**Course on Principles of Communication Systems – Part 1**
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**Lecture 44**
**Module 7**
# Companding for Non-Uniform Quantization, Mu-Law Compressor, A-Law Compressor

Hello, welcome to another module in this massive open online course, alright. So we are looking at quantization, alright and we are looking at we looked at several quantizer so we have looked at uniform quantizer and we also looked at the Lloyd-max quantizer which is the optimal quantizer, alright. Now let us look at the different quantization schemes which is termed as companding, alright this is used for non-uniform quantization, alright so let us look at a different quantization scheme which is termed as companding, okay this is termed as companding which is used for non-uniform quantization.

So companding and this is used for non-uniform quantization, correct. So companding is used for non-uniform, okay so this is used for non-uniform quantization and what is the point of companding, the point of companding is that frequently we have well we have signals which takes smaller amplitudes and larger amplitudes, but it takes smaller amplitude with a larger probability, alright or we need a larger resolution right or we greater accuracy at the smaller amplitudes and the larger amplitude which occur with less probability we do not need a very high level of accuracy, alright.

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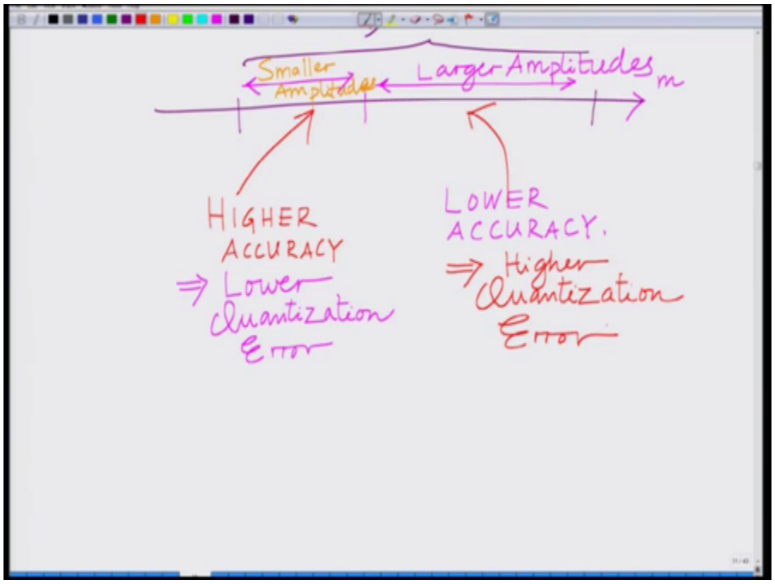


So the point is that we have let say this is the dynamic range of the signal, okay let us look at this schematically this is the dynamic range this is the dynamic range of the signal, okay and this is the increasing amplitude m, correct and if you look at this smaller amplitudes let us call this region as a larger amplitude these are larger amplitudes and this is the region of smaller amplitudes so this is the region of smaller amplitudes we have larger amplitudes, okay.

Now in this region, the smaller amplitude region we need higher accuracy, okay we need a higher accuracy. In this region in larger amplitude region we can afford to be lacks, alright because this occurs with low probability or or the reconstruction is not very sensitive to basically because the amplitude is large the reconstruction is not very sensitive to an error in the amplitude. So one can be relatively more lacks or one can one can reconstruct it or one can tolerate a higher level of error which means the accuracy of quantization or the accuracy of reconstruction in this region of large amplitudes in order to (())(3:54).

So here, one requires a lower accuracy of reconstruction remember lower accuracy means (lo) lower accuracy means you can tolerate so higher accuracy means quantization error has to be lower so lower quantization error in this region we can have a higher quantization error in this region we can have a in this region we can have a higher quantization error, okay. So therefore what we are so the out site what we have to realize is that if you have a signal which is taking samples or values in a certain dynamic range we do not need the same accuracy the same level of accuracy in all the (sig) in all the in all the regions of this signal amplitude, alright.

So for the smaller amplitudes we need a higher level of accuracy, alright for the larger amplitudes we need we can tolerate a higher quantization error or basically we need a lower level of accuracy in reconstruction which naturally means remember we said the quantization error is proportional to the size of the interval that is the larger the interval large number of samples are represented by the same threshold so the quantization error increases.

So larger the interval which means the quantization error is higher, therefore the lower amplitude regions since we need lower quantization error we have lower quantization which means we have to have quantization intervals which have very low quantization interval of low of low size or there is the size of the quantization intervals the length of the width of the quantization intervals has to be smaller at smaller amplitudes, alright which means lower quantization error and the width of this quantization error quantization error can be larger at larger amplitudes which means higher quantization error, that is the point, alright.

