Principles of Communication- Part I
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Module No 7
Lecture 40
Introduction to Quantization, Uniform Quantizer, Mid-Tread Quantizer

Hello welcome to another module in this massive open online course, so in this module let us start looking at another new aspect that is quantization, so we will start looking at the new aspect in this module that is quantization, okay.

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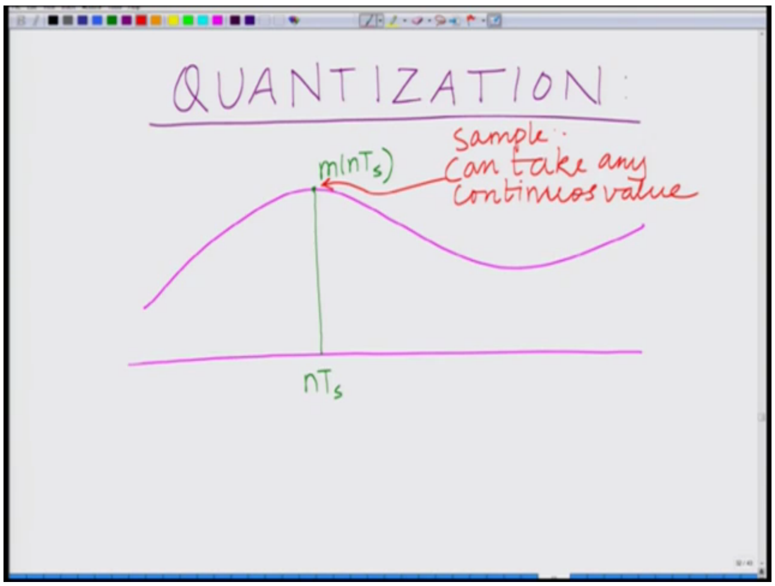

Now what is quantization remember we already know something that is we are sampling signals a discrete time instants, correct? So we have the signal and which we are sampling at the sampling instants the multiples of the sampling instants that this is the sample at mnTs at the nth multiple of the sampling.

Now this sample can still take this is can take any analog value, okay. Can take any continuous continuous so the sample that we have sampled signal the sample can take any value, alright. It can take any value in continuous scale, alright. And so we want to for the purpose of digital communication we want to convert it to a discrete set of values that is belonging to a discrete set so that we can convert it into (discri) information bits and these bits can then be subsequently transmitted over the (())(1:52) or they can be stored, alright. So the purpose of converting is samples which can take values from a continuous set 2 values from a discrete set or 2 levels from a discrete set is termed as quantization.

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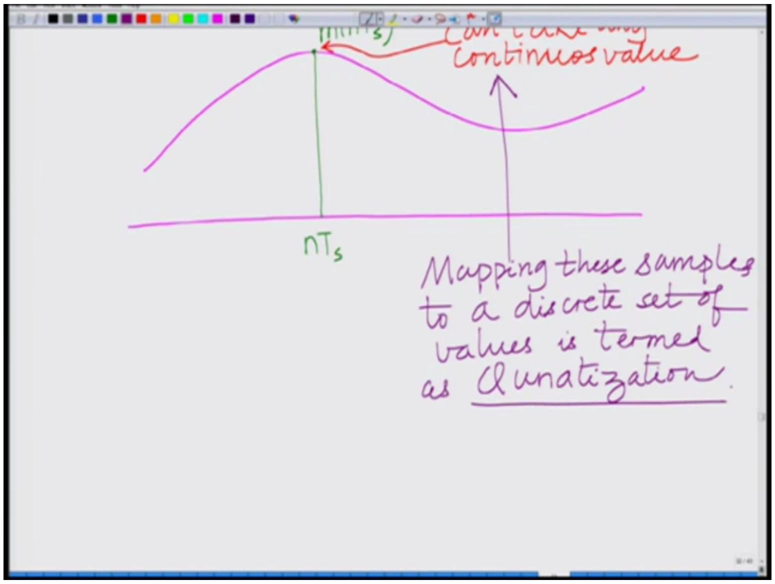

So converting this continuous valued samples conversion of these continuous (())(2:14) to a discrete or mapping these samples to a discrete set of values is termed as is termed as this is termed as quantization, okay. Converting samples this continuous samples which have continuous values to a discrete set of values or a discrete set of levels is termed as this process is termed as quantization, okay.

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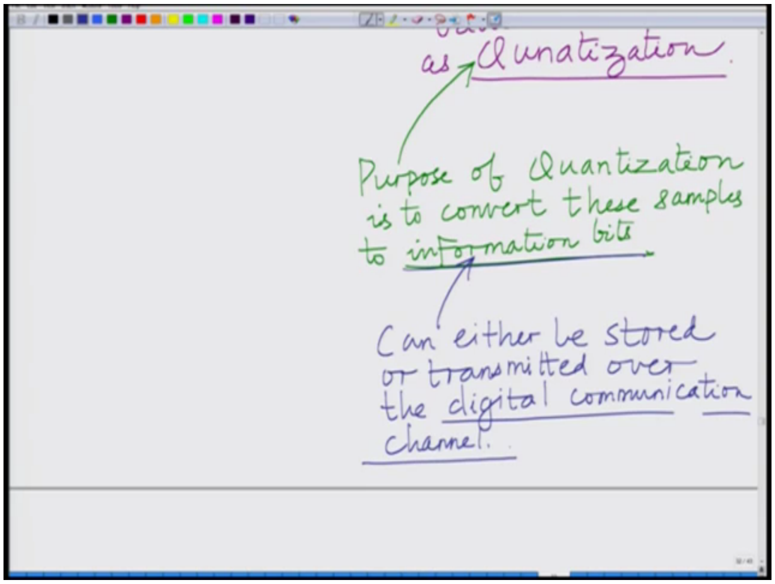

