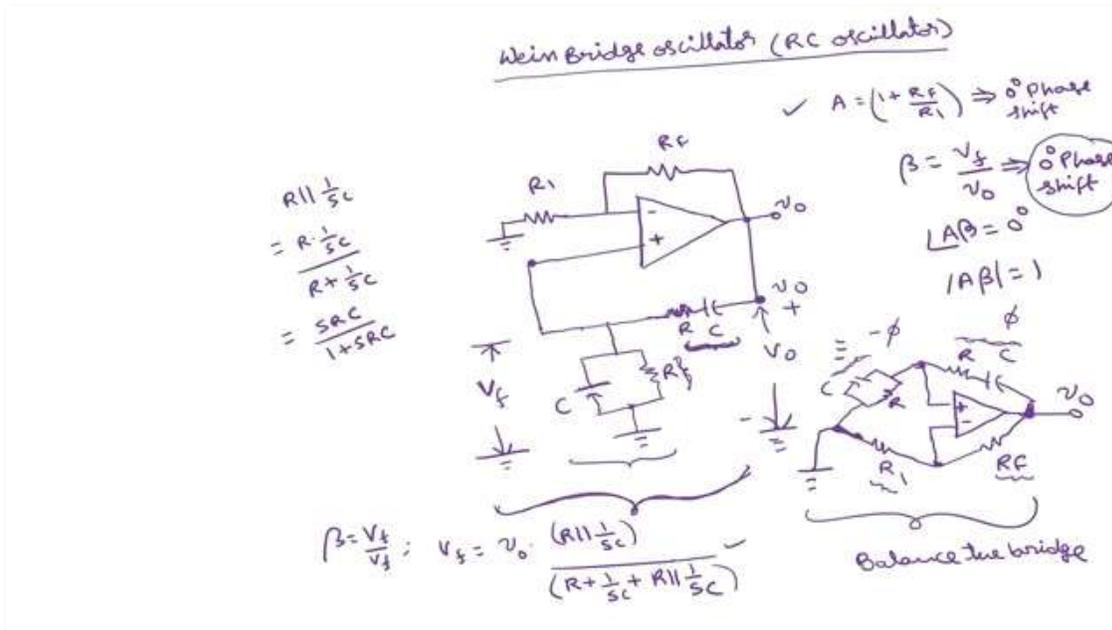


**Integrated Circuits and Applications**  
**Prof. Shaik Rafi Ahamed**  
**Department of Electronics and Electrical Engineering**  
**Indian Institute of Technology, Guwahati**

**Oscillators and Waveform Generators**  
**Lecture - 23**  
**Wien Bridge, Colpitt's and Hartley Oscillators**

So, in the last lecture I have discussed about the RC phase shift oscillator and we have derived the expression for the frequency of oscillations and the condition for the gain to sustain the oscillations. So, today we will discuss another audio frequency oscillator called as a Wien Bridge oscillator. This is also another type of RC oscillator where the feedback circuit consists of R and C elements. So, the circuit diagram of this Wien Bridge oscillator is. So, this is basically a inverting amplifier unlike the RC phase shift oscillator where the amplifier is non-inverting amplifier. So, this is non-inverting amplifier unlike the RC phase shift oscillator where the amplifier is inverting amplifier.

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This is output, this is  $R_F$ , this is  $R_1$ . Basically, the signal here will be amplified with the gain of  $1 + \frac{R_F}{R_1}$ . The gain  $A$  will be  $1 + \frac{R_F}{R_1}$ . So, this will provide 0 degree phase shift because there is no minus sign.

So, the feedback network if I assume that this output voltage here is  $v_f$ . So, this feedback factor  $\beta$  is given by the input here is  $v_o$  with respect to ground,  $\beta$  is  $\frac{v_f}{v_o}$ . So, I am going to derive the expression for this one. So, this also should provide 0 degrees phase shift. So, that overall phase angle will be 0 degrees and magnitude you have to make as unity. This is the condition

for the sustained oscillations. Now, here there is one RC section in series, another RC section in parallel here. So, we have to nullify the phase angle provided by this RC section by this parallel RC section so that the overall resultant phase angle will be 0. So, why the name Wein Bridge? So, if you want to write the equivalent circuit of this, this is equivalent to a bridge type of circuit. Here the output is taken, here we will take four arms of the bridge, this is  $R_1$ , this is  $R_F$ , this is series RC circuit, this is parallel RC circuit and this point will be grounded. Here the output  $v_o$  is taken. So, these two circuits are same this and this. We can see that at the positive terminal. So, one edge will go to series RC, another edge will go to parallel RC, this is parallel RC and from this output of the op-amp we will take the output  $v_o$  and the other end of this output will go to  $R_F$ , this is  $R_F$  whereas the negative terminal will go to  $R_1$ . So, these two are same.

