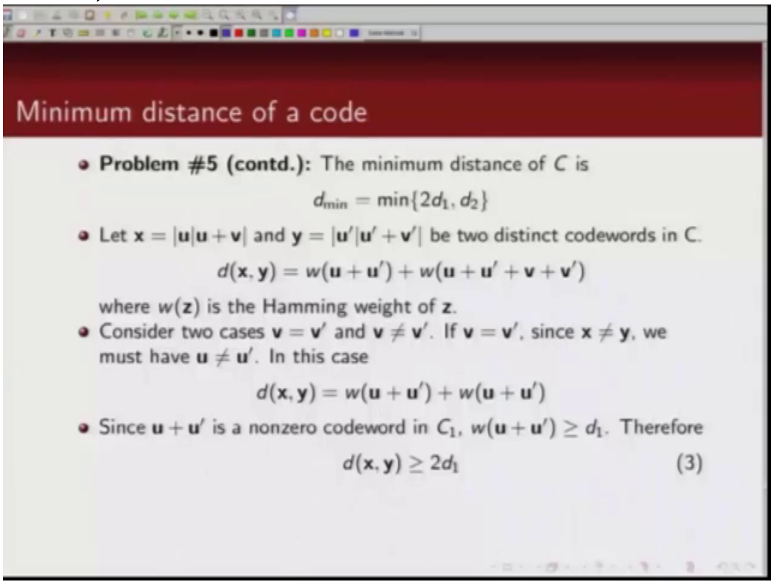

greater than equal to 2 times d 1. So for the case when v is equal to v prime we have shown that minimum distance

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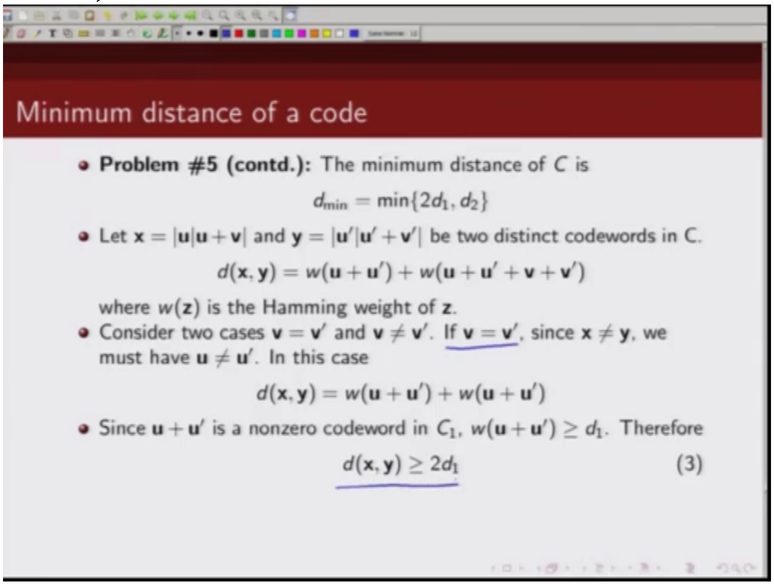

should be at least 2 times d 1.

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Now let us consider the case when

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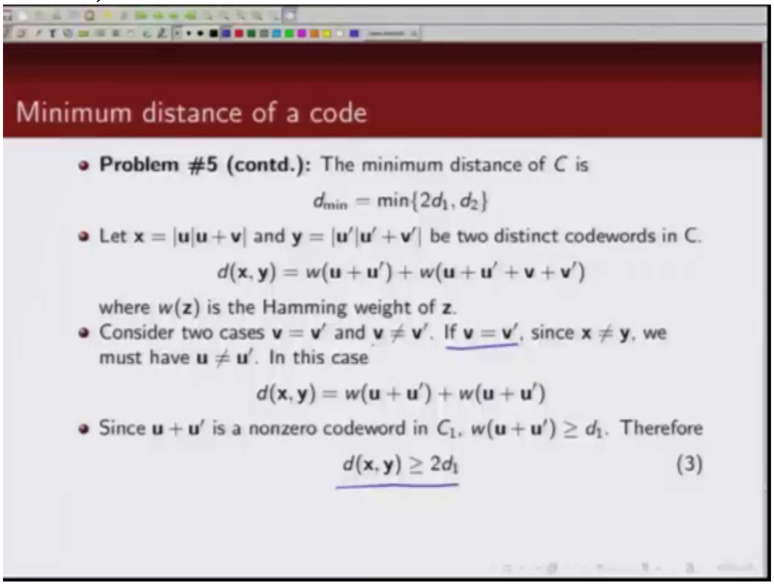

v is not equal to v prime. So before that we will just

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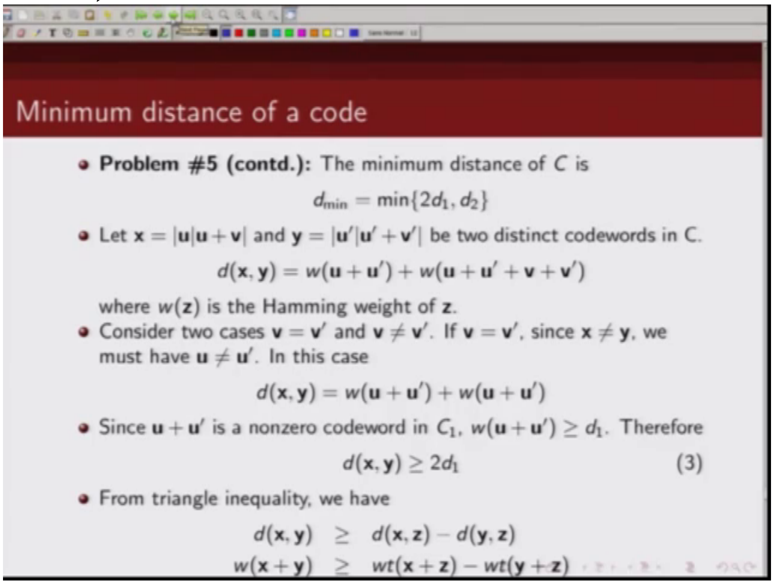

state again the triangular inequality that we are going to use; so from the triangular inequality we know that Hamming distance between x and y is greater than equal to Hamming distance between x and z minus Hamming distance between y and z. And we know the Hamming distance is nothing but Hamming weight of x plus y, Hamming weight of x plus y and Hamming weight of x plus z minus Hamming weight of

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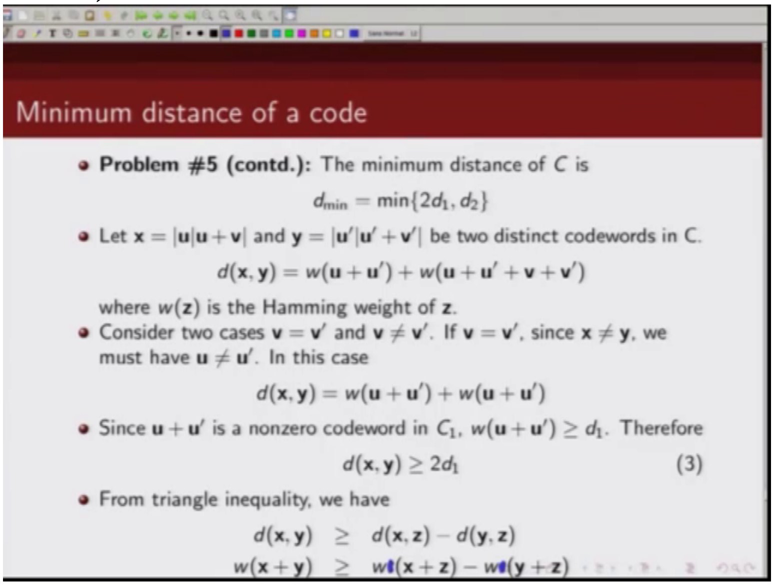

y plus z. So we can write this expression in terms of Hamming distance or we can write in terms of Hamming weight. Now let us take x plus z to be

