

Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur

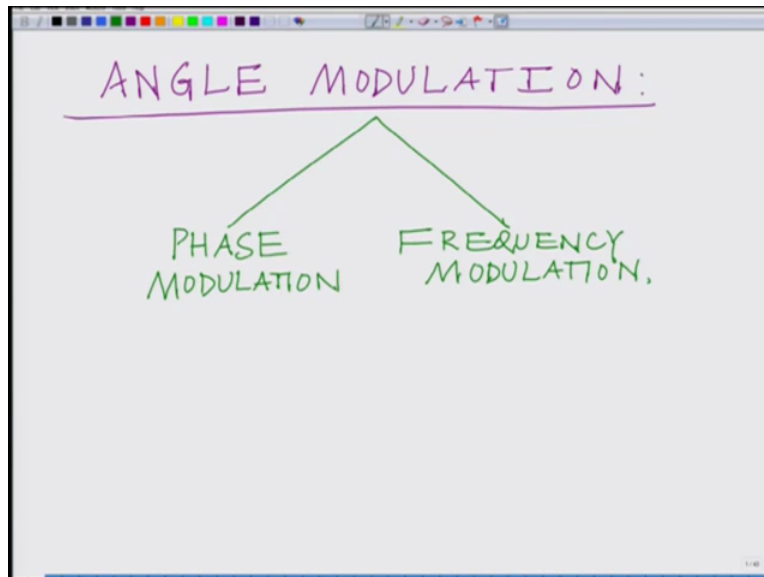
Module No 5

Lecture 28

Introduction to Angle Modulation, Description of Phase Modulation (PM) and Frequency Modulation (FM)

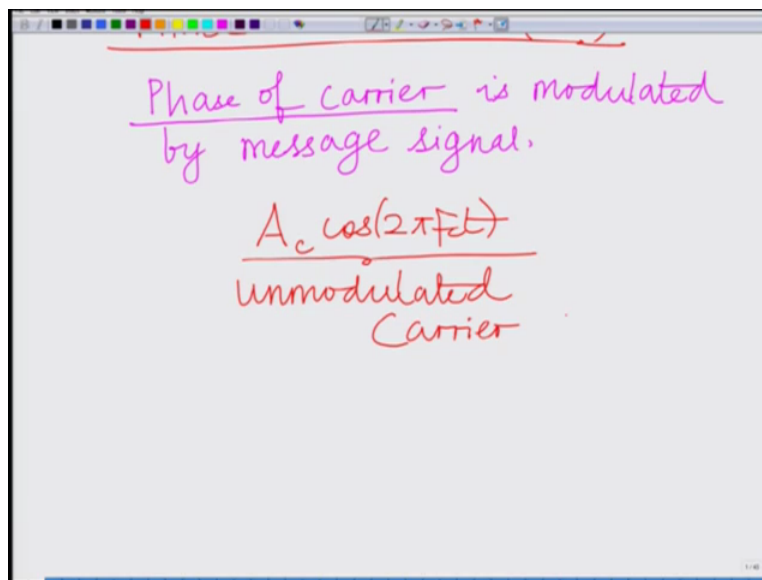
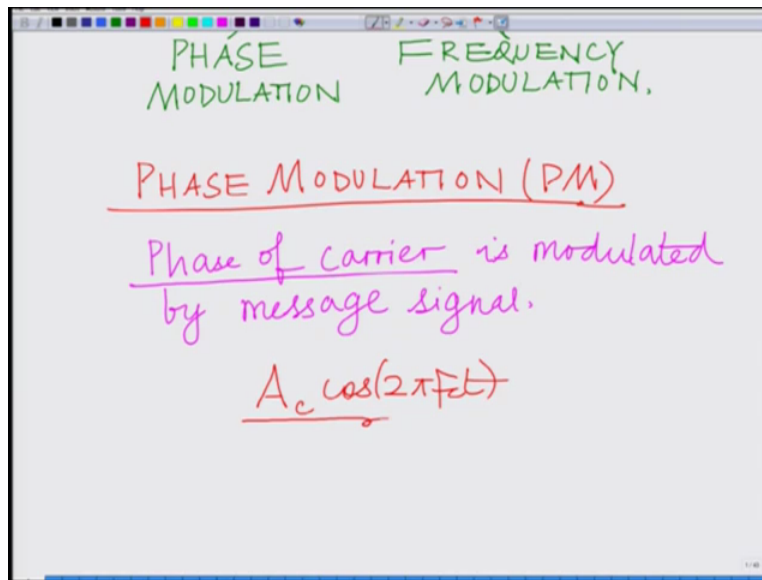
Hello and welcome to another module in this massive open online course, so today we will start looking at a different type of modulation that is angle modulation, alright.

