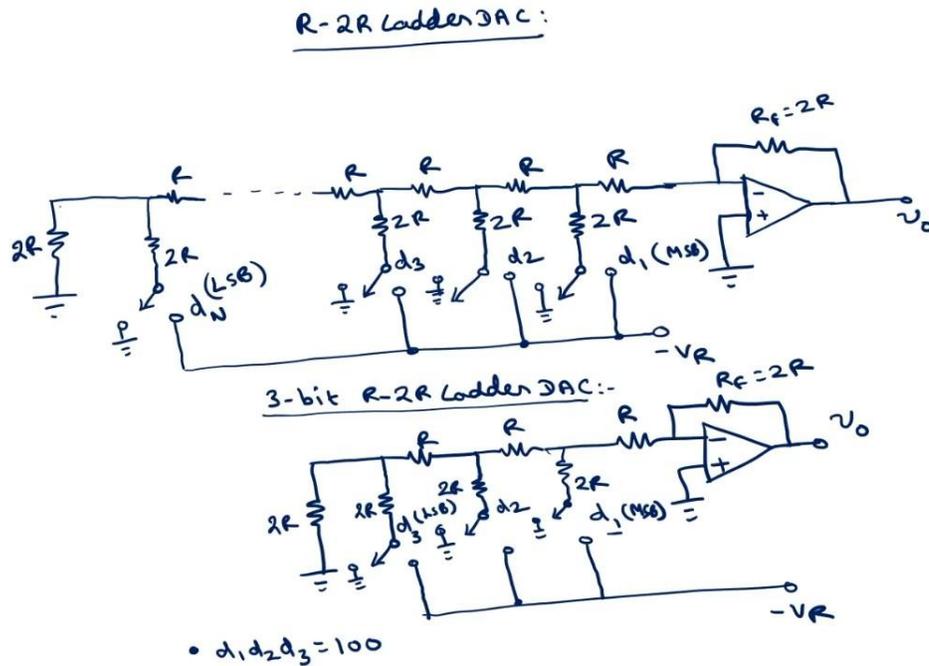


Integrated Circuits and Applications
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Data Converters
Lecture – 38
R-2R Ladder D/A Converter

So, in the last lecture, we have discussed about the weighted resistor D/A converter. The drawbacks of this weighted resistor D/A converter is we require a wide range of resistors. For example, for 8-bit DAC the highest resistor to lowest resistor ratio is 128. So, because of that actually the resistor accuracy is going to affect the overall accuracy of the DAC. And, sometimes it becomes difficult to fabricate the high values of the resistor in IC form also. To avoid this difficulty so, we have the second type of D/A converter, which is called as R-2R ladder Type DAC.

As the name implies, R-2R ladder DAC this consists of only two resistors, R and 2R. So, if we take the circuit diagram of this R-2R ladder DAC, there is a current to voltage converter similar to the weighted resistor type. This is PD back resistor, which was this is equal to twice R. So, that all the resistors will be either R to R.