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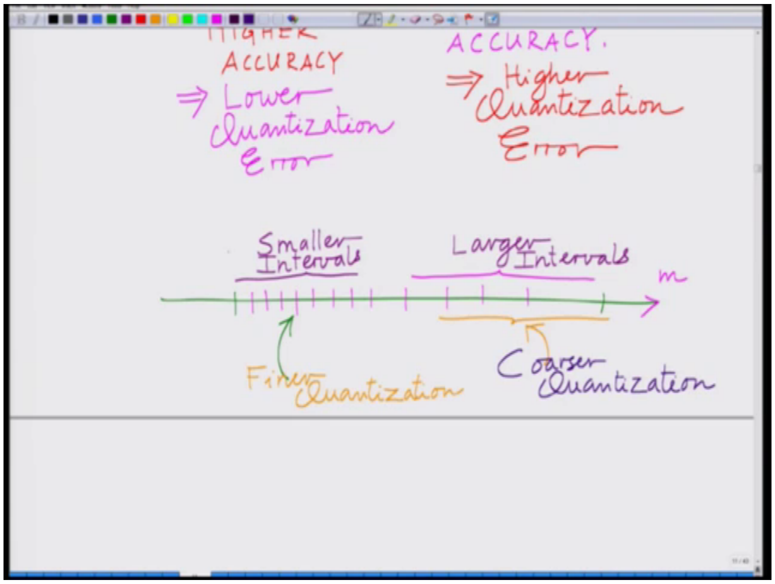


So we have again let us look at the dynamic range of the signal if we place the quantization level this is the alright, we can have quantization width that is quantization intervals smaller and as progressively as the sample value increases, alright the towards the larger amplitudes these can be larger for instance if you can look at this, these are your these are your larger these are the larger quantization intervals, larger intervals and these are smaller intervals.

So this implies this region this is a finer quantization, correct this is quantized very finely so this is a and if you can look at this this is quantized very roughly or this is quantized very coarsely, okay so this is coarser quantization interval so this is a coarser quantization so this is finer quantizer for the smaller intervals and the lower amplitude region which are quantized very finely which means the width of the quantization intervals is small, alright therefore quantization error is lower if you look at the larger amplitude region, then the width of the quantization intervals is large the quantization error is higher therefore these are coarsely or these are roughly quantized, alright.

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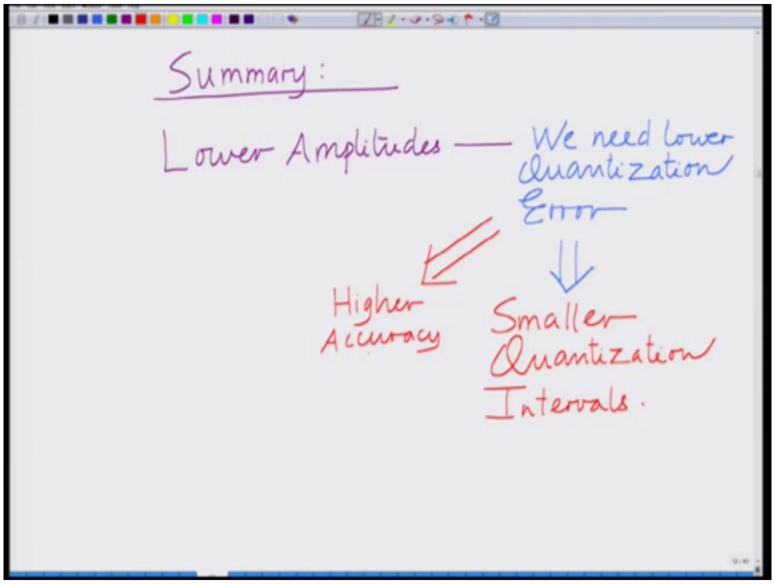