Okay, so the purpose of quantization I already said the purpose of quantization is to eventually quantization is to convert the samples to a these samples to information bits, okay. That is important to binary bits, okay. Information bits which can be stored or transmitted which can be stored these can be stored these information bits can either be stored or transmitted over the channel over the the digital over a digital communication channel, alright. They can be transmitted over a digital communication system or can be stored in a digital storage device, alright.

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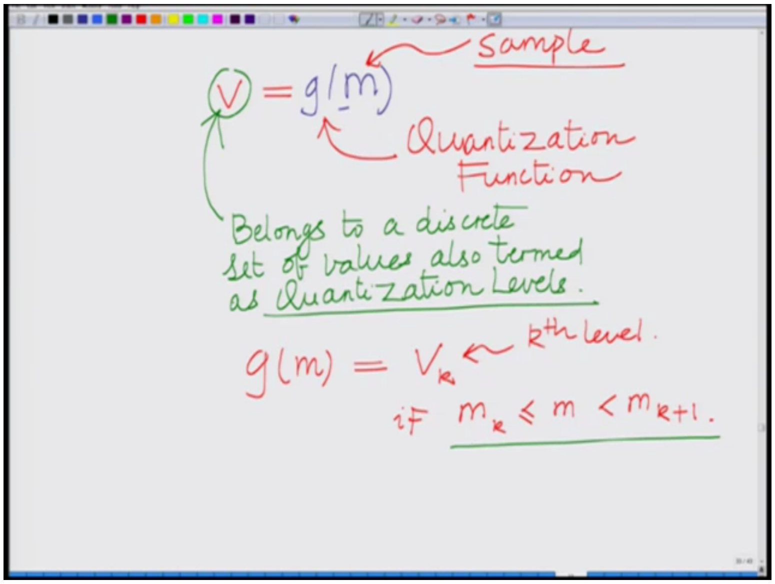

And therefore this process of quantization can be represented as follows now we have the sample m so m is the sample we have the quantization function gm so this maps it to nu, so this is the sample which can take any particular value so this is your sample this is your quantization function this is your quantization function and this bears the quantized this V belongs to a quantized station set.

V is belongs to a this belongs to a discrete set set of values which are also termed as a quantization levels so we have this quantization function g so our (quantiz) m is the sample to be quantized g is the quantization function and V is the output of the quantization function and V this output V belongs to a discrete set of values belongs to a discrete set of values also termed as the quantization levels that is this discrete set of values are termed as quantization level.

So the quantization can be fixed as gm equal to gm the quantizer output, okay g is also the quantization function can be termed as a quantizer equals Vk this is the kth quantizer level kth level if mk less than or equal to be can see mk less than or equal to m less than or equal to less than or (eq) or less than mk plus one. So we can see that this quantizer, the. This quantizer comprises of levels V1 V2 up to Vn such that if the sample m, right? The sample value m lies between mk and mk plus 1 it is quantized to the quantization level, right? The quantizer output is Vk, alright. So V1, V2, Vn are the discrete set of quantization levels or their discrete set of outputs of this quantizer, okay.

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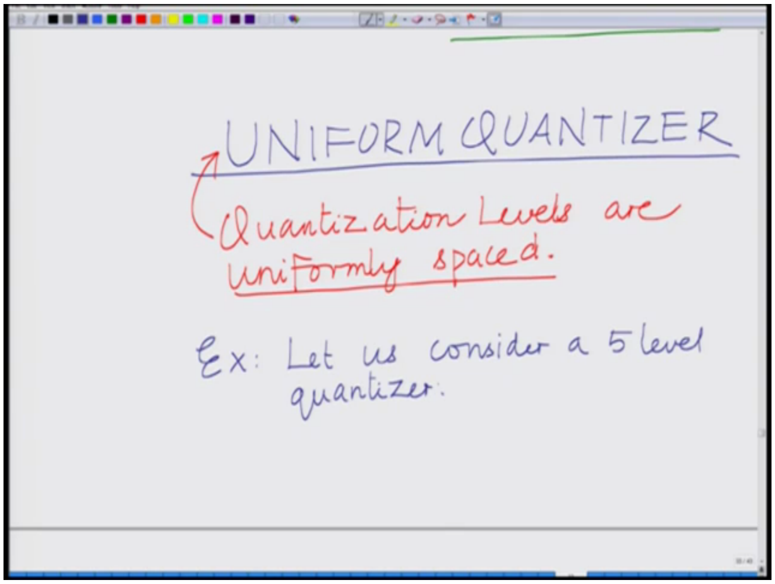

So let us look at some examples of this quantizer a simple quantizer is what is known as a uniform quantizer one of the simplest quantizers is known as a uniform quantizer this is one of the simplest quantizer which is a uniform quantizer as the name implies a uniform quantizer simply means that the quantization levels are uniformly spaced so uniformly quantization in a uniform quantizer as the name implies the quantization are the quantization levels the quantization levels are uniformly these are uniformly spaced for instance.

