Integrated Circuits and Applications
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Oscillators and Waveform Generators
Lecture - 22
RC Phase Shift Oscillator

So, the next application of the operational amplifier is oscillator. So, oscillator is a feedback amplifier in which a part of output is fed back to the input. So, this is a type of feedback amplifier in which a part of output is fed back to input. So, the type of feedback that we are going to use here is positive feedback. You might have studied this negative feedback circuits in your analog circuits course whereas, in oscillators the type of feedback is positive. Here if I properly choose the output signal which is going to be fed back to the input with proper magnitude and proper phase, we can generate the different types of the waveforms.

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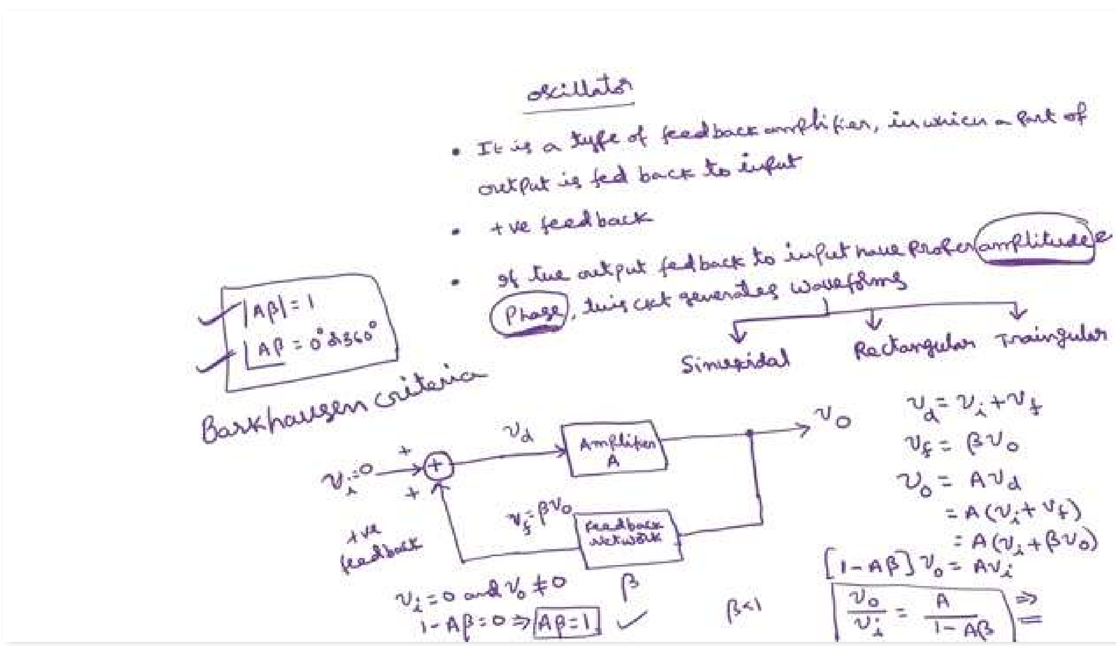

If the output fed back to input have proper amplitude and phase, then this circuit generates waveforms. So, what type of waveforms is basically? It can be sinusoidal waveform, it can be rectangular waveform or it can be triangular waveform. Initially, I will discuss about the sinusoidal oscillators, later I will discuss about the rectangular and triangular waveform generators also. Now, here we are going to derive the condition on this output amplitude and output phase.

So, what is the condition? So, that this feedback amplifier circuit will generate the oscillations or generate the waveforms. So, for that I will consider the block diagram of this oscillator as there is a amplifier circuit in the feed forward path say the gain of amplifier is A. So, the output is a v_o, input is a v_d, then a part of output will be fed back to the input via feedback network. This is feedback network, whose feedback factor is say β. So, the output of this feedback network is if I call this one as $v_f = \beta v_o$ where $\beta < 1$.

Then there will be some input v_i and then this feedback signal will be added with the input that is why the type of feedback is called as positive feedback. So, you see the circuit diagram later we are going to derive the expression for these conditions on amplitude and phase by putting $v_i = 0$. One important point for the oscillator is oscillator does not contain any input unlike any other amplifiers. So, what are the three relations that we have to derive the condition for this amplitude and phase? So, one expression is $v_d = v_i + v_f$, but what is v_f? βv_o and what is v_o? Av_d. So, this is equal to $A(v_d) = A(v_i + v_f)$ and what is v_f? $A(v_i + v_f) = A(v_i + \beta v_o)$.

If we take this v_o terms to one side and v_i terms to other side, $[1 - A\beta]v_o = Av_i$ or what is $\frac{v_o}{v_i}$, the transfer function of this system? is equal to $\frac{A}{1-A\beta}$. This is the transfer function of this circuit. Now, here we are going to make this v_i as 0. This is one of the important property of the oscillator is that the input is 0. So, from this relation $v_i = 0$ and $v_o \neq 0$.