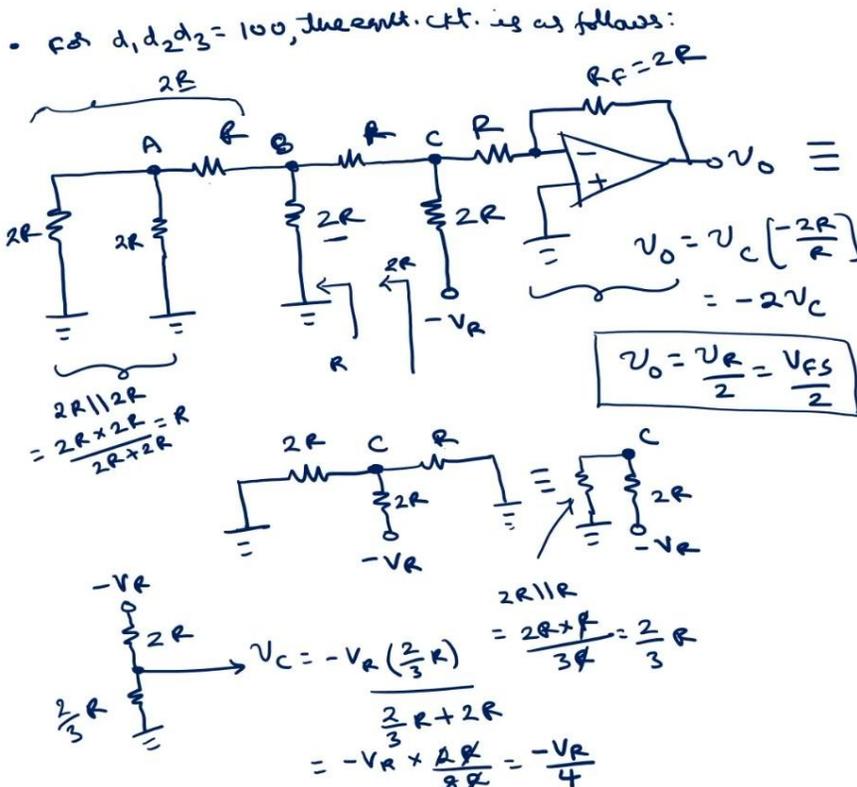


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This is R, this is 2R, this is R, this is 2R. Like that, it will continue, and at the end, we have 2R resistor. This is LSB, and this is d_1 , MSB, this is d_2 , d_3 and so on. So, we can see that overall we will be having two different resistors R and 2R only.

And this also will give the same expression as that of a weighted resistor due to a network where $v_0 = V_{FS} \times [d_1 2^{-1} + d_2 2^{-2} + \dots]$. So, now, to obtain this expression for the output v_0 , I will consider a 3-bit R to A ladder DAC. This will be 2R, this is minus V_R , and here we have one resistor in current to voltage converter. Now, what will be expression for v_0 ? For different combinations of this, d_1 , d_2 and d_3 .

This is d_1 MSB, this is d_2 d_3 LSB. Let us consider first d_1, d_2, d_3 is equal to 1 0 0. Here, if this d value is 1, means it will be connected to minus V_R ; otherwise, it will be connected to ground. So, in 1 0 0, what will be the position? d_1 is connected to minus V_R , d_2 d_3 will be grounded. So, what will be the equivalent circuit? The equivalent circuit will be this LSB d_3 will be connected to the ground, d_2 is also grounded, d_1 is connected to minus V_R , and then we have R, feed by $R_F = 2R$, this is R R R 2R 2R.



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So, what will be expression for the output of this circuit? So, this type of circuits we analyzed in the previous lectures also. So, first, if I find out the voltage here, if

I call this as node C, node B, node A? So, the output voltage v_0 is given by if v_c is the voltage at v_c voltage at node C, v_c into the remaining this circuitry is inverting amplifier with a gain of $-\frac{2R}{R}$, this is equal to $-2v_c$. If I find out the v_c , then you simply multiply with minus 2 to get the output voltage v_0 . Now, what is the expression for v_c ? So, we can see that these two will be in parallel.

So, what will be equivalent resistance $2R$ in parallel with $2R$? In say $\frac{2R \times 2R}{2R + 2R} = R$. Now, including here up to here, what will be this resistance equivalent resistance including this R also? From here to here, this is R , and this is another R . These two are in series. So, up to here, this will be $2R$. Then if you include this $2R$, this $2R$ and this $2R$ will be in parallel.

So, up to here, if I include if I see the resistance looking into this B, this will be again $2R$ in parallel with $2R$, which is R . If I look into here after this R , $R + R = 2R$. Now this will be equivalent to this entire resistance is $2R$, this is C point, this is R , this is $2R$, this is also virtual ground. So, we can call this also at ground only; this is C point. So, to the left side of this C point, total resistance is $2R$, right side, this is R .