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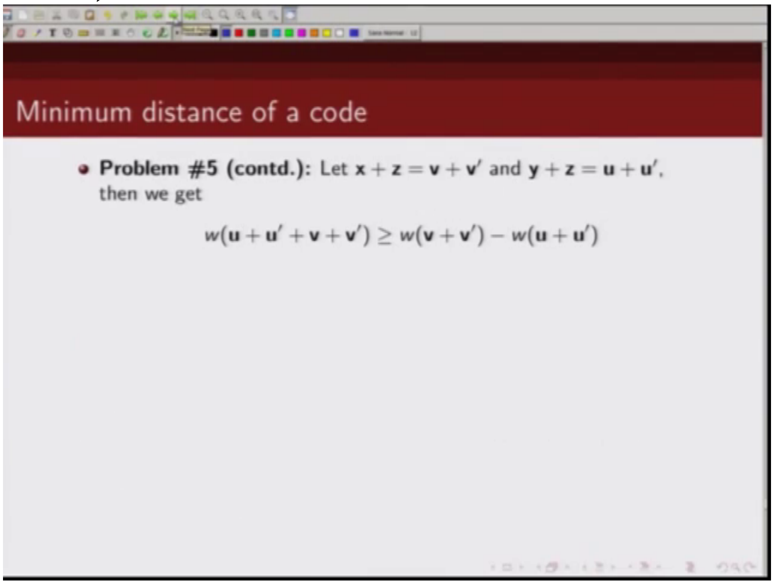

equal to v plus v prime and y plus z as u plus u prime. And we put these values of x

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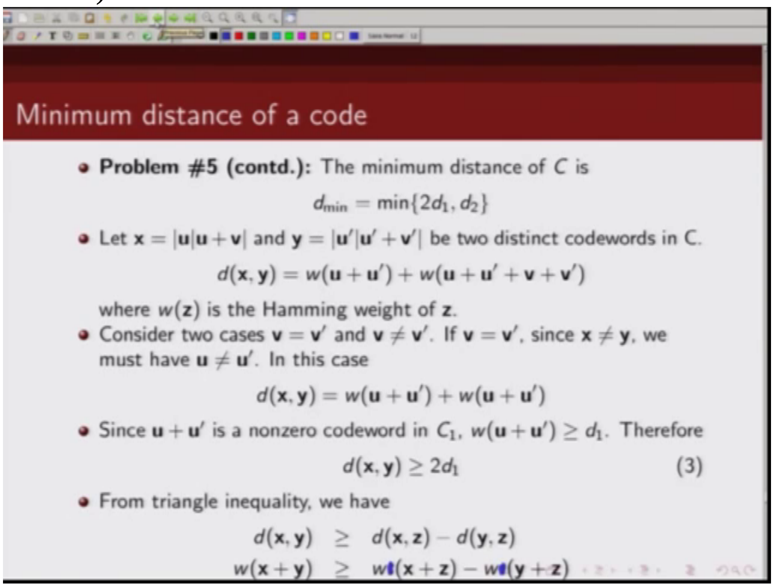

and x plus y and x plus z and y plus z, we put these values

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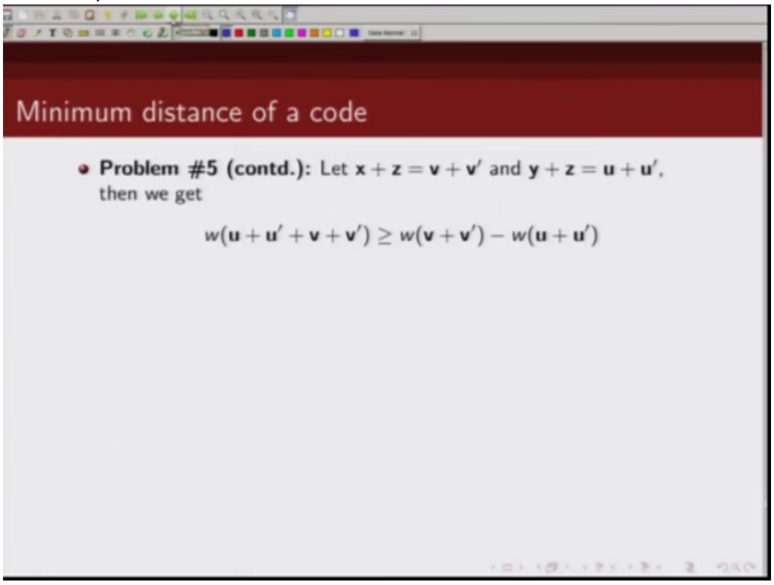

in this expression.

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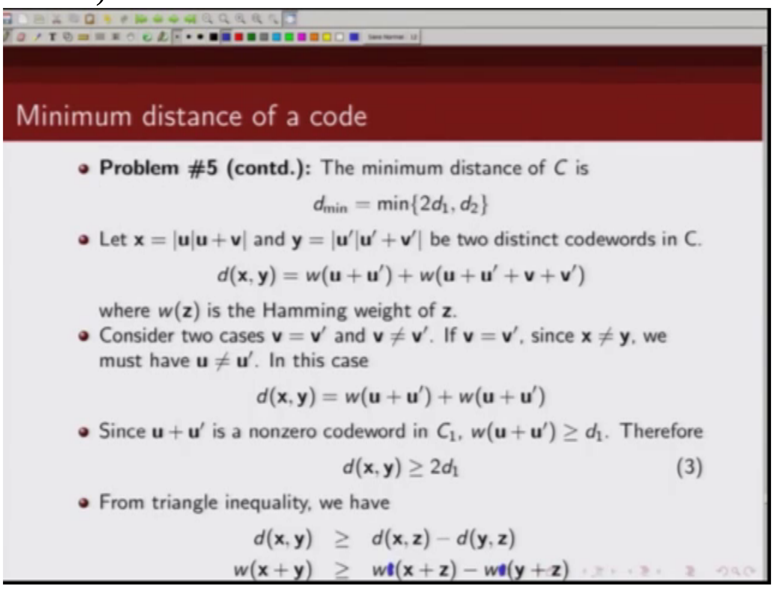

So what is x plus y? x plus y

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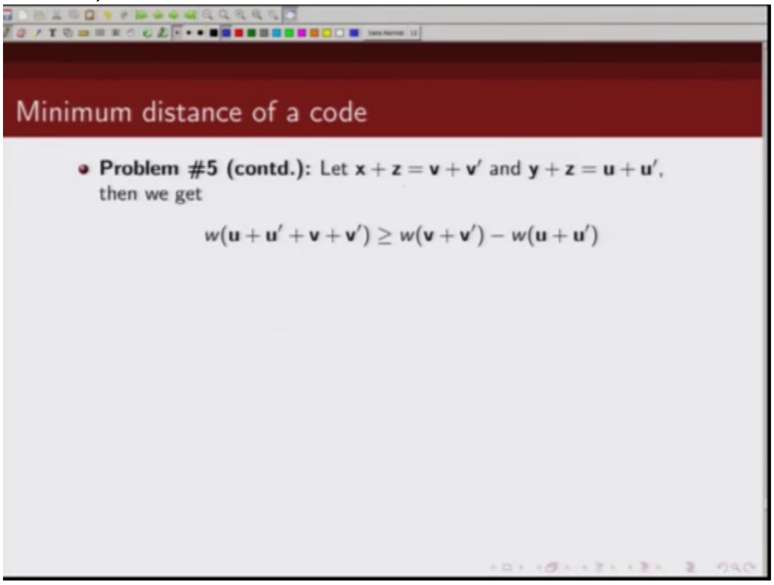

would be v plus v prime plus u plus u prime so x plus y is basically u plus u prime plus v plus v prime. So from here weight of

