Integrated Circuits and Applications
Prof. Shaik Rafi Ahamed
Department of Electronics and Electrical Engineering Indian Institute of Technology, Guwahati

AC Applications of Operational Amplifier
Lecture – 04
Integrator and Diffrentiator

Ok. In the previous lectures we have discussed various DC applications of operational amplifier. The operational amplifier not only processes the DC signals, it can process the AC signals also. There are some AC applications of operational amplifiers. So, we can implement the AC amplifiers, we can have rectifiers ok. So, we have peak detectors, clipper, clampers, logarithmic amplifiers. There are several AC applications of operational amplifiers.

First I will start with the inverting and non-inverting amplifier. First, I will discuss inverting AC amplifier. This is exactly similar to the DC inverting amplifier except for that there will be additional capacitors here, additional capacitors here. This is the circuit diagram of AC inverting amplifier.

This is the output v_o, input v_i. This is capacitance, this is R_1, this is R_F. So, in order to derive the expression for the gain of this AC amplifier. So, we have to use the s domain approach. Here this capacitance will be having capacitive reactance.

$X_c = \frac{1}{\omega c} = \frac{1}{2\pi\pi f c}$. So, in the s domain if you take this v_o becomes $v_o(s)$, v_i becomes $v_i(s)$. Here, $s = \sigma + j\omega$ and if you take the s plane this is σ axis, this is $j\omega$ axis and this is S plane. But for the sake of simplicity normally we will assume that $\sigma = 0$. So, we will substitute $s = j\omega$.

So, here this v_i of s is the $\mathcal{L}[v_i(t)] \rightleftharpoons v_i(s)$. Similarly, $\mathcal{L}[v_o(t)] \rightleftharpoons v_o(s)$. So, the gain as a function of s is given by output Laplace transform by input Laplace transform. If we derive the expression for this one. So, we have the two assumption that the voltage at inverting and non inverting are same this is 0 volt.

So, this is also 0 volts and here the current is 0, here the current is 0. So, if I assume that this is the current i. So, the same i will flows through R_F. So, at the input circuit what is the expression for i? This is v_i, this is 0 divided by the impedance. This will be lags are now impedance Z.

So, what is that impedance Z here? $Z = R_1 + \frac{1}{j\omega c}$. So, $\frac{v_i(s)-0}{R_1+\frac{1}{sc}}$. This is expression at)this. So, what is the expression for the i? $\frac{0-v_o(s)}{R_F}$. So, what is the gain $A(s)$? Is

$\frac{v_o(s)}{v_i(s)}$.

So, if we take

$$\frac{v_o(s)}{v_i(s)} = -\frac{R_F}{R_1 + \frac{1}{sc}}$$

$$A(s) = -\frac{R_F}{R_1}\left(\frac{1}{1 + \frac{1}{sR_1c}}\right)$$

If I take R₁ as common in the denominator, we will get an extra term $1 + \frac{1}{sR_1c}$. This is the expression for the voltage gain of AC inverting amplifier. Now clearly this $A(s)$ or $A(j\omega)$ varies with frequency. So, you can plot as a function of the frequency how does this A of $j\omega$ varies.

So, in order to obtain this DC suppose if $v_i(s)$ is DC then this you have to short circuit. If you short circuit this capacitance then what will be equivalent circuit this is same as inverting amplifier whose gain is simply R_F by R_1. So, say about the AC inverting amplifier. So, here the gain varies with the frequency. So, depends upon the requirement you can design this AC operational amplifier.