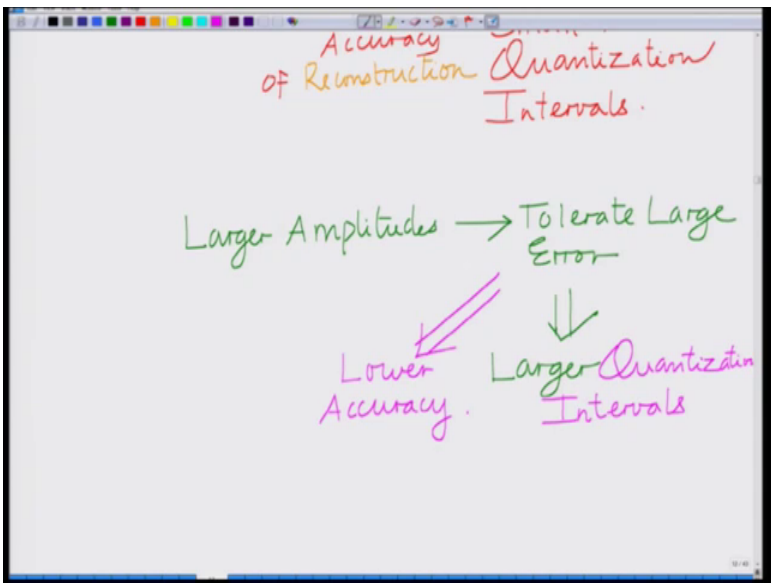

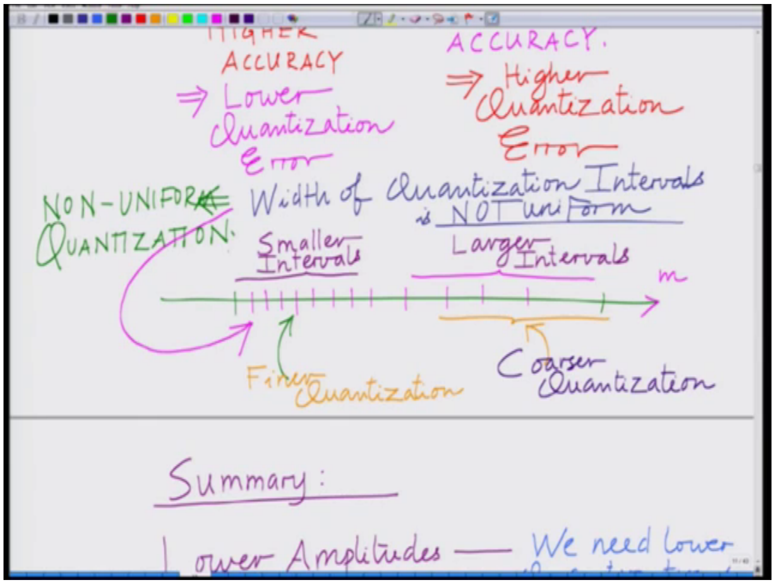


So in summary, at lower amplitudes we need lower quantization error this implies lower quantization errors implies smaller quantization we need smaller quantization interval this implies higher accuracy of reconstruction when you say higher accuracy automatically it means higher accuracy of reconstruction. On the other hand at larger amplitudes we need we can tolerate, right? Quantization error tolerate implies we need to have larger quantization implies we need to have larger quantization intervals, alright.

Which means lower accuracy of reconstruction which means we can have lower accuracy, okay. Now, therefore now if we can see therefore what we have therefore if we have if you can look at this the (quantizat) the width of the quantization in now if you look at this figure you can see that the width of the quantization intervals is not uniform you can see width of the quantization intervals is not therefore this is basically this implies this is non-uniform quantization this is a technique of non-uniform quantization because the width of the quantization intervals is different, alright different intervals are different widths this is non-uniform quantization.

Naturally we said that because the quantization intervals in the lower amplitude region they need to be final they need to have low widths alright and the quantization interval intervals is of in the larger amplitude in the large amplitude region need to have larger widths, alright coarse quantization, alright so this we need non-uniform quantization and companding is a way, right? Comapnding is a way to achieve this non-uniform quantization.

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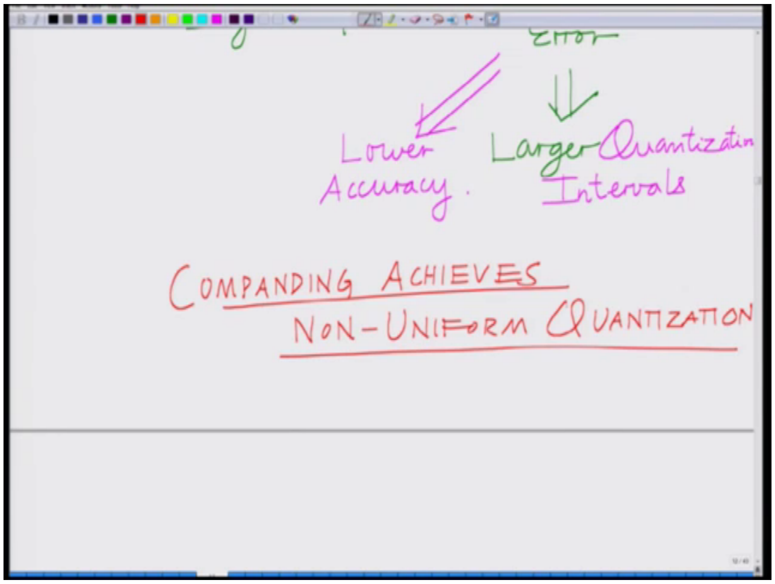


So companding achieves non-uniform quantization, okay so this is what companding achieves so companding achieves non-uniform quantization, okay.

So for companding, alright companding which achieves (uni) non uniform quantization one can employ the mu law compressor, alright this is one of the techniques to compand so let us look at this mu law the mu law compressor, okay so let us look at the mu law compressor for companding, alright and the mu law that is an input output (())(13:15) remember this companding is nothing but an input output mapping it is nonlinear mapping form the input to the output such that the smaller amplitude region is mapped to the mapped to a larger width, alright and the larger amplitude region corresponding to the larger amplitude is compressed, alright that is the process of companding, alright.

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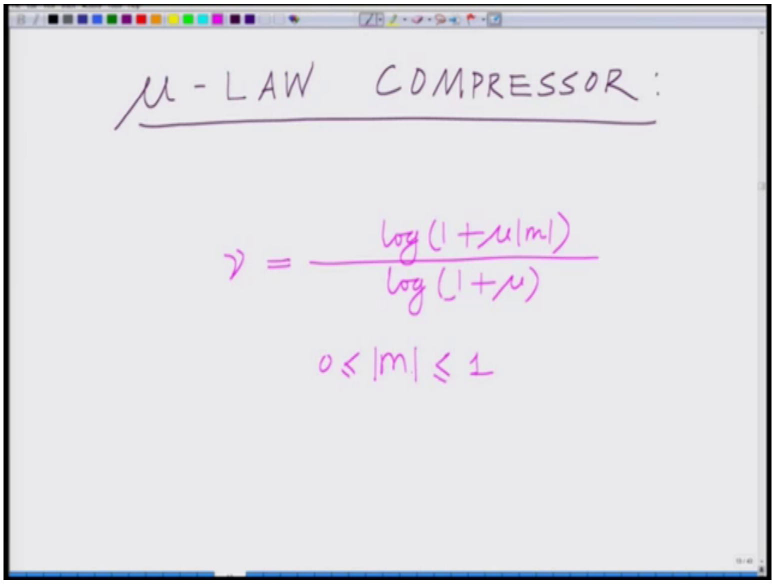


So this is a transformation and the transformation for the mu law compander or the mu law compressor this is given as nu equals well log 1 plus mu times mod m divided by log 1 plus mu and m lies in the range 0 less than equal to or 0 less than equal to mod m less than equal to 1. So the dynamic range of the signal is normalized to one that is (lyi) lying between 0 and 1, alright the basic meaning of that is whatever is the dynamic range of m it is normalized that is by dividing m by the maximum value of m that is the maximum value of the amplitude you can always normalize the samples to lie in the dynamic range 0 to 1 and then you can apply this transformation.

