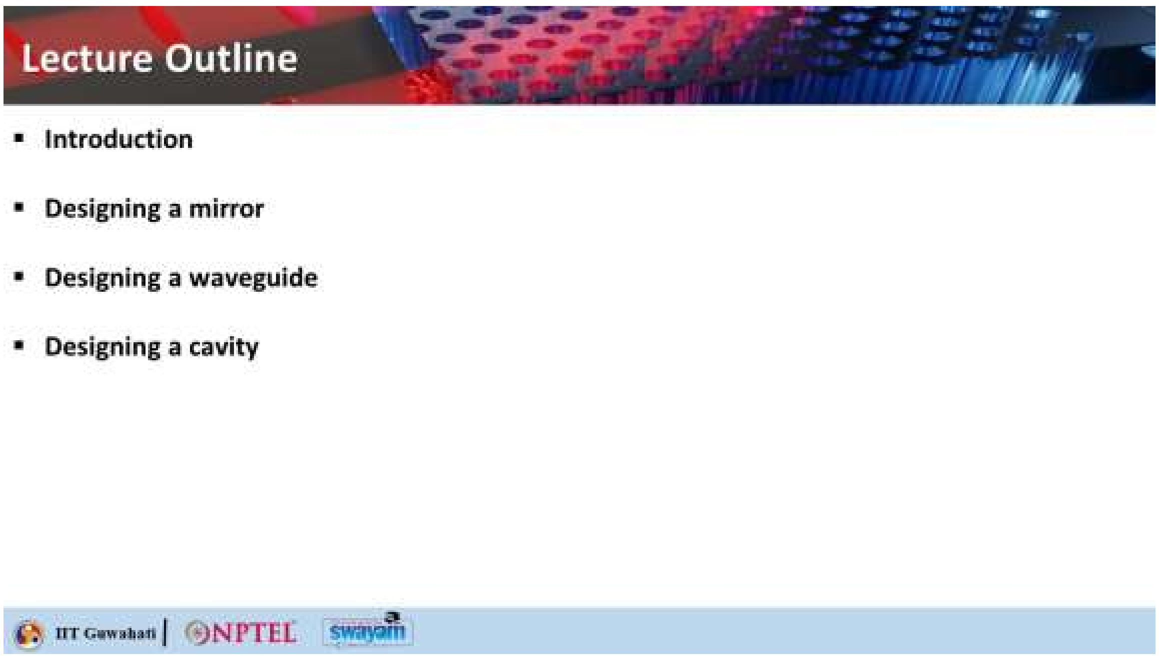
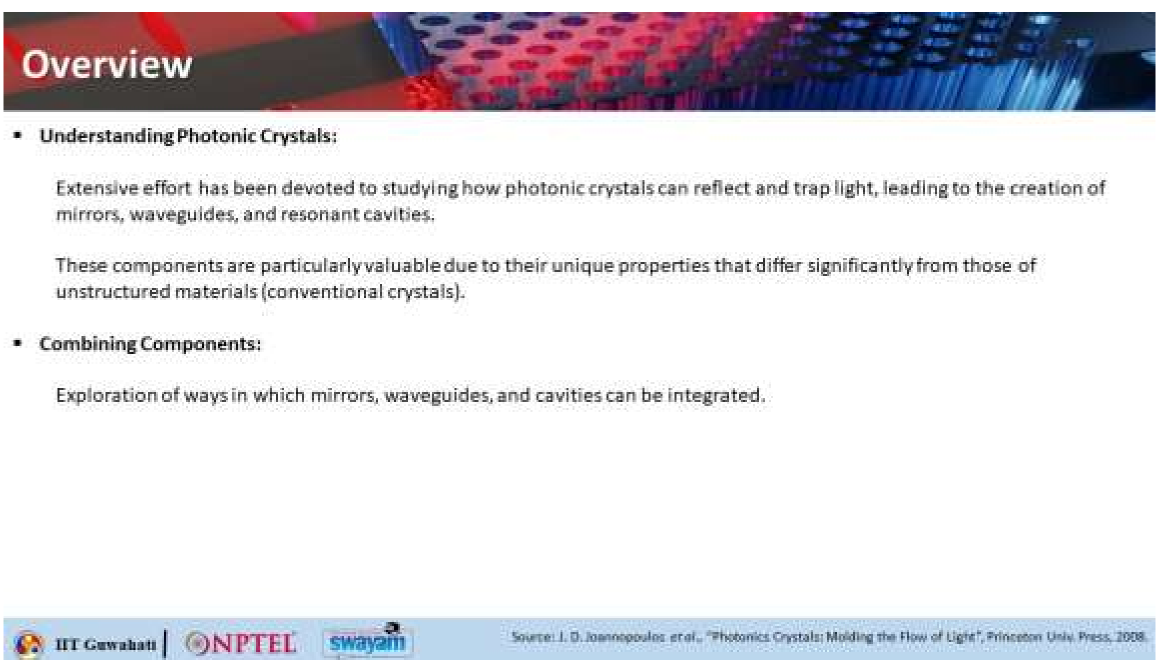
Lec 31: Designing a mirror, waveguide, a cavity

Hello students, welcome to lecture 31 of the online course on Photonic Crystals Fundamentals and Applications. Today's course we will be discussing about designing a mirror, waveguide and a cavity based on photonic crystals. So, here is the lecture outline. We will briefly introduce the agenda for this particular lecture and then we will be taking up an example to design a mirror, then a waveguide and then a cavity using photonic crystal slabs. So, in the beginning we have already seen that you know couple of theoretical tools that allows us to understand the properties of photonic crystals.



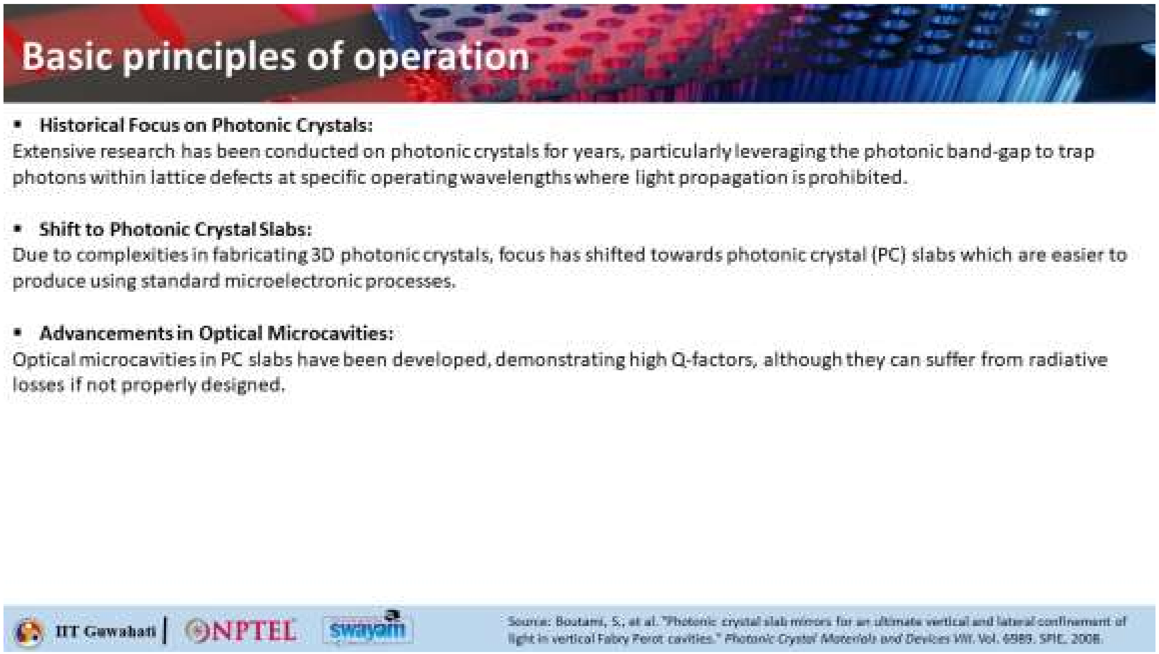
Now extensive effort has also been devoted for understanding how photonic crystals can reflect and trap light and these effects of reflecting and trapping light actually gives rise to the devices like mirrors and resonant cavities. And when you are able to trap light along a corridor where light can be transported from one place in the circuit to another place, that is where the waveguides are formed.

So these components are basically important and they are valuable because of their unique properties which are significantly different from those of the unstructured materials. So if you consider conventional crystals, they do not have all these exciting properties, okay, that can be easily tuned by just changing the structure or some parameters or by introducing some defects into them. So when you move towards integrated circuits the important work would be to combine the different components together something like mirrors, waveguides and cavities all needs to be integrated.



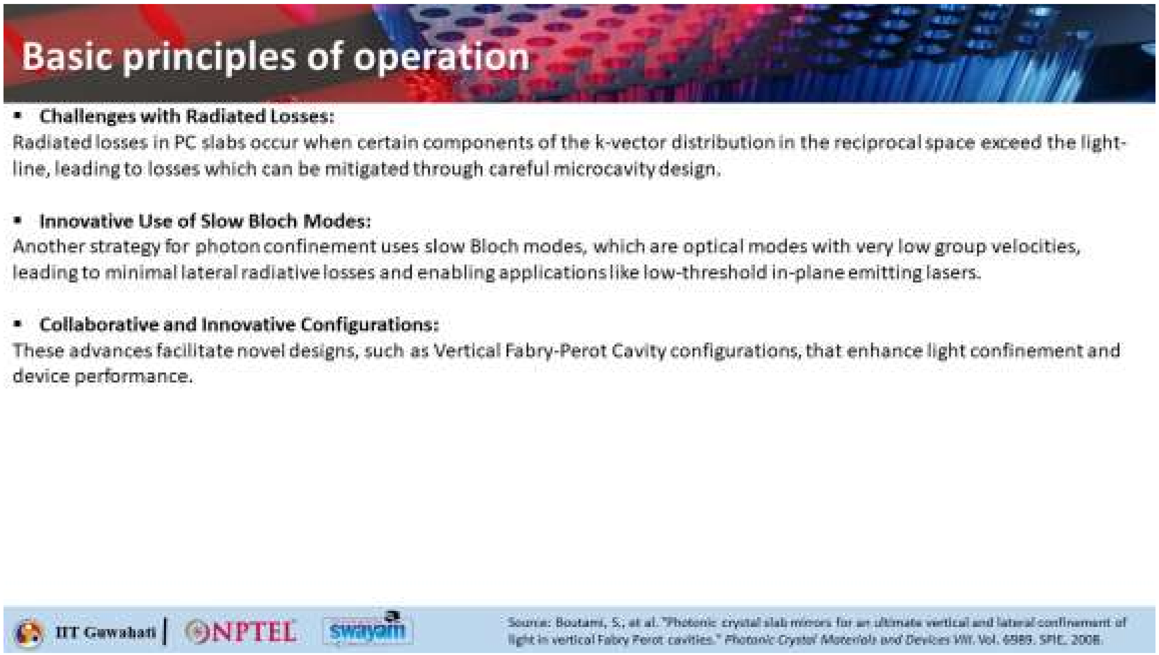
So, we will consider in this particular lecture ok, a slightly different question will not handle the integration part. We will first go and look into the fact that how do you design each of these components from photon and crystals ok. So, let us first take up this particular example of designing a mirror. So, you know the basic principles of operation in this case.