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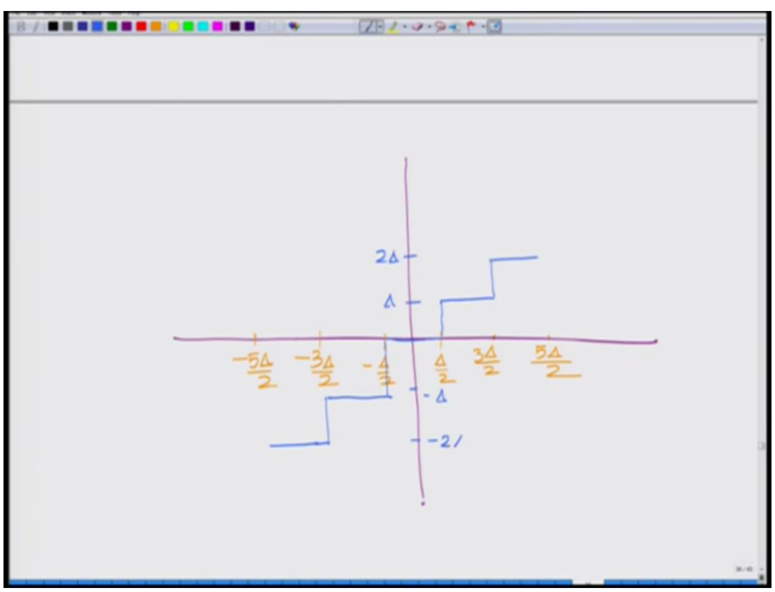

Let us consider a 5 level for example let us consider a 5 level quantizer, okay. So which means there are 5 quantization intervals and correspondingly there will be 5 quantization levels, we. So let us say the quantization interval of is of width Delta, so there is one interval from delta by minus delta by 2 to delta by 2 another from delta by 2 to 3 delta by 2, 3 delta by 2 to 5 delta by 5 delta by 2 r minus 3 delta by 2 minus 5 delta by 2, okay. And now between minus delta by 2 is m lies between minus delta by 2 to delta by 2 it is mapped to 0.

If m lies between 3 delta by 2 to delta by 2 it is mapped to Delta if m lies between 3 delta 2 by 2 to 5 delta by 2 it is mapped to 2 delta that is the output is mapped to 2delta similarly if it lies between minus delta by 2 to minus 3 delta by 2 it is mapped to output is mapped to minus delta, if it lies between minus 3 delta by 2 to minus delta by 2 it is mapped to minus 2 delta this is minus 2 Delta. So we have as you can see we have let us look at this quantizer again.

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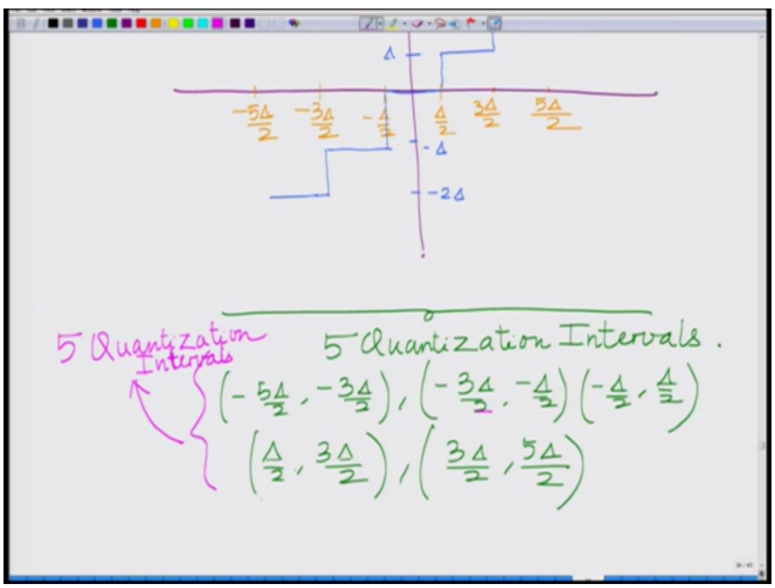

We have well, 5 quantization intervals the 5 quantization intervals are minus 5 delta by 2 to minus 3 delta by 2 minus 3 delta by 2 to minus Delta by 2 minus delta by 2 to delta by 2, delta by 2 to 3 delta by 2, 3 delta by 2 to 5 delta by 2, okay. So these are your 5 quantization intervals, okay. These are 5 quantization intervals and we are calling this as a uniformly spaced uniform quantizer because note that the widths of this quantization interval are uniform.