Once we apply this transformation quantize and obviously while reconstruction you multiply back by the maximum value m so that you can reconstruct it to the original dynamic range, alright so this formula applies for a dynamic range of the signal level that is amplitude to 0 and 1 that is between (ma) magnitude of m lies between 0 and 1 m lies between minus 1 and 1, okay. Now, this is basically your mu law compressor, okay and the mu law compressor if we plot the mu law compressor, right? If we plot the mu law compressor, right.

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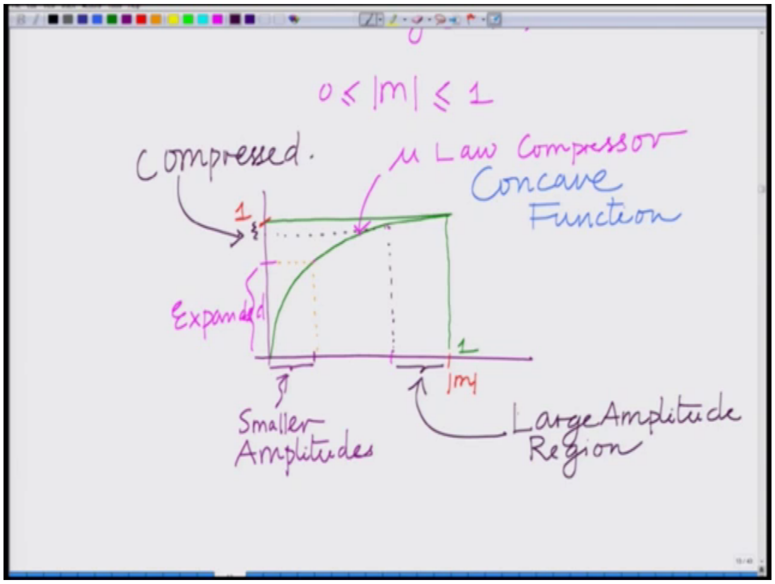


So if we plot the mu law compressor this is log 1 plus mu m of course you can see at m equal to 0 the it is not defined at as m tends to 0, right? So now as m tends to 0 it tends to 0 at m equal to 0 this is 0, correct now we are plotting this for m so this is m or let us say magnitude of m equals 1 this is obviously at magnitude of m equal to 1 this is 1 so the maximum value so it basically remaps magnitude of m lying in this interval 0 to 1 it remaps it to this interval 0 to 1 and the mapping is the log function it is a concave mapping which is given as follows.

It is a log function, okay so this is the mu log quantizer or the mu law compressor this is the mu law compressor and from this so this is basically the log function, okay which is a concave function you can see that this is a just write this clearly this is a concave this is a concave function and you can see from this that is if for instance if you look at this (ampli) this region, correct which corresponds to the smaller amplitudes.

Now this is mapped you can see this region is mapped to this, so this corresponding region, alright you can see this is expanded, right? You can see that this small amplitude because of the concave nature of the log function this is mapped this is expanded at the output, alright that the output of this compressor it is spread or basically it is expanded to a much larger therefore it will contain (())(17:45) when you quantize it when you quantize the output of this mu law compressor at this region, right? Contains a large number of we contain a large number larger number of quantization intervals, therefore which implies that this amplitude region this region corresponding to smaller amplitude will be quantized much more finely, alright.

On the other hand this entire region if you look at this entire region corresponding to the larger amplitude so this is the region corresponding to the smaller amplitude which is expanded and this is the region corresponding to the larger amplitude if you look at this region corresponding to the larger amplitude this entire region is mapped to this small so this is compressed, correct so this is compressed this is the large amplitude region this large amplitude region this is compressed is a smaller amplitude region smaller amplitude this is expanded, alright.

So this expanded which means it contains a it will contain a large a larger number of quantization intervals hence it is fine it is going to be finely quantized once you quantize the output of this compressor. Similarly this larger amplitude region which is compressed to a smaller region in the output will contain much fewer number of quantization intervals, alright. And therefore the resolution is going to be much smaller, alright which means it is basically coarsely quantized that is the whole point, alright.