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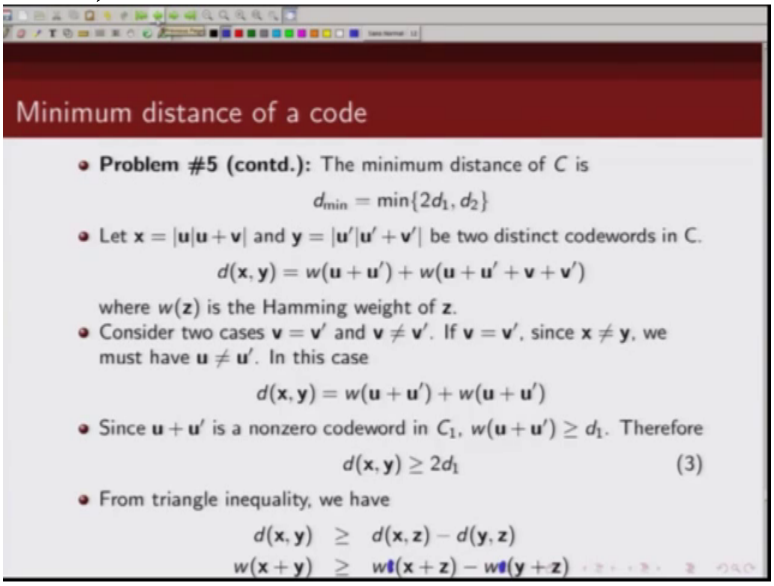

x plus y is given

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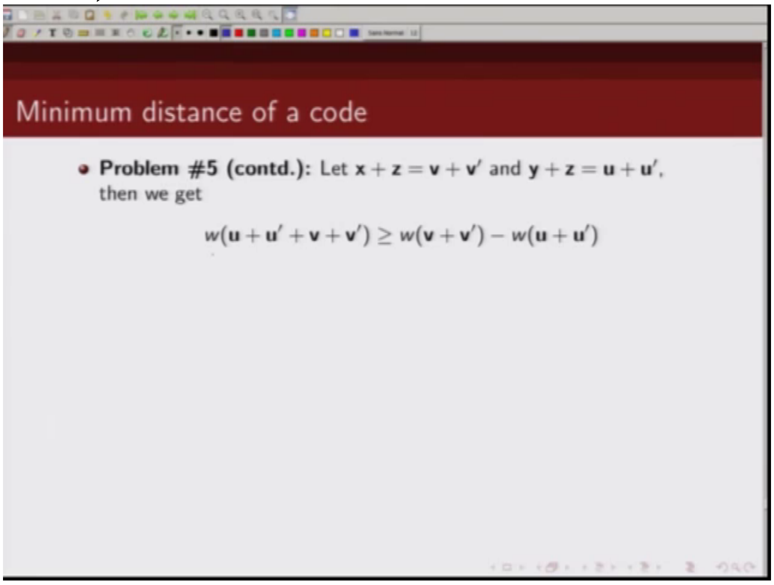

by this.

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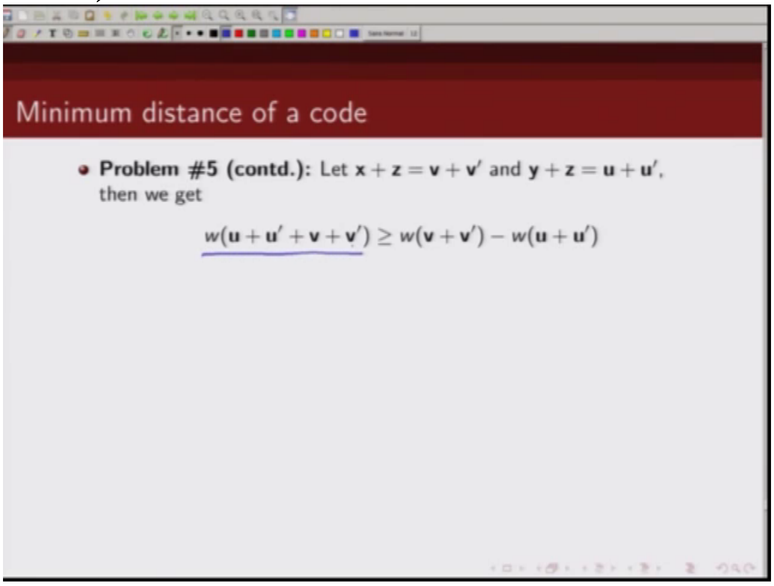

Next what we had was weight of

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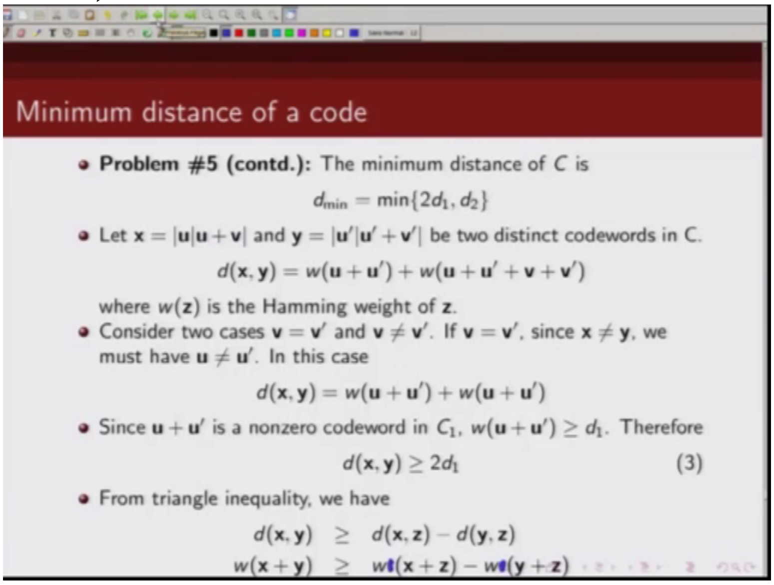

x plus z. What is weight of x plus z?

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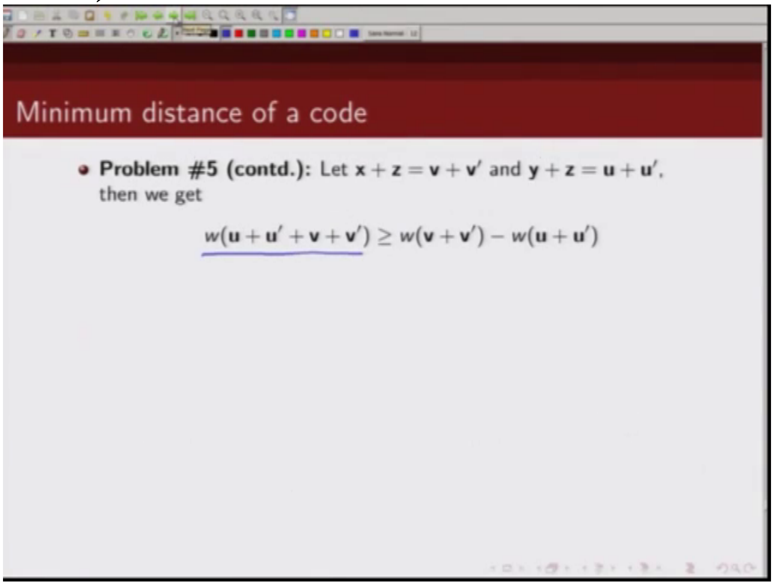

Weight of x plus z is weight of v plus v prime and similarly weight of y plus z is given by this, Ok. So this is upper bounded, this is lower bounded by this quantity. This is lower bounded by this quantity. Now go back and