So, in order to get 0 phase shift in the feedback path, you have to balance the bridge. In the sense this  $R_1$  will provide 0 phase shift,  $R_F$  also provide 0 phase shift. If this provides phase shift of 5, then this parallel RC section has to provide the phase shift of minus 5 so that these two will be get cancelled, then the bridge is said to be balanced that is why the name Wein Bridge Oscillator. So, you can take either of the circuit to derive the expression for the frequency of oscillations. So, what is the gain of this amplifier  $A$  is equal to  $1 + \frac{R_F}{R_1}$  and what is  $\beta$ ?  $\frac{v_f}{v_o}$ . So, from this circuitry what is  $v_f$  in terms of  $v_o$ ?  $v_f$  is equal to  $v_o$  into R in parallel with  $\frac{1}{sC}$  is this impedance divided by. So, this impedance will be total impedance, this will be  $R + \frac{1}{sC} + R || \frac{1}{sC}$ . This is series combination, this is parallel combination. We are taking the output voltage across the parallel combination so that  $v_o$  into parallel combination impedance divided by the total impedance, series combination impedance plus parallel combination impedance. So, what is R in parallel with  $\frac{1}{sC}$  is equal to  $\frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}}$ . So, this is equal to  $sC$ ,  $sC$  will get cancelled and then  $sRC$  in the numerator  $1 + sRC$ .

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$$v_f = v_o \cdot \frac{\left(\frac{sRC}{1+sRC}\right)}{\frac{(1+sRC)}{sC} + \frac{sRC}{1+sRC}} = v_o \cdot \frac{\left(\frac{sRC}{1+sRC}\right) \frac{sC(1+sRC)}{(1+sRC)^2 + sRC^2}}{\frac{(1+sRC)}{sC} + \frac{sRC}{1+sRC}}$$

$$\Rightarrow v_f = v_o \cdot \frac{s^2 RC^2}{s^2 R^2 C^2 + 2sRC + 1 + s^2 RC^2} \quad s^2 = -\omega^2$$

$$\therefore \beta = \frac{v_f}{v_o} = \frac{s^2 RC^2}{s^2 R^2 C^2 + 2sRC + 1 + s^2 RC^2}$$

$$s = j\omega$$

$$\beta = \frac{-\omega^2 RC^2}{-\omega^2 R^2 C^2 + 2j\omega RC + 1 - \omega^2 RC^2}$$

So, if I substitute this here what will be  $v_f$ ?  $v_f$  is equal to  $v_o$  into the parallel combination of  $R$  with  $\frac{1}{sC}$  is  $\frac{sRC}{1+sRC}$ . This is  $\frac{sRC}{1+sRC}$  divided by  $R + \frac{1}{sC}$  series combination plus parallel combination is  $\frac{sRC}{1+sRC}$ . Again, here this will become  $sRC$  whole divided by  $sC$ , this can be simplified as  $\frac{1+s}{sC}$ . Now, again here the LCM is  $sC(1 + sRC)$ .

So, this will be equal to  $v_o \left( \frac{sRC}{1+sRC} \right)$  is the numerator, the numerator will be. So, this will be reverse  $s$ ,  $sC(1 + sRC)$  will be in the numerator, denominator will be  $(1 + sRC)^2 + s^2RC^2$ . So, if you further simplify. So, this  $(1 + sRC)$ ,  $(1 + sRC)$  will get cancelled and the numerator will be  $v_f$  is equal to  $v_o s^2 RC^2$  divided by  $s^2 R^2 C^2 + 2sRC + 1$  which is  $(sRC + 1)^2 + s^2 RC^2$ , okay is it correct. So,  $v_o \left( \frac{sRC}{1+sRC} \right)$  is the numerator, denominator will be  $\frac{sC(1+sRC)}{(1+sRC)^2 + s^2 RC^2}$ .

So, therefore,  $\beta = \frac{v_f}{v_o}$ , this is equal to  $\frac{s^2 RC^2}{s^2 R^2 C^2 + 2sRC + 1 + s^2 RC^2}$ . So, in order to find out the frequency of oscillations  $s$  should be  $j\omega$ . So, if I substitute  $s = j\omega$  here,  $\beta$  becomes  $s^2$  becomes  $(j\omega)^2$  which is equal to  $-\omega^2$ ,  $\frac{-\omega^2 RC^2}{-\omega^2 R^2 C^2 + 2j\omega RC + 1 - \omega^2 RC^2}$ . Now, this  $R$  parallel with  $\frac{1}{sC}$  will be  $R$  in parallel with  $\frac{1}{sC}$  will be  $\frac{R \frac{1}{sC}}{R + \frac{1}{sC}}$ . So, which is equal to  $sC$  will get cancel. So, we will get  $\frac{R}{1+sRC}$ . So, if I substitute this value here and here and if you simplify this expression for  $v_f$ .