So, this will satisfy from here by making this $1 - A\beta = 0 \Rightarrow A\beta = 1$. So, what is the magnitude of $A\beta$? Is equal to unity and what will be phase angle of $A\beta$? This should be either 0 degrees or 360 degrees. Then only this condition $A\beta = 1$ will be satisfied. So, these are the two conditions one on the magnitude another on the phase. As I have defined this oscillator as the amount of the output that we are going to fed back is having some proper amplitude and proper phase. So, what is that proper amplitude is this proper phase is this and this particular criteria is called as Barkhausen criteria. So, this is the basic principle of the oscillator.

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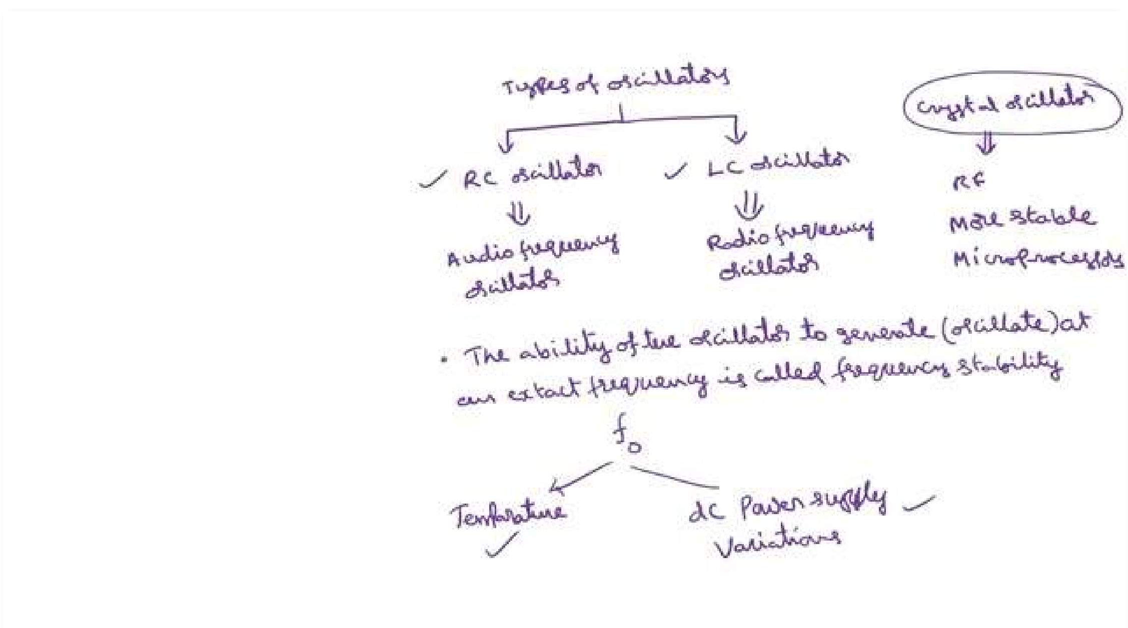

Now, there are different types of the oscillators. So, one is called as RC oscillator. The feedback circuit is going to be constructed using R and C components and if the feedback is constructed using L and C components the corresponding oscillator is called as LC oscillator.

So, there is one more different types of the oscillator which we are not going to develop by using the operational amplifier. So, that is called as Crystal oscillator. So, this RC oscillator will be basically used for the generation of audio frequency signals 20 to 20 kilo Hertz normal range and this is for the radio frequency RF. This is also radio frequency, but the principle of the Crystal oscillator is different from the that of RC and LC oscillators. Here we are not going to use any RC or LC circuit, but here there is a Crystal.

So, if I apply some force on the Crystal this will vibrates and it will generate the oscillations. The advantage of this Crystal oscillator is this is more stable when compared with RC and LC oscillators. The stability of oscillator is defined as the ability of the oscillator to generate an exact frequency or to oscillate at an exact frequency is called as frequency stability. What are the factor that are going to affect the frequency of oscillations? Let f_o is the frequency of oscillations. There are two parameters which affect this frequency of oscillations one is temperature and other is DC power supply variations.

So, for the amplifier there will be no AC source, but there will be some DC power supply. So, if the DC power supply voltage changes then the f_o also will changes. So, whereas, for this Crystal oscillator regardless of this temperature variations or DC power supply variations the frequency of oscillations is almost constant. That is why this Crystal oscillator will be normally used in microprocessors to generate the clock. So, in this course we will discuss about the different types of RC and LC oscillators.

