Integrated Circuits and Applications
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AC Applications of Operational Amplifier
Lecture – 07
Logarithmic and Anti-logarithmic Amplifiers

So, the next application of this diode is logarithmic amplifier. Interestingly this op-amp can be used to produce logarithm of the input operation. As the name implies logarithmic amplifier the output is logarithm of the input voltage. So, if I take the circuit diagram this one, here the input voltage is applied simply a diode is placed in the feedback path. If this is I, this is also I, let the V_F is the voltage forward voltage across the diode, this is R. Then what is the current through the diode? You might have studied this diode in your second-year semiconductor device course.

So, the current through the diode is given by $I = I_S[e^{\frac{V_F}{\eta V_T}} - 1]$. This is the standard diode current expression, where I is the forward current through the diode and V_F is the forward voltage across diode and V_T is thermal voltage. This is constant which is 26mV at 300 degrees Kelvin or 27 degrees centigrade and η is a constant which is equal to 1 for silicon and 2 for germanium. I_S is reverse saturation current.

Now, here if you assume that $V_F > \eta\, V_T$, $\eta\, V_T$ is very small of the order of mV. If the forward voltage across the diode is greater than $\eta\, V_T$, then this can be approximated as $I_S \times e^{\frac{V_F}{\eta V_T}}$, this one you can neglect. Now, if you take the logarithm to the base E on both sides, $lnI = lnI_S e^{\frac{V_F}{\eta V_T}}$. This ln is actually here, this is log_e. So, lnx means log_e^x.

So, we can have the base to the base 10 also, but here I am taking to the base e. So, $logAB = logA + logB$. So, this is $logI_S + ln_e^{e^{\frac{V_F}{\eta V_T}}}$. This is equal to LN I S plus this is one of the important formula in terms of logarithm is this is $ln_e^{e^X}$ is nothing, but X only. So, this is equal to $\frac{V_F}{\eta V_T}$ or what is V_F? $V_F = \eta V_T$.

If I take this logarithm of I minus logarithm of IS, this will be expression for V_F. But what is v_o? This is with respect to 0, this is with respect to this ground, this is plus minus. So, this V_F is this plus minus. So, you can easily see that this is also ground point virtual ground. So, you can easily see that $v_o = -V_F$.

Therefore, what is v_o? $v_o = -V_F = -\eta V_T[lnI - lnI_S]$. But what is current I? This is 0 ground, this is v_i, $\frac{v_i - 0}{R}$=I. So, simply $\frac{v_i}{R}$. Therefore, $v_o = -\eta V_T[ln\frac{v_i}{R} - lnI_S]$. This is the expression for the output of this circuit and you can see that output v_o is proportional to logarithm of v_i that is why the name logarithmic amplifier.

