Integrated Circuits and Applications
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Data Converters
Lecture – 37
Weighted Resistor D/A Converter

In the last lectures, we have discussed about various analog integrated circuits. There are some integrated circuits which consists of both analog circuits as well as digital circuits. They are called as a mixed signal circuits. So, the examples of the mixed signal circuits are analog to digital and digital to analog converters together. They are called as data converters. They are basically analog to digital converter and digital to analog converter. So, before going to discuss the various A to D and D to A converters, I will first discuss the need of this A to D and D to A converters. If you see the most of the real world signals, they are analog in nature.

Current voltage, temperature, pressure, even if you take the audio signal that is the output of the microphone, video signal that is the output of the camera. So, all these signals are analog in nature and we can process these signals in the analog domain. Processing in the sense we can amplify, we can filter, we can transmit. We have discussed about the various analog amplifiers, analog filters etc.

Then, what is the need of converting the analog signal into digital signal? There are several advantages of the digital signal processing over the analog signal processing. ASP is analog signal processing, DSP is digital signal processing. One is, digital signal processors are less sensitive to component tolerances. So, analog signal processing you can implement by using resistors, capacitors, operational amplifier, transistors etc. If you take the resistor, you might have seen that the resistor value, say *10k*, will be given some *1%* tolerance, means this value can vary from $10k\Omega \pm 1\%$.

This type of tolerances are not present in the DSP systems. The basic building blocks of the DSP system are basically adder, multiplier and then delay element. These are the three basic building blocks of DSP processor. So, they do not have any such analog components. Because of that; this will not have any changes in the component values with the temperature and the other effects. The second advantage is DSPs are more immune to noise.

Data Converters

A/D converter
D/A "

Most of real-world signals such as

- current
- Voltage
- Temperature
- Pressure
- Audio signal
- Video signal

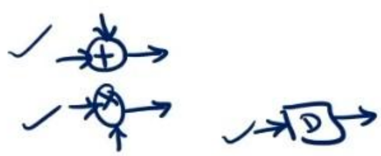
} Analog in nature

Processing
→ Amplify ✓
→ filter ✓
→ Transmit

Advantages of Digital Signal Processing (DSP) over ASP :-

(i) DSP's less sensitive to component tolerances

$R = 10k\Omega \pm 1\%$.



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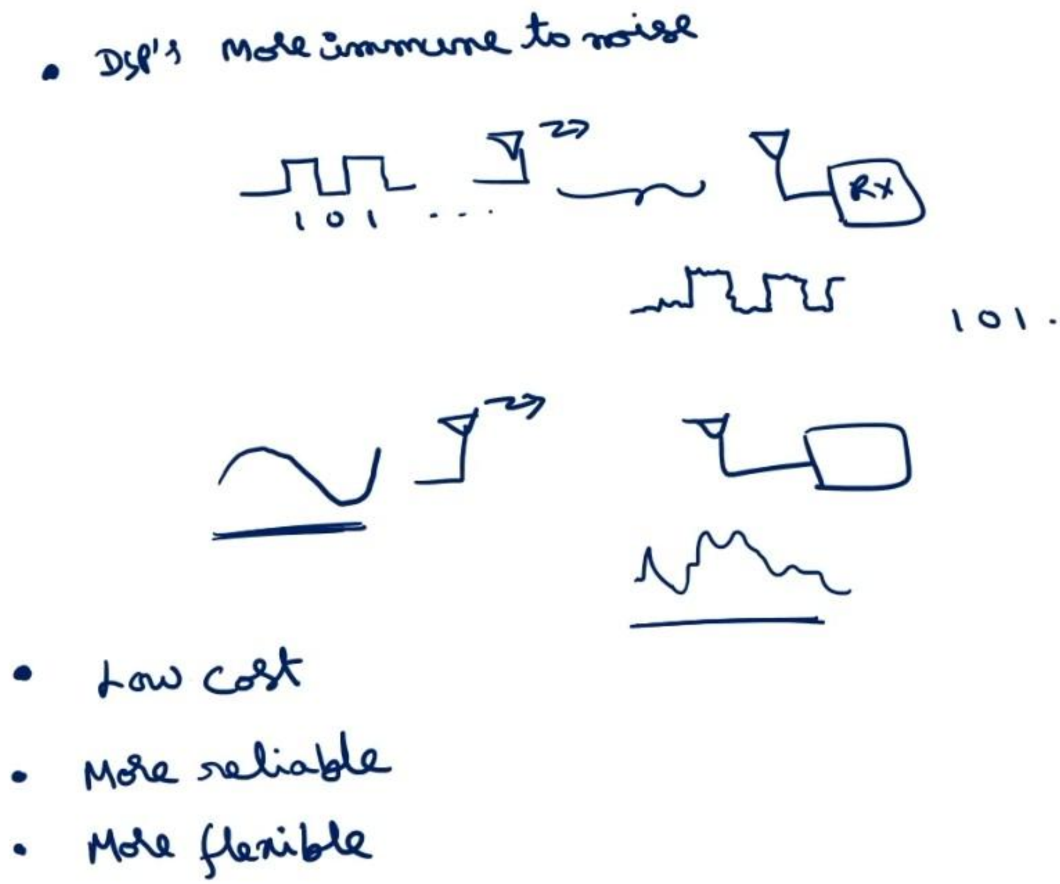
So, in digital signal processing we are going to send 1s and 0s. This is 1 0 1 so on. If you transmit this signal in the channel, if you get some any disturbance. So, at the receiver, you may receive this signal as something like this. Still, you can recover the original data bits as 1, 0, 1 etc.

