Course on Principles of Communication Systems – Part 1
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Lecture 42
Module 7
Introduction to Lloyd-Max Quantization Algorithm, Optimal Quantizer Design

Hello, welcome to another module in this massive open online course, so in this module so we are looking at quantization alright let us continue our discussion on quantization and this module we already looked at uniform quantization. In this module let us look at the different algorithm for the design of a quantizer which is termed as a Lloyd max quantizer, okay.

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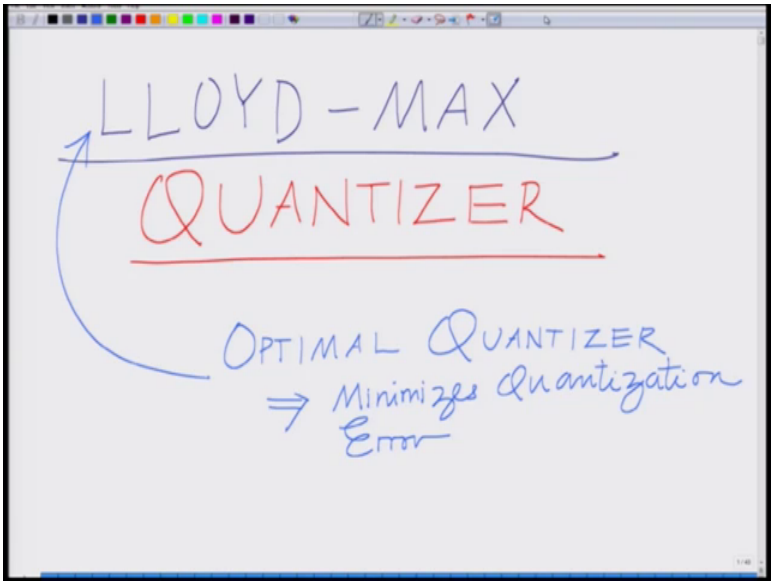

So you want to look at the Lloyd max the Lloyd max quantizer which is really a scheme to design the optimal quantizer. So the Lloyd max quantizer gives what is known as the optimal quantizer optimal meaning, it minimizes the error quantization error.

Optimal meaning, It minimizes right? It gives the lowest quantization error, okay. So for our optimal quantizer now remember a uniform quantizer is a very simple quantizer which simply chooses the (unif) intervals of uniform length and then places the quantization level in the middle or the mid-point of the each interval. Now for any optimal quantizer to (data) to design a better quantizer one has to optimally choose these different quantization intervals which are characterized by the boundaries of the intervals and of course the level in each interval.

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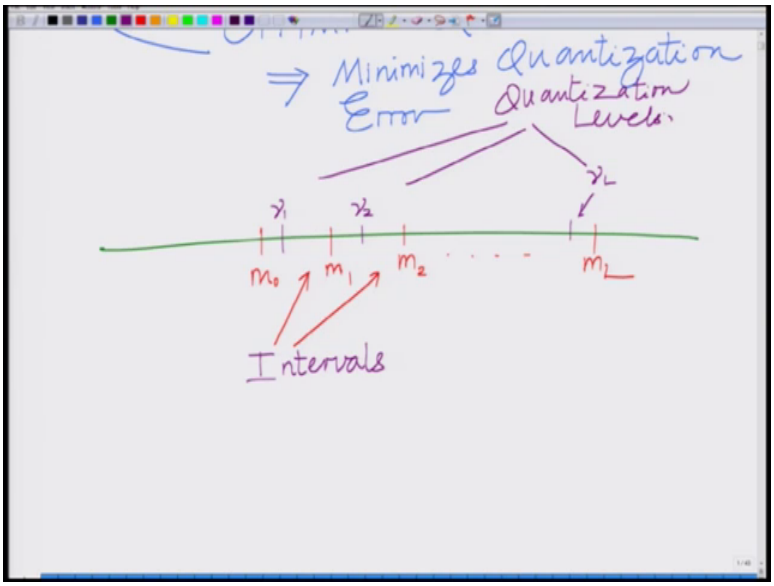