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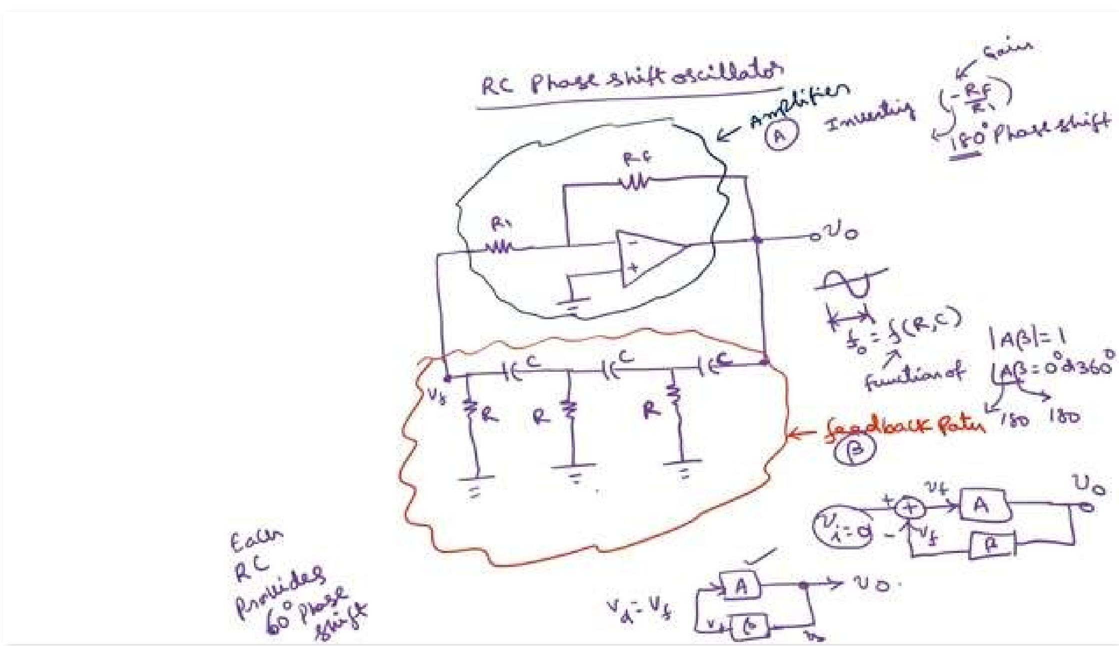

So, the first RC oscillator is a RC phase shift oscillator. So, the circuit diagram of this RC phase shift oscillator is this consisting of some amplifier part and feedback part. This is amplifier basically inverting amplifier non-inverting terminal will be grounded and the part of output is fed back to the input through RC network. There are three sections of RC network here. This is one section, this is second section, this is third section and this will be fed back to the input. Input is applied at inverting terminal this is R_1, R_F. So, the amplifier part here will be this. This is your amplifier part and feedback path will be this. So, if I take this the block diagram that we have discussed in the last slide. This is the block diagram of oscillator.

This is amplifier with A, this is feedback path with beta and here we will get v_o output. This v_i we are going to have 0, this is v_f and this is v_d. This is plus plus because v_i is equal to 0 you can remove this. So, you can make this circuit as simply $A\beta$ this is feedback v_d is equal to v_f this is output. So, this A part is this is going to provide A and this feedback path is the network which provides a feedback factor or feedback gain of β. And here we are going to take the output v_o and input is 0 there is no input here. Because the amplifier is inverting amplifier whose gain is $-\frac{R_F}{R_1}$ this minus sign represents 180 degrees phase shift. So, the condition for the oscillations is $A\beta$ magnitude should be 1 or in general it can be greater than 1 also. Phase angle should be 0 degrees or 360 degrees. Because here the A part is giving 180 degrees phase shift the remaining 180 has to be provided by β.

So, that β is three RC sections has to provide total of 180 degrees phase shift. So, each RC section will provide 60 degrees phase shift. So, with this circuit you can generate here sinusoidal signal without any input. Now, what is the frequency of this sinusoidal signal? If I call the frequency of oscillations as f_o this f_o is a function of R and C this is a function of this R and this C. And what is the expression for this frequency of oscillations of this RC phase shift oscillator? In order to derive this expression, we will consider first the feedback path. Here the output that is given is v_o here the output that is taken is v_f because the output of this β is v_f input is v_o. So, I will just consider only the β network separately.

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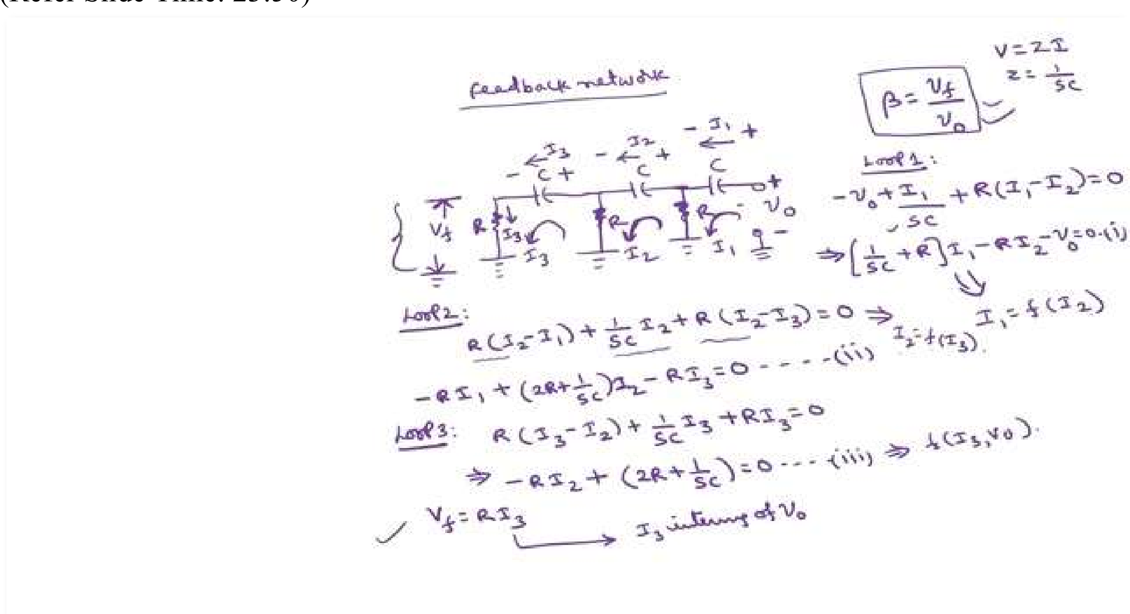