Whereas, in analog signal processing if you transmit a signal like this using the transmitting antenna. If you receive a signal which is corrupted with the noise something like this, this becomes very difficult to reconstruct the original signal from this signal. So, in that way, we can call this DSP systems are more immune to noise. And DSPs can be low cost. Digital signal processing are low cost because of the advancements in the Verilog integration.

In Verilog integration so, billions of the transistors are fabricated on the single silicon chip because of that low cost. And more reliability and more flexible in the sense if you want to change the design, we can simply change the program which will be used to implement the DSP algorithms. Whereas, in analog signal processing you have to pick up the old components and you have to insert the new components. So, there are plenty of applications of the digital signal processing, because of that, most of the signal processing will be done in the digital domain. So, in order to process the signal in digital domain, but the original real world signals are in analog in nature.



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So, we need to convert the analog digital and digital to analog converter. If you take the block diagram of this digital signal processor. So, there is a plant here, I want to control the plant. I will take this various parameters of this plant such as temperature, pressure by using transducer or can also called as sensor. So, transducer or sensor is a device which converts the non electrical quantity into electrical quantity.

Here we are applying non electrical quantity such as temperature, pressure etc. Then, at the output of the transducer, we will get a proportional electrical quantity. And the electrical signal that is available at the output of the transducer is very weak of the order of millivolts or micro volts. So, you have to first amplify the signal. Then, you have to limit the bandwidth of the signal using anti-aliasing filter.

This will limit the bandwidth of signal. For example, if I take the audio signal, the audio range is $20 - 20kHz$. So, we are going to band limit the signal to maximum of $20kHz$. So, all the frequencies beyond this $20kHz$ will be eliminated. Then you have to sample at hold.

So, the signal here will be analog in nature. You have to sample according to the sampling theorem, and then you have to hold the value till the next sample is taken. Then, you have to apply to the A to D converter. Then, we will get digital signal here. This will be processed by a digital signal processor.

After processing again, this will use digital signal. This will be converted back to analog signal using D to A converter. The output of the D to A converter will be a staircase type of the waveform. So, this will be having quantization errors. To avoid this quantization errors, we will use a smoothing filter.

Then, the processed analog signal will be given to the plant. This is the overall block diagram of digital signal processing. We are taking the analog signals or the non electrical signals. We are converting first into analog signals, then we are going to sample and quantize and then convert into digital signal. We will process the signals in the digital domain.