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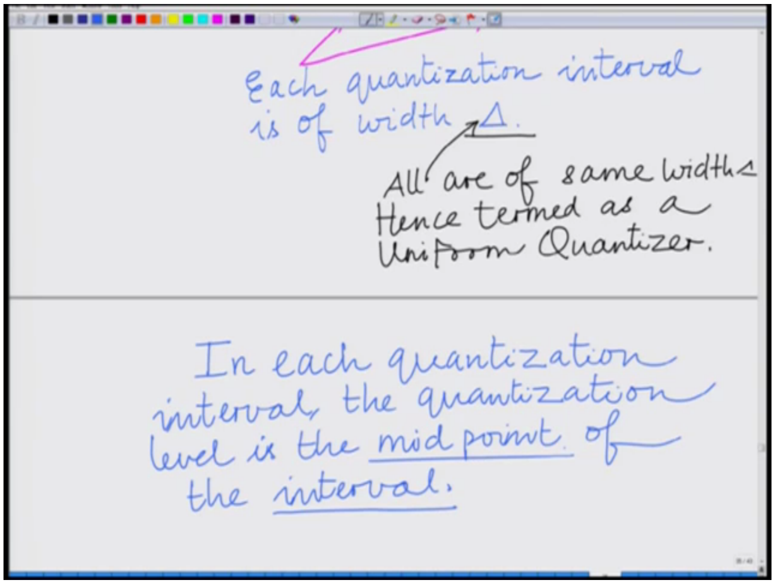

Each quantization interval is of with Delta for instance you can see 5 Delta 2 by 2 minus 3 delta by 2 equals, so you can see each quantization interval all the 5 each quantization interval is of width each quantization interval each quantization interval is of width Delta therefore this is known as a this is termed as a uniform quantizer each quantization (()) (12:40) all or of the same width Delta that is all or of same width Delta or uniform with Delta, hence termed as a uniform quantizer as a hence this is termed as a uniform quantizer, okay that is the first point so therefore this is termed as a uniform therefore this is termed as a uniform quantizer, okay that is the first point.

Now the second aspect is that, now if you look at the quantization level for instance if you look at the quantization level in 3 delta by 2 to (del) 5 Delta by 2 quantization level is the midpoint, alright. So for instance in 3 in each quantization in each quantization interval, correct? In each quantization interval to the quantization level is the image quantization interval the quantization level is the midpoint of the interval, alright.

It's not necessarily always true it's only true because this is a uniform quantizer that is in each quantization interval, alright. In each quantization interval the quantization level is the midpoint of the (())(14:37) for instance in 3 delta by 2 to 5 delta by 2 it is 2 delta in delta by 2 to 3 delta by 2 it is Delta, alright which means that if the signal sample lies between m that is m lies between 3 delta by 2 to 5 delta by 2, alright. It is quantized to 2delta, alright it is mapped to 2 Delta.

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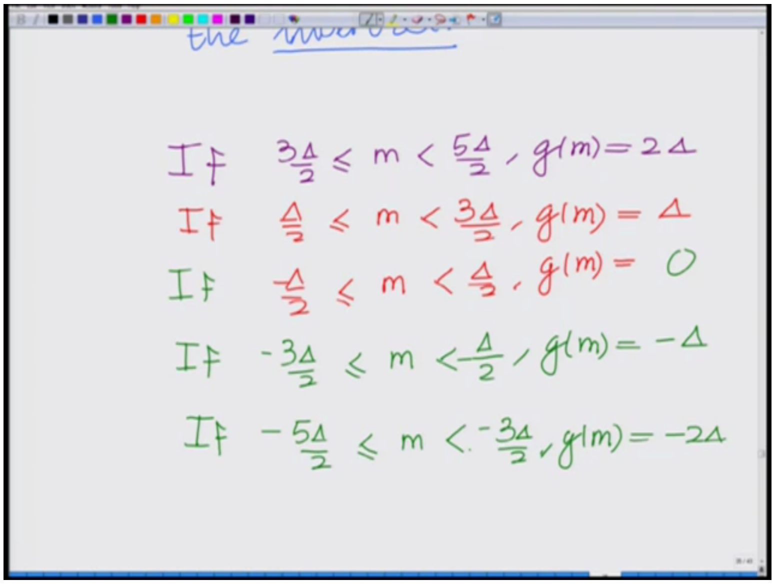

So which means the quantizer can be described as follows if for instance let us say 3 delta by 2 less than equal to m less than 5 delta by 2 g of m is equal to midpoint that is 2 Delta similarly if delta by 2 less than or equal to m less than or equal to 3 Delta by 2 g of m is equal to the midpoint is minus delta by 2 less than or equal to m less than delta by 2 gm is equal to minus delta.

Similarly if minus 3 delta by 2 less than or equal to M less than minus delta by 2, gm is equal to minus well this in this case gm is equal to 0 minus delta by 2 to less than equal to m less than equal to Delta by 2 it is the midpoint 0 if minus 3 delta by 2 less than equal to m less than minus delta by 2, gm is equal to minus delta finally if minus 5 delta by 2 less than or equal to m less than or equal to minus 3 delta by 2 then gm is equal to minus 2 delta, alright.

So this is the your basically the representation of the this is basically the overall representation of the quantizer, alright. Now you can observe that this quantization characteristic for all signal samples, right that lies that lie between for instance 3 delta by 2 to 5 delta by 2 all the samples are mapped to the quantization level delta which means it is a many to one mapping, right? You can see that in finite set of values belonging to minus belonging to 3 delta by 2 to 5 delta by 2 are mapped are mapped to 2 Delta, right?