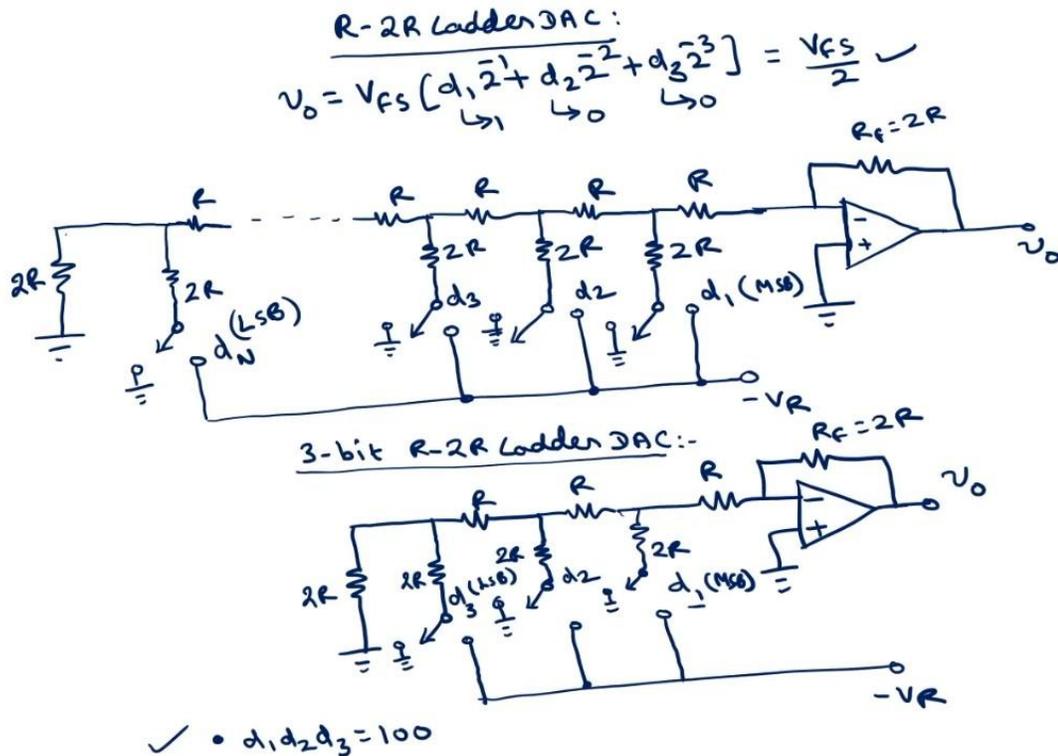
Because between this point and ground, we have $2R$, between this point and ground we have R . So, this is again equivalent to $2R$, this is $-V_R$, this is $2R$ in parallel with R , this is $-V_R$, this is $2R$, this is C point. This is equal to $\frac{R \times 2R}{R + 2R} = \frac{2}{3}R$. So, what is voltage divider formula now? This is equal to $-2R$, and taking after $2R$ you are taking this voltage is v_c , and below this up to ground, you have got $\frac{2}{3}R$.

So, according to voltage divider $v_c = \frac{-V_R \times \frac{2}{3}R}{\frac{2}{3}R + 2R} = -V_R \times \frac{2R}{8R} = -\frac{1}{4}V_R$. Therefore, $v_0 = -2v_c$, so minus into minus becomes plus. So, you will get $\frac{V_R}{2}$. If $V_R = V_{FS}$ and this is equal to $\frac{V_{FS}}{2}$. So, what is the general expression for this v_0 of a 3-bit weighted resistor R $2R$ circuit, I have taken this one as $v_0 = V_{FS} \times [d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$.

The same expression is valid here also. So, the expression that we have derived for the weighted resistor is equal to $V_{FS} \times [d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$. So, in this case, for d_1, d_2, d_3 is equal to $1\ 0\ 0$. So, this is 0 , this is 0 , this is 1 , this will be equal to $V_{FS} \times [1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3}] = \frac{V_{FS}}{2}$.

Similarly, we can take for the other possibility of $d_1\ d_2\ d_3$, which is somewhat complicated than this, then we can generalize the expression as $v_0 = V_{FS} \times [d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$. I will take the another case of $0\ 0\ 1$. The equivalent circuit will be these two are grounded. This is LSB, LSB is 1 . So, this will be

connected to minus V_R . The remaining 2 bits will be connected to logic 0 means grounded this is also grounded this is also grounded.