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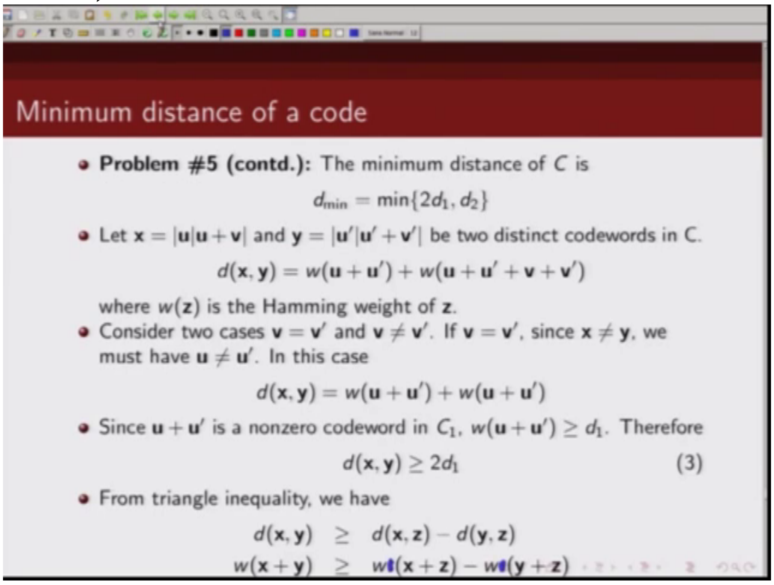

see what is our minimum distance between x and y? Minimum distance between x and y is given by

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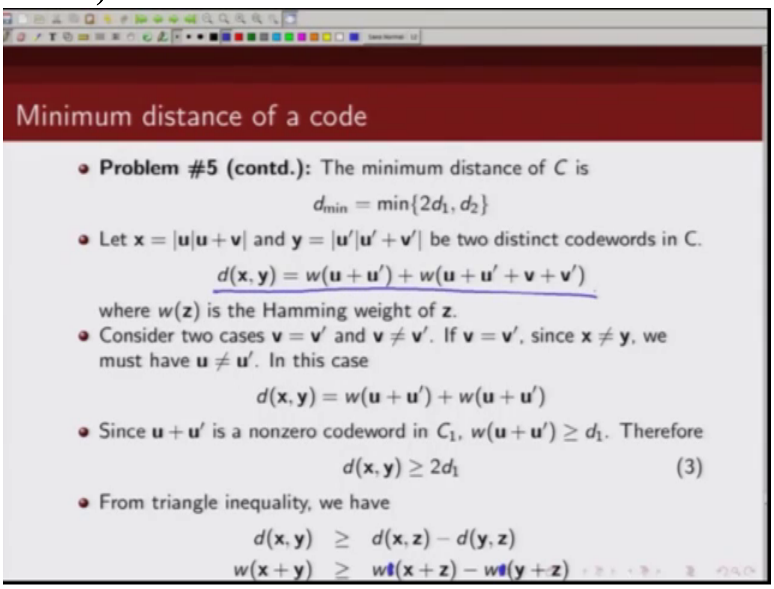

this expression. It's Hamming weight between u and u prime

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plus Hamming weight of u plus u prime plus

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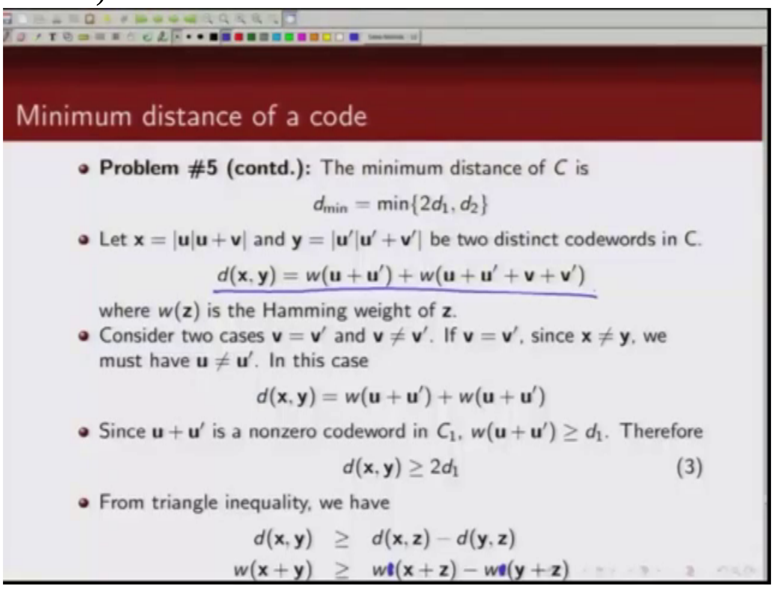

v plus v prime. And what we did just now is we lower bounded this. So then Hamming distance between x and y

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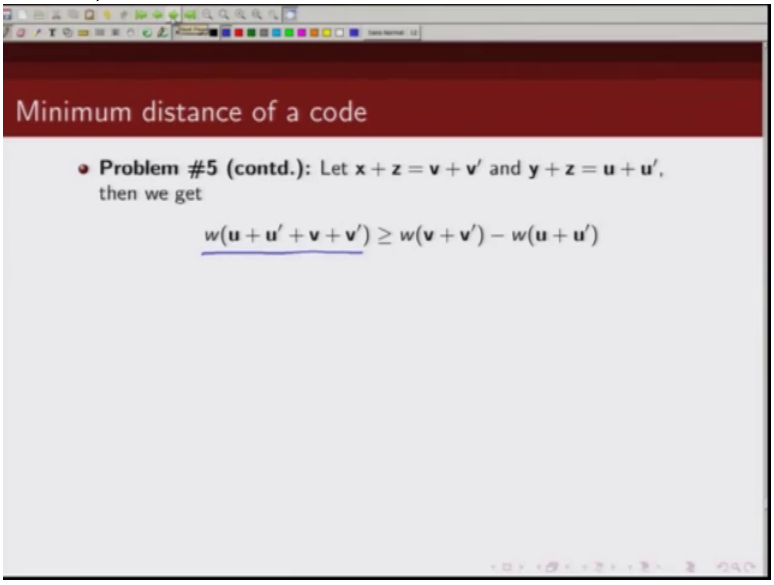

can be, so

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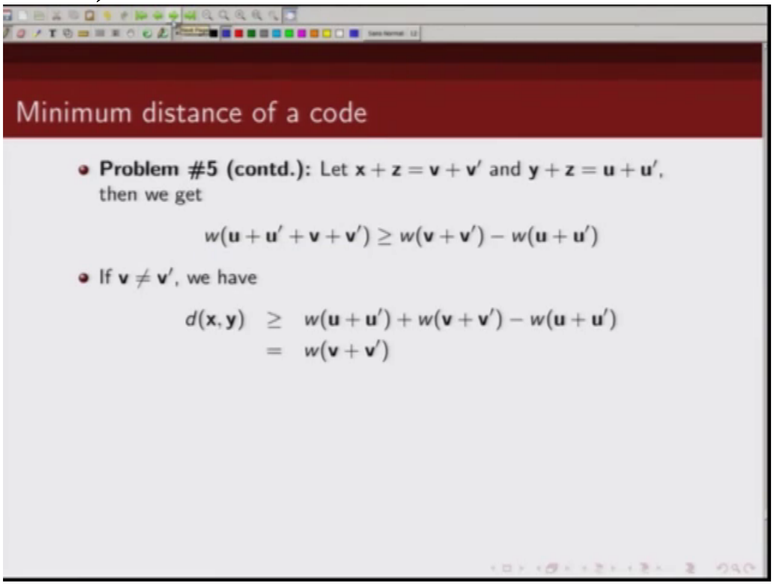

this basically this we lower bounded by this quantity.