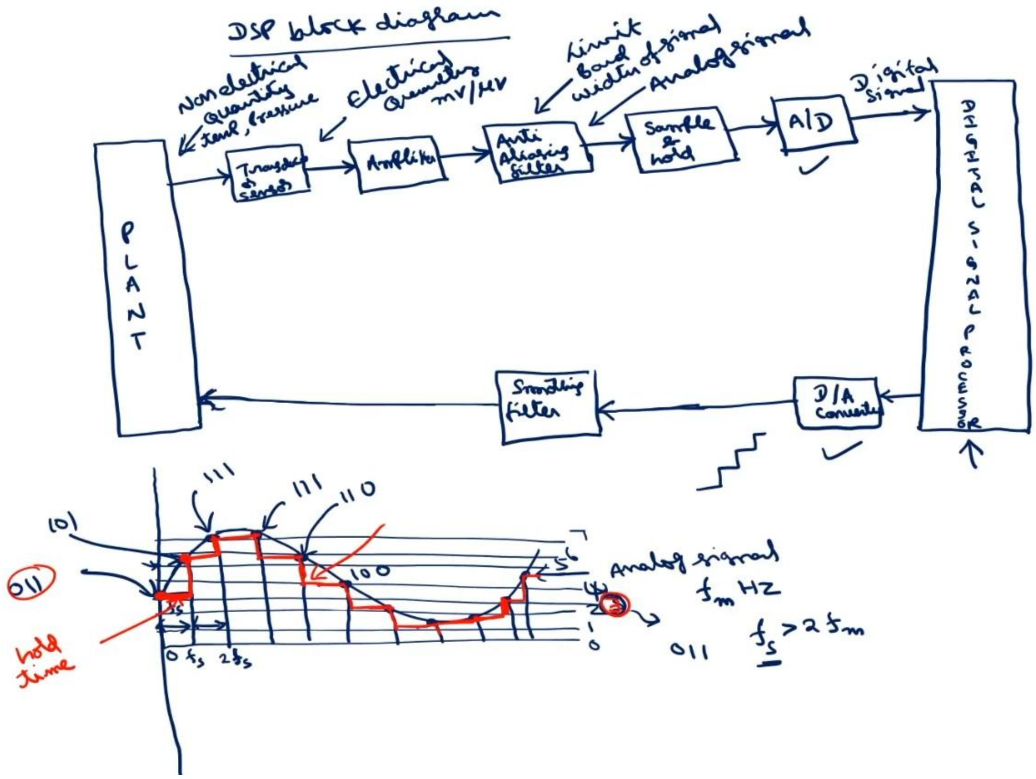
Again we will convert back into analog and the processed signal will be used to control the plant. Here, how to perform this A to D conversion, how to exactly convert this analog to digital signal conversion. For example, if this is the analog signal, if the maximum frequency say f_m Hz. Then, first, you have to sample according to the Nyquist theorem, which states that if you sample the signals at a sampling rate which is greater than twice the maximum frequency. So, the samples will completely describe the signal, the samples will describe the signal behavior and we can reconstruct the original signal from the samples taken at a rate of f_s.

Suppose if I take the first sample here, second sample here, this distance is f_s, another f_s we will take the next sample. Like that, for every f_s, we are going to take the samples.

This f_s should be at least twice the maximum frequency component, this is what is called the sampling. Even if you take the samples without this intermediate portions, this will completely describe the signal according to the Nyquist theorem. Now, what we will do is we will quantize the signal in the sense we will have some finite quantization levels. This is a quantization level 0, level 1, level 2, level 3, level 4, 5, 6, 7 say.

Then the first sample is near to this quantization level 3. So, what is the binary equivalent of 3 0 1 1. So, in order to send this sample we will send 0 1 1 and this sample is near to this fifth sample. So, this is fifth level. So, this sample will be transmitted by using 1 0 1, 1 0 1 is the binary equivalent of decimal 5.

If this sample is above the center of this two consecutive levels, we will take the upper level, if it is below we will take the lower one. If it is exactly at the center, we can take any of these levels, but in that case, the quantization error will be maximum. Now, this is near to 7. So, this will send as 1 1 1, this is also near to 7 will send as 1 1 1, this is near to 6 1 1 0, this is exactly on 4. So, 1 0 0 like that, we are going to convert this analog signal into digital signal.



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So, in the conversion of the analog, digital signal, first, you have to sample, and then we will hold this value until the next sample is taken. You can see that this is the first sample you have taken we will hold this until the next sample is taken, then we will hold this until the next sample is taken, we will hold this until the next sample is taken. This is the output of sample hold circuit, this is the output of sample hold circuit, we will sample and then hold during this hold time, we will convert this analog value into digital this analog value of say 3 into digital 0 1 1. So, the minimum amount of this hold time should be equal to the conversion time of A to D converter. The time taken to convert the digital signal is, say T seconds, then we have to hold for T seconds.