So, there has been a history of extensive research on photonic crystals particularly that leverages the photonic band gap to trap photons with let within the lattice defects at specific operating wavelengths where the light propagation is basically prohibited. So, that is where the photonic band gap comes into the picture. Now, if you actually shine light which belongs to the wavelength or frequencies of the photonic band gap of a photonic crystal, that light is not allowed to enter the crystal. So, where does light go? It will simply get reflected. And if you remember when we actually discussed about generation of photonic band gap, we considered the band gap to be in complete band gap means for all the values of wave vector k.



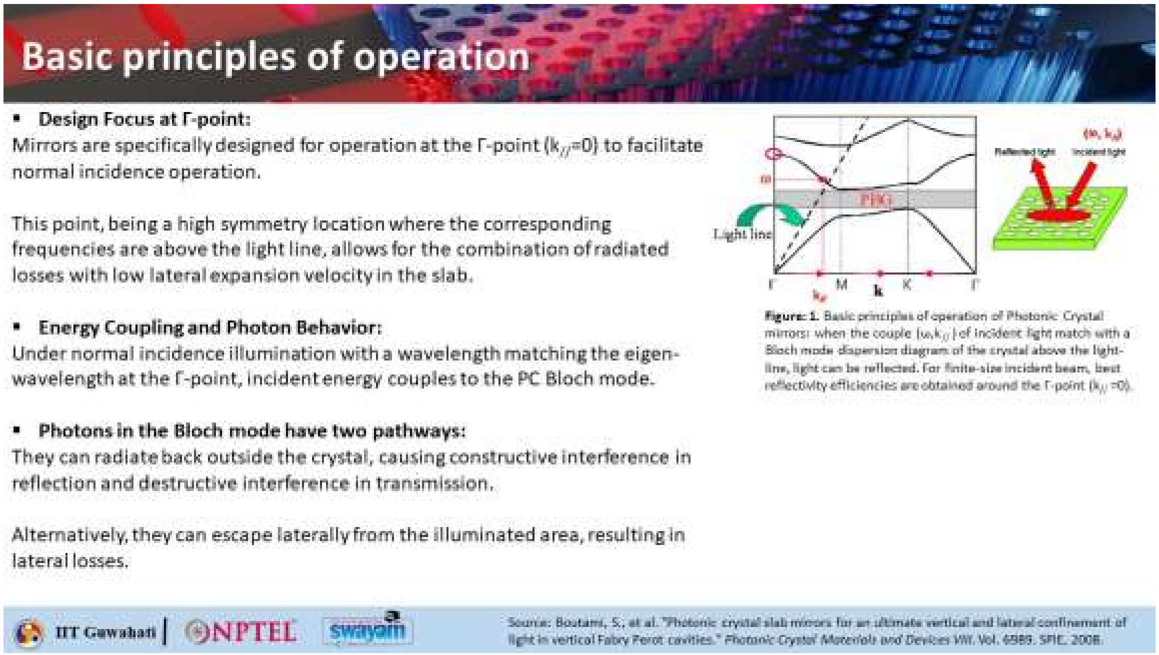
That means in any direction if you shine light the crystal is not going to let that light propagate through it, it is going to be rejected or reflected back. So, that is where we go to photonic crystal slabs. Now, why we go to photonic crystal slabs? Because of the complexity in fabricating 3D photonic crystals, okay. So, photonic crystal slabs are basically 2D structure with finite height. So, those are much more easier to fabricate using the standard microelectronic processes.

okay and we have also seen some advancements in optical micro cavities in photonic crystal slabs that have been developed. So, this kind of cavities they are basically demonstrating high quality factor and they can be very effective one but they will suffer from radiative loss it means the mode can will basically leak out if you do not design them properly. So, there are a couple of important aspects in the designing. So, there is a challenge with these radiative losses. So, radiative losses in photonic crystal slabs occur when certain components of the k vector distribution in the reciprocal space extends or you can say exceed the light line.



So, whenever it goes into the radiating zone, okay that means it can couple to the radiative modes so that will lead to losses okay and that has to be mitigated through careful design of the micro cavities. So one method would be to use the slow block modes okay so that is another strategy for photon confinement. So, where you can use the slow block modes. So, these are basically the optical modes with very low group velocities. So, that leads to minimal lateral radiative losses and that enables applications something like you know low threshold in plane emitting lasers.

So, in towards collaborative and innovative configurations, you can think of some novel designs something like vertical fabric-barrow cavity configuration. So these are also useful in enhancing the light confinement and the overall device performance. Another aspect is design focus at the gamma point. So if you remember this photonic bandgap structure, these are the important points in the brilliance zone, right? So mirrors are specifically designed for operation at gamma point. So that is the point where your K parallel vector is basically zero.



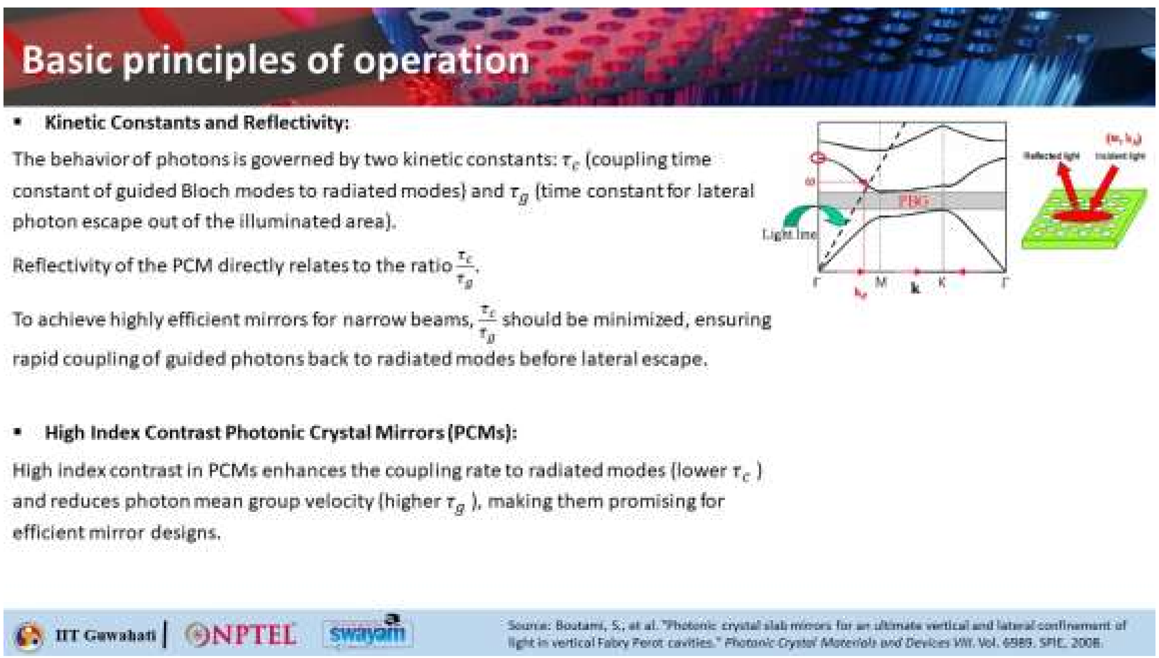
and it will facilitate the normal incidence operation. Now, this point being a highly symmetry location where the corresponding frequencies are basically above the light line. So, this is the light line and that allows so, you can see here this particular frequency is basically above the light line. So, that allows for the combination of radiated losses with low lateral expansion velocity in this particular slab. So, let us first go into this particular diagram.

So, this tells you the basic principle of operation of photonic crystal mirrors ok. Sometimes we will be referring them as PCM also in this particular lecture. So, this happens when the couple that is omega k parallel of the incident light will match with a block mode dispersion diagram of the crystal above the light line and that is the case where the light will be reflected. So, for finite size incident beam that you can see here this ellipsoid or ellipse shown here in red that shows the finite beam size ok. So, that shows you that the best reflectivity coefficients are basically obtained around you know gamma point that is when k parallel is equal to 0.

So, that is where you are basically here. right. So, this is the incident light and this is how the reflection will take place. So, under normal incidence illumination with a wavelength matching that of the Eigen wavelength which you found at gamma point the incident energy could couple to the photonic crystal block mode. So, that is where the coupling takes place.

So, the photons in the block mode have basically two pathways. They can radiate back outside the crystal causing constructive interference in reflection and thus destructive interference in transmission. Alternatively, they can also escape laterally from the illuminated area and those are counted as lateral losses. So, these are the two pathways or means by which photons in block mode can actually go. So this gives us an estimate of the kinetic constant and the reflectivity.

So the behavior of photons are basically governed by two kinetic constants. So you can take τ_c which is basically the coupling time constant of the guided block modes to radiated modes and τ_g , this is the time constant for lateral photon that escapes out of the illuminated area. So, if you consider the reflectivity of the photocrystal mirror that will directly relate to this ratio of $\frac{\tau_c}{\tau_g}$. So, if you want to achieve a high efficient mirror for narrow beams, what do you want to do? You want to minimize this particular ratio. So, that will ensure that rapid coupling of the guided photons back to the lateral modes happen, radiated modes happen before they could escape laterally.