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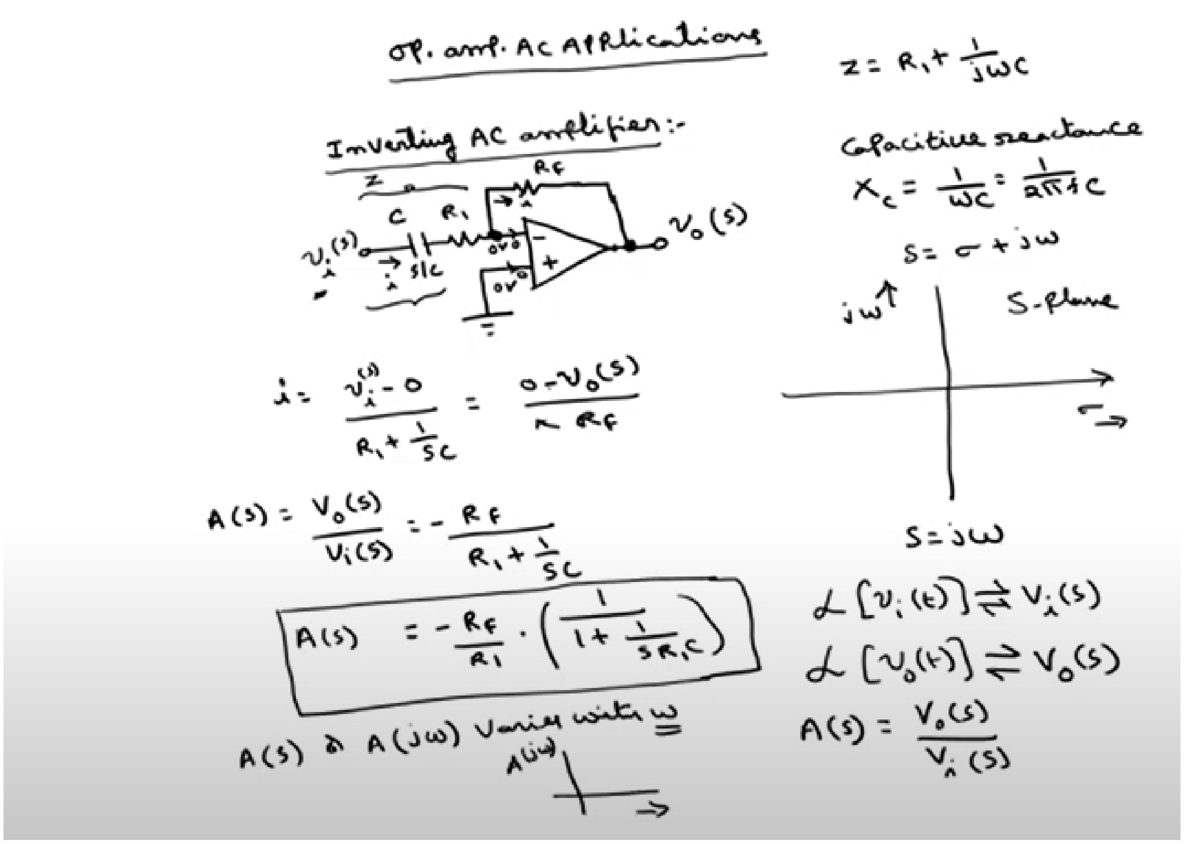

Then non inverting AC amplifier. Now inverting terminal will be grounded input will be applied to the non inverting terminal via capacitance here this capacitance is extra. This is $v_i(t)$ in terms of time domain or in frequency of this s domain $v_i(s)$ and this will be in s domain $\frac{1}{sc}$. This is $v_o(t)$ in time domain or $v_o(s)$ in s domain this is R_1 R_F. Here actually this has to block the DC this will block the DC because for DC $f =$ 0.

So, what happens to the inductive reactance $\frac{1}{2\pi f c} = \infty$ means this will acts as open circuit this is OC means open circuit. So, in order to block this DC we have to connect one additional resistance here this is some R_2. So, what is the expression for the voltage gain $A(s)$ which is defined as $\frac{v_o(s)}{v_i(s)}$. So, if I find out this voltage as say v_1. So, what is $v_1(s)$ voltage divider? $v_i(s)$ is this voltage and you are taking the voltage across the resistor.

$$A(s) = \frac{v_o(s)}{v_i(s)}$$
$$v_1(s) = v_i(s)\frac{R_2}{R_2 + \frac{1}{sc}}$$
$$v_o(s) = v_1(s)[1 + \frac{R_F}{R_1}]$$
$$= \left(\frac{R_2}{R_2 + \frac{1}{sc}}\right)[1 + \frac{R_F}{R_1}]\ v_i(s)$$

In case what is

$$A(s) = \frac{v_o(s)}{v_i(s)} = (1 + \frac{R_F}{R_1})\left(\frac{R_2}{R_2 + \frac{1}{sc}}\right)$$

So, this is the non-inverting AC amplifier. So, in order to get DC voltage gain. So, you have to short circuit this capacitance. If I short circuit this capacitance what will be equivalent circuit minus plus this v_i will be short circuited there will be some resistance this $v_i(s)$ and here this will be same as this. This will be grounded because here there is no current 0. So, the voltage drop across R_2 is also 0. So, you can neglect this you can remove also this one. Now, this is exactly same as non-inverting DC amplifier whose gain is $\frac{v_o(s)}{v_i(s)}$ this is of course, independent of $s = 1 + \frac{R_F}{R_1}$.