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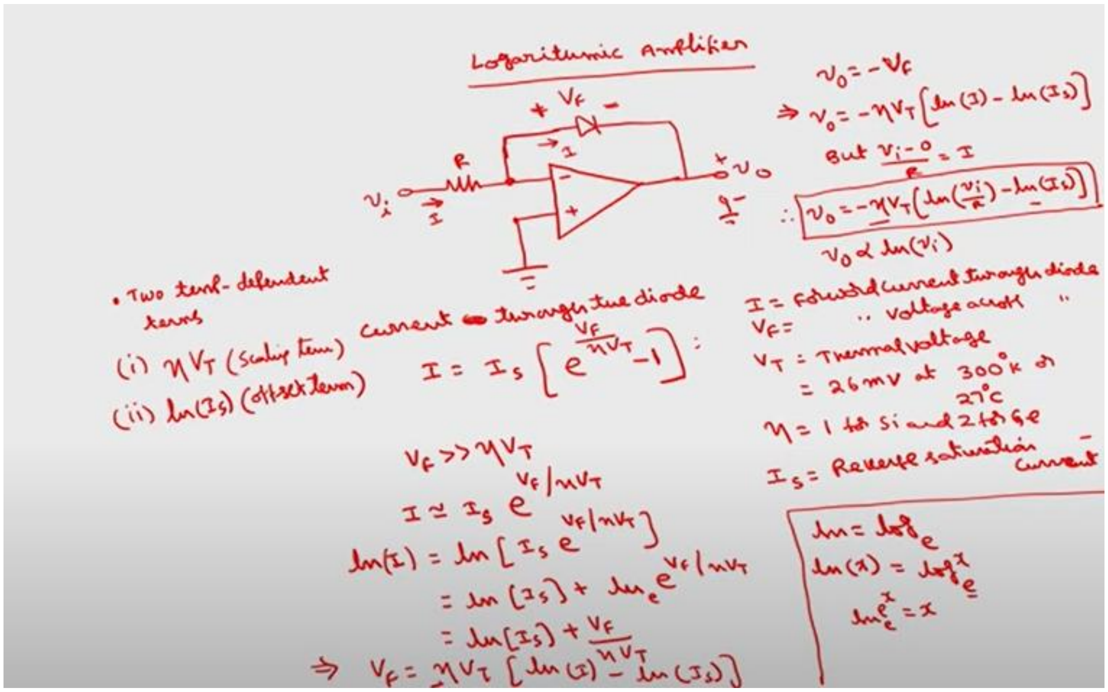

But there are some drawbacks of this logarithmic amplifier. There are two temperature dependent terms, temperature dependent terms. One is eta V_T which is the scaling factor here and second one is this offset term, this is scaling term and another is log or simply lnI_S, this is in fact, offset term. So, because of this temperature dependent terms, so the output will vary with the temperature. So, in order to avoid this problem, so we will modify this logarithmic amplifier which can cancel the temperature effect that is called temperature compensated logarithmic amplifier.

I will call it as just log amp. Here we are going to use two identical diodes to cancel the temperature effect. This is the amplifier A_1. Then here we are going to use one more diode which will be R_T which is thermistor basically, R_T is thermistor. This is same as this R, this is R_F feedback resistance, this is final output v_o, this is v_i.

Let us call this output as v_{01} plus minus this is V_{F1} is the forward voltage across diode D_1, across diode D_2, V_{F2}, these two are identical diodes. Now, we know that v_{01}, we have derived the expression for $v_{01} = -\eta V_T[ln\frac{v_i}{R} - lnI_S]$, just now we have derived this expression this. Now, talk about the expression for V_{F2}, the voltage across the diode D_2. So, current is here this current is 0, so the entire current I will flows through this one and reverse saturation current is I itself. So, this is equal to because this is plus minus this will be $\eta V_T[lnI - lnI_S]$, this is the forward voltage across this diode D_2.

So, what is if I call this one as say v_1. So, what is the relation between v_{01}, V_{F2} and v_1. So, this v_{01} is with respect to plus minus this is minus plus, this is plus minus. So, this is minus to plus, minus to plus and plus to minus. So, KVL is $v_{01} + v_{F2} = v_1$.

So, knowing this v_1 the remaining circuit is this A_2 is inverting amplifier, non-inverting amplifier this is say this is minus and this is plus. This is non-inverting amplifier whose gain is output what is $v_o = v_1[1 + \frac{R_F}{R_T+R}]$. But what is $v_1 = v_{01}$, this plus this equal 1 and 2 this is 1 plus 2 this is 1 this is 2. So, $-\eta V_T ln\frac{v_i}{R}$ $+\eta V_T lnI_S$, is the first expression $+\eta V_T lnI$ $-\eta V_T lnI_S$. This is plus and this is minus these two will get cancelled.

So, this will be $-\eta V_T$ this is $\frac{v_i}{R}$ this is I. So, this becomes $ln\frac{v_i}{IR}$ because $\ln A - lnB = ln\frac{A}{B}$. If I take $-\eta V_T$ outside this will be plus and this will be minus. So, $ln\frac{v_i}{R}$ - lnI which is equal to $ln\frac{v_i}{IR}$. This is the expression for $v_1 = -\eta V_T ln\frac{v_i}{IR}$.