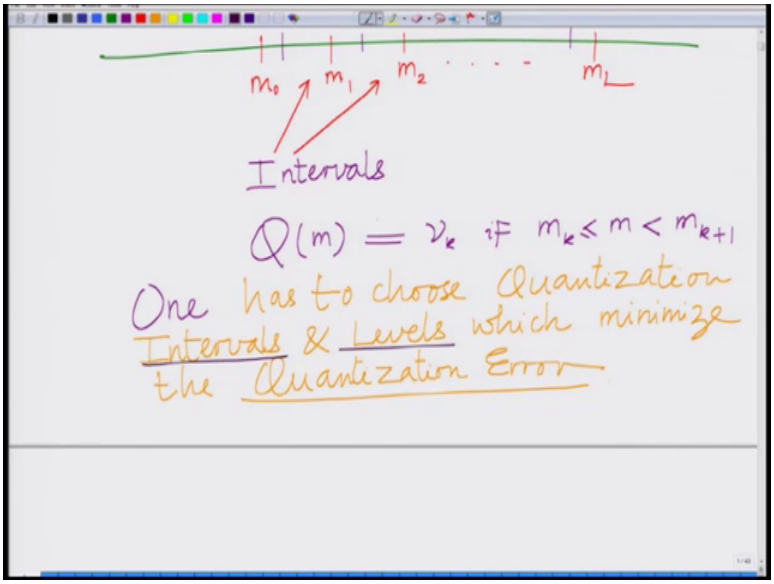

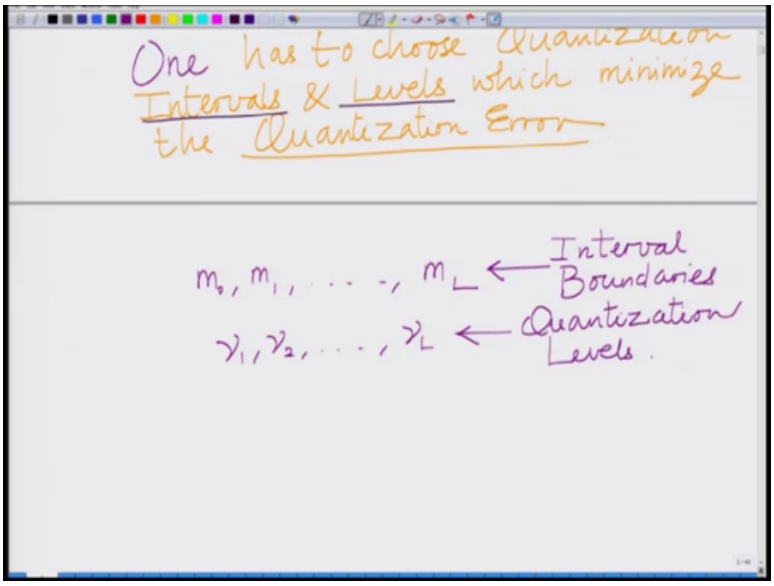

So we have to choose so for an optimal quantizer we have to choose these intervals m0, m1, m2 so on up to mL, correct? So these are for instance each of these are your intervals, correct? So we have to choose to design an optimal quantizer we have to intelligently choose these intervals and in each interval you have the quantization point which need not necessarily be placed in the middle that is it need not necessarily be placed in the middle of (ea) of each levels so I have nu1, nu2 for instance nu3 so on up to for instance nuL.

So I have the quantization intervals and these nu1, nu2, nuL are these are your quantization levels. And from the quantization intervals and the quantization levels one can determine the quantizer because remember the quantizer is if the sample m lies between mk and mk plus 1 so this is equal to nuk if mk less than or equal to m less than mk plus 1, okay. So intervals and quantization levels these determine the quantizer so determine a better quantizer or an improved quantizer we have to optimally choose the quantization levels and the (quan) the quantization intervals and the quantization levels which minimize the quantization error.

So to design an optimal quantizer we have to choose or one has to quantization intervals and levels which minimize the one has to choose the intervals and levels which minimize the intervals and levels which minimize the quantization error. So how do we choose, so the design of the quantizer is nothing but well you have to come up with the boundaries of the intervals m0, m1 up to mL these are your interval boundaries and the quantization levels these are the quantization levels so you have to choose the interval boundaries and the quantization levels, alright?

So that constitutes a complete quantizer because once it determines the interval boundaries mk mk plus 1 when the signal sample lies m lies between mk and mk plus 1 your quantizing it to nuk, alright. So once

you design once we give the set of these characterize the set of these boundaries, alright the interval boundaries and the levels in each interval in each interval quantization interval we have completely characterize the quantizer.

So we have to optimally choose these quantites, okay. Now how does the Lloyd max algorithm optimally design these or optimally choose these quantities. For that we have to start with the probability density function of the signal sample m, alright.