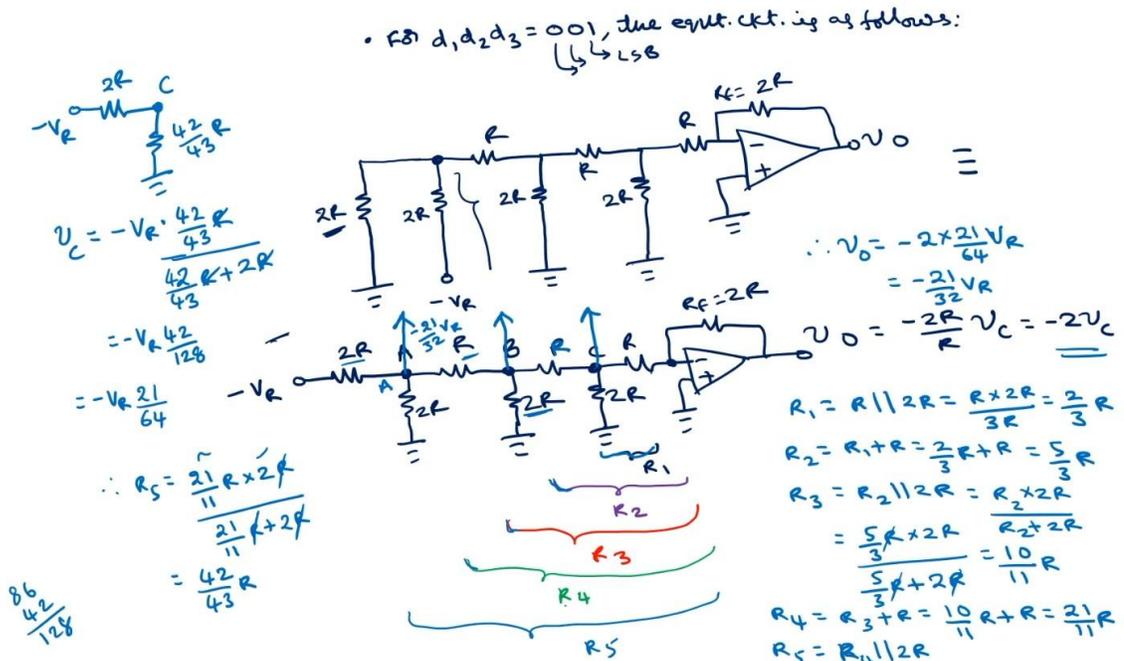
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And here, we have this resistor and inverting amplifier. Now, what is the expression for v_o of this circuit? So, this can be equivalently written as this $V_R \cdot 2R$ if I take to this side minus V_R this $2R$ this branch I have taken here, then this $2R$ is between this point and ground. So, this $2R$ is between this point and ground. This is this $2R$, and after that, the circuit is $R \cdot 2R$ rounded, $R \cdot 2R$ rounded and $R \cdot 2$ inverting amplifier. Here, also, you call this one as node A, node B, node C. If you find out the node C voltage, then you have seen that output $v_o = -\frac{2R}{R} \times v_c = -2v_c$.

Now, to first find out v_c . So, what is v_A , what is v_B and what is v_c ? We will go in a step by step manner. So, first, I will find out v_A . So, in order to find out v_A , what will be the overall resistance? This also will acts as a ground point. This R and this $2R$, if I call this equivalent resistance from here to here as R_1 , say including this R_1 up to here if I call this as R_2 .

If I consider up to this $2R$ including this $2R$ as R_3 including this R if we call as

R_4 including this $2R$ also if I call as R_5 . Then, you can find out v_A using voltage divider. Again, you can apply another voltage divider to find out v_B another voltage divider formula to find out the v_C . Now, what are the expressions for R_1 , R_2 , R_3 , R_4 and R_5 ? $R_1 = R || 2R = \frac{R \times 2R}{R+2R} = \frac{2}{3}R$. Then $R_2 = R_1 + R = \frac{2}{3}R + R = \frac{5}{3}R$. Then $R_3 = R_2 || 2R = \frac{R_2 \times 2R}{R_2 + 2R} = \frac{\frac{5}{3}R \times 2R}{\frac{5}{3}R + 2R} = \frac{10}{11}R$ and R_4 is you have to add R up to here; this is R_3 , and if I add this R also, then this will be R_4 . $R_4 = R_3 + R = \frac{10}{11}R + R = \frac{21}{11}R$ and R_5 will be, finally, $R_5 = R_4 || 2R = \frac{\frac{21}{11}R \times 2R}{\frac{21}{11}R + 2R} = \frac{42}{43}R$.