So, what is the final output by v_{01} plus means this will be $[\frac{R_T+R+R_F}{R_T+R}]$ v_1, $v_1 = -\eta V_T ln\frac{v_i}{IR}$. So, final output of this temperature compensated logarithmic amplifier is $-\eta V_T[\frac{R_T+R+R_F}{R_T+R}]ln\frac{v_i}{IR}$. So, here also the output is proportional logarithm of the input. But the advantage of this one is here this temperature dependent value ηV_T is going to get cancelled. So, as I have told there are two terms in the uncompensated logarithmic amplifier one is ηV_T another is logarithm of I_S.

So, this logarithm of I_S is going to be cancelled by using these two terms will get cancelled. So, the ln I_S is not present here whereas, this ηV_T effect can be nullified by using this thermistor. R_T is a thermistor the principle of the thermistor is resistance varies with the temperature. So, you have to choose this thermistor such that it will compensate the effects of the variations of ηV_T with temperature. So, this ηV_T variations can be nullified by using thermistor and this ln I_S was cancelled by two identical diodes D_1 and D_2.

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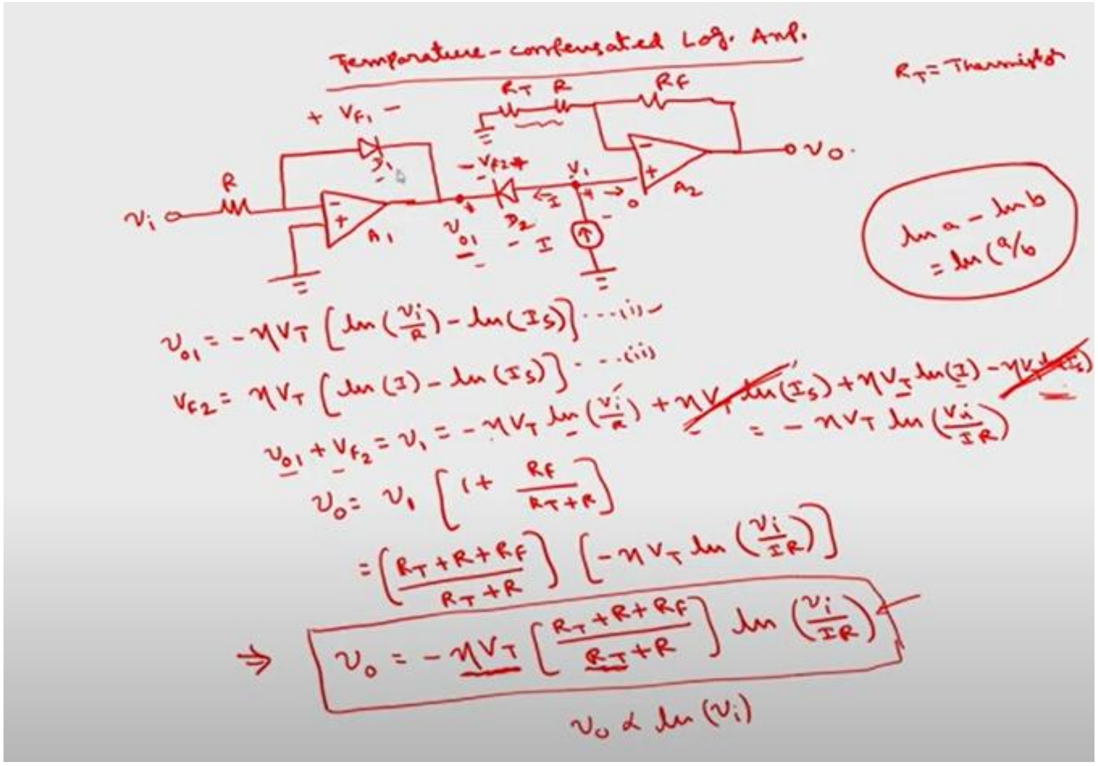