So, see how we can convert the analog into digital signal. Now, the important blocks of this analog digital converter are D to A and A to D converters. So, we will discuss the circuitry of A to D and D to A converters. First, I will consider D to A converter. There are basically three types of the D to A converters, one is called weighted resistor, another is R-2R ladder, and another is inverted R-2R ladder.

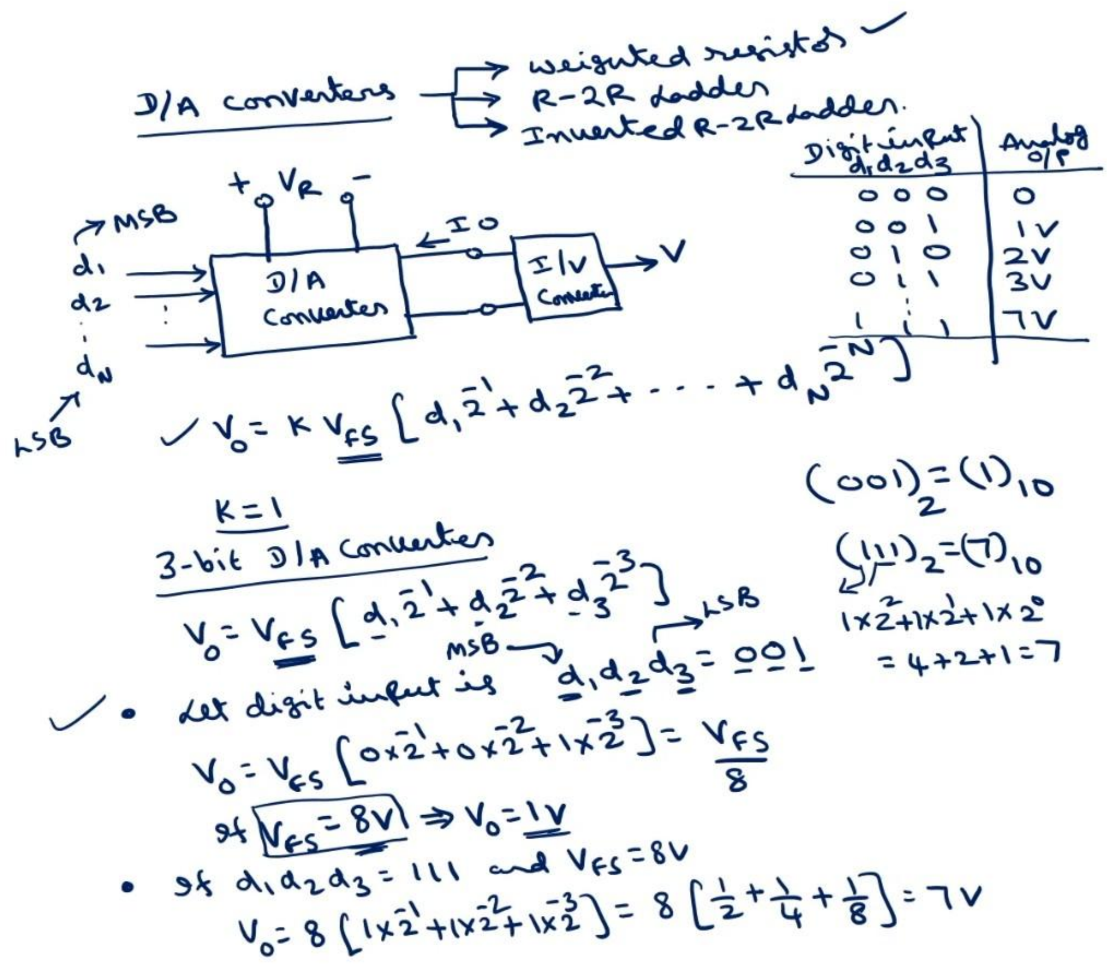
Before going to discuss this D to A converters, first I will take the general theory of D to A converter. This is a D to A converter, this will have some reference voltage V_R and then you have to take the output here, and this is the output current. So, the output of D to A converter is current; we have to connect to the voltage to current to voltage converter to convert into voltage. This will get voltage by converting this current into voltage, and of course, this current will be given to I to V converter to get the corresponding voltage. The input will be digital say n bit digital to analog converter d_1, d_2 so on up to d_n.

d_1 will take as MSB most significant bit, d_n will take as least significant bit. Then the expression for this output voltage will be in the form of some $KV_{FS}[d_1 2^{-1} + d_2 2^{-2} + \cdots + d_n 2^{-N}]$. So, normally the $K=1$. So, if you take, say, a 3-bit D to A converter and if you take $K=1$. So, $V_0 = V_{FS}[d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$.