So, you are actually able to get more reflection. So that is how you can get good mirrors okay. So high contrast, high index contrast photon crystal mirrors can actually enhance the coupling rate to the radiated modes. So that means you will require lower tau c okay and that will also reduce the photon mean group velocity that means you will have higher tau g. And these two cases will make a promising recipe for efficient mirror design.

Now, let us quantify the time constant tau g and the group velocity vg. So, around this point gamma point a parabolic approximation could be applied to the dispersion characteristics and you can write $\omega(k_{//}) = \omega_0 + \frac{1}{2}\alpha \cdot k_{//}^2$. So, alpha here is basically the curvature of the band that is near this gamma point. So, you can see from here also. So, each in plane wave vector component that is k parallel of the block mode matches with one of the incident waves.

Basic principles of operation

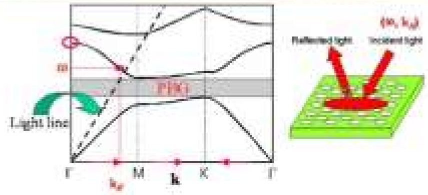
- **Quantifying Time Constant τ_g and Group Velocity:**

Around the Γ-point, a parabolic approximation applies to the dispersion characteristics:

$$\omega(k_{//}) = \omega_0 + \frac{1}{2}\alpha \cdot k_{//}^2$$

Where α is the curvature of the band near the Γ-point

Each in-plane wave-vector components $k_{//}$ of the Bloch mode matches with one of the incident wave.



Source: Boutami, S., et al. "Photonic crystal slab mirrors for an ultimate vertical and lateral confinement of light in vertical Fabry Perot cavities." *Photonic Crystal Materials and Devices VIII*. Vol. 6989. SPIE, 2008.

So what you can obtain from here, the mean parallel wave vector of the block mode is then inversely proportional to the lateral size of the incident beam. So that is another important point. So if you consider S, capital S to be your incident beam area, so you can say that $k_{//\text{avg}}$ is basically $\frac{1}{\sqrt{S}}$, okay? So, that is how they are related. So, the mean group velocity of the block mode you can express as Vg average will be $\left(\frac{d\omega}{dk_{//}}\right)_{k_{//\text{avg}}}$. So, you are here you are considering basically the value when it is k parallel average and that comes out to be $\frac{\alpha}{\sqrt{S}}$.

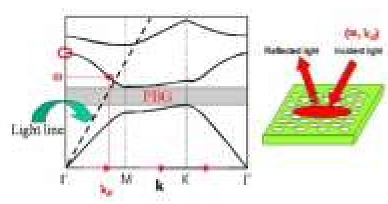
Basic principles of operation

- The mean parallel wave-vector of the Bloch mode is inversely proportional to the lateral size of the incident beam, that is, if S is the incident beam area:

$$k_{//\text{avg}} = \frac{1}{\sqrt{S}}$$

- The mean group velocity of the Bloch mode can thus be expressed as:

$$V_{g_{\text{avg}}} = \left(\frac{d\omega}{dk_{//}}\right)_{k_{//\text{avg}}} = \frac{\alpha}{\sqrt{S}}$$

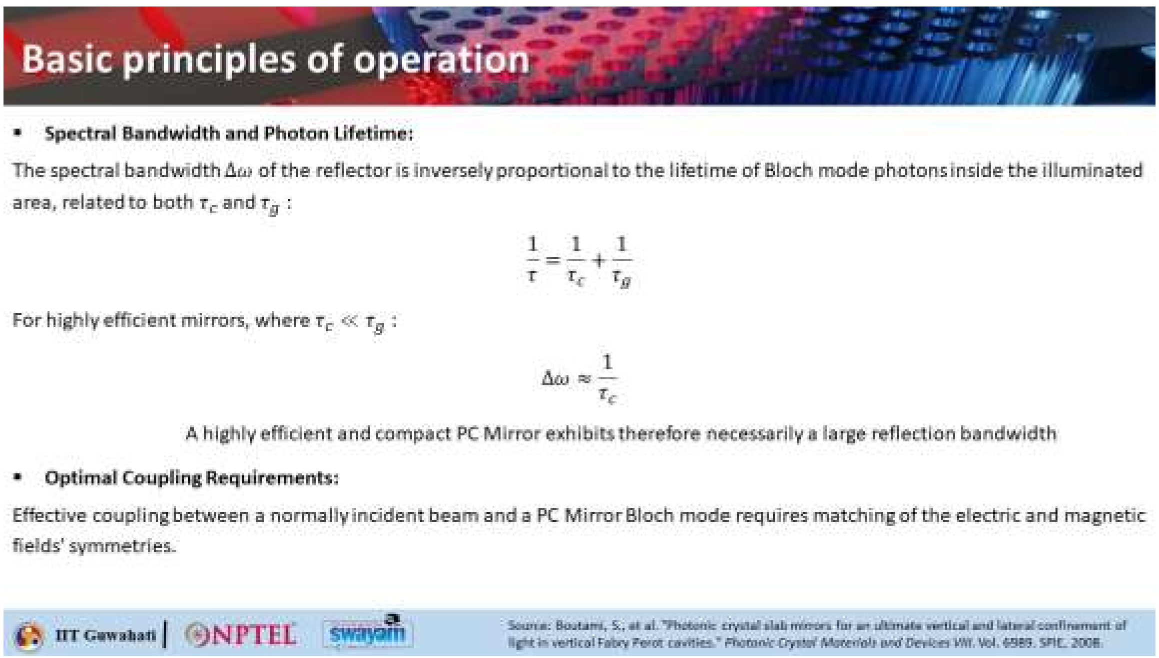


- Finally, the time related to the lateral escape of photons out of the illuminated area is given by:

$$\tau_g = \frac{S}{\alpha}$$

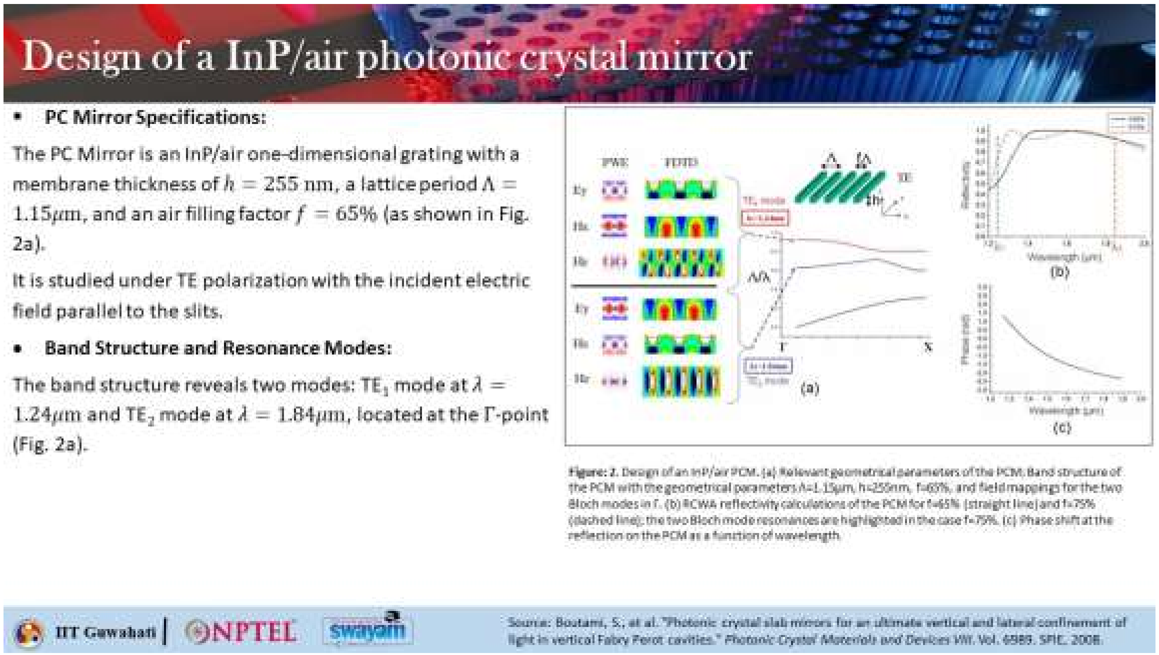
Source: Boutami, S., et al. "Photonic crystal slab mirrors for an ultimate vertical and lateral confinement of light in vertical Fabry Perot cavities." *Photonic Crystal Materials and Devices VIII*. Vol. 6989. SPIE, 2008.

So, finally, the time related to the lateral escape of the photons out of this illuminated area can be given by τ_g that is $\frac{S}{\alpha}$. So, these are the important parameters that can tell you that you want the radiation to be reflected first before it could escape to the lateral modes. So, now we can estimate the spectral bandwidth and the photon lifetime. So, the spectral bandwidth delta omega of the reflector would be inversely proportional to the lifetime of the block mode photons inside the illuminated area and they are both related to both τ_c and τ_g by this $\frac{1}{\tau} = \frac{1}{\tau_c} + \frac{1}{\tau_g}$ okay and if you consider highly efficient mirrors as we mentioned that $\tau_c \ll \tau_g$ okay.



That means the bandwidth will be mainly governed by τ_c and it will be delta omega which is inverse of τ_c. So, a highly efficient and compact photonic crystal mirror will exhibit therefore, necessarily a large reflection bandwidth because you want your τ_c to be small. So, that way you get a large bandwidth as well. Now what are the requirements for optimal coupling? So efficient coupling between a normal incident beam and a photonic crystal mirror block mode requires matching of the electric and magnetic field symmetries. So here we can see the basic principles of this photonic crystal mirrors are applied by you know in order to get a broadband mirror in the near infrared.

So, as you can see we have considered here indium phosphide air photonic crystal mirror. So, it is basically an indium phosphide and air one dimensional grating. So, you can see here in the figure. The membrane thickness is h which is 255 nanometer and the lattice period capital lambda is 1.15 micron and you can see the air filling fraction is around 65 percent.



So, you can see the structural parameters from here okay. Now this is studied under TE polarization with the incident electric field being parallel to the slits. So now we can study the band structure and the resonance mode. So the band structure basically reveals two modes that you can see here.

The first one is TE_1 mode, this one. And it can be seen as at lambda 1 equals 1.24 micron and the other mode is this blue color one which is TE_2 that is 1.84 micron and these are located at the gamma point as you can see, okay.