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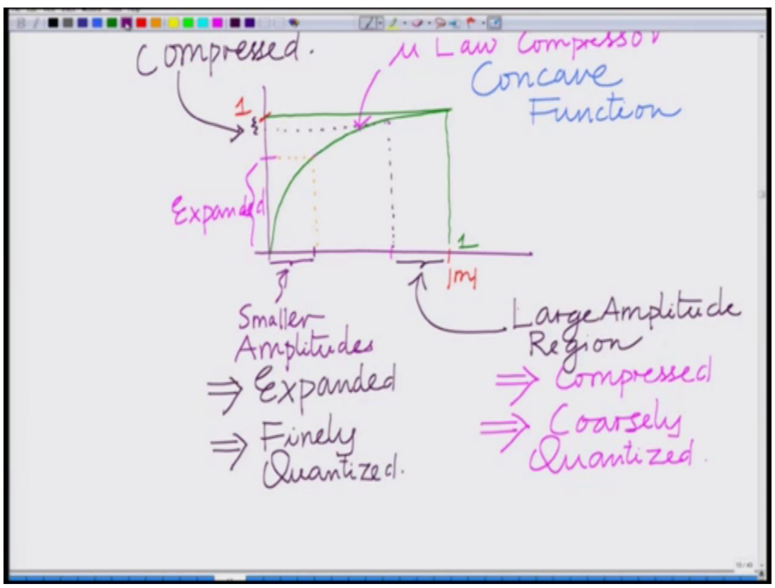


So this implies this is expanded this region is expanded implies this is well this region is going to be finely quantized. This region is large amplitude region this is compressed this implies this is coarsely quantized, okay.

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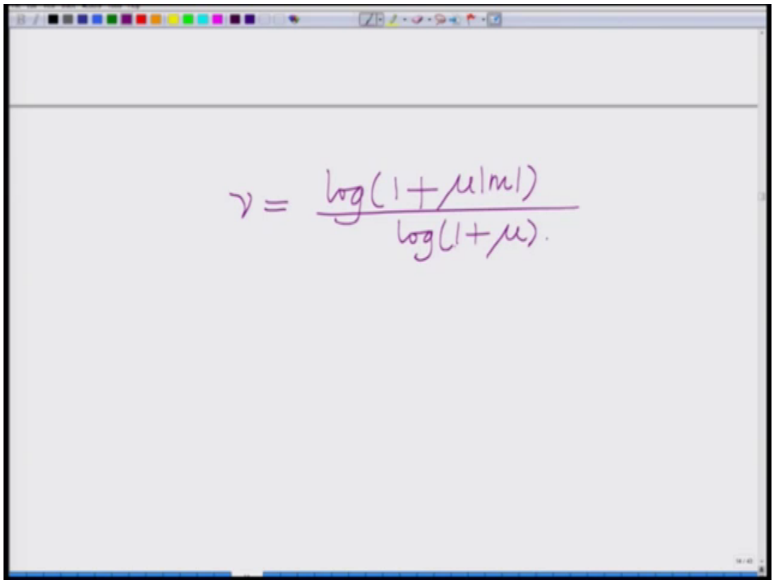


Now, alright this is coarsely quantized, okay and further now if you look at this interestingly if you look at this characteristic for nu that is let us write this again nu equals log 1 plus mu magnitude of m by log 1 plus mu, alright now as mu tends to 0 of course this becomes undefined because this is log 1 which is 0 divided by log 1 so if simply substitute mu equal to 0 then it becomes log 1 divided by log 1 which is 0 over 0 that is indeterminate.

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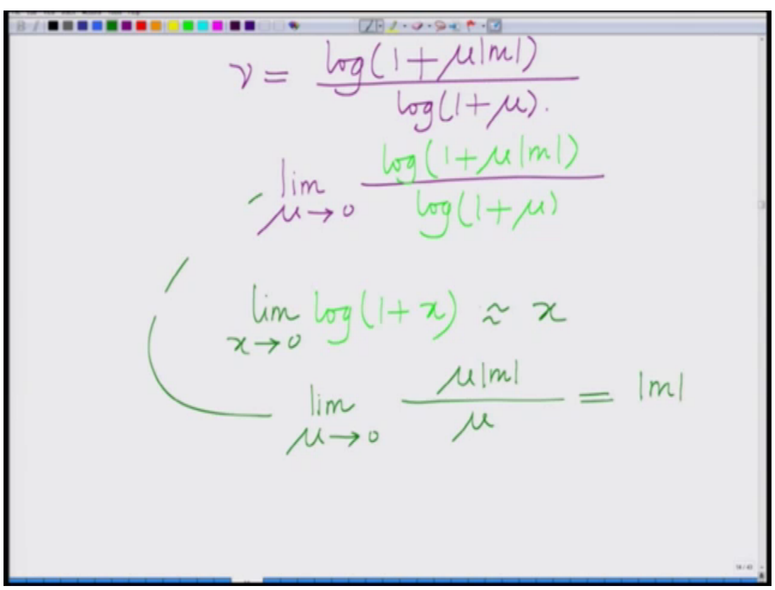