So therefore it is a many to one mapping its lossy mapping, alright. And therefore there is by and therefore by definition there is going to because you are mapping an infinite set of values to represent a finite value, alright. There is going to be an error in the wave, alright. For instance if m is exactly equal to 2 delta when you map it to 2 delta there is not going to be an error but if m is for instance let us say m is for instance let us say 5 delta by 2,okay. And you map it 2 delta then you are going to incur a quantization error of delta by 2, alright. And you can see depending on where it lies in this quantization interval minus 3delta by 2 to 5 delta by 2 you can incur a quantization error that lies between minus delta by 2 to delta by 2, alright.

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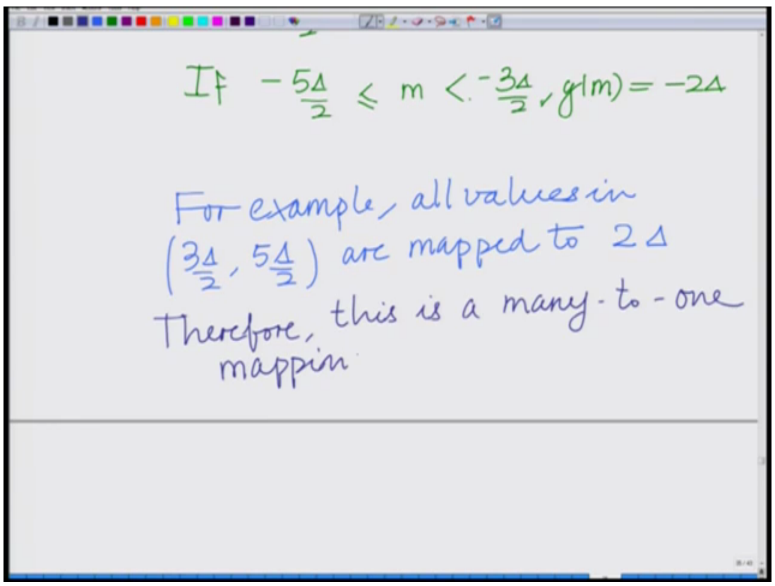

So this quantization error for example all values in all values in 3delta by 2 to 5 delta by 2 are mapped are mapped to 2 Delta therefore therefore there is going to be a quantization error.

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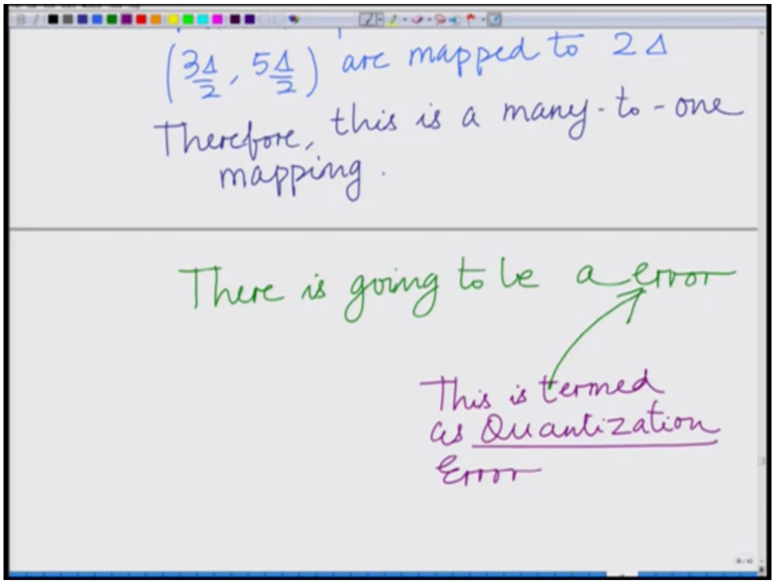

Therefore this is a first of all this is a many to one mapping there is a first point, second there is going to be a quantization error by definition there is going to be a there is going to be an error this is this is a many to one therefore it is a non-invertible mapping this is termed as quantization error this is termed as quantization error for instance, correct?

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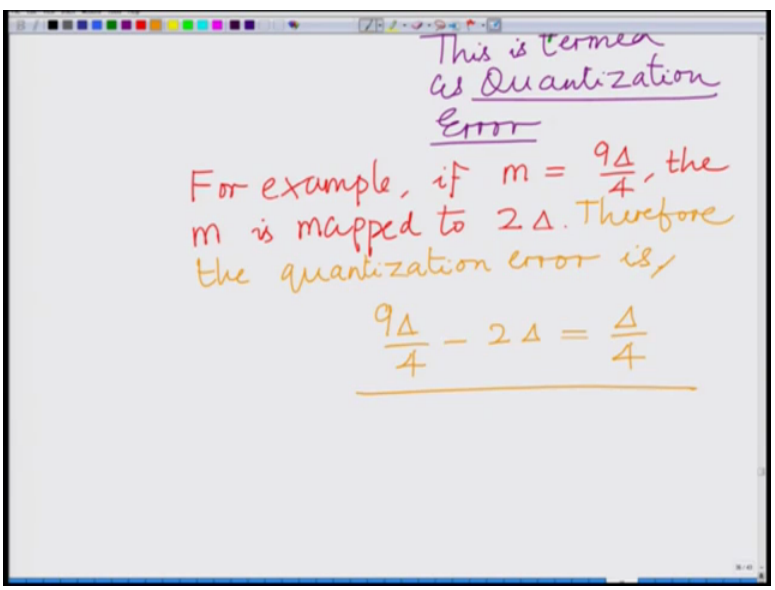

For example if for example if m is equal to let us say you have your 2 delta plus Delta by 4 that is 9 delta by 4 then m you can see lies between 3 delta by 2 to 5 delta by 2 therefore m is mapped, correct? To m is mapped to 2 delta which means the error therefore therefore there is a quantization error and the quantization error is 9 delta by 4 minus 2 delta which is equal to Delta by 4, alright.