So, there are three RC sections. And, here we are going to take the output v_f and here the voltage is with respect to this ground v_o. Now, what is β is nothing, but $\frac{v_f}{v_o}$. Once if I know this β then I can find out the frequency of oscillations. So, in order to derive the expression for this β I will consider the three loops. This is loop 1 let the current in the loop 1 is I_1 current in loop 2 is I_2 current in loop 3 is I_3. So, you apply KVL in loop 1. Here this current through this capacitor is in this direction.

So, this is plus minus this is I_1 whereas, in this resistor the current is I_1 flows in this direction this is plus to minus I_2 flows in the opposite direction. So, the resultant voltage is $(I_1 - I_2)R$. So, I am taking the convention as plus to minus as positive value then the minus to plus becomes minus, $-v_o$ plus I_1 what is the voltage across this capacitor I_1 into. So, we know that $V = ZI$. Z is the impedance. So, in case of capacitor what is Z? Is $\frac{1}{sC}$.

So, here the current is I_1 so, $\frac{1}{sC}$. So, this will be I_1 into $\frac{1}{sC}$ and the direction of this current through this R is R times I_1 is in positive direction I_2 in negative direction is equal to 0 this is the Kirchhoff voltage law in the first loop. So, if I simplify this in terms of I_1 I_2 what is the factor of I_1 this is $\frac{1}{sC}$ is here and here this is R times I_1 and what is the factor of I_2 minus $RI_2 - v_o$ is equal to 0 this is equation 1. Now, let us take loop 2. So, in this loop this current is I_2.

So, this direction is plus to minus now in this resistor R this will be $I_2 - I_1$ because now we are taking the I_2 in the positive direction and I_1 will be in the negative direction whereas, in this R again $I_2 - I_3$. So, what will be expression? This $R(I_2 - I_1) + \frac{1}{sC}I_2 + R(I_2 - I_3) = 0$. So, if you simplify this if I take all I_1 terms to one side I_2 terms to other side I_3 terms to one more side then what will be this result what are the I_1 factor $-RI_1$ and I_2 will be there are three terms I_2 terms this is one this is one this is one. So, this is R R becomes 2R, $+(2R + \frac{1}{sC})I_2$ and then I_3 term is only one term which is $-RI_3$ is equal to 0 this is expression 2. Then loop 3 here this is plus minus this is I_3 and this current through this resistor is $I_2 - I_3$.

So, $R(I_3 - I_2) + \frac{1}{sC}I_3$ plus in this only I_3 current flows RI_3 is equal to 0. So, if I take I_1 I_2 I_3 terms separately what is I_1 there is no I_1 term and I_2 term is $-RI_2$ and I_3 term will be $(2R + \frac{1}{sC})$ is equal to 0 this is expression 3. Now, from this equation 1 2 3 basically I want $\frac{v_f}{v_o}$. So, I have to express this I_1 in terms of I_2 I_2 in terms of I_3 I_3 in terms of I_o. So, what is the expression for v_f from here? So, this current through this one is I_3 voltage across this resistor is v_f resistance is R.

So, $V = RI$. So, $vf = RI_3$. So, basically, I am going to derive this I_3 in terms of v_o. So, that if I take the ratio of v_f to v_o you will get the β. So, in order to express this I_3 in terms of v_o first I am going to express from here from equation 1 you express I_1 as a function of I_2 and you substitute this I_1 here and express I_2 as a function of I_3. Then from this third expression I_2 I_1 everything will be in function of I_3. So, we will be having here this will be a function of only I_3 and v_o. So, finally, I will get the expression for the β. So, I will start with the equation 1 I will express I_1 in terms of I_2.