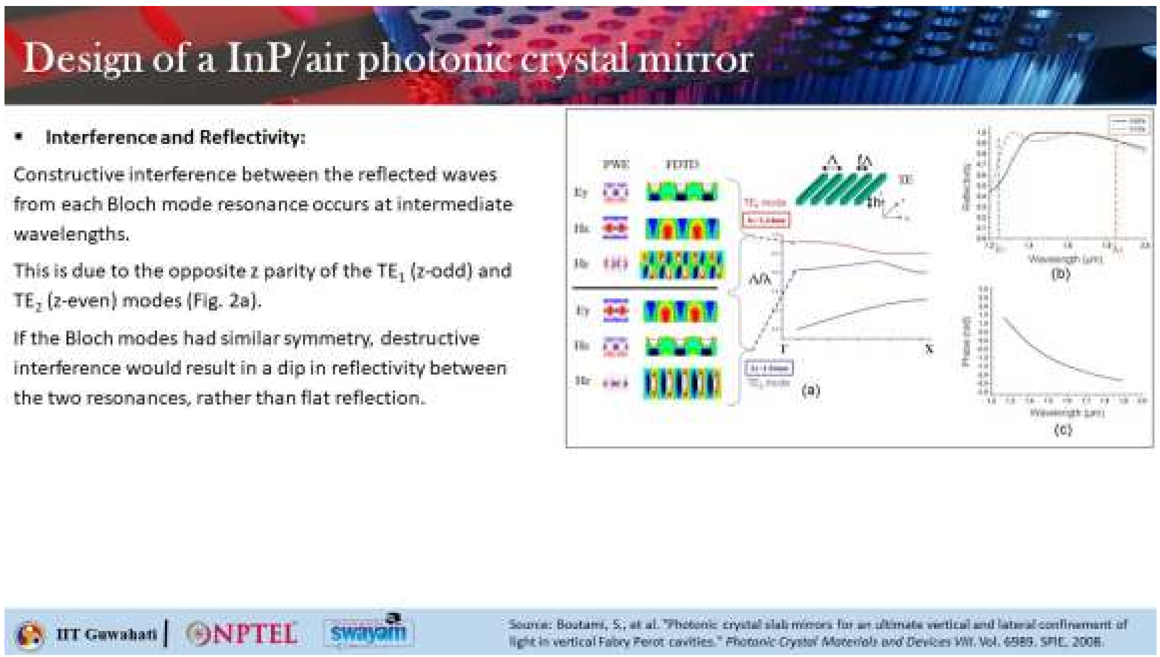
So, these are the different points. So, this is gamma, this is x, okay. And what is important here? So, if you see this is basically the design that we are considering. I have also already mentioned what are the structural parameters. Now, the other things that you can see here are the reflection characteristics and the simulation findings. So, RCWA analysis this is basically rigorous coupled wave analysis that has this is a method of simulation.

We will not cover that in this particular course, but this is a numerical technique that can allow you to calculate the reflection from this particular structure. And it demonstrates that you are getting broadband reflection due to the spectral overlap of the two block mode resonances, okay. So, if you modify so, you can actually see from here ok. So, this is for the filling fraction of 65 percent and when you change it to filling fraction air filling fraction basically of 75 percent this is how the resonances will change. So, when you say air filling fraction is 75 percent it means your indium phosphide slabs are actually getting narrower right.

So, at the normal air filling fraction of 65 percent, the reflection spectrum is basically characterized by two distinct maxima that can actually have almost 100 percent reflectivity, right and this occurs at 1.4 and 1.6 micron, okay.

So, you can see there are two maxima. So, one is at 1.4 another is 1.6. So, they are slightly offset from the resonance Eigen wavelength.

So, ideally they should have been this ones. So what is the main reason for the reflection? It is basically constructive interference between the reflected waves from each block wave resonance that occurs at intermediate wavelength. And this is basically due to opposite z parity of the TE_1 and TE_2 mode. So if you see that TE_1 is basically z- odd. ok and TE_2 is basically z- even. So, here you can see this is TE_2 and this one is TE_1.



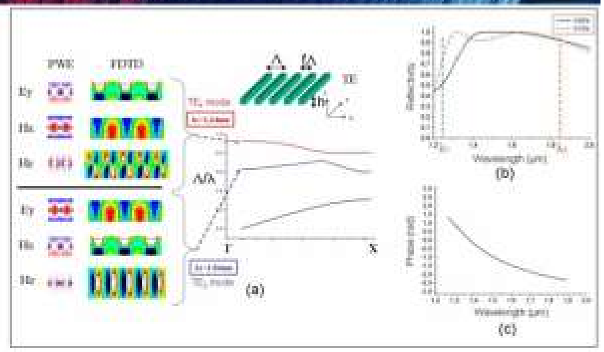
So, this is plane wave expansion method, this is FDD method, this again from this method and what they are plotting? They are basically plotting different components of the electric and magnetic field vectors ok. So, that you can see this odd even parity ok. So, if the block modes have similar symmetry, so destructive interference would result in a dip in the reflectivity between the two resonances rather than giving you a flat reflection okay. So, the peak reflectivity at each wavelength is basically influenced by excitation of both block modes while they still have some differing contributions. So, the reflectivity minimum maximum that you see at lambda equals 1.4 micron is basically coming from the TE_1 mode okay this one and the other one that you see at 1.6 is basically coming from TE_2, though they are saying it is predominantly that particular mode. There is a slight difference in this resonance peak and the Eigen mode wavelength. So, you can understand that these are not purely TE_1 and TE_2 configuration, but predominantly these modes are only creating this maximum and another important point is that the two reflectivity peaks are all almost equal in the amplitude.

Design of a InP/air photonic crystal mirror

- **Reflectivity Association with Bloch Modes:**

The peak reflectivity at each wavelength is influenced by the excitation of both Bloch modes, but with differing contributions.

The reflectivity maximum at $\lambda = 1.4\mu m$ is predominantly due to the TE_1 mode, while the maximum at $\lambda = 1.6\mu m$ is largely influenced by the TE_2 mode.



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Source: Boutami, S., et al. "Photonic crystal slab mirrors for an ultimate vertical and lateral confinement of light in vertical Fabry Perot cavities." *Photonic Crystal Materials and Devices VIII*. Vol. 6989. SPIE, 2008.

So, this equivalence is mainly due to that TE_2 mode having a lower tau g and that you can see from the higher band curvature for the TE_2. But as compared to the first case that is TE_1 okay, but it has got a lower tau c okay. That means it corresponds to a higher spectral bandwidth and that you can also see in this particular case and the ratio $\frac{\tau_c}{\tau_g}$ is very critical for indicating the lateral losses and this remains roughly constant for both block modes despite there are differences. So, this is how you can actually obtain very efficient mirrors based on photonic crystal slabs.

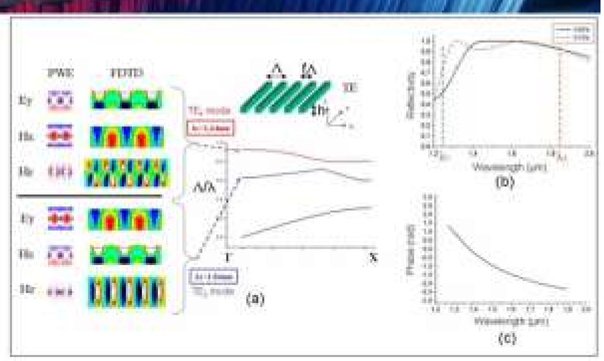
Design of a InP/air photonic crystal mirror

- **Reflectivity and Mode Dynamics:**

The two reflectivity peaks are approximately equal in amplitude.

This equivalence is due to the TE_2 mode having a lower τ_g (indicative of higher band curvature as shown in Fig. 2a) but also a lower τ_c (corresponding to higher spectral bandwidth as depicted in Fig. 2b).

The ratio $\frac{\tau_c}{\tau_g}$, which is crucial for indicating lateral losses, remains roughly consistent for both Bloch modes despite these differences.



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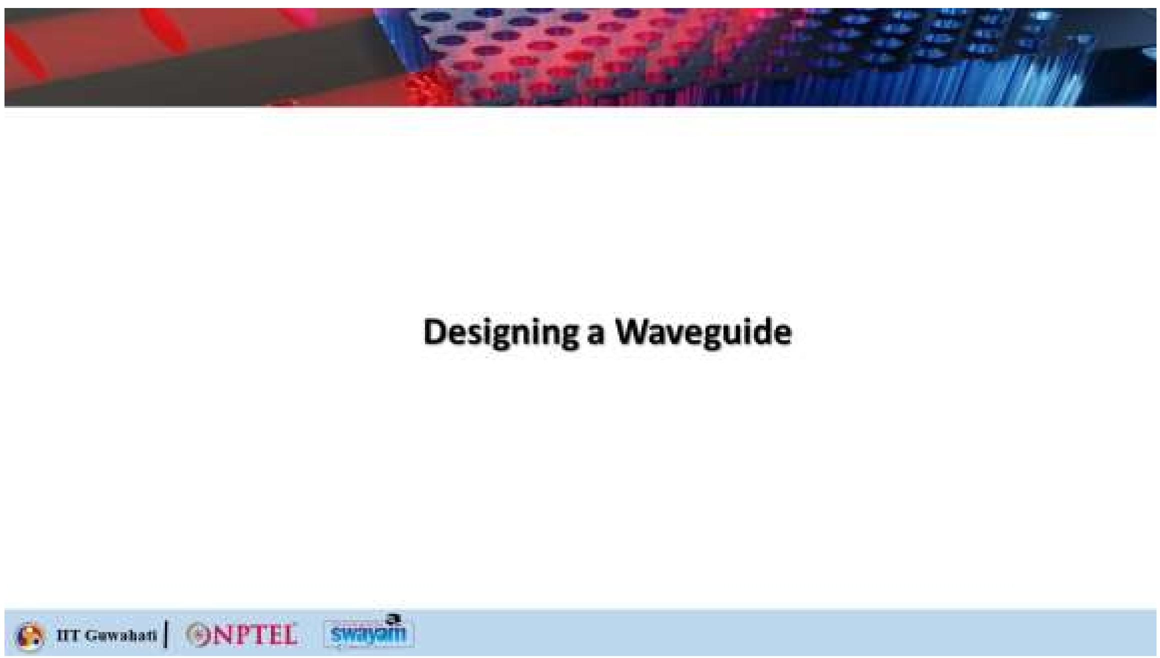
Source: Boutami, S., et al. "Photonic crystal slab mirrors for an ultimate vertical and lateral confinement of light in vertical Fabry Perot cavities." *Photonic Crystal Materials and Devices VIII*. Vol. 6989. SPIE, 2008.