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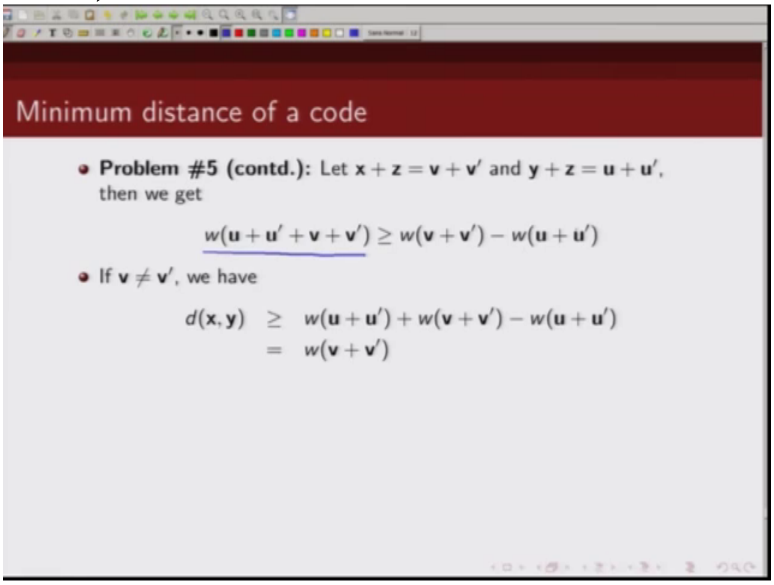

If we plug that in here, what we get here is greater than equal to.

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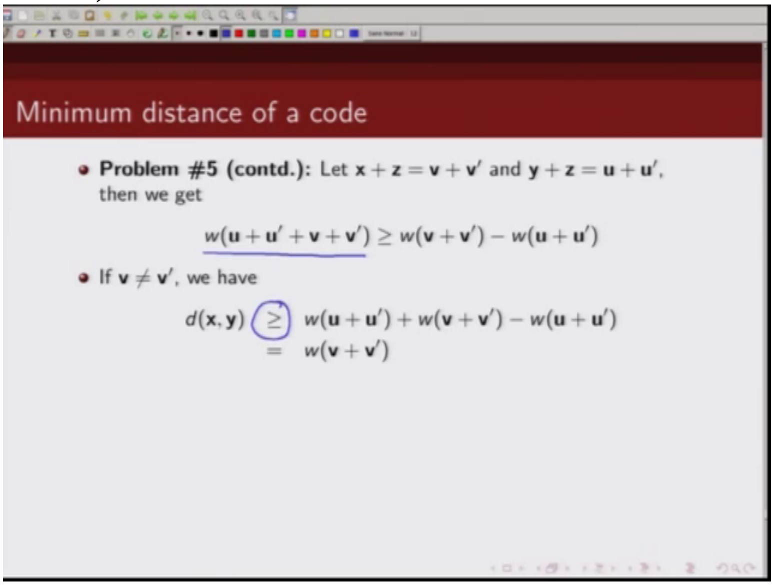

So what we can write is the Hamming distance between x and y is then greater than or equal to this term comes

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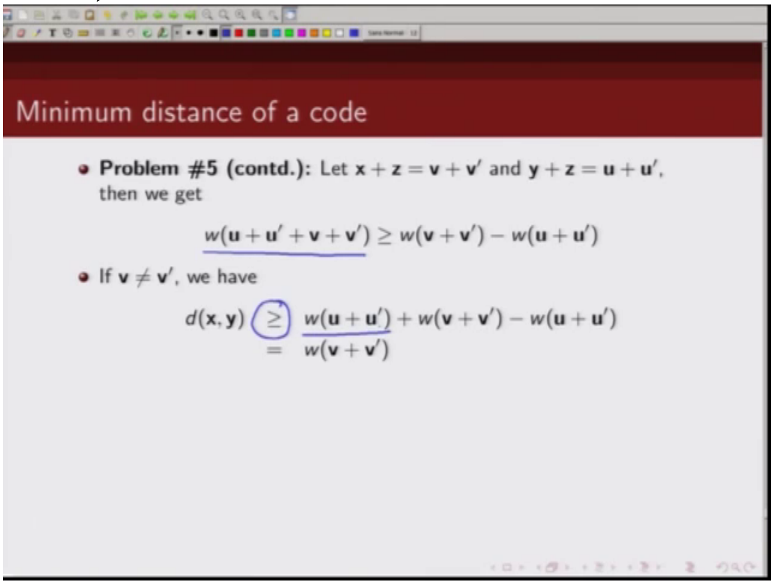

from here, this term

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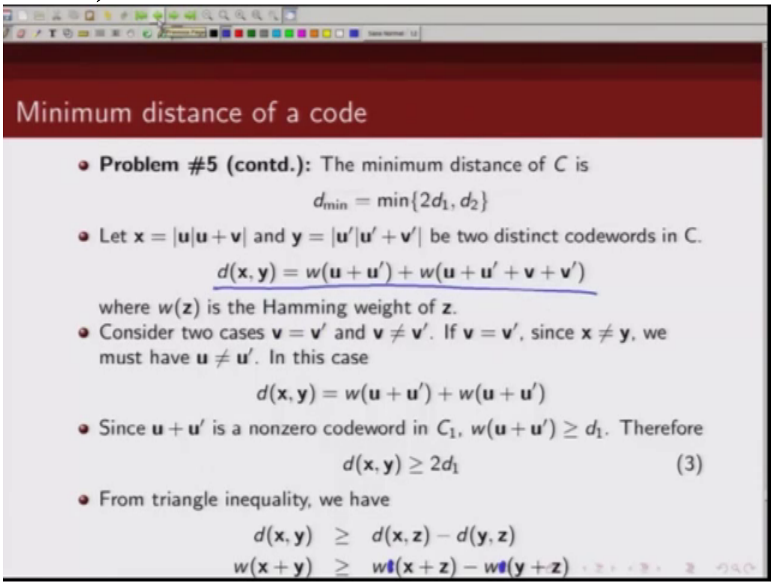

and this term is lower bounded by this term

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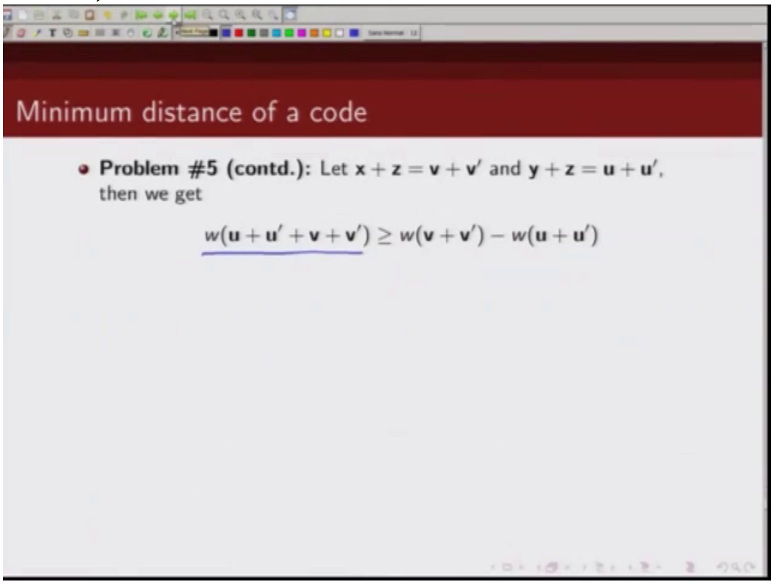

here. So we write it

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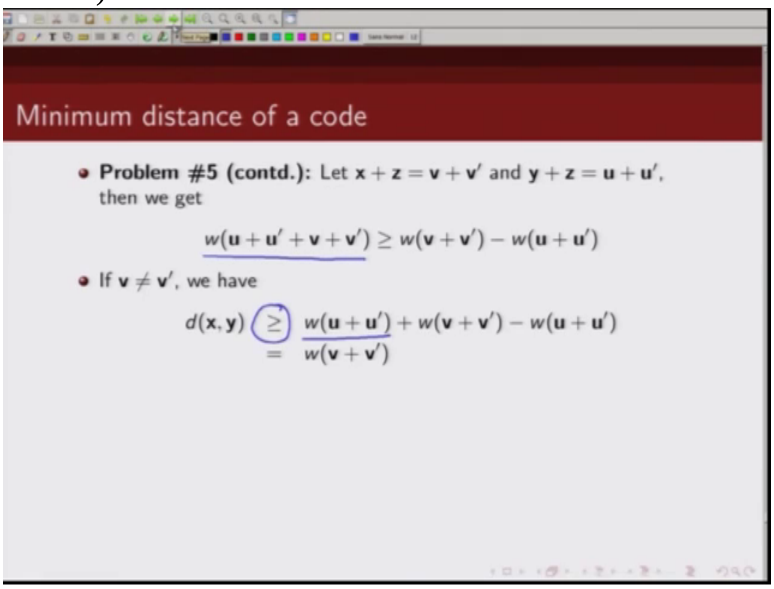

here, now this can be further written as Hamming weight of v plus v prime because these two cancel out. So what