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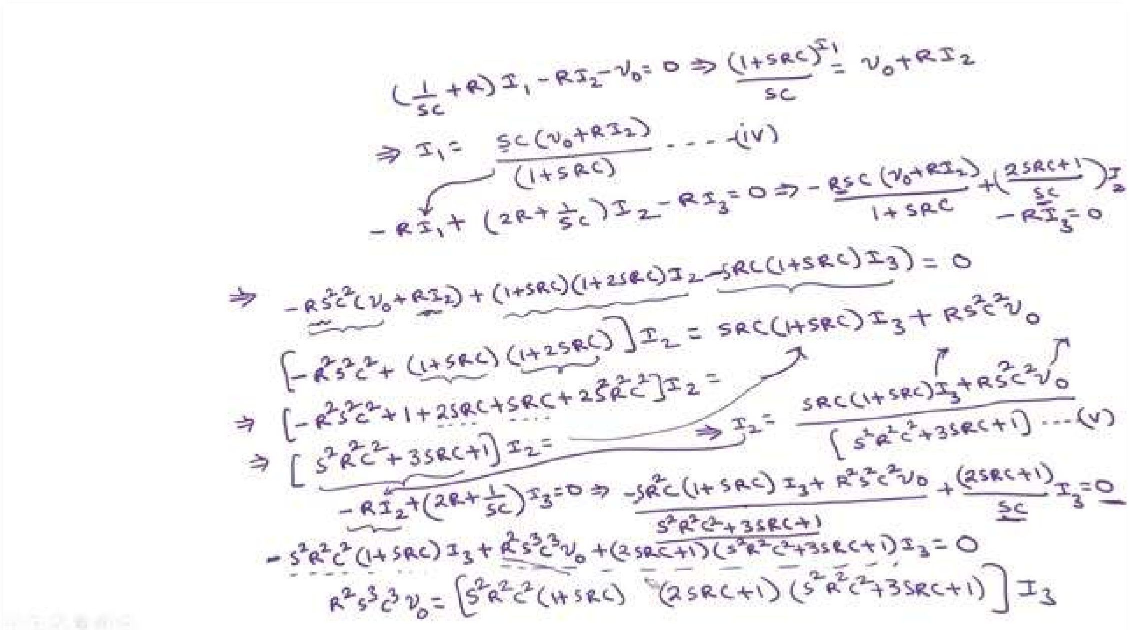

So, what is expression 1? $\left(\frac{1}{sC}+R\right)I_1 - RI_2 - v_o = 0$. So, implies what is I1? If I take LCM as sC. So, $\frac{(1+sRC)}{sC} = v_o + RI_2$ or implies what this into I_1 implies what is I_1? is equal to $\frac{sC(v_o+RI_2)}{(1+sRC)}$ this is equation 4. Now, we will substitute this equation 4 in equation 2. So, that I can eliminate this I_2 this I_1. So, $-RI_1 + \left(2R+\frac{1}{sC}\right)I_2 - RI_3 = 0$.

So, I will substitute this I_1 here now. So, what happens now implies $\frac{-RsC(v_o+RI_2)}{1+sRC}$ is the first term and the second term also if I take sC as LCM this is $\left(\frac{2sRC+1}{sC}\right)I_2 - RI_3$ is equal to 0 implies overall LCM will be sC times $1+sRC$ and what will be the numerator? Here this will be multiplied with this sC. So, this becomes $-Rs^2C^2(v_o+RI_2)$ plus here sC is this you have to multiply this with $1+sRC$ $1+sRC$ into $(1+2sRC)I_2$ minus RsC. So, $sCR(1+sRC)I_3$ is equal to 0. So, this if I take to the other side this total LCM becomes 0 as a result of that this will be this expression will holds. Now, what is the simplification? We have to basically express this I_2 in terms of I_3 and v_o.

So, what are these I_2 terms? This is one I_2 term another I_2 term is this and here there is no I_2 term. So, what are the total I_2 terms? This R into R becomes R^2. So, $[-R^2s^2C^2 + (1+sRC)(1+2sRC)]I_2$ this is the factor of I_2. And what is the factor of I_3 is this. If I take to the other side this minus becomes plus $sRC(1+sRC)I_3$ plus if I take this v_o term to other side this minus becomes $+Rs^2C^2v_o$.

So, what is the simplification of this I_2 coefficient? This is $-R^2s^2C^2$ plus if I multiply these two terms you will get four terms. So, first term is 1 into 1 1 plus $2sRC + sRC$ becomes $3sRC + 2s^2R^2C^2$ into I_2 is equal to same thing. So, what is the simplification here now implies this is $+2s^2R^2C^2$ this is $-R^2s^2C^2$ becomes $1 + R^2C^2s^2$ plus this $2sRC$ this one sRC becomes

$3sRC + 1$ into I_2 is equal to same thing. So, implies what is I_2? Here RHS side by this factor. So, what is RHS side $\frac{sRC(1+sRC)I_3+Rs^2C^2v_o}{[s^2R^2C^2+3sRC+1]}$ this is the expression for I_2 in terms of I_3 and v_o this we will call as equation 5.

This 5 will be substitute in equation 3. What is equation 3? $-RI_2 + \left(2R + \frac{1}{sC}\right) I_3$ is there. This $2R + \frac{1}{sC}$ the factor is I_3. So, $\left(2R + \frac{1}{sC}\right) I_3$. If I substitute this I_2 here and this implies $-R$ times this everything which is becomes now $\frac{-sR^2C(1+sRC)I_3+R^2s^2C^2v_o}{s^2R^2C^2+3sRC+1}$ is this $R\,I_2$ plus remaining term is if I take sC as common here this will be $\frac{(2sRC+\ )}{sC}$ is equal to 0. So, from here we have to express I_3 in terms of v_o.