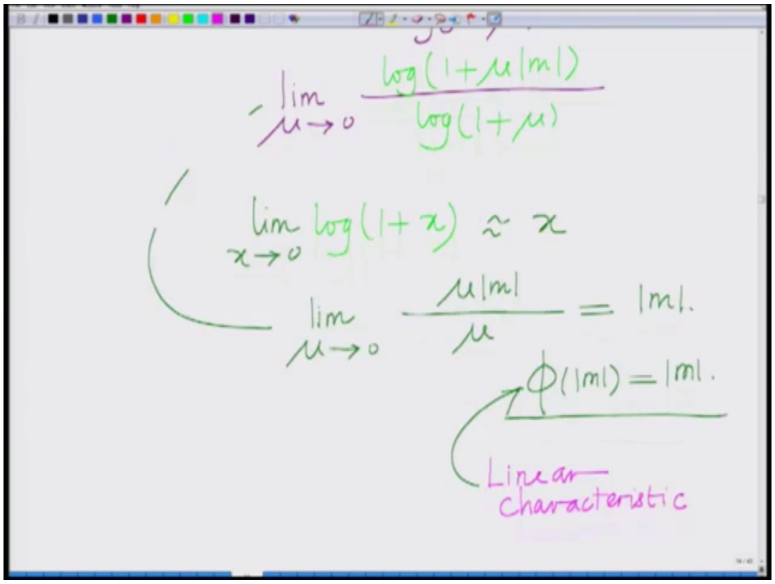


So therefore now we consider the limit mu tending to 0 or limit mu tending to 0 of this that is for very small values of mu this becomes log 1 plus mu magnitude m 1 plus mu well we are going to use the approximation log 1 plus x limit x tending to 0 or equals x or log 1 plus x limit x tending to 0 log 1 plus x is approximately equal to x. So this becomes limit mu tend to 0 log 1 plus mu mod m is approximately mu magnitude of m divided by log (())(22:02) is mu which is basically now we can say this limit is basically simply magnitude of m.

Which means that the characteristic is simply that is your phi of magnitude of m phi is the compressor is simply magnitude of m which means this characteristic is a linear (charac) is simply a straight line, okay magnitude of m is simply (mag) magnitude of m, right? Which means that the (mag) or the samples are un alter, alright. So the samples are un altered, okay and therefore this is simply a linear characteristic, now if you look at this for mu equal to 0 this reduces to a linear characteristic, okay.

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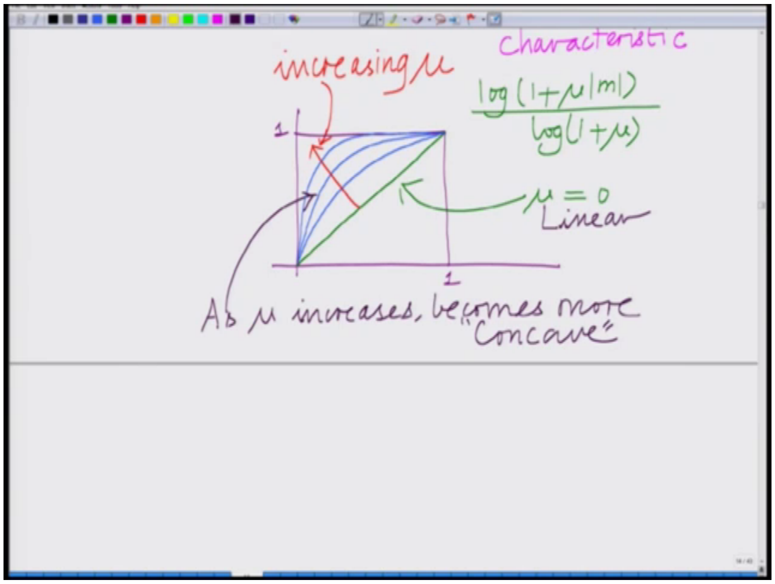


Therefore if you plot this, correct again plot this thing this box remember, right? This is your plotting log 1 plus mu magnitude m by log 1 plus mu now if you plot this what you will observe is that for mu equal to 0 it is a straight line this is for mu equal to 0. As mu increases it becomes more and more so this is basically the direction of increasing mu, as mu increases it becomes more and more concave, alright it becomes more and it becomes (mo) as mu increases it becomes more concave, alright.

Which means basically this smaller amplitude region is mapped to an ever increasing, right? Ever increasing portion of the output and the larger amplitude region is progressively compressed even more and more, right? As the characteristic becomes more and more concave the smaller amplitude region is expanded more and more and the larger amplitude region is compressed more and more, alright. So the resolution that is the number of bits or the number of intervals in smaller amplitude or the finer amplitude improves progressively at mu increases and the resolution or the accuracy of the larger amplitude region that progressively shrinks as mu increases, alright okay.

So that is the mu law compressor, alright for so for mu equal to 0 this is a linear so mu equal to 0 the characteristics is a linear characteristic that is what we have already seen, okay it is a linear (())(25:26).

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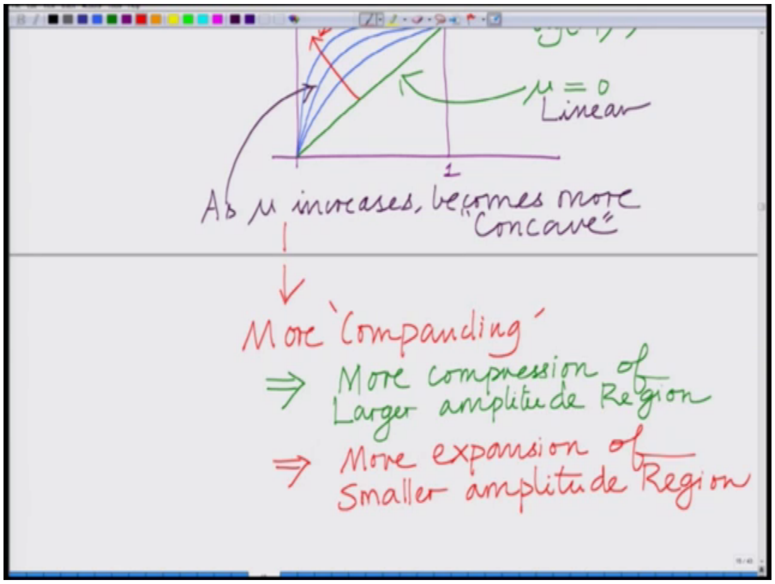