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$$v_f = v_o \cdot \frac{R}{1+sRC} = v_o \cdot \frac{\left( \frac{R}{1+sRC} \right)}{\frac{R + \frac{1}{sC} + \frac{R}{1+sRC}}{1}}$$

$$= v_o \cdot \frac{R}{(1+sRC)} \cdot \frac{sC(1+sRC)}{sRC + s^2 R^2 C^2 + 1 + sRC + sRC}$$

$$= v_o \cdot \frac{sRC}{s^2 R^2 C^2 + 3sRC + 1}$$

$$\Rightarrow \beta = \frac{v_f}{v_o} = \frac{sRC}{s^2 R^2 C^2 + 3sRC + 1}$$

For oscillation,  $\beta$  should be real  
 $\Rightarrow \omega^2 R^2 C^2 = 1$   
 $\Rightarrow$  frequency of oscillations  $\omega_0^2 = \frac{1}{R^2 C^2}$

Replace  $s = j\omega$   
 $s^2 = -\omega^2$   

$$\beta = \frac{j\omega RC}{-\omega^2 R^2 C^2 + 3j\omega RC + 1}$$

$$= \frac{\omega RC}{-j \{-\omega^2 R^2 C^2 + 3j\omega RC + 1\}}$$

$$= \frac{\omega RC}{j\omega^2 R^2 C^2 + 3\omega RC - j}$$

$$= \frac{\omega RC}{3\omega RC + j(\omega^2 R^2 C^2 - 1)}$$

So, what will be this  $v_f$  now?  $v_f$  is equal to  $\frac{R}{1+sRC}$ ,  $\frac{R \frac{1}{sC}}{R + \frac{1}{sC} + \frac{R}{1+sRC}}$ . So, if I take again LCM as  $(1 + sRC)sC$  what will be this? This is equal to  $v_o \left( \frac{R}{1+sRC} \right)$  divided by here the denominator is  $sC(1 + sRC)$  is LCM. So, this will be  $R$  into  $R$  by  $1$ .

So, we will get both the terms  $sC(1 + sRC)$  plus 1 here  $sC$  is there. So, it will get  $1 + sRC$   $1(1 + sRC)$  plus R,  $(1 + sRC)$  is there. So, we will get  $sC$ . So, this will be  $v_o(\frac{R}{1+sRC})$  and if we take this into numerator this will go to numerator this will become the denominator. So,  $sC(1 + sRC)$  divided by.

So, what is the denominator? this is  $sRC$  into 1 is  $sRC$  plus  $sRC$  into  $sRC$  is  $s^2R^2C^2 + 1 + sRC + sRC$ . So, this  $(1 + sRC)$   $(1 + sRC)$  will get cancelled this is equal to  $v_o sRC$  divided by total  $3sRC s^2R^2C^2 + 3sRC + 1$ . So, implies what is  $\beta$ ?  $\frac{v_f}{v_o}$  this is equal to  $\frac{sRC}{s^2R^2C^2 + 3sRC + 1}$ .

This is expression for the  $\beta$ . Now, in order to derive the expression for the frequency of oscillations. So, we have to make the imaginary part of  $\beta$  equal to 0 similar to the analysis that we have done in RC phase shift oscillator.

So, in order to obtain this real and imaginary part we will replace  $s$  with  $j\omega$ . So, what happens to  $s^2$ ?  $j^2\omega^2$  which is just simply  $-\omega^2$ . So, if I substitute this here. So, what will be  $\beta$ ?  $\frac{j\omega RC}{-\omega^2R^2C^2 + 3j\omega RC + 1}$ . So, this  $j$  if I take to the denominator becomes  $-j$ .

So, this is equal to  $\omega RC$  divided by  $-j\{-\omega^2R^2C^2 + 3j\omega RC + 1\}$ . Or this is equal to  $\omega RC$  divided by this  $-j$  into minus becomes  $+j j\omega^2R^2C^2$  and this  $-j$  into  $+j$  becomes  $-j^2$  which is equal to  $+1 + 3\omega RC$  plus this becomes  $-j$ . So, what is the imaginary part? and what is the real part?  $\omega RC$  divided by real part is  $3\omega RC$  plus imaginary part is  $j$  if I take as common  $(\omega^2R^2C^2 - 1)$ . For sustained oscillations what is the condition?  $A$  is real. So,  $\beta$  also should be real implies imaginary part is 0 means  $\omega^2R^2C^2 = 1$  implies what is the frequency of oscillations?  $\omega_o^2 = \frac{1}{R^2C^2}$ .