So, again if I take the LCM which is sC into this one and if I take to other side this becomes 0. So, what will be the factor of the first term? This sC will be multiplied here. So, this becomes $-s^2R^2C^2(1 + sRC)I_3$ plus here also sC. So, this becomes $R^2s^3C^3v_o$ and here already $2sRC + 1$ is there this you have to multiply with $(2sRC + 1)(s^2R^2C^2 + 3sRC + 1)$ is equal to 0. So, the first term and the last term are I_3 terms and whereas, this one is v_o term.

So, if we take these two terms to other side then what happens? What is the total I_3 coefficient? $R^2s^3C^3v_o$ is equal to this minus becomes plus this plus becomes minus if I take to other side. So, this will be $s^2R^2C^2(1 + sRC)$ is I_3 coefficient plus here this will be $(2sRC + 1)(s^2R^2C^2 + 3sRC + 1)$ whole thing into I_3, but this plus becomes minus because I am taking this to other side.

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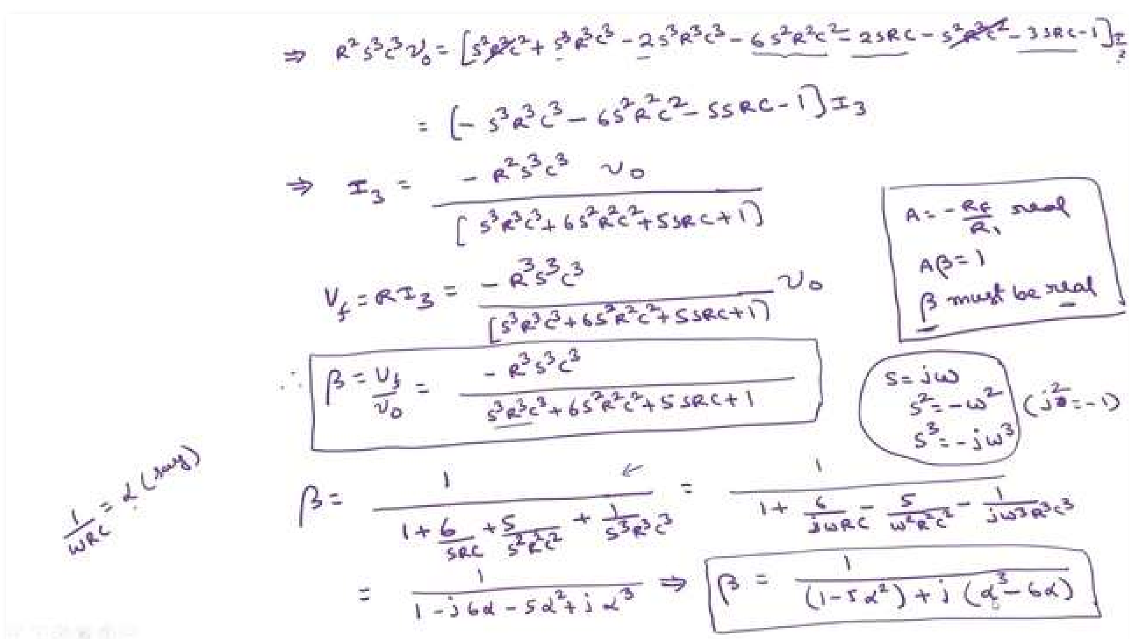

So, after simplification what will be expression for I_3 in terms of v_o? $R^2s^3C^3v_o$ is equal to what is the simplification? $s^2R^2C^2$ into 1 becomes $s^2R^2C^2 + s^3R^3C^3$ this into this and here this will be $2s^3R^3C^3$ with minus sign. Then this will be 6 times $s^2R^2C^2$ and this will be $-2sRC$

and then with 1 we will get the same term $-s^2R^2C^2 - 3sRC - 1$ this whole thing into I_3. So, what is the further simplification? How many $s^2R^2C^2$ terms are there? 6 $s^2R^2C^2$.

So, this $s^2R^2C^2$ this will get cancelled then we have one 6 times $s^2R^2C^2$ and how many $s^3R^3C^3$ terms is this is minus 2 plus 1 is minus. So, this is $-s^3R^3C^3 - 6s^2R^2C^2$ and sRC will be this is $-2\ -3\ -5sRC$ then -1 into I_3 or implies what is I_3? This minus sign if we take to other side $\frac{-R^2s^3C^3v_o}{[s^3R^3C^3+6s^2R^2C^2+5sRC\quad]}$. Now, we know that $v_f = RI_3$. So, implies this is what is v_f? R^2 into R becomes R^3 $\frac{-R^3s^3C^3}{[s^3R^3C^3+6s^2R^2C^2+5sRC+1]}v_o$.

Therefore, $\beta = \frac{v_f}{v_o}$ this is equal to $\frac{-R^3s^3C^3}{s^3R^3C^3+6s^2R^2C^2+5sRC+1}$. This is the final expression for the β of this feedback network.