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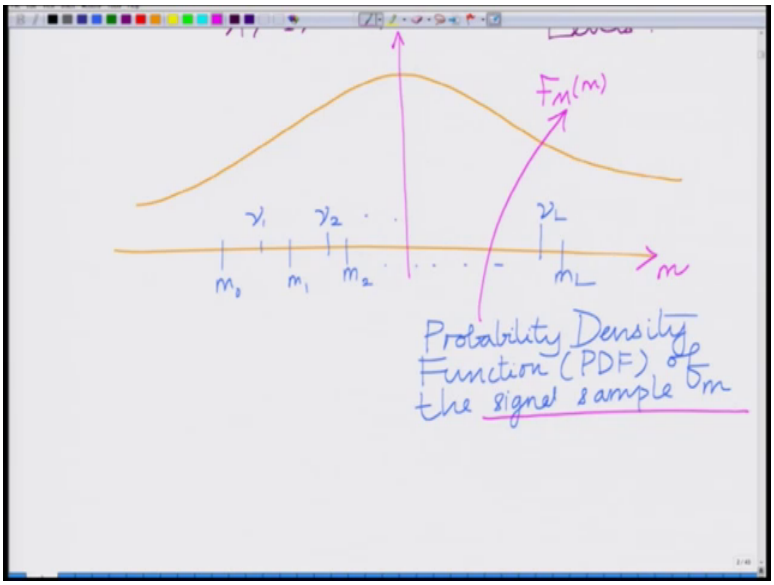

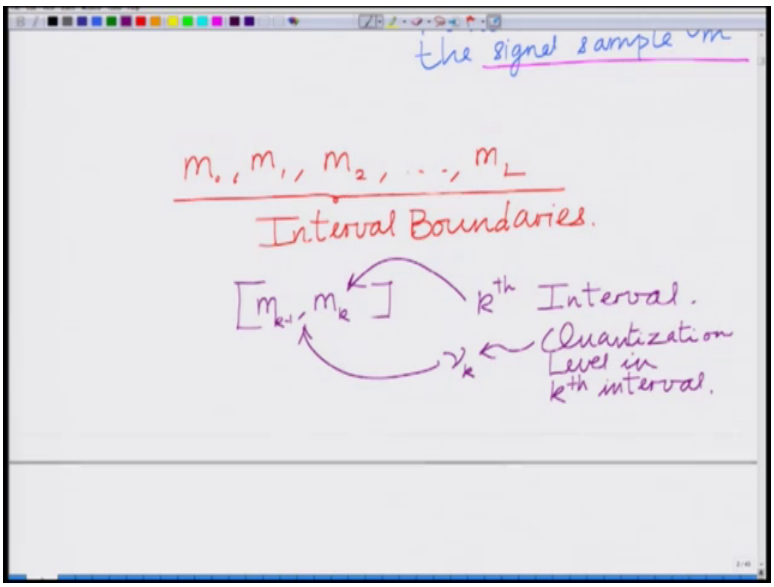

So to start with the Lloyd max quantizer we have to first consider we have to consider Fm(m) or let us make it symmetric this is your Fm(m) which is the this is your probability density function this is the we have to start with the Probability Density Function (PDF) of the PDF of the signal sample m, okay.

So let us consider the signal sample is random in nature that is m the signal sample is random in nature so and it is characterized by this probability density function Fm(m). And now we have to place these intervals m0, m1, mL and the quantization levels nu0, nu1 up to or nu you have to place these levels nu1, nu2 up to nuL and we will use the probability density function so we have to consider the probability density function of the signal sample m, alight this is the probability density function of the signal sample m.

Now m0, m1, m2 are the interval boundaries we already know that m0, m1, m2 these are the interval boundaries, okay. Now let us denote by the kth interval mk to mk plus 1 let us call this as a kth interval this is your kth interval and the quantization level in this is nuk, okay. So let us call this as the kth or let us call mk minus 1 to mk as the kth interval mk minus 1 to mk mk minus 1 to mk this is the kth interval nuk is a quantization level this is the quantization level nuk is a quantization level in the kth interval, okay.

Now need we need as I have already told you we need to find both this quantization levels quantization intervals and the quantization levels alright to determine the optimal quantizer we have to find both the quantization intervals and quantization levels.

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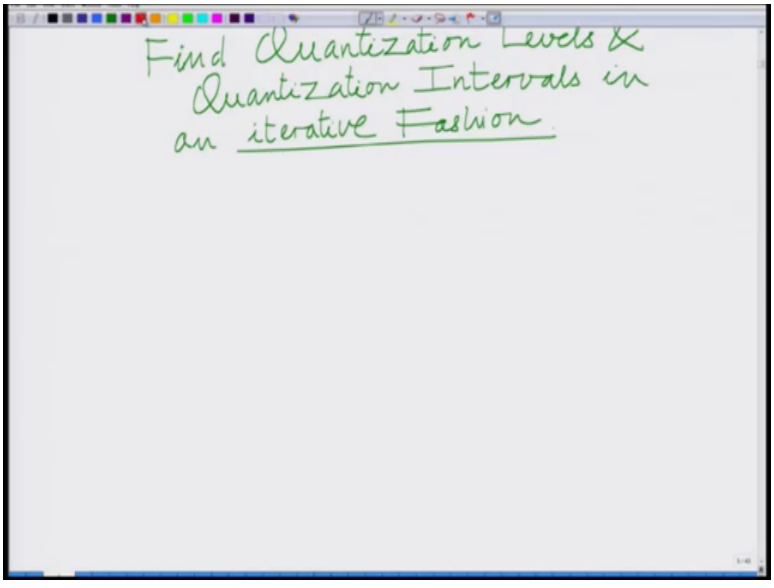