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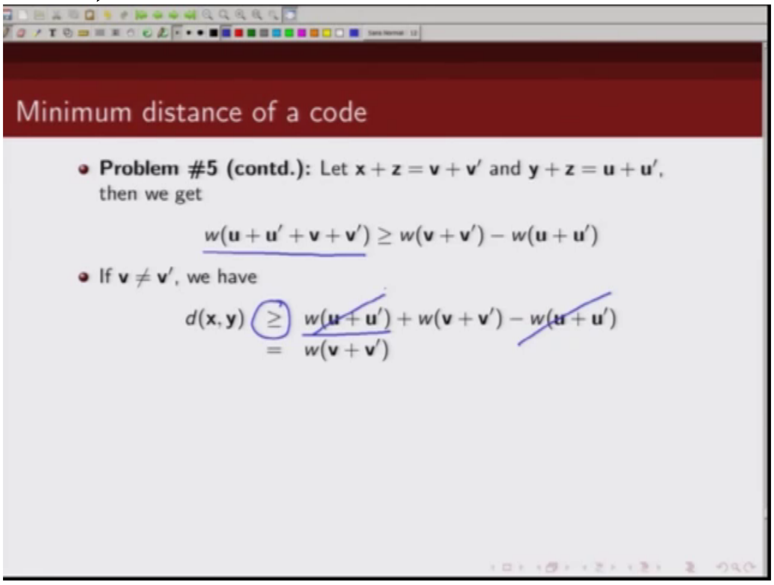

we have shown is when v is not same as v prime the Hamming distance between

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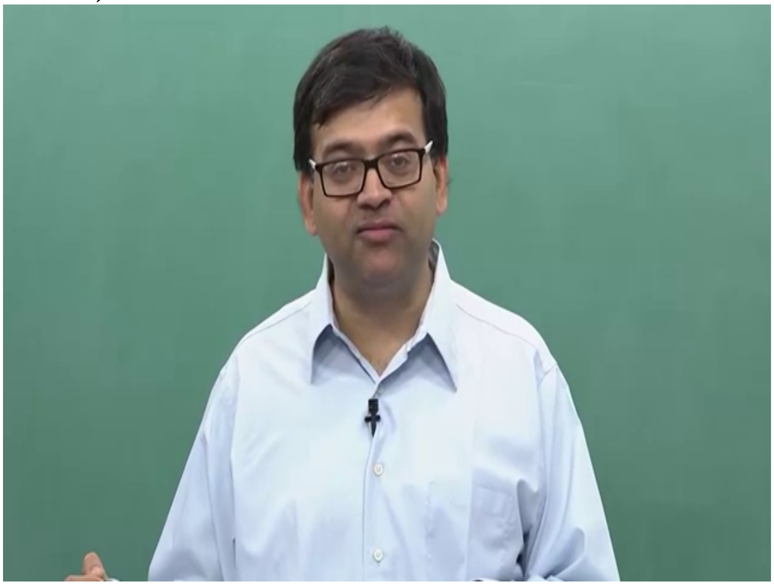

x and y is greater than equal to Hamming weight

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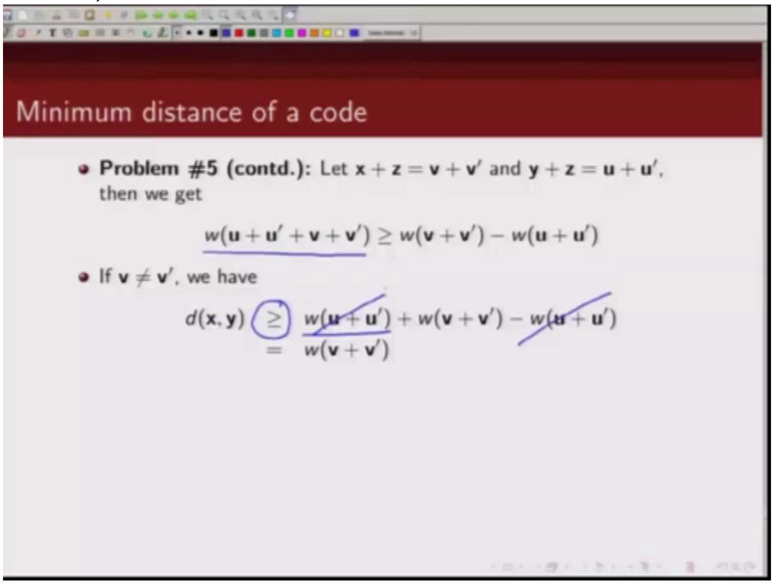

of v plus v prime. And is v plus v prime? v and v prime are valid codewords in linear block code C 2 with minimum distance d 2 so v plus v prime will be another valid codeword in C 2

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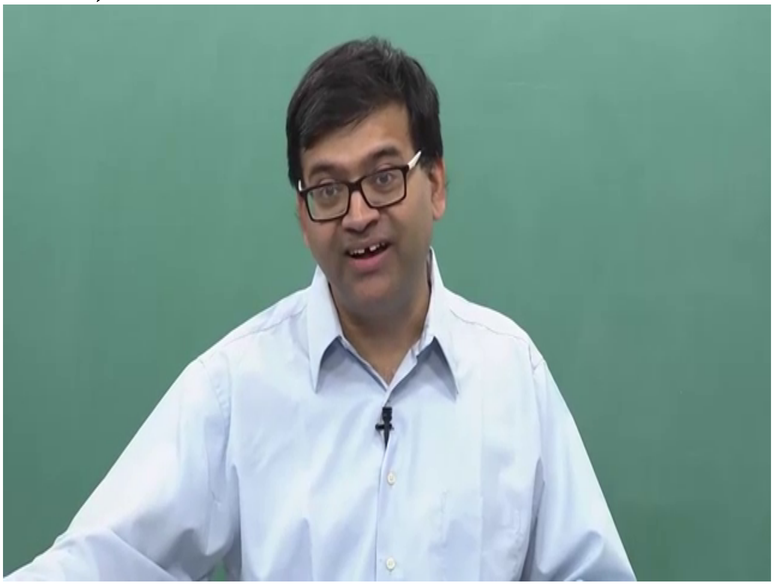

whose minimum distance is d 2. So then we can

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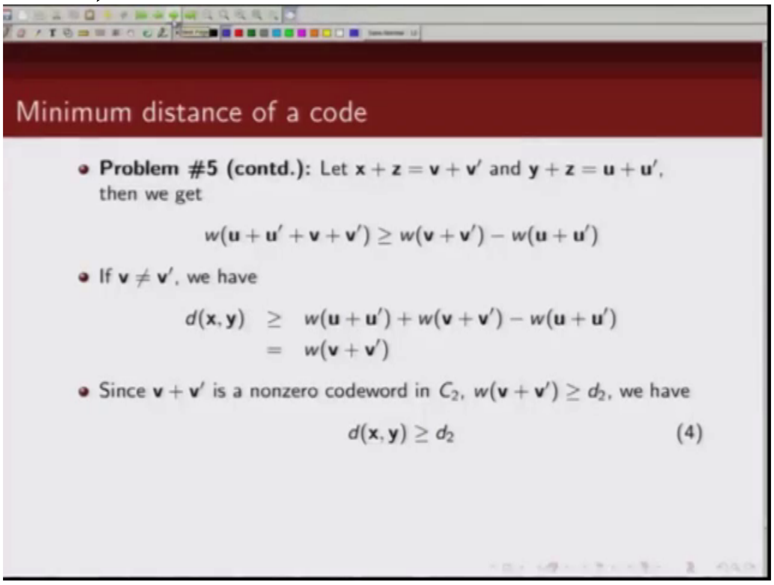

write this as Hamming distance between x and y is greater than equal to d 2. So we look, comparing equation number 4 and equation