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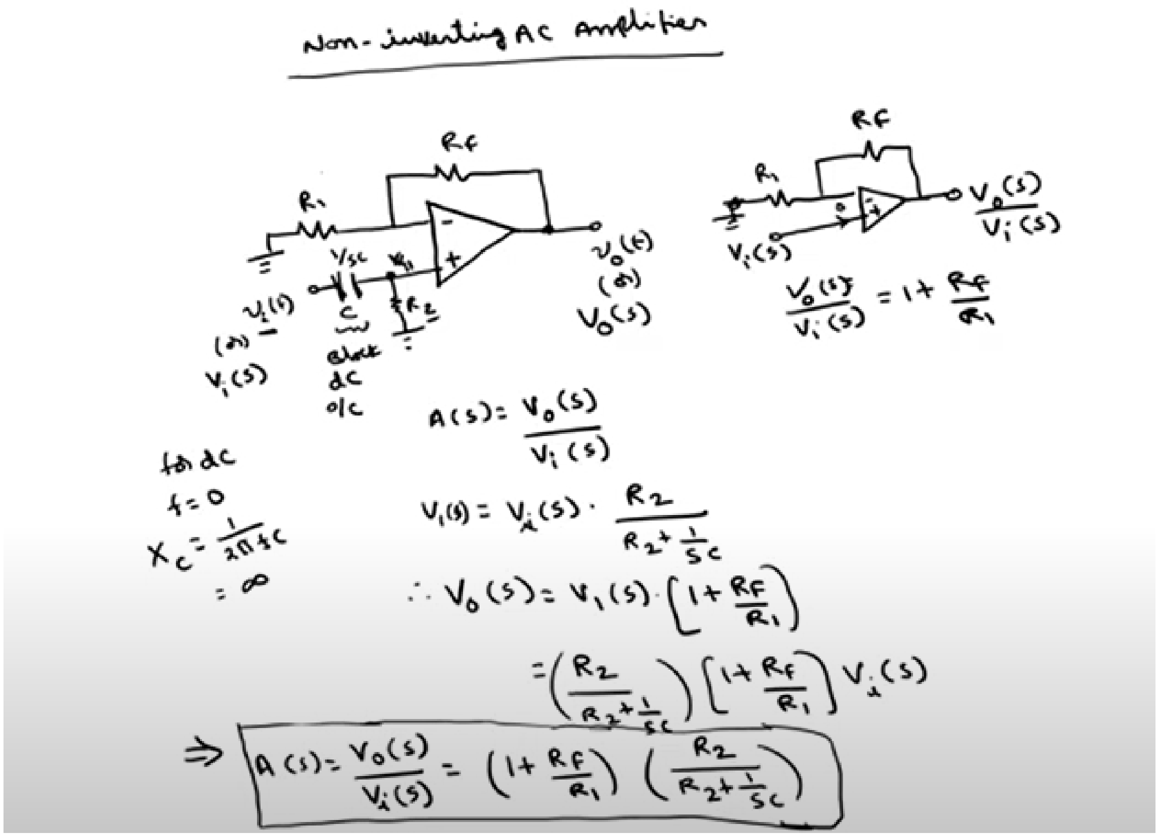

So, the next AC application is we can perform the integration using operational amplifier op-amp integrator. This is the circuit diagram of op-amp integrator, this is the input this is $v_i(t)$ in time domain or $v_i(s)$ in S domain. This is$v_o(t)$ in time domain $v_o(s)$ in S domain this is R and this capacitance is C and capacitive reactance is $\frac{1}{sc}$. Then how to find out the expression for the output we are going to show that output is integration of the input. I can perform this either in the time domain or S domain.