If digital input is d_1, d_2, d_3 is equal to, say 0 0 1 this is least significant bit, this is most significant bit. Then what will be the output voltage $V_0 = V_{FS}[0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}] = \frac{V_{FS}}{8}$. If I take $V_{FS} = 8V \Rightarrow V_0 = 1V$. So, the decimal equivalent of 0 0 1 if you take the binary what is the decimal value 1 in the decimal?

If I take, say, 1 1 1 as d_1, d_2, d_3, what is the decimal equivalent is 7? How to convert the binary into a decimal? So, this 1 you have to multiply with the weight 2 bits are left 2 squares plus second 1 into to the power of 1, third 1 into to the power of 0 this is equal to 4 plus 2 plus 1 is 7. So, we can easily see that from this expression if V_{FS} is equal to 8 volts, then output will be 7 volts correspond to 0 0 1 you are getting 1 volt. If d_1, d_2, d_3 is equal to 1 1 1 and V_{FS} is equal to 8 volts V_{FS} we are fixing at $8V$. Then, what will be this one, $V_0 = V_{FS}[1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}] = 8\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right] = 7V$. So, here, if I take

V_{FS} as $8V$ for this DBT D/A converter, if you form the table digital input analog output d_1, d_2, d_3 0 0 0, you will get 0 volts only because this is 0. This is 0. This is 0. So, 0 into V_{FS} is 0 0 0 1. As you have seen here, this is 1 volt 0 1 0. You can easily see in a similar manner, 2 volts 0 1 1; you will get 3 volts. So, on up to 1 1 1, you will get $7V$. This is how we can convert this digital to analog, ok. So, whatever the decimal equivalent of this digital will be the output of D/A converter.



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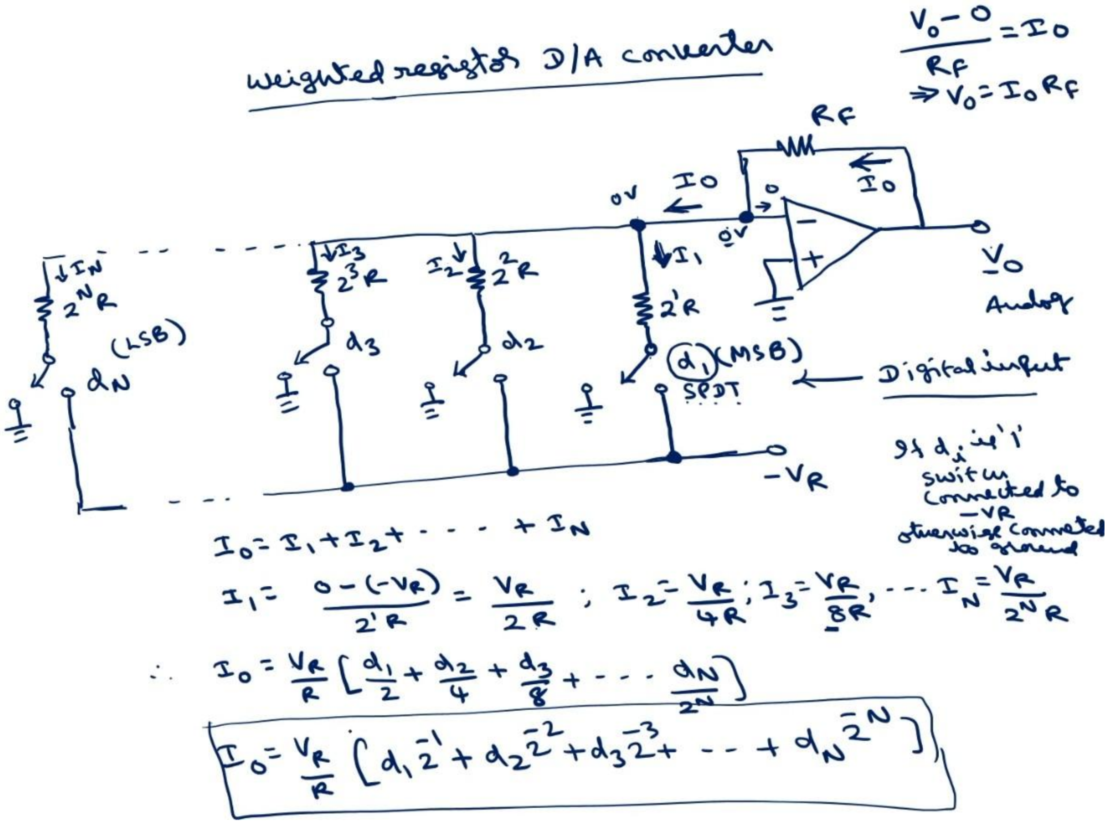
Now, how to implement this D/A converter? If I consider the first type of D/A converter weighted resistor as the name implies we are going to use the resistors which are powers of 2 whose I mean weights are powers of 2. There will be a switch which can be either connected to ground or it can be connected to minus V_R voltage. The weight of this resistor is 2^1R this will be connected to the voltage current to voltage converter with say feedback resistance R_F, this is the analog output voltage. So, if this current is I_0 here, no current flows. If assume that the half amp is ideal, the entire I_0 will flow here. Let us call this current I_1. This is d_1-bit MSB. Then, we will connect here another such type of the switch. This is grounded. This is connected to minus V_R, this is d_2, and this will be 2^2R.

