Integrated Circuits and Applications
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Active Filters II
Lecture - 20
Design of Band Stop Filter

Till now we have discussed about the active low pass filter, active high pass filter, active band pass filter. The next one is active band reject filter or band stop filter. This is also called band stop or band reject or band attenuation. As the name implies here it will reject a band of frequencies. This is the ideal response of band reject filter. It allows all the frequencies except a range of frequencies will be between f_l and f_h eliminated, remaining will be passed with gain of unity.

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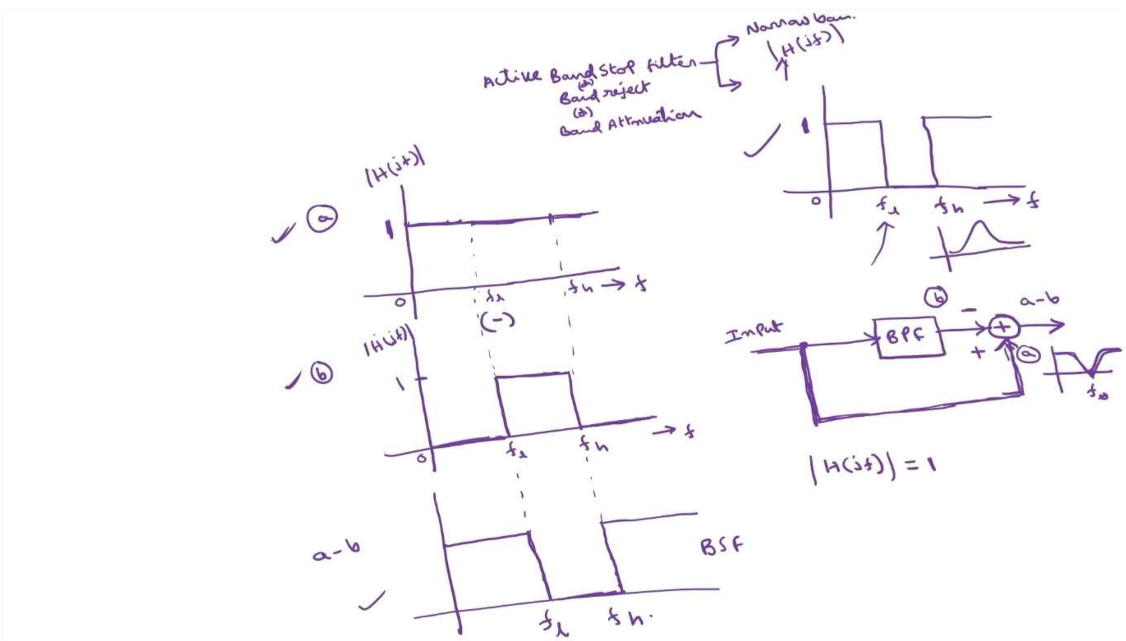

0 to f_l, f_h to infinity will pass with unity gain, between f_l and f_h it rejects. We can obtain this band stop filter from band pass filter by a simple manipulation. If I take the band pass filter, this is some input. This response of this one ideally is this. Now, if I take the same input. If I connect to the subtractor what will be the output? So, what is this signal? If I call this as b is the output of band pass filter, a is the direct input. So, a is nothing, but the input. If I assume that input the characteristics of this one, this is the frequency response. So, if I consider from here to here, output is equal to input means transfer function will be $H(jf)$ is equal to if I consider this as unity feedback path, this is simply 1. So, whatever the input will pass to this feedback path and it will reaches to the input of this adder.

So, what will be the gain of this one unity. So, frequency response is 1 means. So, this a response will be flat response assuming that this is unity. So, this will be unity and what about b response which is the output of band pass filter? It passes a band of frequencies and reject the remaining frequencies. This is a band of frequencies f_l to f_h with unity gain and then 0 below f_l above f_h. This is a and this is b. So, we are going to subtract this because the output is here is $a - b$. So, if I subtract here, this is f_l, this is f_h between 0 to f_l, this is unity, this is 0. So, 1 minus 0 becomes 1. The output $a - b$ plot will be up to f_l, this is 1 minus 0 is 1.