So, this is temperature compensated logarithmic amplifier. So, similar to this logarithmic amplifier. So, we can also construct the anti-logarithmic amplifier using operational amplifier ok. So, anti logarithmic amplifier. So, in the diagram of this consists of two diodes there will be current source here and there is a voltage divider here the input voltage v_i is applied whose anti logarithm operation has to be taken is grounded let us call this as I_F and the feedback path the diode is connected let us call this intermediate output as v_{01} and there is one more diode is connected here both will be symmetrical and here the feedback resistance R_F is connected and here output v_o is taken let us call this current as I and this voltage is v_1 .

Now, we have to perform the analysis and we have to show that output v_o is proportional to the anti logarithm of the input v_i anti logarithm in the sense. So, that exponential function basically we call this as D_1 D_2 D_1 D_2 are identical diodes. So, first I will find out what is v_{01}, this v_{01} is basically due to the current source I_F and the input voltage source v_i. So, this and this both will contribute to this v_{01} say the contribution of I_F let us call as $v_{01}{}'$ and the contribution due to v_i let us call as $v_{01}{}''$ then $v_{01} = v_{01}' + v_{01}{}''$. Now, what is v_{01} which is the contribution due to I_F? So, we have to set $v_i = 0$ to obtain $v_{01}{}'$ set $v_i = 0$.

So, if you ground this v_i if this v_i is grounded then this R_1 R_1 will be parallel between this v_1 and this ground. So, because no current flows through this one due to ideal op-amp there will be a resistance across this point and ground point that normally we will use for compensating the offset voltages. So, no need to consider that one. So, remaining thing is this entire I_F will flows through the diode because the current here is 0. So, what will be expression for this v_{01}, which we have already derived for this logarithmic amplifier as $-\eta V_T[lnI_F - lnI_S]$, I_S is the reverse saturation current this is $v_{01}{}'$ and to obtain $v_{01}{}''$ set $I_F = 0$ implies what happens if this $I_F = 0$.

So, this also will be 0 as a result of that if 0 current flows through the diode this voltage is v_1 this is also v_1 positive voltage is applied to this diode. So, this will add short circuit thereby this v_1 will be the output $v_{01}{}''$. So, what is v_1? $v_1 = v_i \frac{R_2}{R_1+R_2}$, because the circuit will acts as a voltage follower this itself is $v_{01}{}''$. Therefore, what is $v_{01} = -\eta V_T[lnI_F - lnI_S] + \frac{R_2}{R_1+R_2} v_i$, this is one expression. Now, we can also find out this v_{01} this is ground because of the virtual ground concept.