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So in today's in this lecture will start looking at so far we have looked at amplitude modulation we will now look at different form of modulation that is termed as angle modulation, okay. An angle modulation can belong to 2types either it can be either phase modulation. So angle modulation can either be phase modulation or it can be, frequency modulation or it can be frequency modulation and we will start looking at both these different branch of modulation. So angle modulation can either be of the type phase modulation for frequency modulation frequency modulation, so first let us start looking at phase modulation, okay.

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So now in phase modulation obviously as the name implies, so you phase modulation which can be denoted by PM. So in phase modulation we have the phase of a carrier, so phase of carrier is modulated by the message is modulated or varied by the message signal. So the phase of the carrier is modulated by the message signal for instance let us say your carrier signal is $A_c \cos(2\pi f_c t)$ this is your un-modulated carrier signal, what we are going to do is we are going to we are going to modulated the phase of this.

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by message signal.

$$A_c \cos(2\pi f_c t)$$

Unmodulated Carrier

Message Signal.

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

Modulated Phase.

So if I take the phase that is θ_i of t and if I modulated it as follows that is this is the unmodulated phase that is $2\pi f_c t$ plus, now modulated using a message signal that is $m(t)$ where $m(t)$, remember $m(t)$ is the message signal so this is $\theta_i t$ this is a modulated phase or this is basically your this is the modulated phase, so the phase of the modulated message signal is $2\pi f_c t$ plus k_p times $m(t)$. You can see the phase is modulated by the message signal $m(t)$. So therefore the carrier with its phase modulated by this message signal is given as.

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Modulated Phase.

$$s(t) = A_c \cos(\theta_i(t))$$

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

Phase Modulated (PM) Signal.

Now the modulated carrier is given as or the modulated signal is given as $A_c \cos(\theta(t))$ which is basically $A_c \cos(2\pi f_c t + k_p m(t))$ so this is a phase modulated signal. So this is your phase modulated signal where the phase of the signal is modulated by the message signal $m(t)$, okay. And what you can see is because of this message signal where the phase is modulated by the message.

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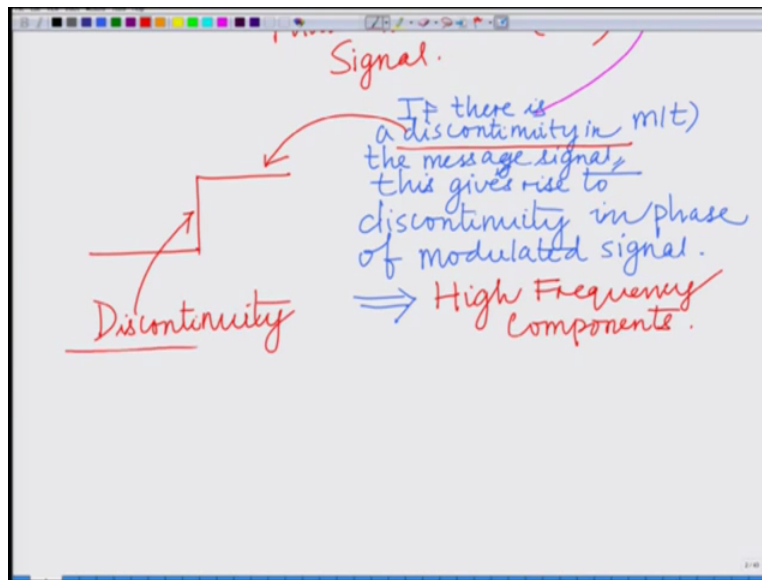
$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

Phase Modulated (PM) Signal.