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The image shows a handwritten derivation of the frequency and gain conditions for a Wein Bridge oscillator. The steps are as follows:

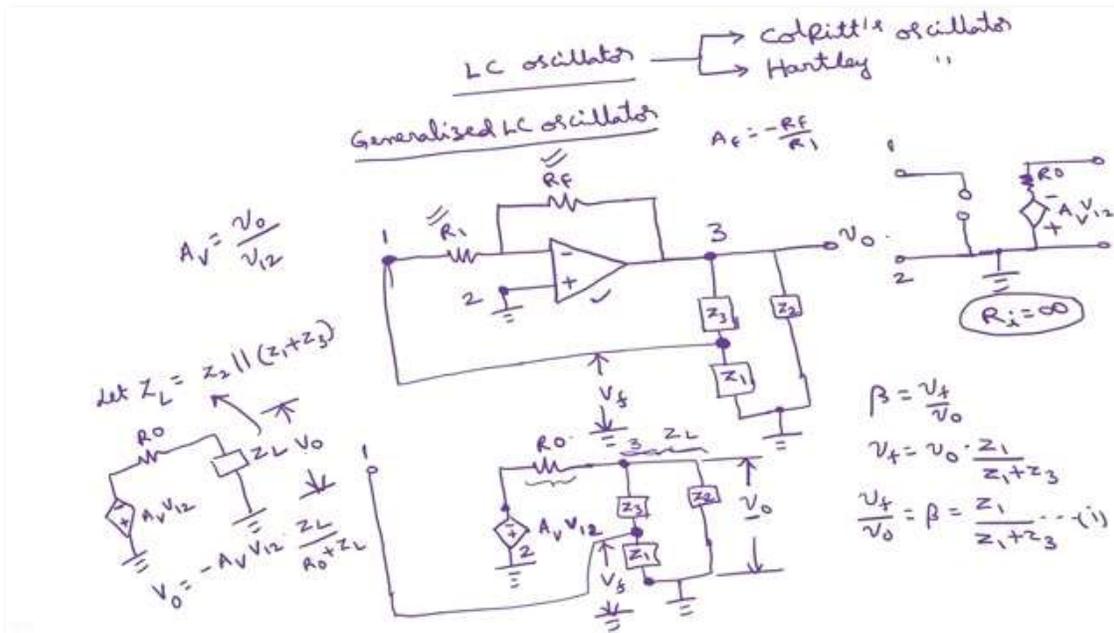
- $\omega_o = \frac{1}{RC} \Rightarrow f_o = \frac{1}{2\pi RC}$  (boxed)
- $\Rightarrow \omega RC = 1$
- $\Rightarrow \beta = \frac{1}{3}$
- $|A\beta| \geq 1 \Rightarrow |A| \geq 3$
- $A = 1 + \frac{R_F}{R_1} \geq 3 \Rightarrow \frac{R_F}{R_1} \geq 2 \Rightarrow R_F \geq 2R_1$  (boxed)

Or  $\omega_o = \frac{1}{RC}$  or  $f_o = \frac{1}{2\pi RC}$ . This is the expression for the frequency of oscillations of Wein Bridge oscillator. So, at this frequency of oscillations what will be the value of  $\beta$ ?  $\beta$  is this. So,

this becomes 0 and this  $\omega RC$  becomes 1 from this  $\omega RC$  is 1. So, if I substitute that so, this  $\omega RC$  is 1. So, what happens to  $\beta$  this is 1 this is 1 and this is 0.

So,  $\frac{1}{3} \times 1$  is  $\frac{1}{3}$ . So, implies  $\beta = \frac{1}{3}$ , but we know that for sustained oscillation  $|A\beta| \geq 1$  implies  $|A|$  should be  $\geq 3$ . So, what is  $A$ ? is equal to  $1 + \frac{R_F}{R_1}$  should be  $\geq 3$  implies  $\frac{R_F}{R_1}$  should be  $\geq 2$   $3 - 1$  is 3 minus this 1 implies  $R_F \geq 2R_1$ . So, these are the two design expressions to design the Wein Bridge oscillator. This is second type of RC oscillator. So, these RC oscillators are suitable for audio frequency applications. So, for radio frequency applications we have to go for the LC oscillators.

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There are two types of LC oscillators similar to the RC oscillator one is called Colpitts oscillator and the second one is called Hartley oscillator. So, before going for this Colpitts and Hartley oscillator first I will derive the expression for the generalized LC oscillator. Then I will substitute the values of this impedances in case of a Colpitts oscillator in case of Hartley oscillator okay. First, I will take the generalized LC oscillator. So, the circuit diagram of a generalized LC oscillator is this is also type of inverting amplifier. So, the feedback path here will be consisting of three impedances. Later I will replace these impedances with L and C elements so that we can derive the Colpitts oscillator. So, here the output  $v_o$  is taken and this is  $v_f$  a part of this output will be fed back to the input and this we will call as  $Z_1, Z_2, Z_3$  this is  $R_1, R_F$ .