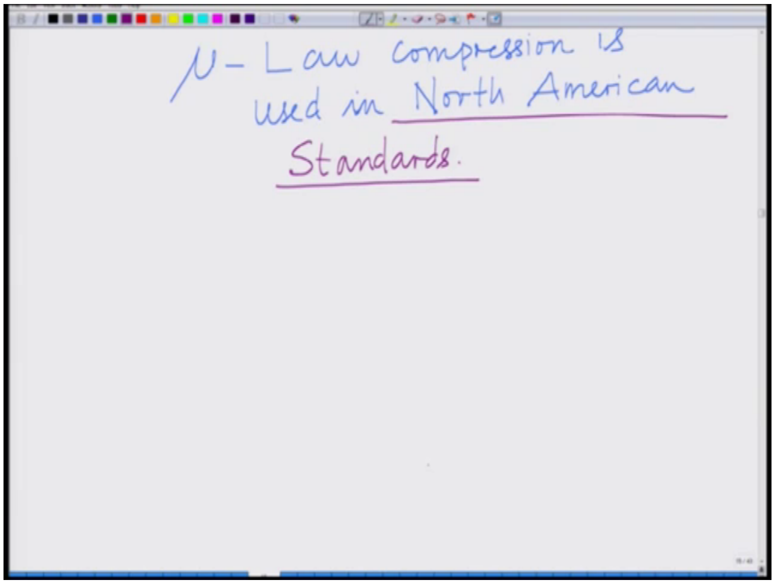


As mu increases it becomes more (())(25:28) as mu increases which means more companding as mu increases there is more companding there is more compression of the larger amplitudes this implies more compression of larger amplitude region and also more expansion of the more expansion of the smaller amplitude region, okay.

And also it is worth noting that this mu law compression this compressor for quantization is used in the North American standards mu law compression is used in North American standards this is used in North American standards, okay so the mu law compression is used in North American standards, okay and in comparison in contrast there is also another compression standard another compression or competing compression this is known as the A law compressor.

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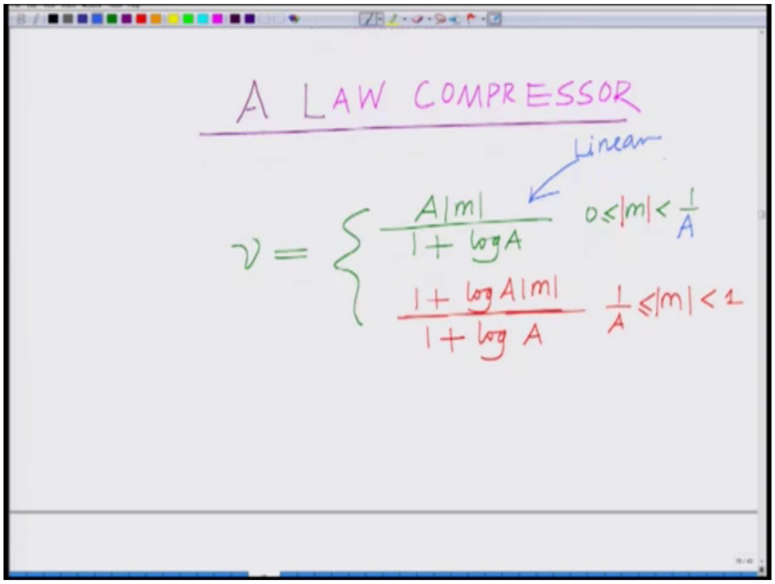


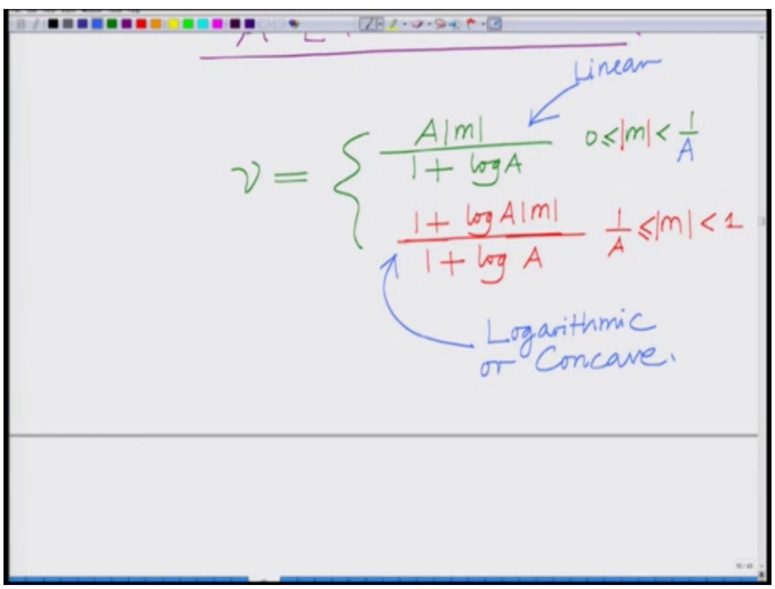


So there is a mu law compressor and we also have a A law we also have an A law compressor, okay and the A law compressor the expression is given as follows the A law expression for the A law compressor that is nu equals A magnitude m divided by 1 plus log A where 0 less than equal to m less than 1 by A and it is 1 plus log A magnitude m A magnitude m 1 plus log A 1 over A less than equal to magnitude m less than, okay.