So, that is one practical example. So, now we will move on to designing a waveguide. So, we have been discussing this for some time that we can introduce a line defect in a photonic crystal slab and we can propagate or you can actually allow a wavelength that falls within the photonic band gap of the substrate or the surrounding medium slab and incident that particular wavelength into the defect waveguide and that will be happily propagating through it without any losses. So, let us look into that into more details. So, we will analyze the band structure of square and rectangular lattice photonic crystals and that will help us to identify the optimal design parameters something like what should be the inter hole spacing, the hole radius and the slab thickness. So what is the impact of finite slab thickness? Because we are considering photonic crystal slab as I mentioned that these are easy to fabricate, right? So unlike 2D models, finite thickness slabs can support higher order vertical oscillations of light modes.



Theoretical Analysis

- **Objective of Theoretical Analysis:**
Analyze the band structure of square and triangular lattice photonic crystals to identify optimal design parameters like interhole spacing (a), hole radius (r), and slab thickness (d).

- **Impact of Finite Slab Thickness:**
Unlike 2-D models, finite-thickness slabs can support higher-order vertical oscillations of light modes.

Excessive thickness may lead to the closing of the bandgap, making the slab thickness a crucial parameter to optimize

So, excessive thickness in that case may lead to closing of the bandgap making the slab thickness to be a very important parameter that you need to optimize, okay. So, the structure that we will first analyze in a silicon slab, we will consider the structure pattern with square or triangular lattice of holes and surrounded by air okay like this. And then if you consider the thickness of this silicon slab to be around d equals 0.55a and you can consider the radius of the holes to be equal to 0.4a okay and we consider the refractive index of silicon to be 3.5 okay.

Theoretical Analysis

- The structure that we analyze first is a silicon slab patterned with square or triangular lattice of holes and surrounded by air (Fig. 3).
- The thickness of the Si slab was equal to d = 0.55a and the hole radius was r = 0.4a.
- The index of refraction of Si is assumed to be n_{Si}=3.5.

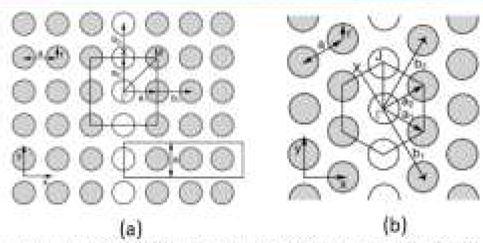


Figure 3. (a) Si slab perforated with 2-D square lattice of holes (left). Lattice vectors are $a_1 = a \cdot (1,0,0)$ and $a_2 = a \cdot (0,1,0)$. Reciprocal lattice vectors are $b_1 = (2\pi/a) \cdot (1,0,0)$ and $b_2 = (2\pi/a) \cdot (0,1,0)$. (b) Si slab perforated with 2-D triangular lattice of holes (right). Lattice vectors are $a_1 = a \cdot (\sqrt{3}/2, -1/2, 0)$, and $a_2 = a \cdot (\sqrt{3}/2, 1/2, 0)$. Reciprocal lattice vectors are $b_1 = (4\pi/a\sqrt{3}) \cdot (1/2, -\sqrt{3}/2, 0)$, and $b_2 = (4\pi/a\sqrt{3}) \cdot (1/2, \sqrt{3}/2, 0)$.

IIT Guwahati | NPTEL swayam Source: ar, Marko, et al. "Design and fabrication of silicon photonic crystal optical waveguides." *Journal of lightwave technology* 18.10 (2000): 1402.

So, what are the other parameters that you see in these two figures? So, this one is basically for the 2D square lattice of holes and this one is for the 2D triangular lattice of holes. So, a here basically shows a_1 and a_2 basically showing you the lattice vectors and b_1 and b_2 are basically the reciprocal lattice vectors which are also mentioned.

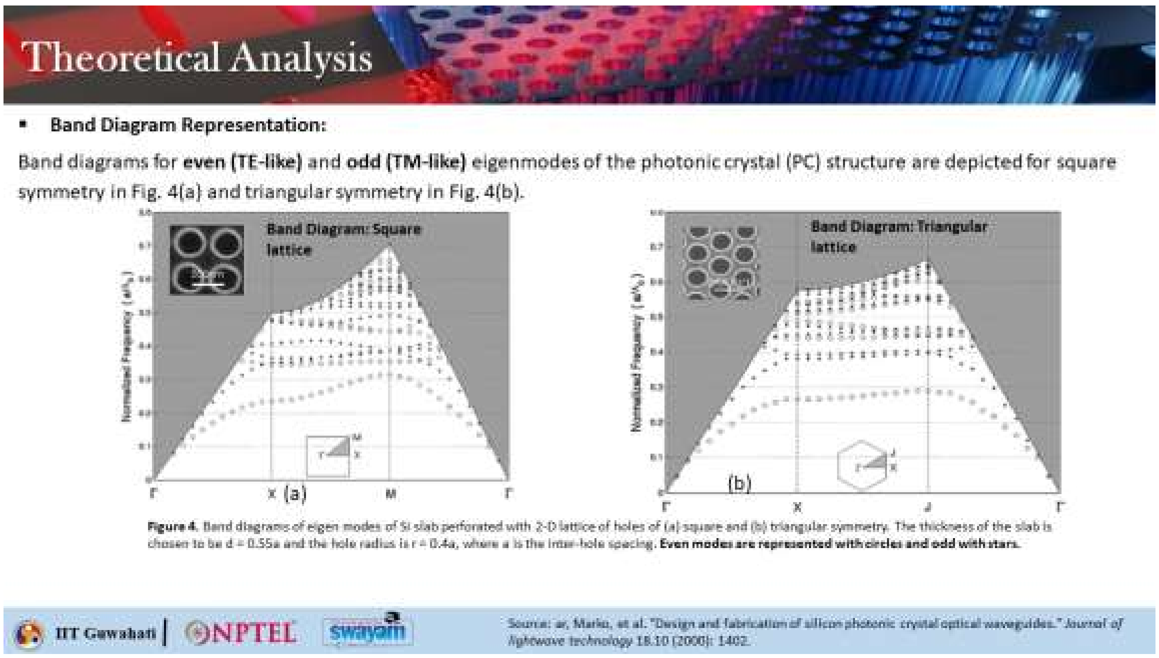
So, here just for a change because this design has been taken from this particular journal. There the authors have used different kind of notations for the points of the irreducible Brillouin zone. So, they are calling this as gamma m and x and for this case they are calling it as gamma x and j. So, this is slightly different from the traditional convention that we used earlier in this course. But for the discussion in this lecture because they have calculated the band diagrams based on those values, so you can actually see, okay.

So, gamma xm, so gamma xm gamma, so that is how you traverse the entire Brillouin zone boundary. Here also you can do that gamma xj gamma. So, what they have done? They have calculated the band diagram of the eigen modes of the silicon slab which are basically perforated with 2D lattice of holes. In this case, this is square lattice and this is a triangular lattice.

The slab is of finite thickness as you can see it is d equals 0.55a and the hole radius is 0.4a that is the inter where a is basically the inter-hole spacing. So, what you see here the even modes are basically represented with circles and the odd modes are shown with stars. So, just to remind you what is this x, x is this basically discuss about different directions in the reciprocal lattice and what is the y axis that is giving you the normalized frequency or you can say normalized wavelength a by lambda, where lambda will be the wavelength in air or vacuum.

You can see the solid lines here. These are basically the light line dispersion relation for the photons in air where the momentum is normalized to 1 on the x-axis. And then what you can also see here, the modes are occurring at frequencies above this line will be prone to leak the energy into air and those are basically the leaky modes which are causing the primary loss mechanism for the waveguides. above the light line if there is any mode ok that will be able to couple to the continuum as you can see these are basically continuum of radiative modes means for every frequency there will be a mode that is supported. So, it is a continuum ok and then you will actually not get complete band gap ok. So what you also see by comparing the two cases of square and triangular is that the triangular lattice can basically feature a wider band gap than the So here you can, if you look into the circles, so here is a narrow band gap, but here is pretty wide, okay? So this lines and this line, so you can see the gap, it's pretty, pretty wide band gap, okay? And that is because of the greater symmetry and a smoother Brillouin zone, right? So if you remember that this has got much better, higher symmetry than the square lattice.

okay and that is why you know it gives a more robust option for making photonic crystal waveguide okay and especially by considering the fabrication tolerances that might close the band gap in the case of square structure lattice because this gap is very narrow. This is wide so even if something goes wrong during fabrication you will still get band gap. okay if you consider this one fine. So, I think we have already discussed this we have already seen that the even modes are basically the TE like modes and the odd modes are basically the TM like modes of the photonic crystal structures and they are basically shown for both square and triangular lattice right. And here is the table that shows the parameter or you can say figure of merit of the two samples of photonic crystals.

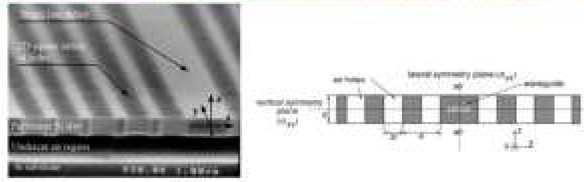


So, this is the square lattice, it is giving a band gap of 0.031, a by lambda. So, here you can see the starting and the end values, but here you can see it is a much wider band gap. okay and mid gap frequency here is 0.33 here it is 0.365 okay and these are the lattice parameters. So, a and then in terms of a you are getting r the whole radius and also you are getting the thickness okay. So, you can see that you will still require slightly thicker slab in the case of triangular lattice and this is how you know. the fabricated square lattice waveguide will look like, okay? So, this is the cross-section of the fabricated square lattice waveguide that is suspended in air, okay? And this basically shows when you look it from the end, okay? So, this is the view of end on, okay? So, the reflection symmetry planes here can also be seen, which are basically this sigma xy okay. So that is a vertical symmetry plane okay and then you have this one this is called sigma yz that is the lateral symmetry plane. So why these planes are important that actually helps you to identify those odd even modes that we have discussed earlier as well.