So, the gain of this amplifier is  $R_F$  is equal to  $-\frac{R_F}{R_1}$  into of course, this voltage okay. This is nothing, but  $v_f$ . So, in order to derive the expression for the frequency of oscillations first I will draw the equivalent circuit. So, in the earlier lectures we have discussed the internal circuitry of the operational amplifier okay. So, that is nothing, but here we will be having input

resistance, then there will be a output resistance, there will be a voltage source this will be grounded which is minus plus.

If I call this as terminal 1, terminal 2 this is  $R_i$ , this is  $R_o$ ,  $A_v$  is the say voltage gain into  $v_{12}$  this is the output voltage. Now, here if I assume that the operational amplifier is ideal  $R_i$  will be infinite. And in the equivalent circuit for the sake of simplicity I will just not show this  $R_1$  and  $R_F$ . So, what will be the equivalent circuit of this operational amplifier we are going to replace with this equivalent circuit with  $R_i$  is equal to infinite means this will acts as a open circuit this will acts as a open circuit here. So, the equivalent circuit will be now if I call this as a terminal 1 and this as terminal 2, this as terminal 3.

So, terminal 1 this is connected to the ground, this is connected to this point, then there will be no connection between this terminal 1 and this point except for that here this ground. So, there will be a voltage source with ground, this will be your terminal 2, this terminal 2, this is minus plus, this is  $A_v$  times  $v_{12}$  and this is output resistance, this output resistance  $R_o$  and this will be connected to this point, this is terminal point 3. So, where this is connected to  $Z_3, Z_1$  and then  $Z_2$ . So, this overall this one will be grounded and here of course, we will take the output.

This is  $Z_2, Z_1, Z_3$  this is  $R_o$ , this is terminal 1 and this point will be connected to here, see here this one will be connected to here, so a central point between  $Z_1$  and  $Z_3$ . So, this is the equivalent circuit okay. Now, this will be your  $v_f$  with respect to the ground, this will be  $v_o$ . So,  $\beta$  is nothing but  $\frac{v_f}{v_o}$ . So, what is  $v_f$  in terms of  $v_o$ ?  $v_o$  into this is  $v_o$  between this point and this point, there is a voltage divider.

So, we are taking  $v_f$  across  $Z_1$ . So,  $\frac{Z_1}{Z_1+Z_3}$  implies  $\frac{v_f}{v_o}$  is equal to  $\beta$ , this is equal to  $\frac{Z_1}{Z_1+Z_3}$ , this is one expression. Then what about this output voltage  $v_o$ ? So, the overall gain will be here, this is  $v_o$  and this is  $v_{12}$ . So, overall gain  $A_v$  is given by  $\frac{v_o}{v_{12}}$ . So, in terms of  $v_{12}$  what is  $v_o$  first of all? So, this  $v_o$  in terms of this  $v_{12}$ . So, you can see that this voltage  $A_v$  into  $v_{12}$  is the total voltage between this point to ground, this ground also of course.

And there is a resistance here and there will be parallel combination of this and we are taking this output across this parallel combination. So, let this parallel combination as  $Z_L, Z_L$  is equal to  $Z_2 \parallel (Z_1 + Z_3)$ , this entire circuitry this you are calling as  $Z_L$ . Then what will be equivalent circuit?

So, here this is voltage source minus plus this is grounded, this is  $A_v$  into  $v_{12}$ ,  $A_v$  into  $v_{12}$  and this is  $R_o$ . Now, this will be total  $Z_L$ , this is also grounded, this  $Z_L$  is this, this is  $R_o$  and here we are taking output  $v_o$ . So, what is  $v_o$  is equal to this is minus sign because plus is grounded. So, this is of opposite polarity minus  $A_v$  into  $v_{12}$  and we are taking the output across  $Z_L$  means  $\frac{Z_L}{R_o+Z_L}$ .

So, this is if I substitute this  $Z_L$  also here, A becomes this A if I call this overall gain as A, this is  $v_o$  is the output voltage  $v_{12}$  is the input voltage. So,  $\frac{v_o}{v_{12}}$  from here  $\frac{v_o}{v_{12}}$  will be your A that is equal to  $-A_v$  times this  $\frac{v_o}{v_{12}}$  is equal to  $-A_v \frac{Z_L}{R_o+Z_L}$ . Here  $Z_L$  is  $Z_2 \parallel (Z_1 + Z_3)$ , this is equal to  $\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}$ . If I substitute this here implies what is A?  $-A_v$  times this is  $Z_2, Z_L$  is  $Z_2, Z_2(Z_1 + Z_3)$

divided by  $Z_1 + Z_2 + Z_3$  is the numerator is  $Z_L$  whole divided by  $R_o$  plus again  $Z_L$  is same thing  $Z_2(Z_1 + Z_3)$  divided by  $Z_1 + Z_2 + Z_3$ .