We have 2^3R, this is ground this is connected to -V_R, this is I_3, and so on. For n bit D/A converter the last one is 2^nR this will be d_n^{th} bit LSB this is grounded this will be connected to V_R. Let us call this current as I_2 this current as I_3 this current as I_N. Now, you can derive the expression for the output of this D/A converter, which is analog signal. The input is here. These are the digital input, this is digital input. So, at this node, if I consider the currents leaving or this is I_1, this is I_2, this is I_3.

So, only current that is entering is I_0 the remaining all currents I_1 I_2 I_3 so on up to I_N are there. So, $I_0 = I_1 + I_2 + \cdots + I_N$. What is I_1? This is at ground potential because of the virtual ground. This is also a 0 potential, this is the direction of the current. So, this $I_1 = \frac{0-(-V_R)}{2^1R} = \frac{V_R}{2^1R}$. Similarly, $I_2 = \frac{V_R}{4R}$, $I_3 = \frac{V_R}{8R}$, … $I_N = \frac{V_R}{2^NR}$.

Therefore, what is I_0? Some of this. So, $\frac{V_R}{R}$, if we take as common here this switch we are going to use is called single pole double through switch SPDP single pole double through. So, if the bit is 1 it is connected to -V_R if d_i in general I varies from 1 to N, is 1 is logic 1 switch connects to -V_R. If bit is 0, it will be connected to ground. So, we can call this one as d_1. A switch is 1, then only this will connect to -V_R. So, to get this -V_R correspond to the I_1, $I_1 = \frac{V_R}{2R}$, $\frac{V_R}{R}$, we have taken outside.

So, this is $\frac{d_1}{2} + \frac{d_2}{4} + \frac{d_3}{8} + \cdots + \frac{d_N}{2^N}$. So, this is same as the expression that I have given $I_0 = \frac{V_R}{R}[d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3} + \cdots + d_N 2^{-N}]$. See the current expression. So, what about the output voltage? We can see that here this voltage is 0. So, V_0 minus 0 because the direction is here. So, $\frac{V_0 - 0}{R_F} = I_0 \Rightarrow V_0 = I_0 R_F$.