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Now this can be written equivalently as minus V_R is this, this is $2R$ this is C point, and overall resistance here will be total is R_5 , which is equal to $\frac{42}{43}R$. Therefore, according to the voltage divider formula what is $v_C = -V_R \frac{\frac{42}{43}R}{\frac{42}{43}R + 2R} = -V_R \frac{42}{128} = -V_R \frac{21}{64}$, this is v_C and what about v_0 minus twice v_C . So therefore, $v_0 = -2 \times \frac{21}{64}V_R = -\frac{21}{32}V_R$; this is node A , node B ; what is the voltage similar node C , what is the voltage?

So, I will redraw here so that overwriting can be avoided. So, the voltage at this point you have derived as this is node A , this is minus V_R , this is $2R$, this is B , this is C , I have obtained only v_A , not v_C , this is R_5 then what is the voltage here that is v_A this is $-V_R$ $2R$ this is at node A the overall resistance is R_5 which is equal to

$\frac{42}{43}R$, this is node A this is $2R$ this is minus V_R and what is $v_A = -V_R \frac{\frac{42}{43}R}{\frac{42}{43}R+2R} = -V_R \frac{42}{128} = -V_R \frac{21}{64}$; this is v_A , the voltage here is $20 - \frac{21}{32}V_R$. So, now, to find out voltage at B and C, I will redraw this circuit; this is v_A . We have got this v_A as $-\frac{21}{64}V_R$. Now, this is v_A I have to find out the voltage here.

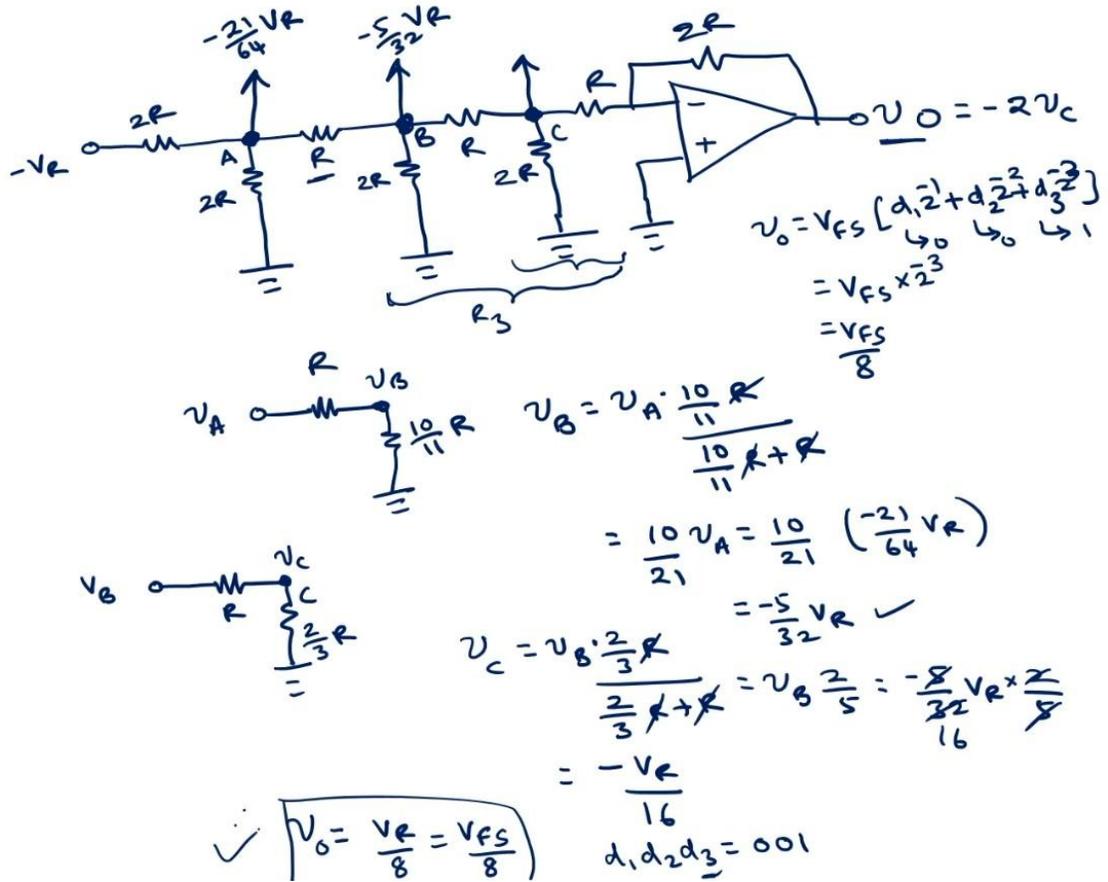
So, v_A is voltage here now this is across this resistor R , and this is B, and the voltage here is say v_B this is B point and this is A point, and what is the overall resistance from here to ground is the resistance up to this $2R$. So, you see nothing, but up to this R is this $R_3 = \frac{10}{11}R$. Now, again voltage divider what is $v_B = v_A \times \frac{\frac{10}{11}R}{\frac{10}{11}R+R} = \frac{10}{21}v_A = \frac{10}{21} \times -\frac{21}{64}V_R = -\frac{5}{32}V_R$, but what is v_A , we have derived as $-\frac{21}{64}V_R$.