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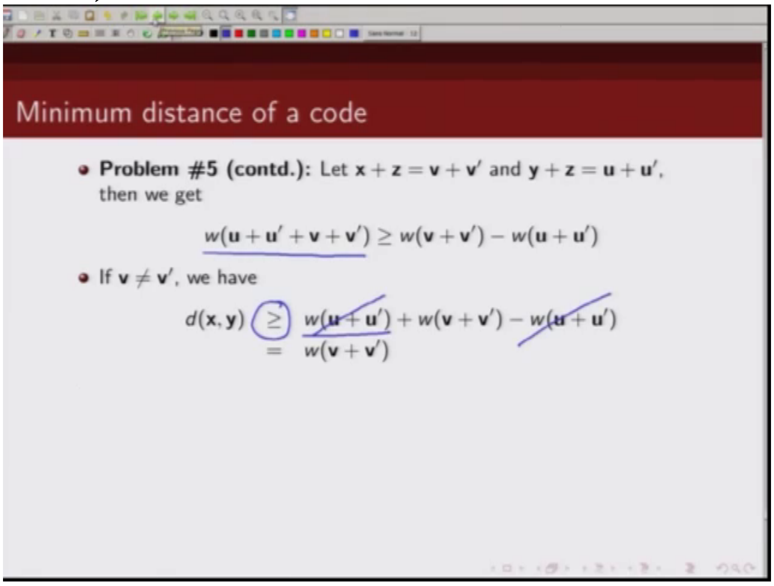

number 3,

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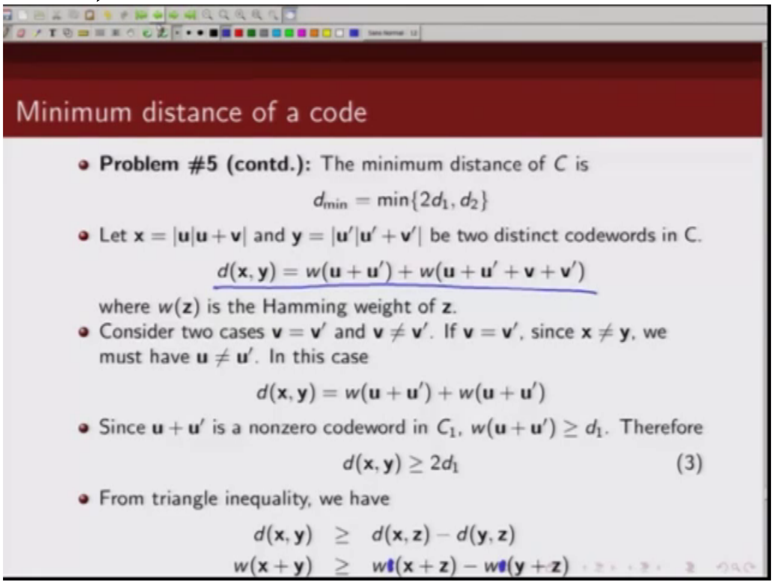

if we compare these two equations we can write that minimum distance of the code is minimum of 2 d 1 or d 2, Ok.

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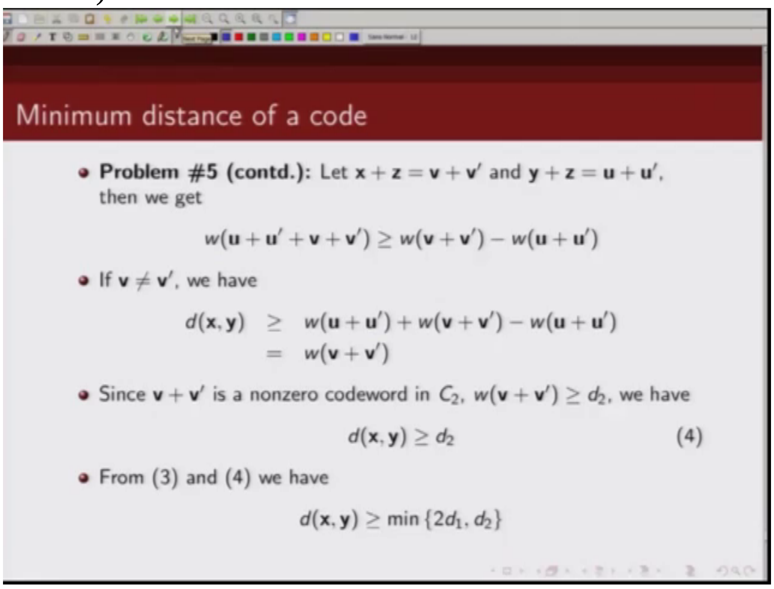

Now let us show that

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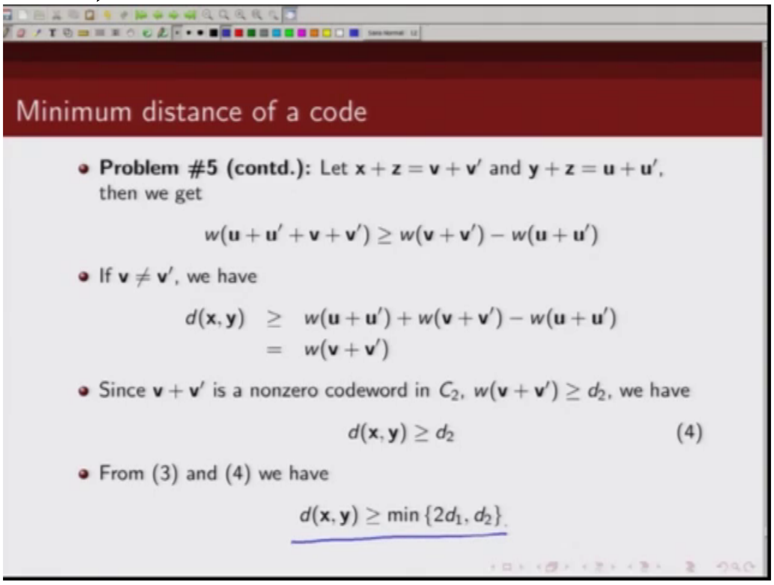

there exists a code

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with, minimum distance of code is indeed

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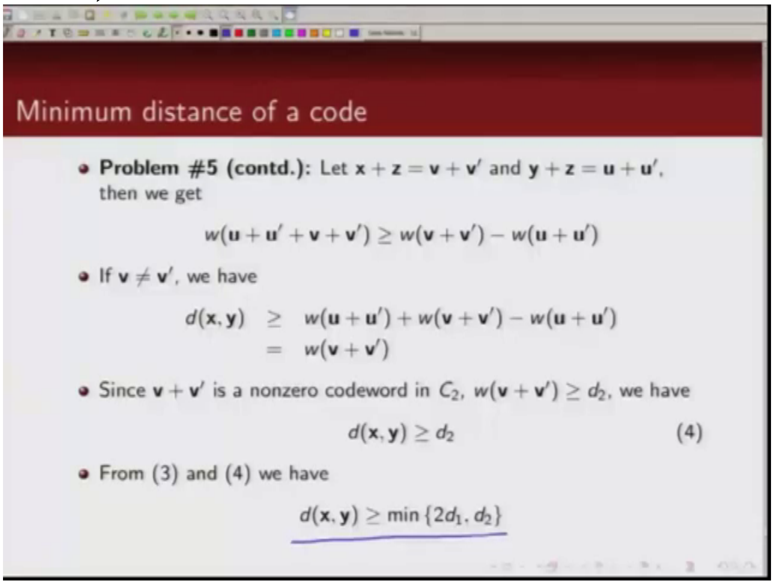