And therefore depending on where the sample lies between 3 delta by 2 to 5 delta by 2 the quantization error can range from minus delta by 2 to 5 delta and if the (quan) the sample is minus 3 3 delta by 2 to 2 delta then of course quantization error is well, the quantization error is 3 Delta by 2 minus 2 delta that is basically the original sample minus this quantized value, correct? Remember how we are calculating this quantization error that is the original sample value minus its quantized value, so 3delta by 2 minus 2 delta that is minus delta by 2, if it is 5 Delta by 2, alright. 5 Delta by 2 minus 2 delta that is delta by 2, so depending on where it lies in the interval the quantization error can range from minus delta by 2 to delta by 2 for example.

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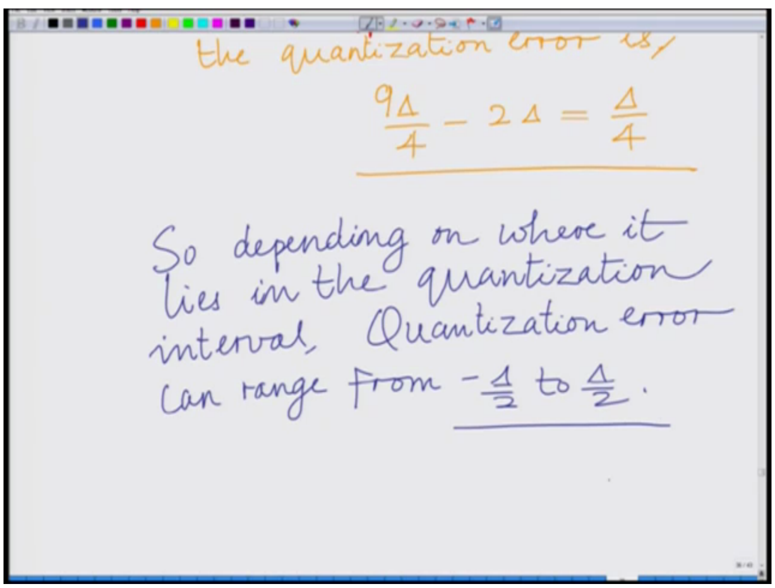

So depending on where it lies in the interval depending on where it lies in the quantization interval the quantization error can range from minus delta by 2 to delta by 2, okay. That is the first that is another point, okay. This quantization error lies in the interval minus delta by 2 to delta by 2. Now the other thing that you can observe is the maximum, since the interval the total quantizer can take input from minus 5delta by 2 to 5 delta by 2, alright. So the minimum sample value can be minus 5 delta by 2 maximum sample value can be 5 Delta by 2 this is termed as the dynamic range of the quantizer, okay.

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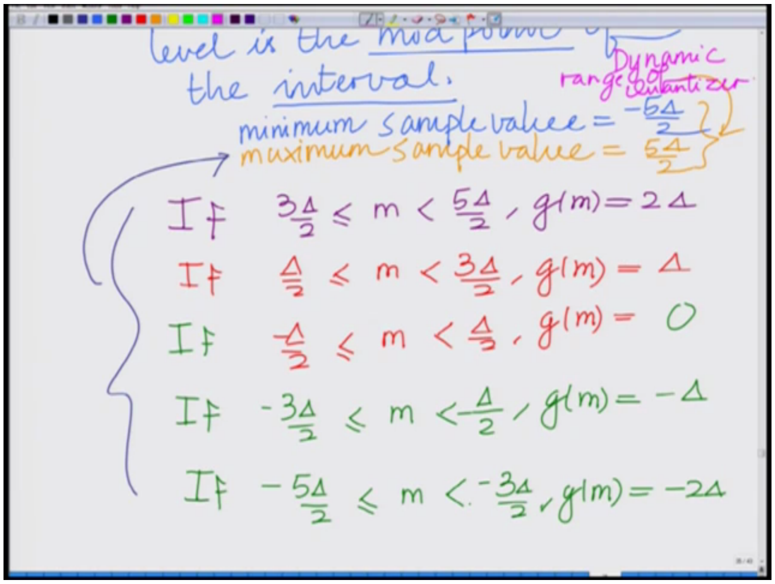

You also noticed from here noticed from here that minimum sample value equals minus 5delta by 2 the maximum this minus 5 delta by 2 to delta by 2 this is termed as the dynamic range of the quantizer, alright. So which means that the quantizer can take only values of samples from minus 5 delta by 2 to 5 delta by 2, so therefore when you design a quantizer one has to take this into account what is the range of the (samp)? So naturally the samples the total range in which the samples (hi) lie has to be captured, correct? By the dynamic range of this quantizer, alright. So they have to be captured by the dynamic range of the quantizer, okay. So that is this minus 5 delta by 2 to 5 delta by 2 can be considered as the dynamic range of the quantizer, okay.