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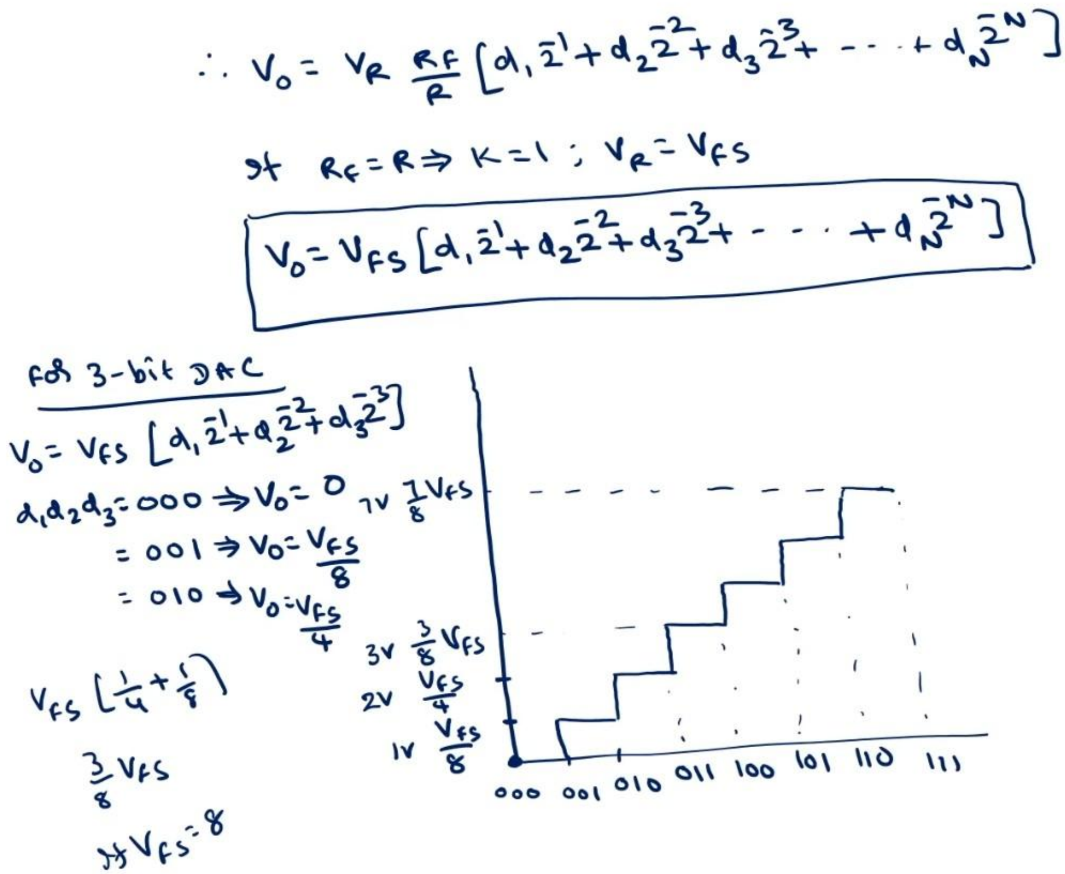
So, here, if I want V_0, I have to multiply with R_F. $\frac{V_R R_F}{R}[d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3} + \cdots + d_N 2^{-N}]$. If I choose $R_F = R \Rightarrow K = 1$, and $V_R = V_{FS}$, then this expression is exactly same as the general expression that I have given $K = 1$, V_{FS} the remaining terms are same. Then $V_0 = V_{FS}[d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3} + \cdots + d_N 2^{-N}]$. So, this is expression for the output of D to A converter. The inputs are the digital bits d_1 to d_N.

And as you have already seen here if it is 3-bit this is the expression. So, correspond to 0 0 0, you will get 0 0 0, 1 you will get 1. The same table is valid here also. So, if I take this input versus output if it is 0 0 0, voltage is 0 correspond to 0 0 1, if I take 3-bit $V_0 = V_{FS}[d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3}]$. If d_1 d_2 d_3 is 0 0 0 implies output $V_0 = 0$, it is equal to 0 0 1. You can see that this is $V_0 = \frac{V_{FS}}{8}$. 0 1 0, 0 1 0 is $V_0 = \frac{V_{FS}}{4}$. So, like that so, this will hold up to this one here this will becomes at 0 0 1, $\frac{V_{FS}}{8}$ and this will hold until the next value this will hold until the next value like that this will continue up to 1 1 1 this is 0 1 0 this is

0 1 1. This is 1 0 0, 1 0 1, 1 1 0, 1 1 1 this voltage levels are this is $\frac{V_{FS}}{4}$, where 0 1 1 this will be $V_{FS}\left[\frac{1}{4}+\frac{1}{8}\right]=\frac{3V_{FS}}{8}$, like that for the last one $\frac{7V_{FS}}{8}$.