If I perform this time domain analysis first, here the current is 0, here current is 0 because this is at 0V this is also at 0V. If I assume that this current is i the entire current v_c will flows through this one also. Let the voltage across this capacitance is $v_v(t)$ in terms of time domain. So, what is the expression for the current through the capacitor i is equal to $\frac{1}{c}\int i(t)dt$. Here the across this capacitor the voltage is $v_c(t)$.

Sometimes this may be written as this integral you can written as $\int_0^T v_c(\tau)d\tau$ ok. You can write directly this $v_c(t)dt$ also. Then what is the relation between the v_0 and v_c? This v_0 is here plus with respect to ground minus and this is plus minus. If we take from here to here, here to here, here to here a loop because this is also ground this point is also ground. So, this plus 2 minus is $v_c(t)$.

So, $v_c(t)$ is plus 2 minus this is also plus 2 minus plus $v_o(t) = 0 \Rightarrow v_o(t) = -v_c(t)$. If you take so, first I will perform the time domain analysis. So, in the time domain analysis we know that the current through the capacitor is I here and the voltage across the capacitor is $v_c(t)$. So, what is the expression for the voltage across the capacitance? $\frac{1}{c}\int i$ of course, this also you can call as $i(t)$ simply i or $i(t) \times dt$. But what is the relation between $v_c(t)$ and $v_o(t)$? If you assume that this is $v_c(t)$ is voltage across the capacitor with plus sign minus sign and $v_o(t)$ is this with plus sign minus sign.

If you take the loop KVL $v_c(t)$ is plus 2 minus and $v_o(t)$ also plus 2 minus plus $v_o(t) = 0 \Rightarrow v_o(t) = -v_c(t)$. So, therefore, here if I substitute $v_o(t) = -\frac{1}{c}\int i(t)dt$. But from this loop what is $i(t)$? This is $v_i(t)$ and this is 0V divided by R, $\frac{v_i(t)-0}{R}$. If you substitute this expression 2 in expression 1, you will get $v_o(t) = -\frac{1}{c}\int \frac{v_i(t)}{R}dt$. This is equal to $v_o(t) = -\frac{1}{Rc}\int v_i(t)dt$.

That is the output of this circuit is integration of the input signal. So, this will acts as a op-amp integrator. So, we can perform this integration by just simply using a resistor and capacitor also. This also will acts as integrator. This is $v_i(t)$, RC, this is $v_o(t)$.

So, what is the advantage of using the operational amplifier to realize the integration? So, as we know that this operational amplifier will provide high input impedance is one. Another is in addition to integration this also will provide some gain also. This is $\frac{1}{RC}$ is the gain factor. If I choose say $C = 1\mu F$ and $R = 1k\Omega$, then what will be $\frac{1}{RC} = 1000$.

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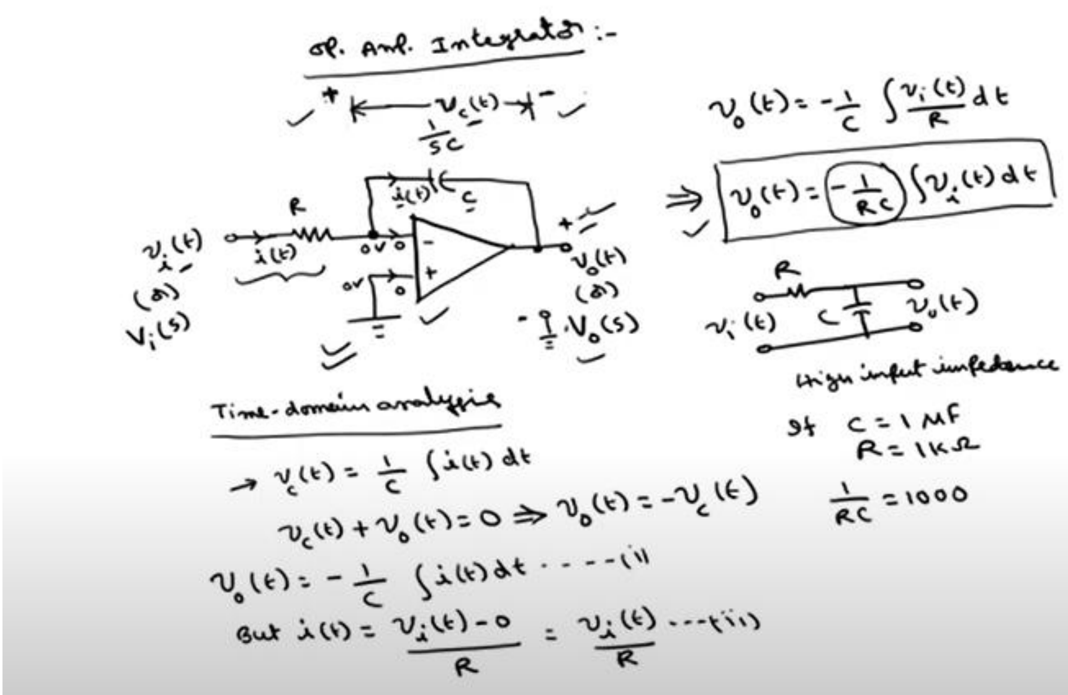