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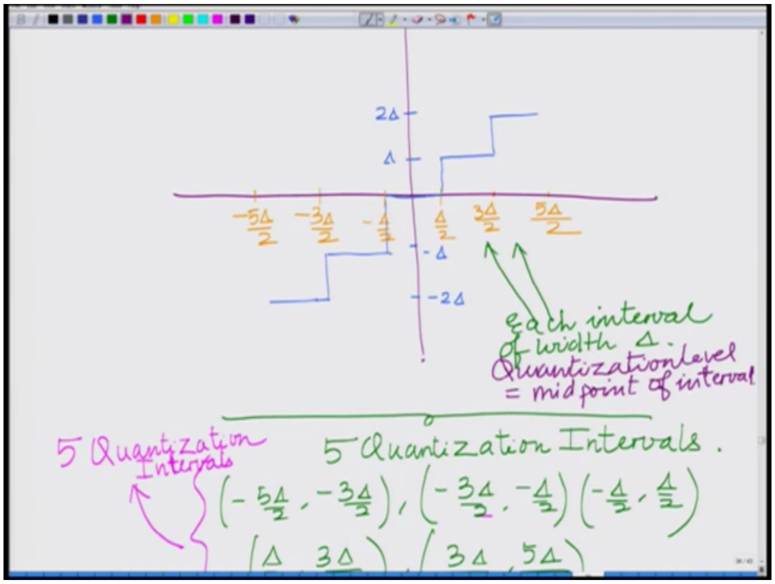

So can be considered as the (min) dynamic range of the quantizer and finally so you can see that you can see various aspects each interval is of width delta, correct? Of width the delta quantization level is the midpoint of the interval quantization level equals the midpoint of the interval and further you can see one more point that is when there are odd number of levels there is going to be a quantization interval from minus delta by 2 to delta by 2 and which corresponds to the quantization level 0.

So there is going to be a quantization level corresponding to 0 this is therefore termed as the mid thread quantizer mid-thread quantizer because in the midpoint of the quantizer the quantization the quantizer function is also termed as the quantizer characteristic is flat this is termed as a mid thread quantizer. So we have a flat quantizer characteristic at midpoint at midpoint this is termed as as a mid thread quantizer, okay.

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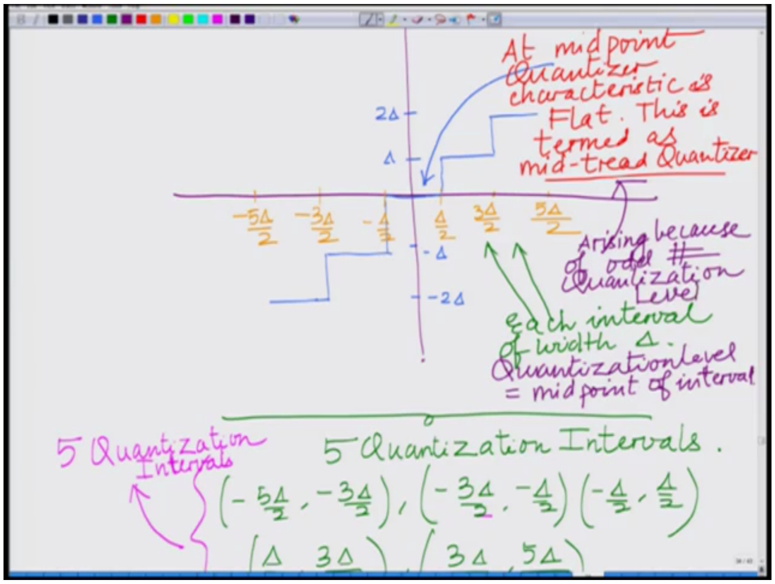

So at midpoint the quantization characteristic is flat this is termed as that is basically between minus delta by 2 to delta by 2 it is mapped to 0 this is termed as a mid thread quantizer, alright and this mid thread odd quantizer is arising because there are (())(26:15) odd number of quantization levels, okay. And this is arising because there are an odd number of quantization levels, okay. Arising because there are odd number of odd number of quantization, so odd number of quantization implies that it is a mid thread quantization, okay.

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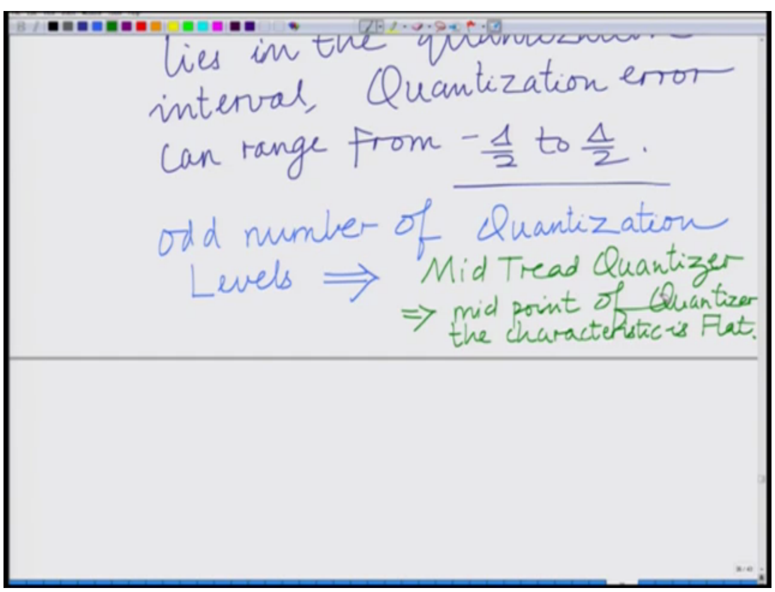