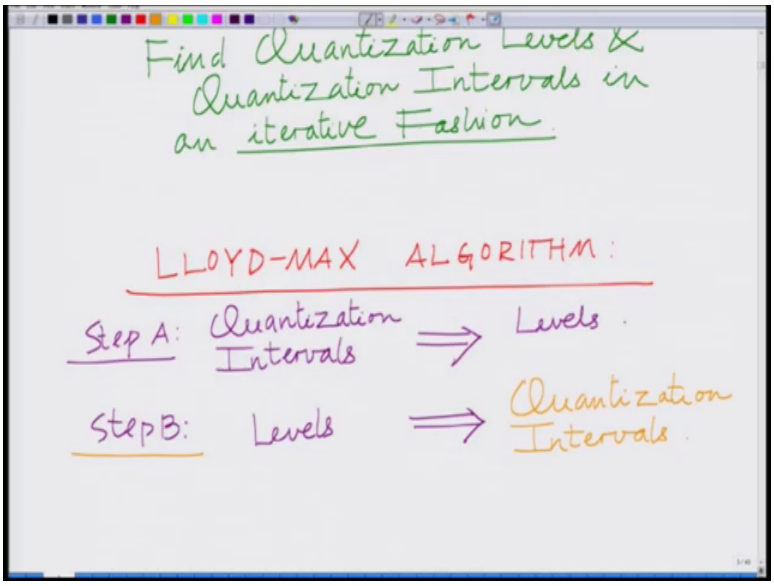

And therefore what we are going to do is we are going to find them in a iterative fashions so Lloyd max algorithms we are going to find in an iterative fashion. Now what do we mean by the iterative fashion given the intervals we will find the optimal quantization levels.

Given the optimal quantization levels we will find the optimization optimal quantization intervals or the quantization interval boundary. So (ou) algorithm the Lloyd max algorithm in summary so what we are going to see in this Lloyd max algorithm is basically given the quantization levels alright or given the (inte) given the intervals we will find the levels and given the levels we will find the quantization intervals. So this is let us say step A this is let say your step B step B is basically given the levels you will find the you will find the quantization intervals.

And then we will repeat steps A and B iteratively, so given the quantization intervals we will find the optimal quantization levels we on the quantization levels we will find the quantization (())(13:11). We will iteratively repeat this until we get the final quantizer, so iteratively repeat these steps A and B until we get the final quantizer.

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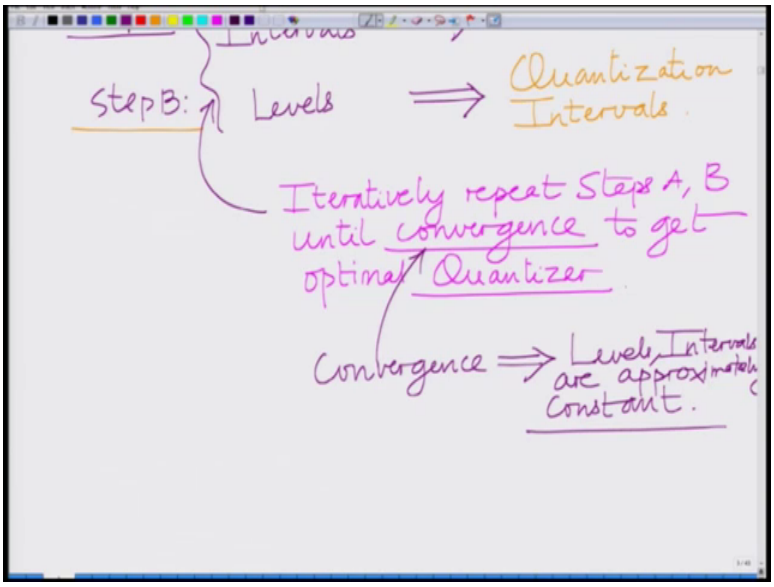

So the algorithm will be iteratively okay iteratively repeats step until convergence okay until non the levels and the intervals and the levels are not changing basically that is what it means until convergence to get the optimal quantizer.

And what do we mean by the convergence by convergence we mean that implies levels comma are not changing are there approximately constant that is once you observe that once you observe that with each iteration, the levels and intervals the quantization intervals are not changing that they are more or less constant than the algorithm is convergence at that point you can output the intervals and output the quantization levels and basically determine the optimal quantizer and use it for the purposes of quantization, okay.

So that in summary is the Lloyd max algorithm, alright and that is basically a sketch of how the Lloyd max algorithm works now what exactly is the working of the Lloyd max algorithm we are going to look at that in the next module, thank you very much.