This will provide a gain of 1000 also. So, you see with the time domain analysis of this operational amplifier. We can perform the S domain analysis also we can show that this is the circuit diagram. This is $v_o(s)$, this is $v_i(s)$, this is $R + \frac{1}{SC}$. So, if you call this current as $i(s)$, all will be S domain now. This current is 0, this current is 0, this is 0V because this is also 0V, this will also $i(s)$.

Now, what is the expression for $v_o(s)$ in terms of $v_i(s)$? So, you can see that this $i_i(s)$ from the input side, this is $v_i(s)$, this is 0V. So, $i(s) = v_i(s)/R$ whereas, the output side this is 0V, this is $v_o(s)$ and we have the capacitive reactance in the feedback path. So, what is $i(s)$? If I take from this 0V to $v_o(s)$, $i(s) = \frac{0-v_o(s)}{\frac{1}{SC}}$. So, if we equate this 1 and 2, we will get $-\frac{v_o(s)}{\frac{1}{SC}} = \frac{v_i(s)}{R} \Rightarrow v_o(s) = \frac{1}{RC}\frac{1}{s}v_i(s) \Rightarrow A(s) = \frac{v_o(s)}{v_i(s)} = -\frac{1}{SRC} = -\frac{1}{j2\pi fRC}$. Now, we know that in the Laplace transform, if $\mathcal{L}[x(t)] = X(s)$, there is a property that in $\mathcal{L}\left[\int x(t)dt\right] = \frac{1}{s}X(s)$ or what is $\mathcal{L}^{-1}\frac{1}{s}X(s) = \int x(t)dt.$

So, if you use this property here. And takes inverse Laplace Transform on both side. So, $v_o(s)$ the input time domain is $v_o(t) = \frac{1}{RC}$ of course, minus sign is there here, this minus sign $-\frac{1}{RC}$. The inverse Laplace transform of this $\frac{1}{s}$ this one is equal to $\int v_i(t)dt,$ this is same as the previous time domain expression. So, this is actually in fact, the ideal integrator, but practically there will be some problem with this type of integrator. So, what happens is if I take from here the gain in S domain, $A(s) = \frac{v_o(s)}{v_i(s)} = -\frac{1}{SRC} = -\frac{1}{j\omega RC} = -\frac{1}{2j\pi fRC}$. At low frequencies if the frequency is less, if you decrease the frequency what happens to this gain? Gain will be increases, thereby it may causes the instability problem.