So, the voltage here will be $-\frac{5}{32}V_R$. Similarly, how to find out the voltage here? If I know this voltage, then output voltage is equal to minus 2 times v_c . So, now, to find out voltage at v_c now v_B would be the input across the voltage divider this is R . So, from here to here, what is the resistance? We see nothing but R to parallel with $2R$, which is nothing, but R_1 only $R_1 = \frac{2}{3}R$.

So, what is v_c according to the voltage divider again? v_B into we are taking across $\frac{2}{3}R$. So, $\frac{\frac{2}{3}R}{\frac{2}{3}R+R} = \frac{2}{5}$. So, this is equal to $v_B \times \frac{2}{5}$, but what is v_B from here? This is equal to $-\frac{5}{32}V_R \times \frac{2}{5} = -\frac{V_R}{16}$, but what is v_0 as we have derived earlier $2v_c$.

Therefore, $v_0 = \frac{V_R}{8}$. If $V_R = V_{FS}$, $\frac{V_{FS}}{8}$. This is for d_1, d_2, d_3 is equal to 001. This is also satisfying that expression, which we have derived in the weighted resistor as $v_0 = V_{FS} \times [d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$. So, in this case, d_3 is 1 remaining to our 0s. This is 0, this is 0, this is 1.

So, we will get this one as $V_{FS} \times 2^{-3} = \frac{V_{FS}}{8} V_{FS}$. So, similarly for all other combinations also you can derive this expression. So, here for this R-2R ladder type of circuit, we will get the expression for $v_0 = V_{FS} \times [d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$, which is same as the weighted resistor R to R weighted resistor D/A converter output. So, both are giving the same outputs, but the advantage of this R-2R ladder DAC is here, we require only two different resistors.



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So, we can have here good accuracy. Still there is some drawback of this type of R-2R ladder DAC also. So, the drawback of this R-2R ladder DAC is here the currents through this resistors varies with the d_1 to d_n combination. So, as a result of that the power dissipation also will increases, power dissipation basically depend upon the change in the current due to the change in the current wherever the change in current is there you will get more power dissipation because this currents changes with the combination of this d_1, d_2, d_3 so on. So, the current also changes. So, due to the current change, you will get power dissipation, and due to power dissipation, heat will be generated that will causes the nonlinearity in the output of the R-2R ladder DAC.

To avoid this we will use inverted R-2R ladder DAC that we will discuss in the next lecture. Thank you.