Theoretical Analysis

Lattice type	Bandgap width (start, end) [a/λ]	Midgap frequency f_0 [a/λ]	Lattice Parameters Theory		
			$a = f_0 * \lambda$	$r = 0.4a$	$d = 0.55a$
Square	0.031 (0.315, 0.346)	0.33	496nm	198nm	272nm
Triangular	0.148 (0.291, 0.439)	0.365	547nm	219nm	301nm

Table 1: Design parameters of the photonic crystal used in order to have bandgap centered around λ = 1.5 μm



(a) (b)

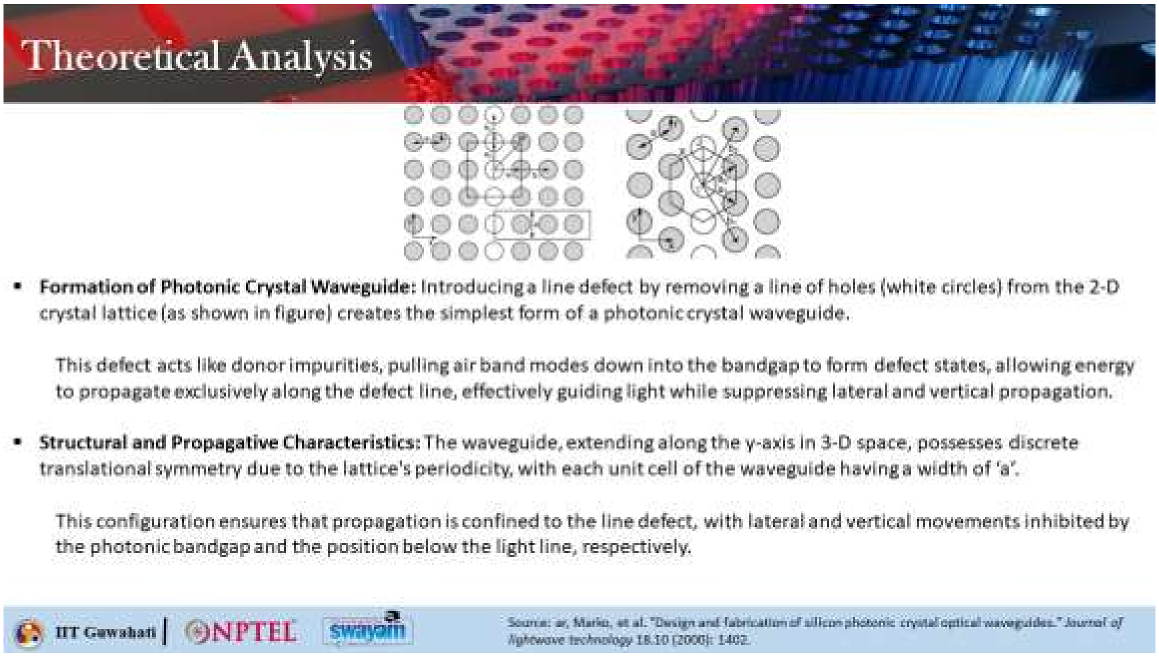
Figure 5: (a) Cross section of the fabricated square lattice waveguide suspended in the air. (b) Schematic of the waveguide viewed end-on. Reflection symmetry planes are: σ_{xy} in the middle of the slab (vertical symmetry plane), and σ_{yz} in the center of the waveguide (lateral symmetry plane).

- From figure we can find the design parameters for each structure so that the bandgap is centered around wavelength of λ= 1.5 μm.
- Using the table, that the lattice constant is 496nm in the case of square lattice, and 547 nm in the case of triangular lattice.

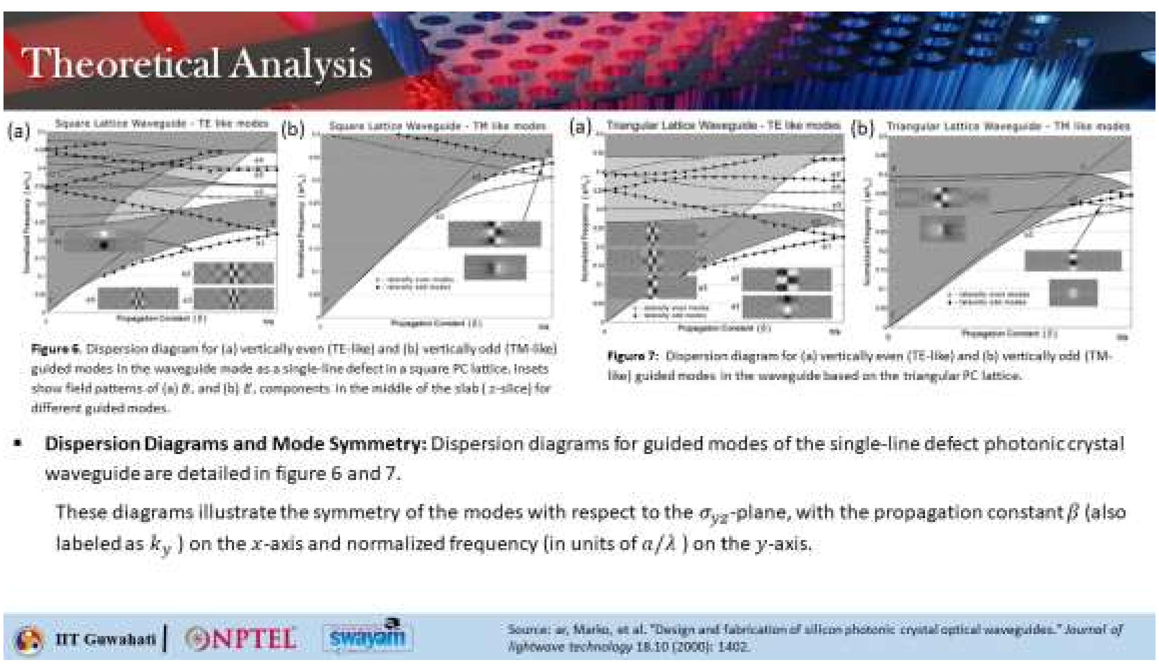
Source: ar, Mario, et al. "Design and fabrication of silicon photonic crystal optical waveguides." *Journal of lightwave technology* 18.10 (2000): 1402.

So, these recent parameters that you have discussed that actually gives you a band gap centered around wavelength of 1.5 micron, okay. And using this table, you can see that the lattice constant is 496 and 547 nanometers in the case of square and triangular lattice. fine.

So, with that if you introduce a line defect by removing a line of holes. So, you what you are doing you are removing this line of holes. So, you actually these are missing holes. So, you can actually represent that them by white circles and this is how the waveguide will look like for the square lattice and for the triangular lattice. So, this defects will act like donor impurities, they will pull the air band modes towards the band gap to form a defect state and that will allow energy to propagate exclusively along the defect line and effectively guiding light while suppressing lateral and vertical propagation. So, what are the structural and propagating propagative characteristics? So, if you see the waveguide extending along the y axis in the 3D space.



This processes discrete translational symmetry which we understood already because of the lattice periodicity where the each unit cell of the waveguide has got a width of a and this configuration ensures that the propagation is confined to the line defect with lateral and vertical movements which are basically inhibited by the photonic band gap. So, it cannot actually go out in on the sides okay and the position below the light line. So vertical movement is because it cannot actually have coupled to any mode that is leaky mode.



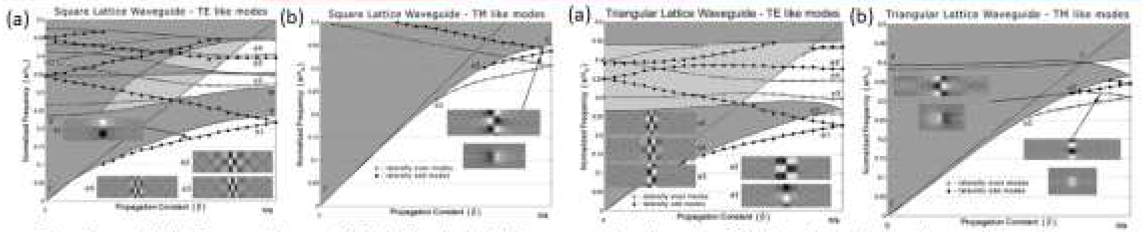
So that will be propagating through this defect only. So here are some analysis of the square lattice waveguides and for the TE like modes and this is for TM like modes. So these are the dispersion diagram for vertically even that is TE like yeah modes and this is for vertically odd so you can see these are the vertically even ones this is how the vertically odd ones look like and both these are for the square lattice. So these are showing the field distribution for different modes.

So this is for E2, E3, and E4. And here, these are like TM-like modes. So these are the odd modes, O1, O2, and so on. And these are the points in the line. So they are plotting it from 0 to pi by a. okay and this is the propagation constant so the you have marked this point that is your gamma and this point is basically x okay the same thing you can do for the triangular lattice okay so here you can see that these are again t like modes so these are having vertically even symmetry, these are having vertically odd symmetry and you can also see the distribution of E1, E2, E3, E4, E5 modes and here you can see O1, O2 modes. Similarly, this for the triangular lattice they have marked the points here gamma, j, x and so on right and one another important thing, if you look here, the solid line that you see is basically marking the light line in each case that has got a slope of one OK, so it's basically from here 0 this is basically pi by a.

So, this point is pi by a comma 0.5. So, this is how you can mark it ok. So, this is also it will go to 0.5 here it is 0.4. So, it looks like that ok. So, ideally all of them should go to 0.5 at pi by 4 it should go to 0.5 right. So, the leaky modes in this case you can see the leaky modes are not shown in the case of TM like modes okay. And here you can see in the insets okay they are basically showing you the fill patterns of the magnetic and electric field components in the middle of the slab that is basically the jet slice. And in this case, the TM-like modes O1 and O2, for them the distribution of EZ component along the cross section of the waveguide is also shown.

So, you can see these ones. So, So the field profile for O2 is basically taken near the cutoff and therefore the field penetrates into the photonic crystal and that can be seen from the inset. okay. So, this we have already discussed. So, what else you can see from these diagrams? The direction of light propagation and how do you analyze the modes? So, the light propagates along the gamma x direction in the reciprocal space in the case of square lattice.