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So, if I take  $Z_1 + Z_2 + Z_3$  as LCM in the denominator, so that will cancel with this one. So, you will get this as  $\frac{-A_v Z_2 (Z_1 + Z_3)}{R_o (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$ . This is your  $A$  expression and  $\beta$  we have derived as  $\frac{Z_1}{Z_1 + Z_3}$ . So,  $A\beta$  is equal to 1 for oscillation the condition is. So, implies  $A$  is  $\frac{-A_v Z_2 (Z_1 + Z_3)}{R_o (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$  into  $\beta$  is  $\frac{Z_1}{Z_1 + Z_3}$  this should be equal to 1.

So, this get cancelled. So, you will get  $A\beta$  is equal to now  $\frac{-A_v Z_1 Z_2}{R_o (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$ . Here  $Z_1, Z_2, Z_3$  are the impedances okay. So, this is of the form of  $Z = j\omega L$  or  $\frac{1}{j\omega C}$ . So, what I will do is I will call this  $Z_1$  as  $jX_1, Z_2$  as  $jX_2, Z_3$  as  $jX_3$  where  $X$  can be  $\omega L$  or  $\frac{-1}{\omega C}$  because  $j$  we have taken the numerator this will equal  $\frac{-j}{\omega C}$ . So,  $j$  if I take as common it will be 1 by  $-1$  by  $\omega C$ . So, the  $X$  we are going to substitute later either  $\omega L$  or  $\frac{-1}{\omega C}$  depends upon the type of impedance that we are going to obtain.

Now, by substituting these values here what is  $A\beta$ ? Therefore,  $A\beta$  is equal to  $-A_v$  times  $Z_1 Z_2$  means  $(jX_1)(jX_2)$  divided by  $R_o$  into  $X_1 + X_2 + X_3$  means  $j$  if I take as common and this will be  $X_1 + X_2 + X_3$  plus we have  $Z_2(Z_1 + Z_3)$ .  $Z_2$  is  $jX_2$ ,  $(Z_1 + Z_3)$  is  $j$  also common here. So, this becomes  $j^2 X_1 + X_3$ . So, this  $j^2$  is  $-1$  and this  $j$  into  $j$  is also  $-1$ .

So, this minus minus becomes plus. So, this will be now  $A_v X_1 X_2$  divided by what is the real part and what is imaginary part  $-X_2(X_1 + X_3)$  is real part  $+jR_o(X_1 + X_2 + X_3)$  is imaginary part. But for the oscillations  $A\beta$  should be real implies imaginary part is 0. This is 0 means  $(X_1 + X_2 + X_3) = 0$ . This is the condition for the oscillations.

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$$\therefore A\beta = \frac{-A_v (jx_1)(jx_2)}{jR_0(x_1+x_2+x_3) + j^2x_2(x_1+x_3)}$$

$\hookrightarrow -1$

$$= \frac{A_v x_1 x_2}{-x_2(x_1+x_3) + jR_0(x_1+x_2+x_3)}$$

for oscillations,  $A\beta$  should be real  $\Rightarrow$   $x_1+x_2+x_3=0$   
 $x_1+x_3=-x_2$

$$A\beta = \frac{A_v x_1 \cancel{x_2}}{\cancel{x_2}(-x_2)} = -A_v \frac{x_1}{x_2}$$

$$|A\beta| \geq 1 \Rightarrow \left| A_v \frac{x_1}{x_2} \right| \geq 1$$

$$\Rightarrow \frac{x_1}{x_2} \geq 1$$

If  $X_1 + X_2 + X_3$  is 0 what happens to  $A\beta$ ? This term becomes 0 and what is  $X_1 + X_3$  from this  $X_1 + X_3$  will be  $-X_2$ .

So, if I substitute this  $-X_2$  minus minus becomes plus. So,  $A\beta$  is equal to  $A_v X_1 X_2$  divided by  $X_2$ ,  $-X_2$  into  $-X_2$  becomes  $+X_2$ . This  $X_2 X_2$  will get cancelled will get this is equal to  $A_v \frac{X_1}{X_2}$ .

Then  $|A\beta| \geq 1$  implies  $A_v$  into  $|A_v \frac{X_1}{X_2}|$  should be greater than or equal to 1 implies  $\frac{X_1}{X_2}$  should be greater than or equal to 1.

These are the conditions for the oscillations. Now, we will substitute here this  $Z_1, Z_2, Z_3$  with either impedances or capacitances. Thereby, we will get this Colpitts and Hartley oscillator okay.