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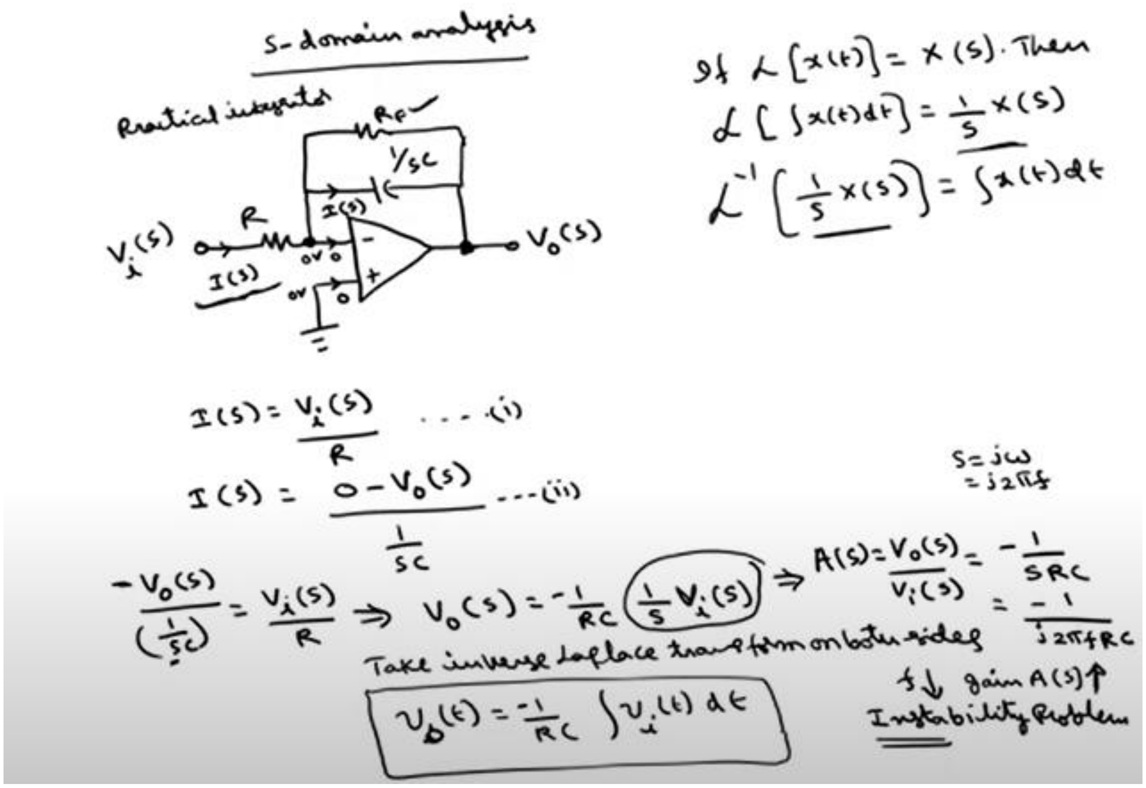

So, if gain is increases there is a possibility that output may drive into the saturation ok. So, this may cause instability problem. To avoid this for the practical integrator this is ideal integrator, if you take the practical integrator here we are going to connect one more resistor say R_F. So, we are connecting this R_F resistor we can avoid the problem of instability or driving the output into the saturation ok.

So, we put this op-amp integrator. So, we can also perform the differentiation operation also op-amp differentiator. Here also you can perform the time domain as well as S domain analysis. So, first I will perform the time domain analysis and the circuit diagram of this differentiator is obtained. The circuit diagram of this differentiator can be obtained by just exchanging the R and C. Now, you have to connect the C here and R and the feedback path.

Now, let us take the time domain analysis. So, everything you call as t this is $\frac{1}{C}$ this is simply capacitance this current is $i(t)$. So, the current here is 0 current here is 0 this voltage is 0V because this is 0V this is R the same $i(t)$ will flows through this. So, in order to derive this again we know that now the voltage across the capacitor I will call as $v_c(t)$ this is plus sign minus sign and $v_i(t)$ is this is with respect to ground this is plus minus. If you take from here to here the KVL $v_i(t) - v_c(t) = 0 \Rightarrow v_c(t) =$

$v_i(t)$. And what is the expression for the voltage across the capacitor $v_c(t) = v_i(t) = \frac{1}{C}\int i(t)dt$.

This I have used in the previous derivation also, but what is $i(t)$? If you take this 0V to this $v_o(t)$ this current is $i(t)$ and the resistance is R, $i(t) = \frac{0-v_o(t)}{R} = -\frac{v_o(t)}{R}$. So, if you substitute this in the previous expression we call this as 2 in 1, $v_i(t) = -\frac{1}{RC}\int v_o(t)dt$. Now, we take differentiation on both sides, if you take $\frac{d}{dt}v_i(t) = -\frac{1}{RC}v_o(t)$ is equal to differentiation and integration will get cancelled. So, we will get simply $-\frac{1}{RC}v_o(t)$ or what is $v_o(t) = -RC\frac{d}{dt}v_i(t)$. So, the output of this circuit is proportional to the differentiation of the input signal.