Theoretical Analysis



- **Direction of Light Propagation and Mode Analysis:** Light propagates along the ΓX direction in the reciprocal space in square-lattice PC waveguides.

  Given the waveguide's periodicity in the y-direction, k_y values range from 0 to π/a.

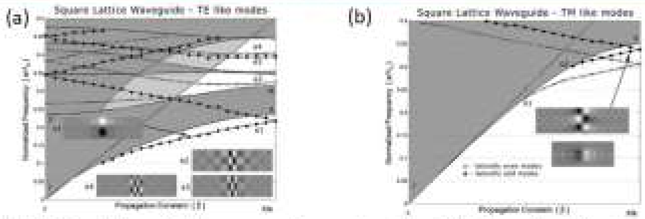
  Not all detected modes are guided; some correspond to modes of the PC slab that either exist in the dark gray regions (modes of the photonic crystal slab) or above the light line in the light gray region, where they will leak energy into the air.

  The white regions in figures indicate areas where 3-D localization of light occurs in the waveguide, free from interference by other photonic crystal modes.

Source: ar, Marko, et al. "Design and fabrication of silicon photonic crystal optical waveguides." *Journal of lightwave technology* 18.10 (2000): 1402.

So, this is gamma and this is x, okay. You can see here and given the waveguides periodicity in the y direction, the ky values will range from 0 to pi by a. And, that is why not all detected modes are guided, some would correspond to the modes of the photonic crystal slab that either exist in the grey dark region, those are basically the modes of the photonic crystal slab or they will be above the light line in the light grey region, where they will basically leak out into air as leaky modes. So, the white regions that you see that is where the 3D localization of light occurs in the waveguide and that those are basically free from interference from any sort of photonic crystal modes. So, the guiding mechanism for the modes situated below all photonic crystal modes something like you know modes E1, O1 and O2. So, they basically perceive the silicon slab perforated with 2D photonic crystals, that is not as a photonic bandgap material, but they will basically see it as an effective medium. which has got a refractive index of N effective and that is lower than the effective index of silicon core that means it is lower than 3.5. So, these modes are basically guided by a total internal reflection and they are basically unaffected by the periodicity of the photon crystal.

Theoretical Analysis



- **Guiding Mechanism and Effective Index:** Guiding occurs for modes situated below all photonic crystal modes (modes e1, o1, and o2 in figure)

They perceive the Si slab perforated with a 2-D photonic crystal lattice not as a photonic bandgap (PBG) material, but rather as a medium with an effective refractive index (n_{eff}) that is lower than the refractive index of the Si core (n_{Si} =3.5).

These modes are guided via total internal reflection, and are unaffected by the periodicity of the photonic crystal, allowing for an unrestricted propagation constant k_y (or β).

Source: ar, Marko, et al. "Design and fabrication of silicon photonic crystal optical waveguides." *Journal of lightwave technology* 18.10 (2000): 1402.

So, that allows for unrestricted propagation constant ky or beta you can say. So, the here the it is not depending on the band gap it is whether you know depending on the total internal reflection phenomena and that is why there is no restriction on the propagation constant in this case. So, what are the strategies for enhancing the coupling efficiency? So, if you want to improve the coupling efficiency. okay, you have to eliminate the leaky modes in the frequency range of the targeted guided modes, okay, that is very important. And this could be achieved by adjusting the r by a ratio of the photonic crystal lattice by narrowing the width of the waveguide, okay, both of which will help to exclude the undesired leaky modes. So, first thing, what could be the mode method for single mode regulation, narrowing the waveguide width will also push some modes back into the air band because they will not be supported within the waveguide.

Theoretical Analysis

- **Strategies for Enhancing Coupling Efficiency:**

To improve coupling efficiency, strategies to eliminate leaky modes in the frequency range of the targeted guided mode are necessary.

This could be achieved by adjusting the r/a ratio of the PC lattice or by narrowing the width of the waveguide, both of which help to exclude undesirable leaky modes.

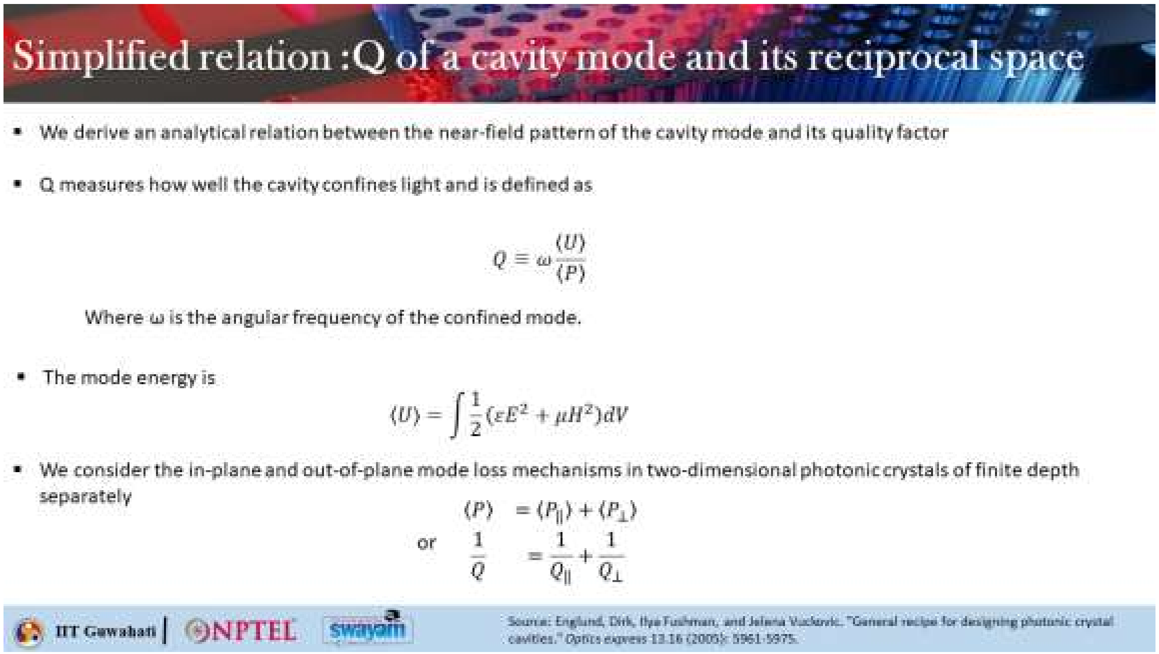
- **Methods for Single-Mode Regulation:**

Narrowing the waveguide width helps push some modes back into the air band, reducing the number of guided modes, potentially increasing the cutoff frequency for TM-like modes while retaining some TE-like modes to achieve single-mode operation.

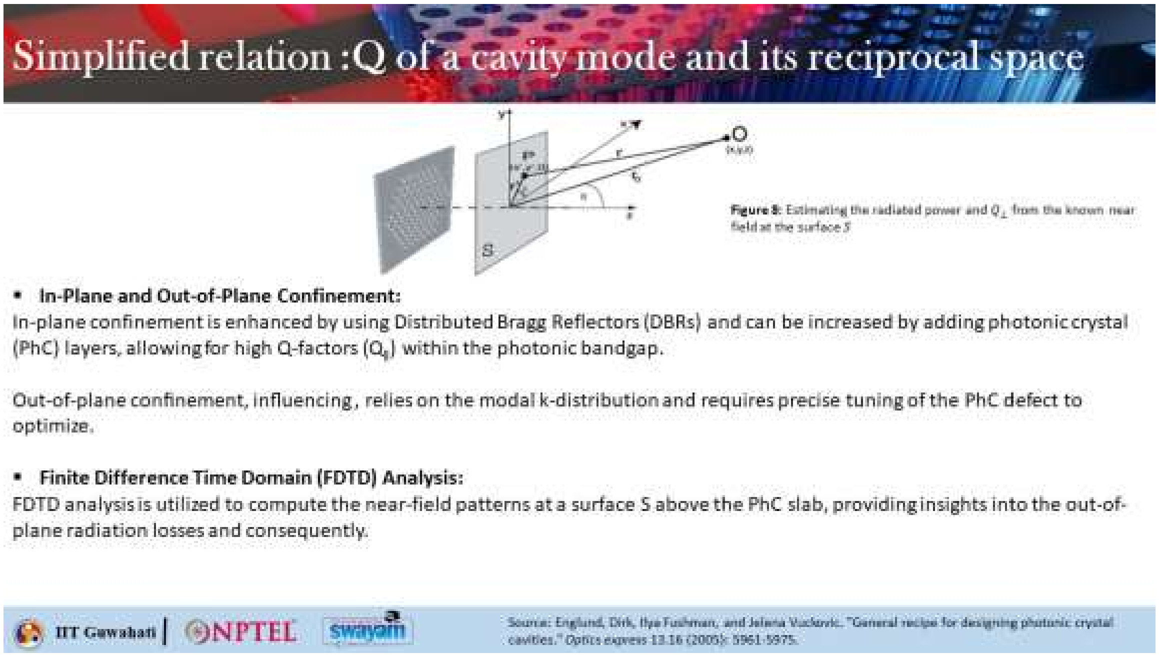
Source: ar, Marko, et al. "Design and fabrication of silicon photonic crystal optical waveguides." *Journal of lightwave technology* 18.10 (2000): 1402.

So, that basically reduces the number of guided modes and potentially that increases the cutoff frequency for TM like modes while retaining some of the TE like modes and that will help you achieve single mode kind of operation. So, that is how you can actually make single mode to propagate with high efficiency in the waveguides. Now, we went to the next and the last topic of this lecture that is designing a cavity in the photonic crystals. So, let us look into the quality factor of a cavity mode and its reciprocal space. So, we could derive an analytical relation between the near field pattern that we see in the cavity mode and its quality factor.