Now, our ultimate goal is to derive the frequency of oscillations of the RC phase shift oscillator okay. So, for that what we are going to do here is. So, A is $-\frac{R_F}{R_1}$ which is real $A\beta$ should be equal to 1. So, in order to satisfy this β must be real. So, based on this condition we will derive the expression for the frequency of oscillations. So, in order to express this beta as real, I will replace s with $j\omega$ okay.

So, before that if I take this $R^3s^3C^3$ as common what will be the β? This is equal to -1 divided by I will take the magnitude only here for the sake of simplicity I will assume the magnitude. So, I am not taking minus sign. So, this becomes 1 because this will be divided with $R^3C^3s^3$ plus 6 times $\frac{s^2R^2C^2}{s^3R^3C^3}$ means this will be sRC plus 5 times $\frac{sRC}{s^3R^3C^3}$ means $s^2R^2C^2$ and this $\frac{1}{s^3R^3C^3}$.

So, this is equal to s is equal to $j\omega$. So, what is s^2 is equal to $-\omega^2$ what is s^3 because $j^2 = -1$. What is s^3? j^2 is -1 another j is there. So, $-j\omega^3$. If I substitute these values here we will get 1 over $1 + s$ becomes $j\omega$. So, $\frac{6}{j\omega RC} + 5$ by s^2 means $-\omega^2$ this becomes minus now $\frac{5}{\omega^2R^2C^2} +$ s^3 means $-j\omega$.

So, this will be $-\frac{1}{j\omega^3R^3C^3}$. Now, let us assume that this angular frequency of oscillations if I define this $\omega 0$ as $\frac{1}{\omega RC}$ as αC for the sake of simplicity. So, what happens to this β now? This will be 1 over 1 j if I take to the numerator it becomes $-j$ and what is $\frac{6}{\omega RC}$ is 6α $-\frac{5}{\omega^2R^2C^2}$ becomes $5\alpha^2$. This is $-j$ if I take to the plus it becomes plus j $\frac{1}{\omega^3R^3C^3}$ will be α^3. So, what are the real terms and what are the imaginary terms in the β? 1 over real term is $(1 - 5\alpha^2) + j(\alpha^3 - 6\alpha)$. This is the expression for the β in terms of α. But what is the condition for this β? β must be real means imaginary part should be 0.

As β is real implies imaginary part must be 0 which implies $\alpha^3 - 6\alpha = 0$. α if we take out common $\alpha^2 - 6$ should be 0 $\alpha^2 = 6$ or $\alpha = \sqrt{6}$. But what is α? We have defined this as $\frac{1}{\omega RC}$. Now, the frequency of oscillations $\omega_o RC$ becomes $\frac{1}{\sqrt{6}}$ or $\omega_o = \frac{1}{\sqrt{6}RC}$ or $f_o = \frac{1}{\sqrt{6}2\pi RC}$.

This is one of the important derivation of the frequency of oscillations of RC phase shift oscillator.

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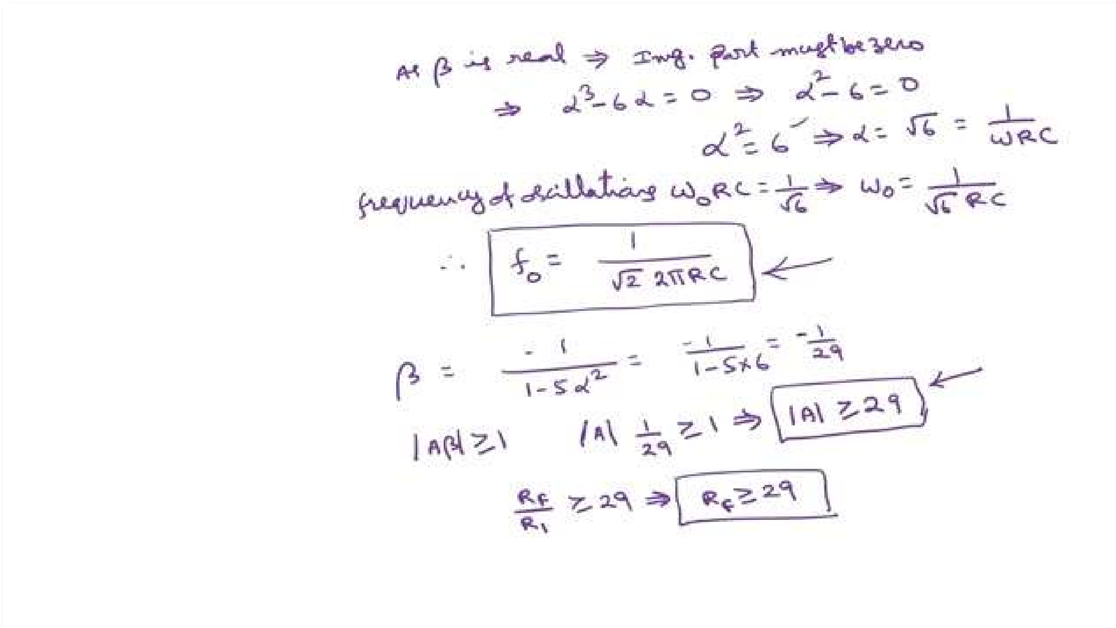

So, at this frequency of oscillations what will be the β value? What will be this β value? This is 0 and this α^2 is you have derived as α^2 is 6. So, if I substitute that what will be β? β becomes 1 by imaginary part becomes 0. So, real part is $1 - 5\alpha^2$ 1 by $1 - 5$ into α^2 is 6. So, 1 over 29. So, what is the condition for the oscillations? $|A\beta|$ should be greater than or equal to 1.