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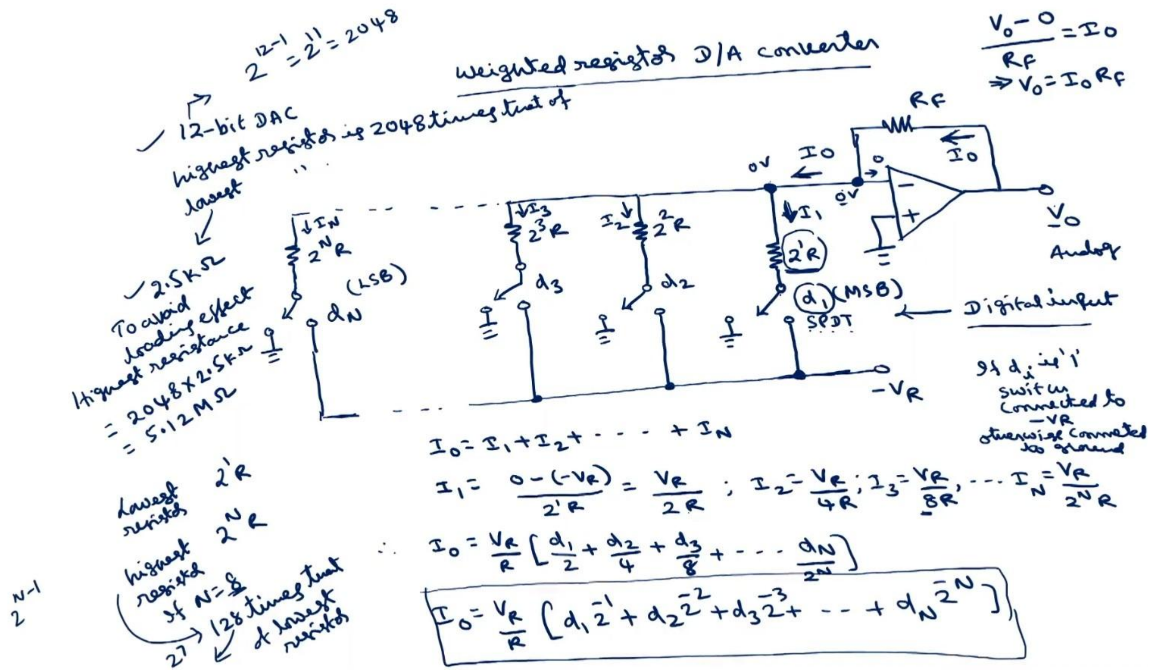
If $V_{FS}=8$ correspond to 0 0, this will be *1V*, this will be *2V*, $\frac{8}{4}=2$V, and $\frac{3}{8}\times 8 = 3V$. So, on out this will becomes *7V*, you can convert the digital into analog. So, this is the first type of D to A converter where you can implement this using weighted resistors, but there are several drawbacks of this weighted resistor due to a converter. One is we require the long range of the resistors. The lower value is 2^1R. If it is the lowest resistor highest resistor will be having value of 2^NR, if $N=8$. So, if lower resistor is *2R* highest resistor is 2^8, means highest resistor will be 128 times that of lowest resistor.

If it is 12 bit DAC highest resistor will be 2048 times this is $128=2^7$, that is $N=8$, 2^{N-1}, but 12 bit this is 2^{12-1}, this is $2^{11}=2048$, this is wide range. So, normally, we will take the lowest resistor, at least $2.5k\Omega$, because this is going to load; otherwise, the operational amplifier input impedance becomes less, thereby it causes the loading effect.

So, normally, you have take at least $2.5k\Omega$ to avoid loading effect. If I take the lowest resistance as lowest resistance as $2.5k\Omega$ for 12 bit DAC, the highest resistance will be is equal to $2048 \times 2.5k\Omega$. This comes to around $5.12M\Omega$. This is a large resistor, it becomes difficult to fabricate in the IC form this is the one of the drawback. Another is the accuracy depends upon the accuracy of the resistors. As I have told the resistor values varies with the various parameters such as temperature, and all if resistance value varies, then the accuracy will be affected.



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So, because of this region will this weighted resistor D to A converter is not popular to avoid this drawback. So, we will consider the another type of a D to A converter, such as R to R, a ladder type D to A converter, that we will discuss in the next lecture. Thank you.