So odd number of quantization levels odd number of remember we have 5 quantization levels odd number of quantization levels implies we have a mid thread (quanti) which implies implies midpoint of quantizer the characteristic is flat that is midpoint not midpoint of quantizer basically the midpoint of the dynamic range that is if you will get the dynamic range minus 5 delta by 2 to delta by 2 5 Delta by 2 at the midpoint at the middle of the quantizer the quantization characteristic, right?

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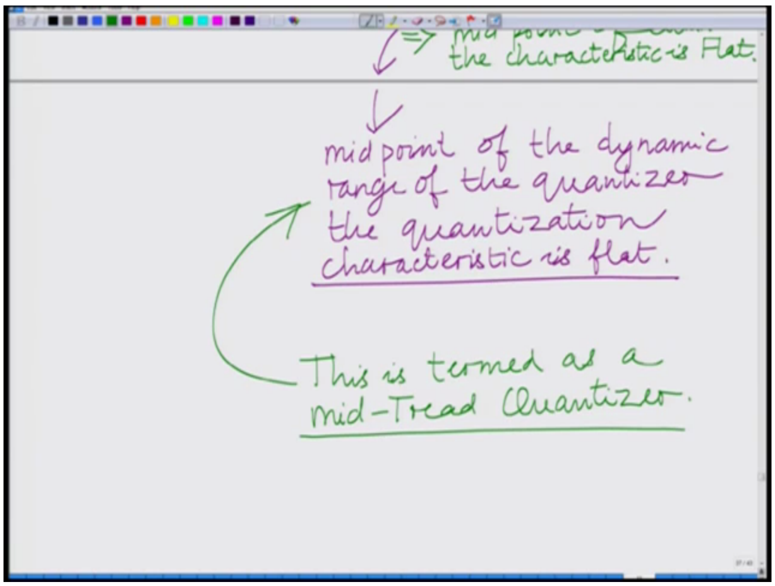

The quantization characteristic is flat that is it is (ba) basically mapped to the 0 between minus delta by 2 to Delta by 2 the quantizer level is 0 this is termed as the mid thread quantizer, alright they can (ser) midpoint of the quantization characteristic a midpoint of the dynamic of the quantizer the quantization characteristic is flat this is termed as mid thread quantizer this is termed as a mid thread quantizer, okay. This is termed as a as a, this is termed as a mid thread this is termed as a mid thread this is termed as a mid thread quantizer, okay.

So what we have seen in this module in this (mi) module we have seen we have seen an introduction to basically the concept of quantization, alright. We have seen what is the definition of quantization? quantization basically maps the continuous valued sample of the signal to a set of discrete values of quantization levels, alright this is termed as quantization, alright and there can be different types of quantizer. One of the simplest type of quantizer is a uniform quantizer which consist of a certain number of levels each of them have all of the with the uniform quantizer is (ca) characterized by uniform spacing of the intervals. All the intervals of the uniform of the uniform quantizer have an equal width, alright.

And the quantization level corresponds to the midpoint of the level of the interval, okay. And we have seen that with the (quanti) the uniform quantizer has an odd number of quantization intervals or corresponding to my odd number of quantization levels the midpoint of the quantizer, alright. The midpoint of the quantizer has is flat this is termed as a mid thread quantizer and more importantly the quantizer not just the uniform quantizer any quantization characteristic is a many to one mapping because all the sample values belonging to an interval are mapped to a signal value that is the quantization level, so naturally this is many to one mapping which means it is not invertible it is a lossy mapping, alright.

So once you have your quantization level you cannot recover the original sample from that because it's a many to one mapping, okay. So therefore there is going to be a quantization error, alright and this quantization error naturally depending on the width of the interval the width of the interval is large then the large number of samples are mapped to a single value therefore the quantization error is going to grow, alright.

So the final the quantization intervals that is the finer the interval that is the smaller the quantization interval the smaller is going to be the quantization error because of quantization error remember, minus delta by 2 to delta by 2 in this case is basically proportional to Delta by 2 to minus minus delta by its delta it is proportional to the width of the quantization interval therefore you can say that if you want to reduce the quantization intervals to make it more accurate you have to make the quantization intervals smaller but if the quantization interval becomes smaller for the same dynamic range then you are going to be a large number of quantization intervals which means large number of quantization levels which means you will need a large number of bits represent the quantization levels of that quantize.

So that is the trade-off it is the quantization error versus the bit rate of the quantizer, alright which is the number of bits that are required to represent the levels of the quantizer, alright. So we will look more at this in the subsequent lectures or the subsequent modules, thank you very much.