If there is a discontinuity in the message signal $m(t)$, this gives rise to discontinuity in phase of modulated signal.

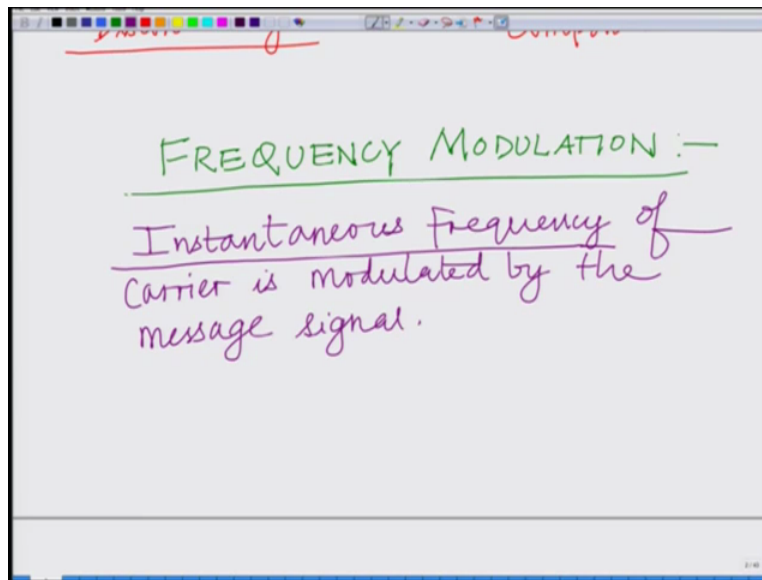
What you can see is if the message has a discontinuity if the message has a discontinuity that is for instance if the message is a step function for instance if the message has a discontinuity then the phase of the modulated signal experiences a discontinuity this gives rise to high frequency components. So what happens the disadvantage with this is that if there is a discontinuity or if there is a discontinuity or let us say this is your $m(t)$ if there is a discontinuity in the message signal this gives rise to this gives rise to discontinuity or this gives rise to discontinuity in the phase of the continuity in phase of the of the modulated signal which leads to and this leads to high frequency components.

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This leads to high frequency components and output, so what we can see is if we have a signal which is discontinuous, right? If there is a discontinuity in the message signal which is something like this. If there is a discontinuity in the message signal, so this is your discontinuity, okay. If there is a discontinuity in the message signal that means there is going to be a discontinuity in the phase of the modulated signal and this discontinuity in the phase of the modulated signal gives rise to high frequency components in the output therefore to avoid this we use another different form of angle modulation that is termed as frequency modulation.

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So frequency modulation is another kind of angle modulation so frequency modulation so let us now come to frequency modulation in the frequency modulation instantaneous frequency of the modulator varied so the instantaneous frequency of carrier is modulated by the message signal. So in this what we have is the instantaneous frequency is modular rather than the phase in the phase modulation the phase of the signal carrier is modulated by the message which results in phase discontinuity.

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$$f_i(t) = f_c + k_f m(t)$$

↑ modulated frequency

$$\theta_i(t) = 2\pi \int_0^t f_i(z) dz$$
$$= 2\pi \int_0^t (f_c + k_f m(z)) dz$$

However in frequency modulation the instantaneous frequency is modulated by the message signal and this can be expressed as follows, so if $F_i(t)$ is the instantaneous frequency so the instantaneous frequency equals F_c that is the steady state frequency, frequency of the modulated carrier plus k_f times $m(t)$. Once again $m(t)$ is the message signal, so this $F_i(t)$ is the modulated frequency modulated frequency or basically the, now you can think of this as the instantaneous frequency of the modulated signal so you can think of a signal carrier wave in which the frequency at every time instant depends on the modulating message signal $m(t)$, okay.