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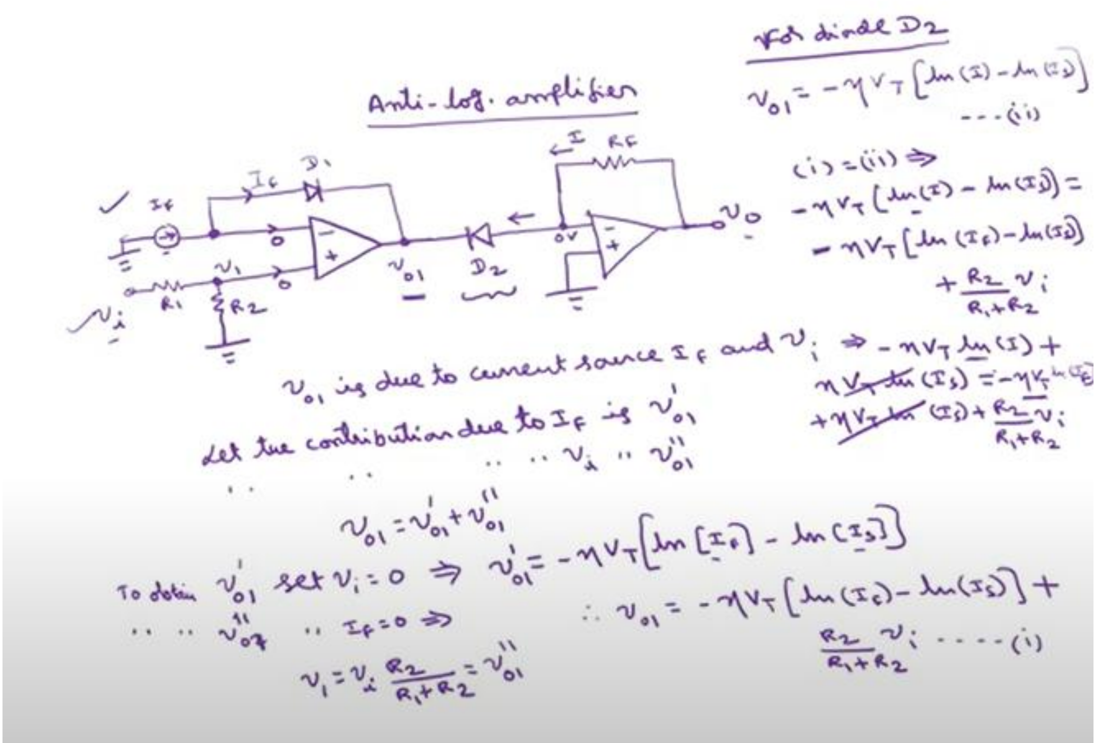

So, the current direction is this and the voltage across this diode is v_{01} with respect to this ground. So, v_{01} can also be written for the diode D_2 as $v_{01} = -\eta V_T[lnI - lnI_S]$, I_S I_S remains same because we are taking identical. This is equation 2 we can equate 1 and 2.

So, that 1 is equal to 2 implies $-\eta V_T[lnI - lnI_S] = -\eta V_T[lnI_F - lnI_S] + \frac{R_2}{R_1+R_2}v_i$. So, this is $-\eta V_T$ that implies $-\eta V_T$ times logarithm of I minus of minus becomes $+\eta V_T$times logarithm of I_S is equal to this is $-\eta V_T lnI_F$ plus this minus minus becomes $+\eta V_T lnI_S + \frac{R_2}{R_1+R_2}v_i$.

So, this will get cancelled this and this will get cancelled. So, what you will get now? If I take this term to the other side this term to other side then it will be plus. So, $\frac{R_2}{R_1+R_2}v_i = -\eta V_T lnI + \eta V_T lnI_F$. So, ηV_T is common this will be $lnI_F - lnI$, we know that $\log A - logB = log\frac{A}{B}$, $\eta V_T \ln(\frac{I_F}{I})$. We want basically the output voltage in terms of the input voltage v_i, v_i is here anyhow.

So, I will find out I in terms of v_o. So, that will get the expression for v_o implies $\ln(\frac{I_F}{I})$ is equal to this ηV_T will go to other side. So, $\frac{R_2}{R_1+R_2}\eta V_T$ or if you want take the reverse logarithm of $(\frac{I_F}{I})$ then there will be negative sign this is equal to $\frac{-R_2}{(R_1+R_2)\eta V_T}v_i$. So, what is $\frac{I}{I_F}$ is anti logarithm this is logarithm if you take this logarithm to the other side this will becomes anti logarithm means this is $ln^{-1}(\frac{-R_2}{(R_1+R_2)\eta V_T}v_i)$. Log inverse is nothing, but e thus this is $ln_e^a = x$, what is $a = e^x = ln^{-1}x$. So, this log inverse also can be written as $e^{(\frac{-R_2}{(R_1+R_2)\eta V_T}v_i)}$, but we want the expression for v_o. So, this implies if I want I then this will be IF times, but I want the expression here as v_o. So, what is the expression for I in terms of v_o? We can see that here this is v_o this is 0 this current is I and this resistance is R_F. So, what is the relation between v_o I and R_F? v_o this is the direction of the current. So, $\frac{v_o - 0}{R_F} = I \Rightarrow I = \frac{v_o}{R_F}$.