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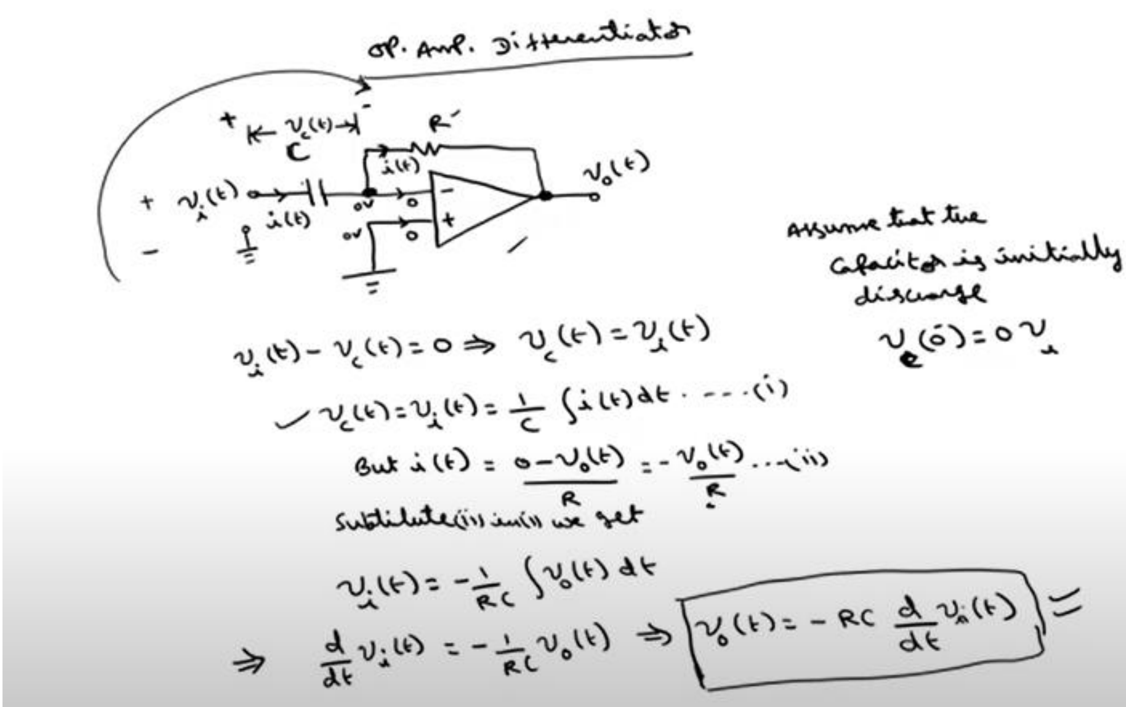

So, this will acts as a differentiator. So, we can derive this in S domain also. So, I will derive this $v_o(s)$ in terms of $v_i(s)$ and we can see the problem with this ideal differentiator similar to the ideal integrator then how to overcome that problem and how to construct the practical differentiator. Now, everything will be in S $v_o(s)$, $v_i(s)$ and this becomes $\frac{1}{SC}$ this current is $i(s)$ this is resistance R this is 0V, 0V this

current is 0 this current is 0 this current is also $i(s)$. So, if you consider this input side. So, what is
$i(s) = \frac{v_i(s) - 0}{\frac{1}{Sc}} = \frac{0 - v_0(s)}{R} \Rightarrow v_0(s) = -RCSv_i(s)$.

There is another property of Laplace transform if $\mathcal{L}[x(t)] = X(s)$ then the $\mathcal{L}\left[\frac{d}{dt}x(t)\right] = X(s) - X(0^-)$ is this initial value before T is equal o. So, if I take the inverse Laplace transform on both sides of this if X of 0 is 0 implies what is $\mathcal{L}^{-1}[SX(s)] = \frac{d}{dt}x(t)dt$. So, if we take inverse Laplace transform LT Laplace transform on both sides $v_o(t) = -RC\frac{d}{dt}v_i(t)$, as I have discussed in the last slide that $v_i = v_c$. So, if I assume that initially this capacitor is discharged initially discharged. So, that v_0 of or $v_c(0^-) = 0$ this is also $v_i(0^-)$.