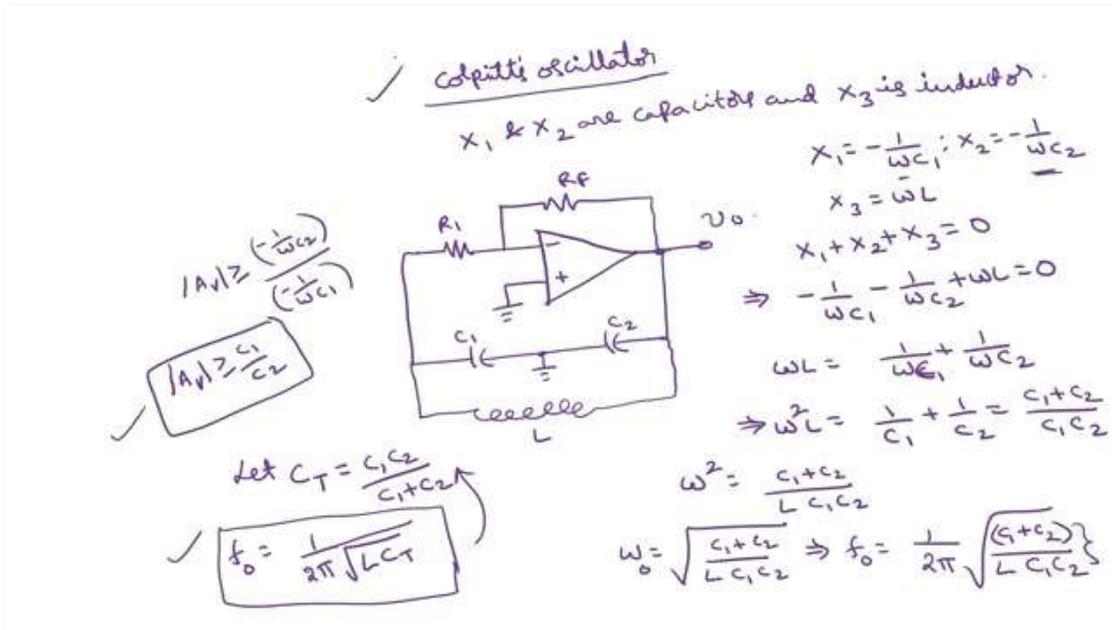
So, Colpitts oscillator first. Colpitts oscillator can be obtained by a replacing  $X_1, X_2$  with capacitor and  $X_3$  is inductor. So, the corresponding circuitry will be this. Here the output is taken, this is grounded. Here in the feedback path, you have two capacitances and this point is grounded and there will be an inductance. This is the feedback network  $L C C R_1, R_F$ . So,  $X_1$  will be  $\frac{-1}{\omega C}$ . If I call this as  $C_1$  and  $C_2$  for example, this  $X_1$  is  $\frac{-1}{\omega C_1}$ ,  $X_2$  is  $\frac{-1}{\omega C_2}$  and  $X_3$  is  $\omega L$ .

If I substitute this, what is the condition for the oscillations?  $X_1 + X_2 + X_3$  should be 0 implies  $\frac{-1}{\omega C_1} + \frac{-1}{\omega C_2} + \omega L = 0$ . So, what is  $\omega L$ ?  $\frac{1}{\omega C_1} + \frac{1}{\omega C_2}$ . This  $\omega$  you can take as common if you take to the other side implies  $\omega^2 L = \frac{1}{C_1} + \frac{1}{C_2}$ .

This is equal to  $\frac{C_1+C_2}{C_1C_2}$  or  $\omega^2$  is equal to  $\frac{1}{L}$  times  $\frac{C_1+C_2}{LC_1C_2}$  or what is  $\omega$ ?  $\sqrt{\frac{C_1+C_2}{LC_1C_2}}$ . This we call as frequency of oscillations implies  $f_o$  is equal to  $\frac{1}{2\pi}$  square root of  $\frac{1}{2\pi} \sqrt{\frac{C_1+C_2}{LC_1C_2}}$ .

If I define the equivalent capacitance as let  $C_T$  total capacitance is  $\frac{C_1C_2}{C_1+C_2}$ . So, then this  $f_o$  becomes  $\frac{1}{2\pi\sqrt{LC_T}}$ . So, this becomes this becomes  $\frac{1}{C_T}$ . So, here  $C_T$  is this.