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The image shows a digital whiteboard with handwritten mathematical equations. The first equation is $\theta_i(t) = 2\pi \int_0^t F_i(\tau) d\tau$. The second equation is $= 2\pi \int_0^t (f_c + k_f m(\tau)) d\tau$. The third equation, enclosed in a green rectangular box, is $\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$.

The image shows a digital whiteboard with handwritten mathematical equations. The first equation is $= 2\pi \int_0^t (f_c + k_f m(\tau)) d\tau$. The second equation, enclosed in a green rectangular box, is $\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$. A red arrow points from the text "Phase of FM signal." to the boxed equation.

So therefore the phase now remember the phase frequency is the rate of change of phase therefore the phase of this is given as $\theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau$ which is basically $2\pi \int_0^t f_c + k_f m(\tau) d\tau$ which is equal to $2\pi f_c t + 2\pi \int_0^t k_f m(\tau) d\tau$ or rather $2\pi k_f \int_0^t m(\tau) d\tau$ that is your θ_i that is your θ_i this is the phase this is a phase of the frequency modulated signal.

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$$S(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$$

The frequency modulated signal or Fm signal therefore and therefore and now you can see and therefore the modulated carrier is given as you are $s(t)$ is basically $A_c \cos$ well, $2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$ this is your frequency modulated signal, okay. So this is the expression for the Fm, so this is $s(t)$ is your frequency modulated signal, okay. So we are saying that the carrier the frequency instantaneous frequency $f_i(t)$ is basically 2π is basically $f_c + k_f m(t)$ and therefore from that by integrating the frequency from 0 to t you can get the phase because the phase is the (rea) because frequency is the rate of change of phase and from the phase you can get basically the modulated carrier, alright. And this gives you the expression of the Fm signal.

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The image shows a digital whiteboard with the following content:

- At the top, the FM signal equation is written:
$$S(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$$
 This equation is enclosed in a yellow box. An arrow points from the text "FM Signal" below to the equation.
- Below that, the instantaneous frequency equation is written:
$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$
 This equation is enclosed in a red box.
- At the bottom, the instantaneous phase equation is written:
$$\theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau$$
 This equation is enclosed in a purple box.

Also now $f_i(t)$ remember $f_i(t)$ equals we you are using this relation we can also get $f_i(t)$ remember this it makes sense remember this relation $f_i(t)$ in the instantaneous frequency is the rate of change of the instantaneous phase that is the $\theta_i(t)$ divided by dt and also we have used basically the relation that therefore follows that $\theta_i(t)$ equals and this has to be there has to be a factor of 1 over 2π and $\theta_i(t)$ is basically $\int_0^t f_i(\tau) d\tau$ that is the that is the expression for the expression for the instantaneous phase, okay.

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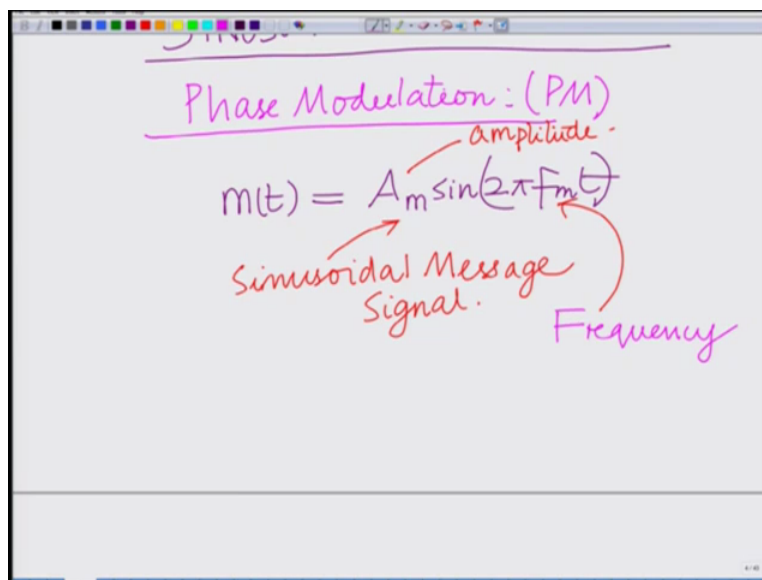
The image shows a digital whiteboard with the following content:

- The text "Max Frequency Deviation" is written in red.
- Below it, the equation $= \max |f_i(t) - f_c|$ is written in red.
- Below that, the equation $= \max |k_f m(t)|$ is written in purple.
- At the bottom, the equation $\Delta F = k_f \max |m(t)|$ is written in purple and enclosed in a yellow box. An arrow points from the text "Peak Frequency Deviation" below to the equation.

That is the expression for the instantaneous phase and further one can define the peak frequency deviation of this the maximum frequency deviation can be defined as well maximum you can see deviation can be defined as this can be defined as maximum of magnitude of $F_i(t)$ minus F_c which is equal to the maximum of magnitude $F_i(t)$ minus F_c you can see that is equal to kF times $m(t)$ which is equal to well, kF times the maximum of magnitude $m(t)$ that is your ΔF .

That is your maximum the maximum frequency deviation or basically the peak frequency deviation, so this ΔF is basically your maximum frequency deviation or basically it denotes the frequency deviation, okay. This is the meaning this is the definition of the quantity ΔF , okay. Now let us consider a simple example let us look at a Sinusoidal modulating signal so let us consider a specific example of Sinusoidal modulation.

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The image shows a handwritten equation on a whiteboard: $m(t) = A_m \sin(2\pi f_m t)$. The word "amplitude" is written in red above A_m with an arrow pointing to it. The word "Frequency" is written in purple below f_m with an arrow pointing to it. The phrase "Sinusoidal Message Signal." is written in red below the equation with an arrow pointing to the entire expression. The title "Phase Modulation: (PM)" is written in purple at the top of the whiteboard.

Let us look at the specific example of Sinusoidal modulation in that let us again once again start with let us once again start with phase modulation that is PM and let the modulating message signal the $m(t)$ modulating message signal is $m(t)$ equals $A_m \sin 2\pi f_m t$ we are considering a Sinusoid modulating signal, correct? We are considering a $m(t)$ we are considering specifically to be a Sinusoidal a Sinusoidal message signal, okay. We are considering the specific case of a Sinusoidal message signal with amplitude A_m this is as you know A_m is amplitude f_m is the frequency of Sinusoidal message signal f_m is the frequency of the Sinusoidal message signal, okay.

And therefore in this scenario remember in phase modulation the message signal modulates the phase of the carrier therefore we have $\theta_i(t)$ is basically $2\pi f_c t$ that is the regular phase $2\pi f_c t$ plus the phase offset which is given by the message that is k_p times $m(t)$.

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Handwritten notes on a whiteboard:

Simusoidal Message Signal. Frequency

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$= 2\pi f_c t + k_p A_m \sin(2\pi f_m t)$$

$$S(t) = A_c \cos(\theta_i(t))$$

$$= A_c \cos(2\pi f_c t + k_p A_m \sin(2\pi f_m t))$$

Therefore we have $\theta_i(t)$ equals $2\pi f_c t$ plus k_p times $m(t)$ but we have $m(t)$ is $A_m \sin 2\pi f_m t$, so this is simply your $2\pi f_c t$ plus k_p times $A_m \sin 2\pi f_m t$.

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Handwritten notes on a whiteboard:

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$= 2\pi f_c t + k_p A_m \sin(2\pi f_m t)$$

$$S(t) = A_c \cos(\theta_i(t))$$

$$S(t) = A_c \cos(2\pi f_c t + k_p A_m \sin(2\pi f_m t))$$

Modulated Signal.

So this is your $\theta_i(t)$ the net modulated message signal is given by $s(t) = A_c \cos(\theta_i(t))$ which is basically your $A_c \cos(2\pi F_c t + k_p A_m \sin(2\pi F_m t))$ this is the modulated signal, okay. $s(t)$ is your $s(t)$ is the modulated signal, okay. Now $F_i(t)$ that is we have $\theta_i(t)$.