So, this $v_i(0^-) = 0$. So, simply this $\mathcal{L}^{-1}[Sv_i(s)] = \frac{d}{dt}v_i(t)dt$. So, this is same expression that we have derived in using time domain analysis. Now, similar to this integrator here also there is a problem with frequencies. So, if I take this voltage gain of this one using this relation what is $A(s) = \frac{v_o(s)}{v_i(s)} = -SRC = -j2\pi fRC$. If we take magnitude of $A(s)$ or $A(j\omega)$ this will be is equal to this j magnitude is unity and minus sign becomes plus.

$$|A(s)| = 2\pi fRC$$

Now, here as *f* increases in case of integrator if *f* decreases gain increases now as *f* increases the magnitude of gain increases. Thereby it may cause instability problem. If the gain is very large even this circuit may produce the oscillations. This is one problem and another problem is here if we take the input impedance here we will call as impedance because capacitance is there Zi this is nothing, but between this input and ground. So, whatever the impedance is there this is the input impedance is $\frac{1}{SC}$ this is equal to $\frac{1}{j\omega C}$ or $\frac{1}{j2\pi fC}$.

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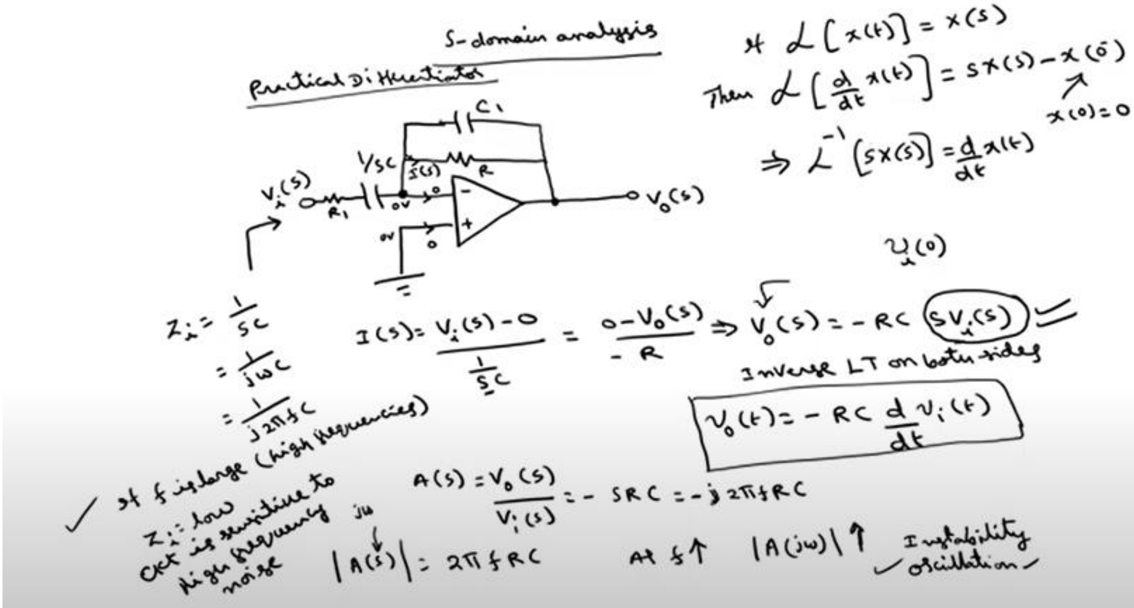

If f is large means at high frequencies Zi becomes low. So, low input impedance means it is more sensitive to the high frequency noise because for high frequency signals it offers low input impedance means the circuit is sensitive to the high frequency noise. CKT is the short form of the circuit high frequency noise these two are undesirable things one side if we increase the frequency. So, this circuit may produce the oscillations other side it is sensitive to the high frequency noise. To avoid this similar to this practical integrator here also we will use practical differentiator here we will connect one capacitance C1 and in addition to that here also we will connect one resistance. So, that these two problems can be avoided this is about this practical differentiator.

So, the precision rectifier that we will discuss in the next lecture. Thank you.