equal minimum of 2 d 1 or d 2. So let us take two minimum weight codewords

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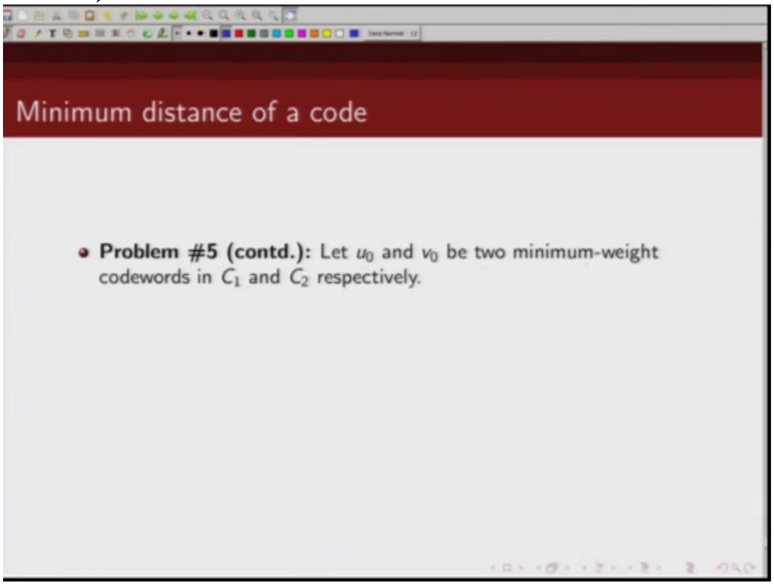

in C 1 and C 2. Let us call them u 0

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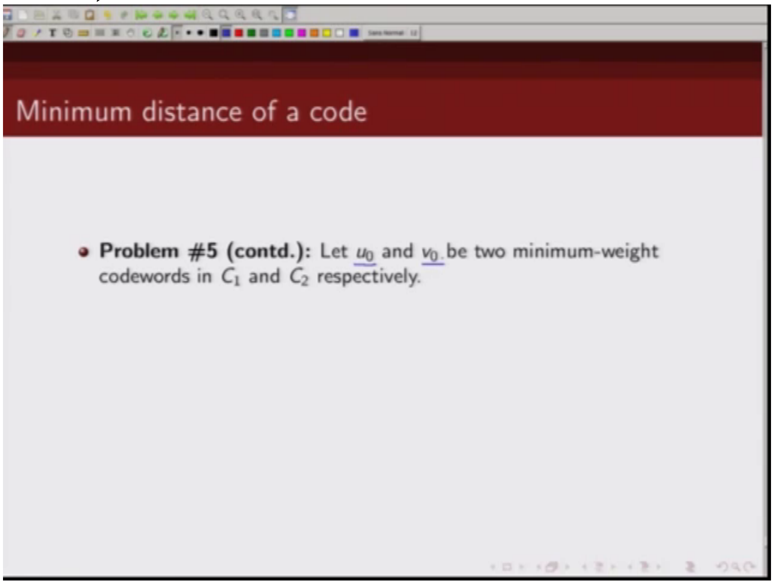

and v 0.

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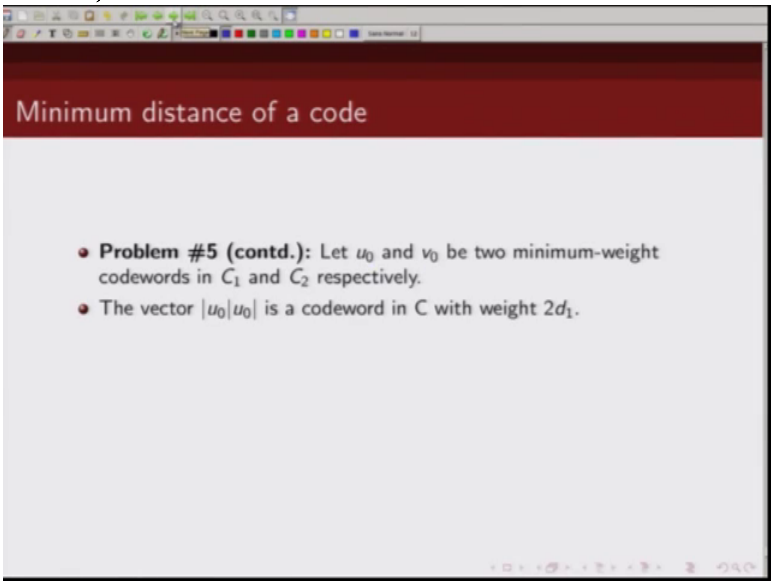

Now this is a valid codeword in C and what's its minimum distance? It is 2 times d 1.

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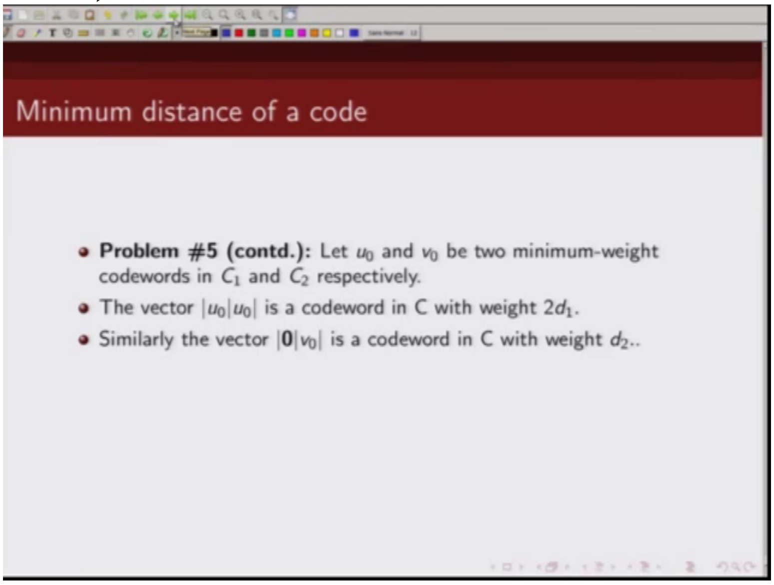

So if we take v 0 to v all zero codeword, what we get is u 0 and u 0, this is a valid codeword in C. And its minimum distance is 2 times d 1. Similarly if we take u 0 to be all zero codeword, then what we get is this codeword 0 and u 0, whose minimum distance is d 2. So hence we have shown that there,

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basically there minimum distance of code, of this code new code c is indeed minimum of 2 d 1 or d 2,

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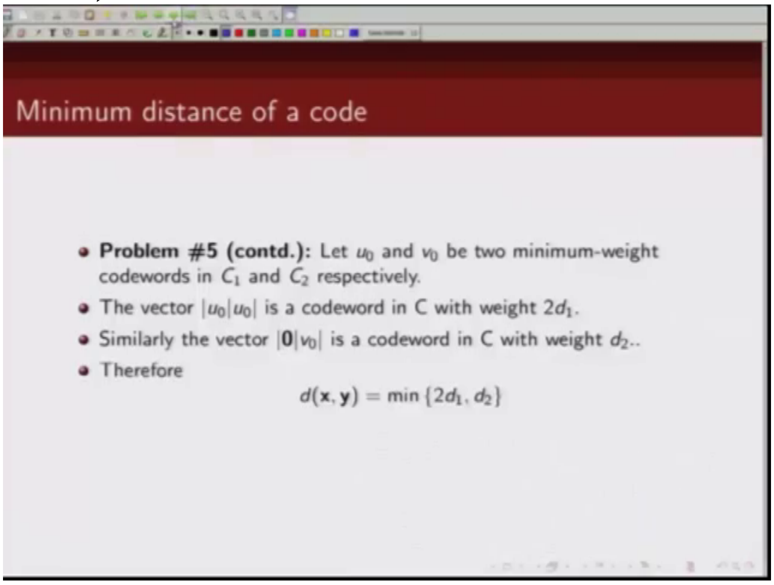

Ok. Thank you