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Handwritten derivation on a whiteboard:

$$F_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

instantaneous frequency

$$= \frac{1}{2\pi} \frac{d}{dt} (2\pi F_c t + k_p A_m \sin(2\pi F_m t))$$

$$= F_c + \frac{1}{2\pi} k_p A_m 2\pi F_m \cos(2\pi F_m t)$$

Now the frequency instantaneous frequency $F_i(t)$ remember, $F_i(t)$ is your instantaneous frequency, okay. So this so the instantaneous frequency is given as $\frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$ which is basically given as which is basically equal to this is equal to F_c , well one over 2π let us write the expression for $\frac{d}{dt}$ of well, $\theta_i(t)$ as we know $2\pi F_c t + k_p A_m \sin(2\pi F_m t)$ which is equal to well, F_c plus well, the derivative $\frac{1}{2\pi}$ times derivative $k_p A_m \sin(2\pi F_m t)$ which is $k_p A_m$ into $2\pi F_m$ into the derivative of $\sin(2\pi F_m t)$ is $\cos(2\pi F_m t)$ and now basically this can be simplified further as F_c plus we have the 2π is cancelling.

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The image shows a whiteboard with the following handwritten derivation:

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} (2\pi f_c t + k_p A_m \sin(2\pi f_m t))$$

instantaneous frequency

$$= f_c + \frac{1}{2\pi} k_p A_m 2\pi f_m \cos(2\pi f_m t)$$
$$f_i(t) = f_c + k_p A_m f_m \cos(2\pi f_m t)$$

instantaneous frequency.

So this is $k_p A_m f_m \cos(2\pi f_m t)$ this is the instantaneous frequency. This is your expression for the instantaneous frequency. This is the expression for the this $f_i(t)$ this is the expression for the instantaneous frequency $f_i(t)$ of this phase modulated signal phase which is modulated by the Sinusoidal message signal $A_m \sin(2\pi f_m t)$, okay.

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The image shows a whiteboard with the following handwritten derivation:

$$\Delta F = \max |f_i(t) - f_c|$$
$$= \max |k_p A_m f_m \cos(2\pi f_m t)|$$
$$\Delta F = k_p A_m f_m$$

Maximum Frequency Deviation

$$\beta = \frac{\Delta F}{f_m} = \frac{k_p A_m f_m}{f_m} = k_p A$$

Modulation index of FM

Now this is $f_i(t)$ let us find the frequency deviation of this remember Delta F frequency deviation is the maximum of $f_i(t)$ minus f_c there is a unmodulated frequency of the

unmodulated carrier which is basically maximum as you can see from above $F_i(t)$ minus F_c is $k_p A_m F_m \cos(2\pi F_m t)$ and therefore the maximum of $k_p A_m \cos(2\pi F_m t)$ the maximum magnitude of this is basically $k_p A_m$ times F_m . So we have ΔF that is the frequency deviation, remember what is ΔF ? ΔF is the maximum frequency deviation, okay.

This is the maximum frequency deviation equals $k_p A_m$ times F_m which implies that beta, now beta can be expressed as therefore beta can be expressed as ΔF by F_c remember beta is the modulation index of F_m which is equal to $k_p A_m$ from the above it's $k_p A_m$ well, ΔF by F_c $k_p A_m$ well, let us write the complete thing $k_p A_m F_m$ divide by F_m which is equal to k_p times k_p times A_m .

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The image shows a handwritten derivation on a whiteboard. At the top, the equation $\beta = \frac{\Delta F}{F_m} = \frac{k_p A_m F_m}{F_m} = k_p A_m$ is written in purple ink. A purple arrow points from the text 'Modulation index of F_m ' below to the β in the equation. Below this, the simplified equation $\beta = k_p A_m$ is written in orange ink and enclosed in a rectangular box. A purple arrow points from the text 'Modulation index' below to the β in the boxed equation.