Designing a Cavity



And why Q is important because it actually tells you how well the cavity is able to confine light. And you can define Q as $\omega \frac{\langle U \rangle}{\langle P \rangle}$ that is basically mode energy divided by the So, in this case ω is basically the angular frequency of the confined mode. u is basically giving you the mode energy that can be calculated as $\int \frac{1}{2}(\varepsilon E^2 + \mu H^2) dV$ and then if you consider the in plane and out of plane mode loss mechanisms in the 2D photonic crystals of the finite depth separately, you will be able to estimate P as P parallel plus P perpendicular. So, you can actually write them in terms of Q as $\frac{1}{Q} = \frac{1}{Q_{\parallel}} + \frac{1}{Q_{\perp}}$. Now, what is that? So, how do you estimate this confinement? So, in plane and out of plane confinement. So, in plane confinement you can actually take this particular diagram and see how we are basically estimating the radiated power and cube perpendicular for the known near field at a surface S.



So, the in-plane confinement as I mentioned is enhanced by using distributed Bragg reflected DBRs and they can be increased by adding photonic crystal layers allowing for high quality factors that is Q parallel within the photonic designs. And the out of plane confinement that is basically influencing that relies on the modal k distribution and it requires precise tuning of the photonic crystal defect for optimization. So, you can do finite difference time domain analysis for computing the near field pattern at a surface S that is above the photonic crystal slab and that will provide insight about the out of plane radiation losses okay. So, the total radiated power P okay that is the loss okay or you can say radiative loss that can be calculated like this. So, the total time average power radiated into the half space above the surface S. So, you can refer to this diagram how that integration is calculated. So, P is considered to be $P = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} d\theta d\phi \sin(\theta) K(\theta, \phi)$. So, if you consider the radiated power per solid or per unit solid angle that is k that has you know kx and ky and this radiated power is derived from the 2D Fourier transform of Hz and Ez at the surface S.

Simplified relation :Q of a cavity mode and its reciprocal space

- The total time-averaged power radiated into the half-space above the surface S is:

$$P = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} d\theta d\phi \sin(\theta) K(\theta, \phi)$$

- **Radiated Power Expression and Fourier Transforms:**

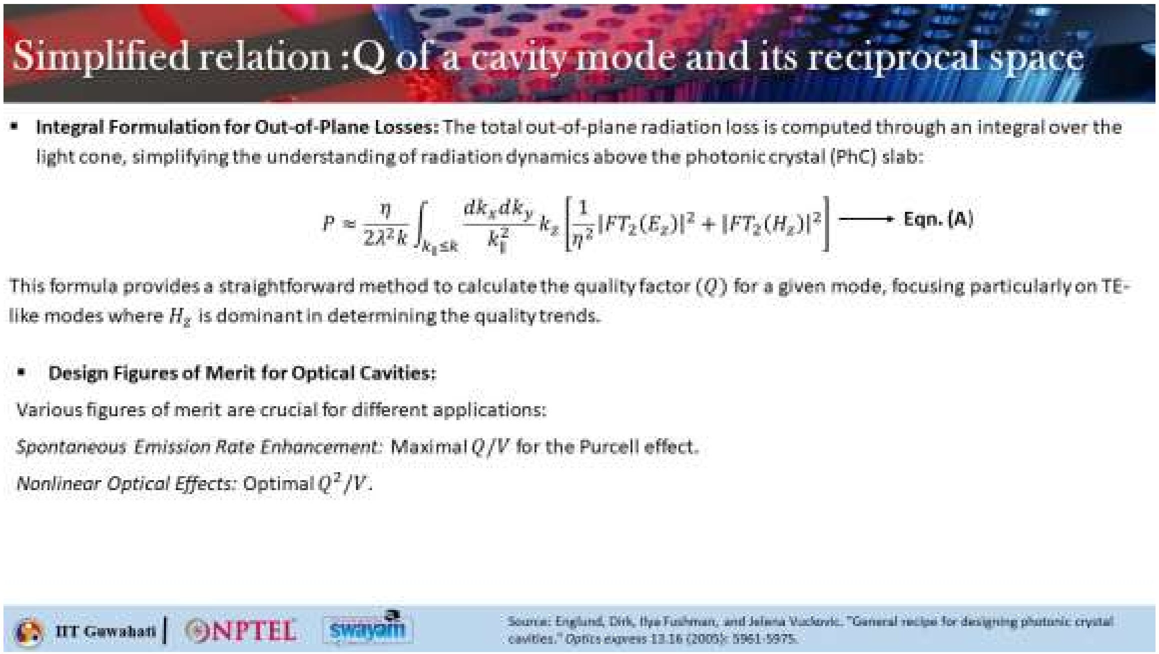
The radiated power per unit solid angle, $K(k_x, k_y)$, is derived using 2D Fourier Transforms (FTs) of H_z and E_z at the surface S, represented as:

$$K(k_x, k_y) = \frac{\eta k_z^2}{2\lambda^2 k_\parallel^2}\left[\frac{1}{\eta^2}|FT_2(E_z)|^2 + |FT_2(H_z)|^2\right]$$

Here, $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, λ is the wavelength in air, and $\vec{k}_\parallel$ and k_z denote in-plane and out-of plane components, calculated using the angular definitions in the coordinate system.

Source: Englund, Dirk, Ilya Fushman, and Jelena Vuckovic. "General recipe for designing photonic crystal cavities." Optics express 13.16 (2005): 5961-5975.

So, this is how you can calculate that. So, k you can obtain from the 2D Fourier transform of the electric and magnetic field vectors along the z direction, their eta is the free space impedance, lambda is the wavelength in air, k parallel and kz are basically in plane and out of plane components and they are calculated using the angular definitions in the coordinate system. Now, you can also find out the same similar formulation for out of plane loss. So, the total out of plane radiation loss can be calculated through an integral over the light cone and that basically simplifies the understanding of the radiation dynamics above the photonic crystal slab.



So, you can compute $P \approx \frac{\eta}{2\lambda^2 k} \int_{k_1 \leq k} \frac{dk_x dk_y}{k_\parallel^2} k_z \left[\frac{1}{\eta^2}|FT_2(E_z)|^2 + |FT_2(H_z)|^2\right]$ and here again you are using the 2D Fourier transform of E z and E z. So, this is similar to this one ok, but then you are integrating over this cases where the k parallel is less than equals k. So this is where it will be restricted. And this formula provides a straightforward method to calculate the quality factor Q for a given mode, focusing particularly on the TE-like modes where H_Z is basically dominant. And that will determine the quality trends.

Now, the design figure of merits of optical cavities is also very important. So, what are the important figure of mandates? The first thing is maximal Q by V that we have seen. this different figure of merits will be important for different applications, something like if you consider spontaneous emission rate enhancement, you will see that maximal Q by V ratio is needed for the Purcell effect. For nonlinear optical effects, you basically want optimal Q square by V ratio and so on. Once you have understood what is the equation for P, what is the equation for U, you can also find out what is your Q, that is the quality factor.

Optimum k-space field distribution of the cavity mode field

$$P \approx \frac{\eta}{2\lambda^2 k} \int_{k_\parallel \leq k} \frac{dk_x dk_y}{k_\parallel^2} k_z \left[\frac{1}{\eta^2} |FT_2(E_z)|^2 + |FT_2(H_z)|^2 \right] \quad Q = \omega \frac{\langle U \rangle}{\langle P \rangle} \quad \langle U \rangle = \int \frac{1}{2} (\varepsilon E^2 + \mu H^2) dV$$

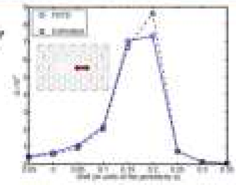


Figure 9: Comparison of Q factors derived from Eq. (A) (squares) to those calculated with FDTD (circles).

- **Applications of Eq. (A) in Photonic Research:**

Theoretical Exploration of Q Factor: Eq. (A) facilitates the investigation of the theoretical limits of the cavity's Q factor and its relationship with the mode volume, enabling deeper understanding and optimization of photonic structures.

- **Validation and Implementation of Eq. (A):**

Comparison with Experimental Data: Application of Eq. (A) to cavities identified through an iterative parameter space search shows a strong correlation with results from comprehensive FDTD simulations, as displayed in figure

Practical Utility in Design: This concurrence validates Eq. (A) as a reliable measure of radiative properties and supports its use in theoretical design approaches and rapid optimization of cavity parameters.

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Source: Englund, Dirk, Ilya Fushman, and Jelena Vuckovic. "General recipe for designing photonic crystal cavities." Optics express 13.16 (2005): 5961-5975.

So, this is that expression. So, what is the application of this expression in photonic research? First thing is that it gives you a theoretical explanation for Q factor because this equation facilitates the investigation of the theoretical limits of the cavity's quality factor and also the relationship with its mode volume that allows deeper understanding of the photonic structures. So, here also you can see a comparison of the Q factors which are basically obtained from the equation A and that is basically the red one and the blue one is basically the FDD simulation and you can see that they are pretty close to each other right. So, you can get a comparable results. kind of matching validates that you know this particular equation is a reliable measure of the radiative properties and basically can support the theoretical design process and rapid optimization of the cavity parameters.

So with that, we will try to conclude. So while designing high quality factor photonic crystal cavities, we will follow the methods like first you consider designing a two-dimensional photonic crystal slab. You demonstrate the effectiveness of this method for designing exceptionally high quality factors something like greater than 10 to the power 6 with very minimal mode volumes. The mode volume is basically lambda by n whole cube. So, that is how you can actually get very good Q by V ratio.

We have also seen that there are analytical approach and this method is purely analytical. You can find more details about this in this particular journal paper. It's called General Recipe for Designing Photonic Crystal Cavities. So this is an open access optic express journal.

So you can download this paper and read their work. So they actually give a detailed recipe. of designing very high quality factor photon crystal cavities. And few important thing is that you do not need to run simulations all the time this particular method is purely analytical. So, you can actually obtain the key parameters such as out of plane radiative losses by doing a simple computation single step. So, this efficiency accelerates the evaluation of Q factors across variety of cavity designs without running numerical simulations.

and you can also go for optimal mode placement to ensure minimal losses. So, why this is important? For achieving high quality cavities with least radiative loss, it is important to center the Fourier transform mode pattern at the extremes of the Brillouin zone and positioning it as far as possible from the light cone. So, that will ensure that it is not going to leak out. So, that will optimize the performance and it is also minimize the energy leakage from the cavity. So with that we will conclude this lecture and we will start the discussion of temporal couple mode theory fundamentals in the next lecture. So if you have got any queries or doubts you can always email to me, mention photonic crystals and the lecture number in the subject line. Thank you.

Thank You