So, you can substitute this $I = \frac{v_o}{R_F}$ in this expression $\frac{v_o}{R_F} = I_F e^{\frac{-R_2}{(R_1+R_2)\eta V_T}v_i}$, implies what is the expression for $v_o = R_F I_F e^{\frac{-R_2}{(R_1+R_2)\eta V_T}v_i}$. This is the expression for the output of the given circuit. Gauss output $v_o \propto e^{v_i}$ this e is nothing, but anti logarithm. So, this is called anti logarithmic amplifier. So, now, by using this logarithmic and anti-logarithmic amplifiers we can implement the multiplication and division multiplier using op-amp. So, I am not showing the entire op-amp of circuit.

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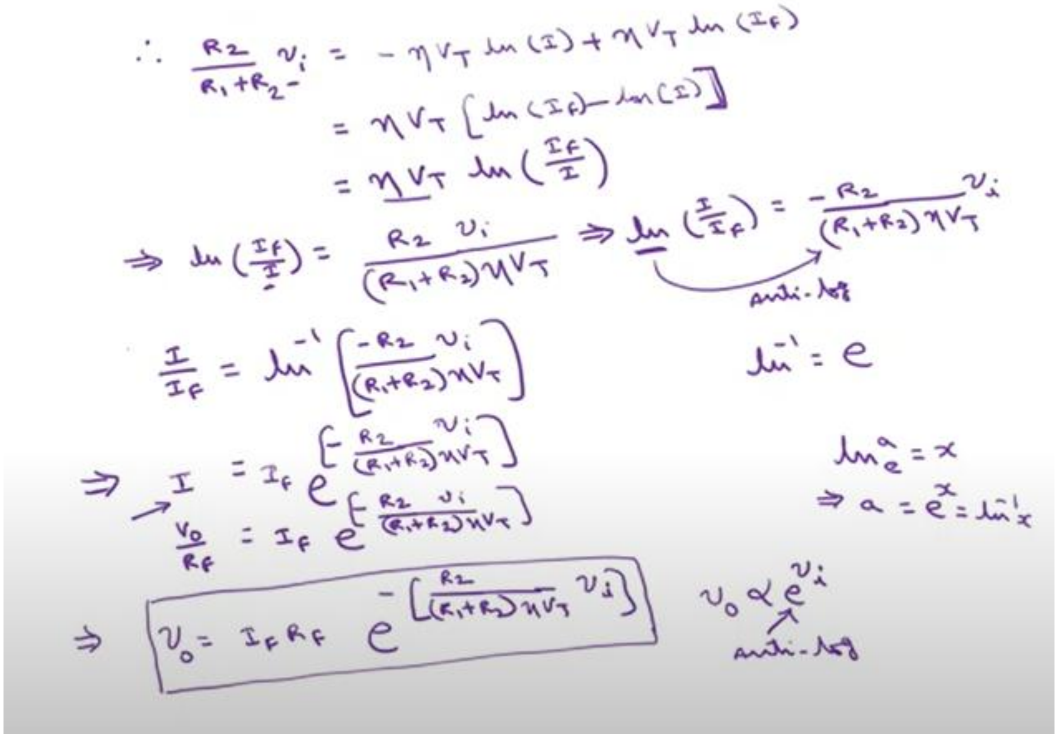

So, I will just show the logarithmic block this is log amplifier if the input is say x what will be the output this is proportional logarithm of x. And, if I give second input y to another logarithmic amplifier then output will be logarithmic of y. If I add these two then the output will be $\ln(x) + \ln(y)$, which is nothing, but $\ln(xy)$. Now, to get xy what you have to do you have to pass through the anti logarithmic amplifier. So, this output will be $ln^{-1}[\ln(xy)] = xy$.