If it is equal to 1 it the oscillation will sustain. If you want to have the sustained oscillations $A\beta$ is equal to 1 is enough for the oscillations, but to get the sustained oscillations $|A\beta|$ should be greater than or equal to 1. So, what should be A now $|A|\frac{1}{29}$ this is of course, $-\frac{1}{29}$ because here we have minus term this minus term was there here okay. But anyhow we are going to take magnitude.

So, magnitude of β becomes $\frac{1}{29}$ plus $\frac{1}{29}$ should be greater than or equal to 1 implies magnitude of A should be greater than or equal to 29 this is another important condition. So, this is the frequency of oscillations provided A has to be greater than or equal to $\frac{1}{29}$.

So, what is A? is $\frac{R_F}{R_1}$. In fact, $-\frac{R_F}{R_1}$ if I take magnitude $\frac{R_F}{R_1}$ this should be greater than or equal to 29 means R_F should be greater than or equal to 29. So, in this circuit this R_F you have to choose greater than or equal to 29 times R_1.

I will take one example design a RC phase shift oscillator which generates 100 Hertz sinusoidal signal. So, f_o is given as 100 Hertz what is the expression for f_o? $\frac{1}{\sqrt{6}2\pi RC}$ implies what is RC is equal to $\frac{1}{\sqrt{6}200\pi}$.

Let $C = 0.1\ \mu F$ implies R is approximately equal to 6.5 $k\Omega$. So, we have to find out the R and C components what are the other components in the RC phase shift oscillator R_1 and R_F okay.

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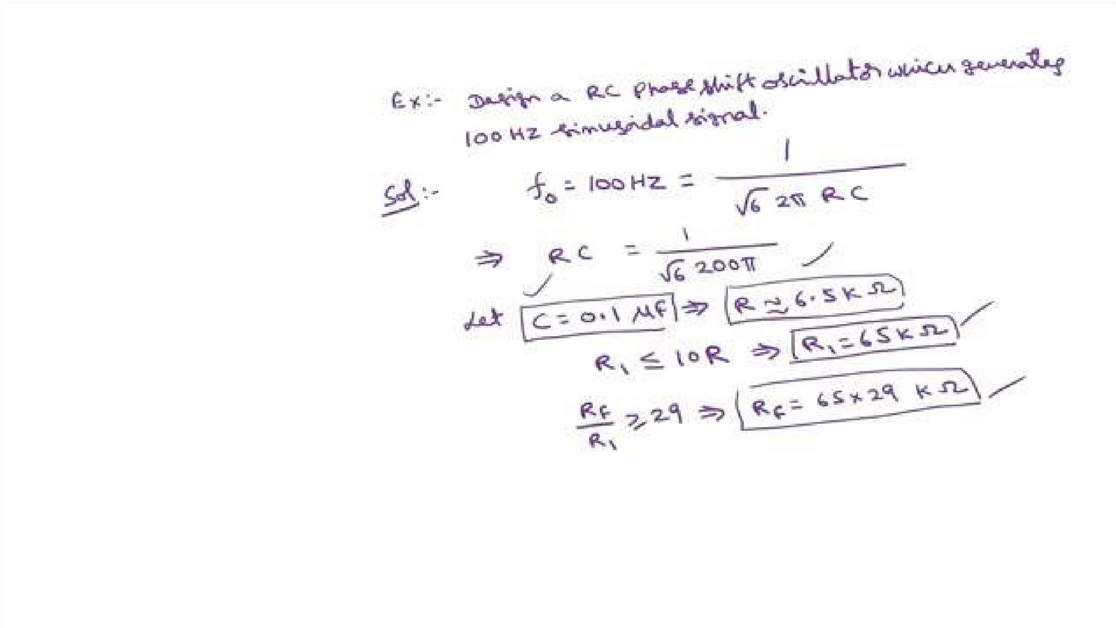

So, for that what is the condition you have to choose R_1 should be less than or equal to 10 times R to avoid the loading effect. So, this R_1 should be now $65k\Omega$ and another condition is $\frac{R_F}{R_1}$ should be greater than or equal to 29 implies what is R_F is equal to 29 times $65k\Omega$. So, this is about the design of RC phase shift oscillator this is R_1 value, this is R value, this is C value.

So, there are the four components in this RC phase shift oscillator the component values are C is this, R is this, R_1 is this, R_2 is R_F is this okay. So, in the circuit we have these four components only this R_F R_1 C and R this is how we can design this RC phase shift oscillator. So, this second type of RC oscillator is called Wien Bridge oscillator that we will discuss in the next lecture. Thank you.