Therefore beta this is the modulation index equals k_p times A_m and look at this you can see this k_p times A_m this is also equal to the maximum phase deviation.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the word "index" is written in orange. Below it, the phase $\theta_i(t)$ is given as $2\pi f_c t + k_p A_m \sin(2\pi f_m t)$. The text "Phase Deviation" is written in purple. The derivation for $\Delta\theta$ is shown in red: $\Delta\theta = \max |\theta_i(t) - 2\pi f_c t|$, which simplifies to $\max |k_p A_m \sin(2\pi f_m t)|$. The final result, $\Delta\theta = k_p A_m = \beta$, is enclosed in a yellow box.

$$\theta_i(t) = 2\pi f_c t + k_p A_m \sin(2\pi f_m t)$$

Phase Deviation

$$\Delta\theta = \max |\theta_i(t) - 2\pi f_c t|$$
$$= \max |k_p A_m \sin(2\pi f_m t)|$$
$$\Delta\theta = k_p A_m = \beta$$

For instance if you look at if you look at this thing if you look at the phase remember the phase is $\theta_i(t)$ $\theta_i(t)$ equals well, $\theta_i(t)$ equals $2\pi f_c t$ plus $k_p A_m$ times $\sin 2\pi f_m t$, now phase deviation $\Delta\theta$ equals maximum magnitude $\Delta\theta$ $\theta_i(t)$ minus $2\pi f_c t$ which is equal to the maximum of well. Substituting $\theta_i(t)$ minus $2\pi f_c t$ we get $k_p A_m \sin 2\pi f_m t$ which is equal to maximum of $k_p A_m \sin 2\pi f_m t$ is equal to $k_p A_m$, so this is equal to $\Delta\theta$ and you can also see, so this is the $\Delta\theta$ and this is also you can see this is also equal to your β that is the modulation index.

So for Sinusoidal modulation what you can see is β equals that is with phase modulation with Sinusoidal modulating signal with that is where the phase is given by $2\pi f_c t$ plus k_p times $m(t)$ what we have been able to show that is for $m(t)$ equals $A_m \sin 2\pi f_m t$ what we have been able to show that this modulation index β which is defined as peak frequency deviation divided by f_m the frequency of the modulating signal that is equal to k_p times A_m which is also seem to be the maximum phase deviation of the signal, okay.

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Phase Deviation

$$\Delta\theta = \max |\theta_i(t) - 2\pi f_c t|$$
$$= \max |k_p A_m \sin(2\pi f_m t)|$$
$$\Delta\theta = k_p A_m = \beta$$
$$\beta = k_p A_m = \Delta\theta$$

Modulation index

Maximum Phase Deviation

The whiteboard shows a handwritten derivation. At the top, it says 'Phase Deviation'. Below that, it shows the equation $\Delta\theta = \max |\theta_i(t) - 2\pi f_c t|$. The next line is $= \max |k_p A_m \sin(2\pi f_m t)|$. The third line is $\Delta\theta = k_p A_m = \beta$, which is enclosed in a yellow box. The fourth line is $\beta = k_p A_m = \Delta\theta$, enclosed in a purple box. Below this, there are two labels: 'Modulation index' with an arrow pointing to the β in the purple box, and 'Maximum Phase Deviation' with an arrow pointing to the $\Delta\theta$ in the purple box.

So we have beta equal to what we have derived is we have beta equals the modulation index equals k_p times A_m which is also equal to Delta theta which is also basically so your beta is basically the beta is basically the modulation index and Delta theta is basically the maximum phase deviation and Delta theta is the maximum phase deviation, alright. So in this module we have seen an introduction, alright. We have started looking at angle modulation we have said that there are 2 different kinds of angle modulation namely phase modulation and frequency modulation we have described overview we have given an overview of both phase modulation and frequency modulation and we have also seen the specific case of a (phan) phase modulated signal for a Sinusoidal modulating message signal, alright. So we will stop here and continue with other aspects in the subsequent modules, thank you.