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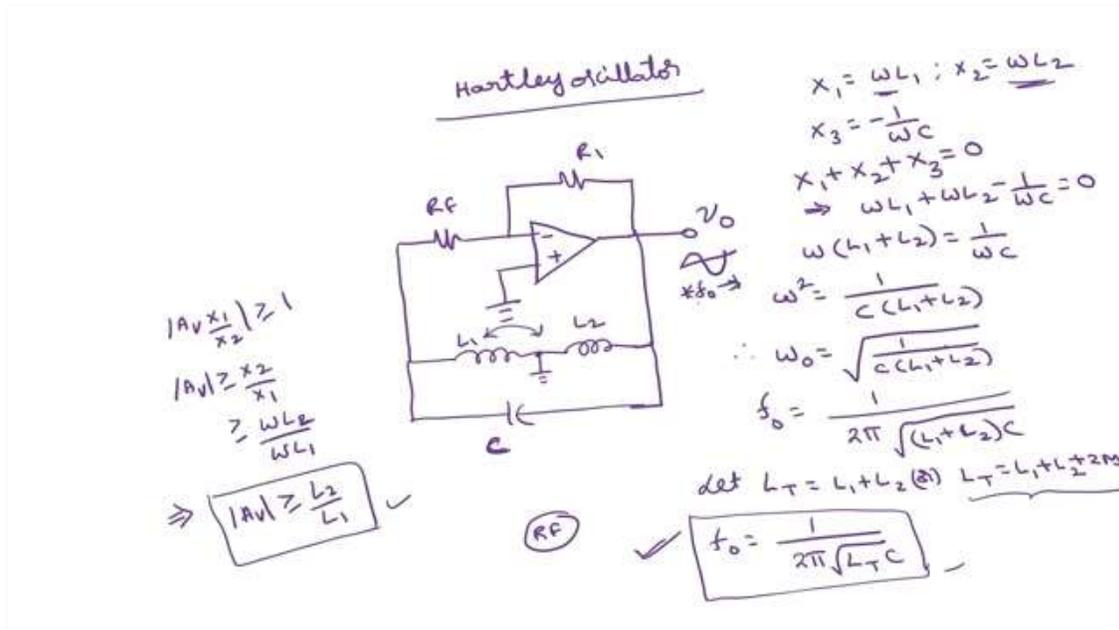
This is the expression for the frequency of oscillations. Now, what will be the condition on the gain? So, here  $|A_v|$  should be greater than or equal to  $\frac{X_2}{X_1}$ . So, here this expression should be  $|A_v|$  should be greater than or equal to  $\frac{X_2}{X_1}$ . What is  $X_2$ ? What is  $X_1$ ?  $X_2$  is this  $X_1$  is this. So, should be greater than or equal to  $\frac{(-1/\omega C_2)}{(-1/\omega C_1)}$ .

So, minus minus get cancelled omega get cancelled. So,  $|A_v|$  should be greater than or equal to  $\frac{C_1}{C_2}$  this is another condition. So, subjected to this condition the frequency of oscillations is  $\frac{1}{2\pi\sqrt{LC_T}}$ . So, this is the frequency of oscillations of Colpitts oscillator.

And the other type of LC oscillator is called Hartley oscillator. Here this is reverse  $X_1, X_2$  is inductors and  $X_3$  is capacitance. This is the circuit diagram of Hartley oscillator. This we call as  $L_1, L_2, C$ . So, what is  $X_1$  is  $\omega L_1$ ,  $X_2$  is  $\omega L_2$ ,  $X_3$  is  $\frac{-1}{\omega C}$ . And the condition for the oscillations which we have derived is  $X_1 + X_2 + X_3$  should be 0 implies  $\omega L_1 + \omega L_2 - \frac{1}{\omega C}$  should be 0 or  $\omega(L_1 + L_2) = \frac{1}{\omega C}$  or  $\omega^2 = \frac{1}{C(L_1+L_2)}$ .

So, therefore, frequency of oscillations  $\omega_o$  is given by  $\sqrt{\frac{1}{C(L_1+L_2)}}$  or  $f_o = \frac{1}{2\pi\sqrt{(L_1+L_2)C}}$ .

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Let  $L_T = L_1 + L_2$ . If I assume that there is no mutual inductance between  $L_1$  and  $L_2$  or if I assume that mutual inductance also there  $L_T = L_1 + L_2 + 2M$ ,  $M$  is mutual inductance. Then what will be frequency of oscillations? is  $\frac{1}{2\pi\sqrt{L_T C}}$ . Here  $L_T$  depends upon the mutual inductance  $L_1 + L_2$  or  $L_1 + L_2 + 2M$ . This is the total inductance of two inductors connected in series with mutual inductance of  $F$ . And what is the another condition  $\left|A_v \frac{X_1}{X_2}\right|$  should be  $\geq 1$  or  $|A_v|$  should be  $\geq \frac{X_2}{X_1}$ .

This is greater than or equal to  $X_2$  is  $\omega L_1$ ,  $X_1$  is  $\omega L_1$ ,  $X_2$  is  $\omega L_2$ ,  $\frac{\omega L_2}{\omega L_1}$ ,  $\omega$   $\omega$  get cancelled. So,  $|A_v|$  should be greater than or equal to  $\frac{L_2}{L_1}$ . So, with this condition we will get the frequency of oscillations as  $\frac{1}{2\pi}$  the resultant inductance times the capacitance. So, here the two types of LC oscillators. So, normally this will be used to design RF frequencies of the order of mega Hertz.

So, this is all about these sinusoidal oscillators which can generate the sinusoidal signal whose frequency is this. We can generate the rectangular waveform triangular waveform also that we will discuss in the next lecture. Thank you.