Now we can see in this A law quantizer that initially for some part in this region basically 0 less than equal to magnitude m less than magnitude m less than 1 over A in this region it is linear, okay and in after a certain threshold that is 1 over A in this portion it is logarithmic or it is concave. So what we have is a linearly increasing characteristic or linear increasing compander so at a certain portion and then it starts becoming concave that is where things starts to be compressed that is the larger amplitude region is progressively shrunk.

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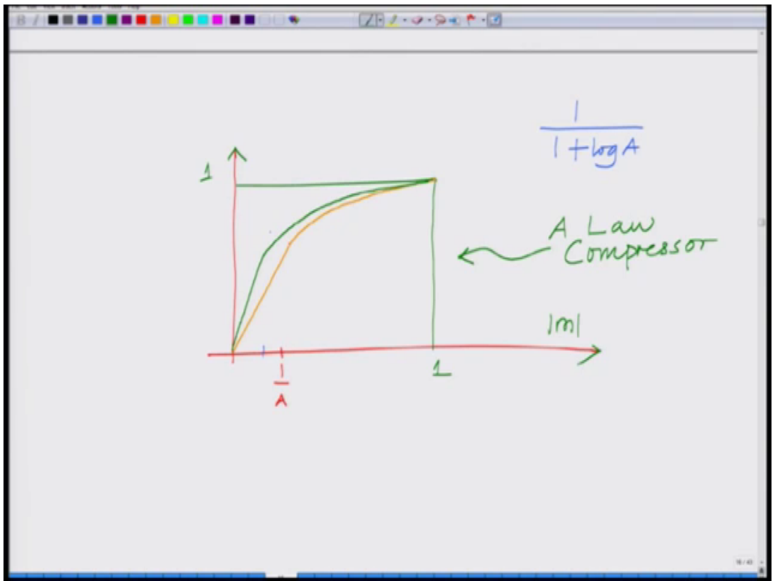


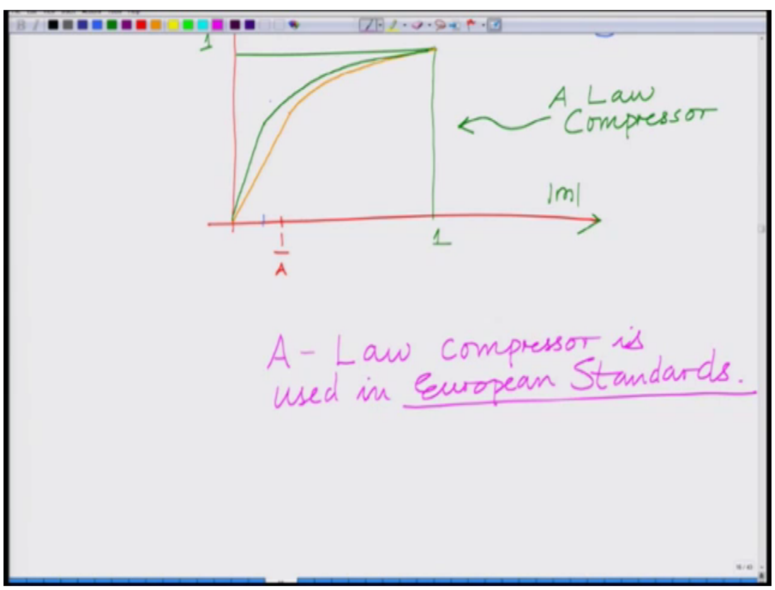


So if you plot this, this looks something like this again in our box corresponding to this 1 comma 1 magnitude of m is 1 magnitude of m is 1, correct now until this point 1 over A where A is suitable chosen, right until this point it is linear so this will look something like this and then it is logarithmic. And as A increases this points progressively shifts, right this 1 over A decreases, correct 1 over A decreases and it progressively shifts.

So this is A times one or so this becomes 1 over 1 plus log so this becomes as this increases this progressively shifts, correct so this becomes so this progressively shifts this point here this is 1 over 1 plus log A, correct when m magnitude m equals 1 over A this is 1 over 1 plus log A therefore this point comes down and it looks something like it looks (som) it looks something like (())(31:33) and so on and so, alright. So as A increases this point progressively shifts to the left and this is basically your a law this is basically your a law compressor.

So we have a mu law compressor we have an A law compressor and it is also again worth noting that this A law compressor is used in the European standards, so the A law compressor is basically used in the is used in the European standards, alright. So basically we have seen companding, right companding is very but we need different resolutions in different regions of in different regions of the dynamic range, alright for the smaller amplitude regions of the samples we need finer quantization and in the larger amplitudes we can tolerate course quantization that is we can tolerate a larger quantization error.

Therefore we choose that is rather than uniform quantization we choose to quantize it non uniformly that is having finer quantization in the smaller amplitude region and coarser in the larger amplitude region for that purpose we imply companding, alright. And we have seen two different kinds of compressors, alright the mu law compressor and the A law compressor, alright and the details has been discussed, thank you very much.