So, log inverse and log get cancel will get simply xy that is the multiplication of the two inputs x and y. Similarly, you can implement the division using log and anti logarithmic amplifier. You have the same diagram, but here instead of plus this you have to minus if you take this as minus then you will get anti logarithmic amplifier then you will get the division operation. This is log amplifier this is another log amplifier x y then you subtract one from the other this is order, but this is plus sign this is minus sign. This is $\ln(x)$ this is $\ln(y)$ output will be $\ln(x) - \ln(y)$.We know that $loga - logb = log\frac{a}{b}$. So, if I pass through the anti-logarithmic amplifier then this will be $ln^{-1}\left[\ln\left(\frac{x}{y}\right)\right]$. So, logarithmic inverse and logarithm get cancel will get $\frac{x}{y}$.

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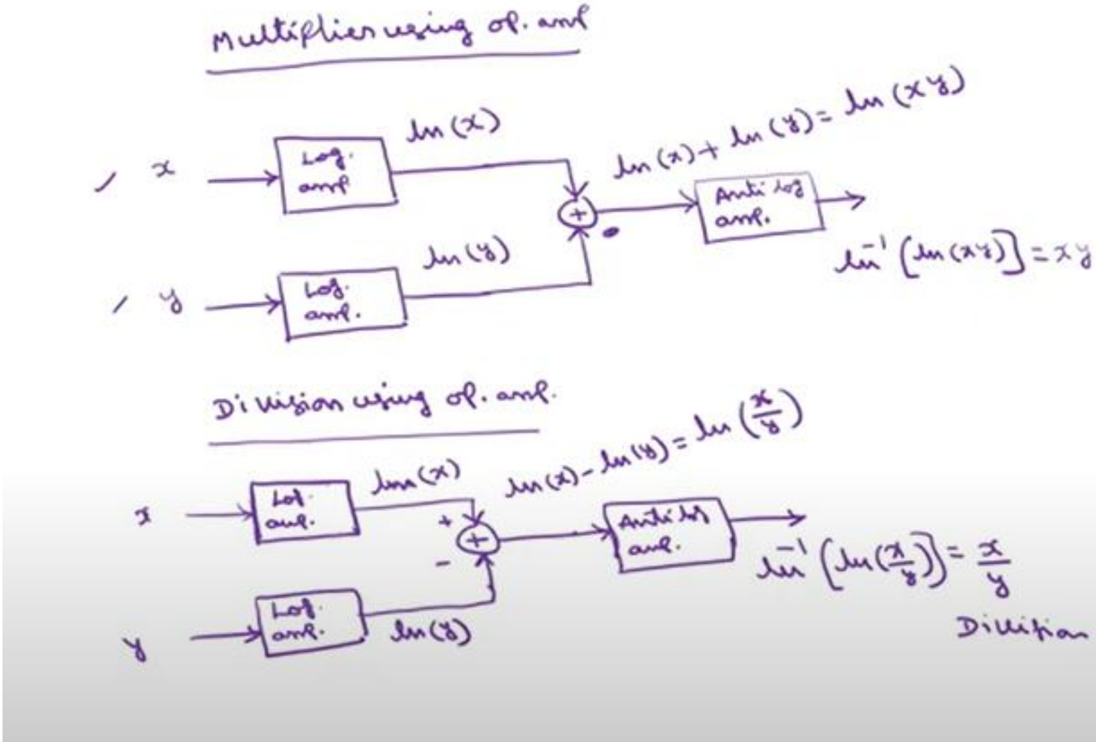

So, you see about this multiplier and division operations using operational amplifier. So, we will discuss about the practical op amp characteristics in the next lecture. Thank you.