Between f_l to f_h this is also 1, this is also 1. So, 1 minus 1 becomes 0 and from f_h to infinity this is 1, this is 0, 1 minus 0 is 1. This is nothing but the response of band stop filter the same response. So, in order to obtain the band stop filter response what you have to do is, so you have to subtract from the input the output of the band pass filter. This is the ideal case.

Practically so, this response will be something like band pass filter, response will be something like this. So, if I subtract from 1, here the output of this 1 will be something like this. So, here this particular frequency this magnitude is 0. So, that particular frequency we can call as notch frequency. So, this is the type of response you will get from the output of this system which is band reject filter. Similar to this band pass filter we have two types of the band stop filter. So, one is wide band another is narrow band.

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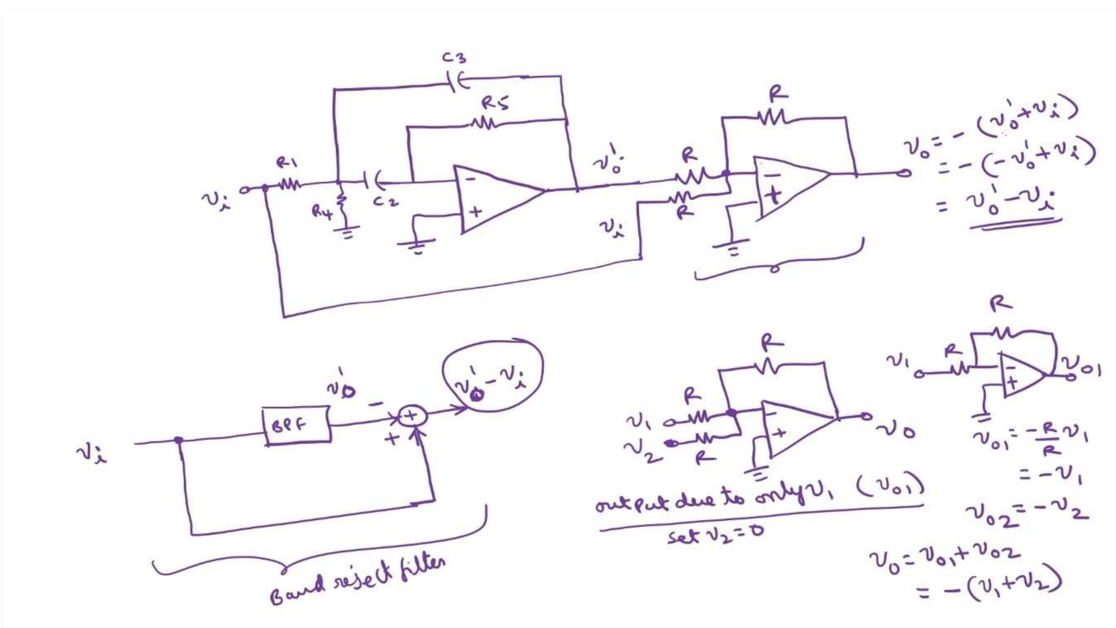

This is circuit diagram of band pass filter which we have discussed in the last lecture. This we have taken as ground. This is v_i, this is v_o of band pass filter. We have taken this one as R_1, C_2, C_3, R_4, R_5. Now as we have discussed here that this band pass filter output, you have to connect to adder. This you have to subtract, this you have to add. This will be the output of band reject filter. So, you have to connect to basically adder circuit.

Adder circuit is nothing but the same input you have to take $v_i, v_i(t)$. So, you have to take from v_i the second connection to the adder. One is this one. So, this is one input for the adder. Another input is you have to take from here.

So, in the earlier lectures we have discussed about this adder circuit. If all these values are R, R, R, if I call as this as v_o' as the output of band pass filter and this is v_i itself. So, output should be $v_i' - v_i$. This will be output of band pass filter, this is output v_o final. So, for this circuit what will be the output? This is basically an adder circuit.

This circuit we have already discussed with the previous class, previous lectures. This is one, this is another one, then we have this one minus plus. This is v_1 say, this is v_2 in general R, R, R. So, what is the output of this circuit? Superposition theorem. First you find out the output due to only v_1, let it be v_{01}.

So, to get output due to only v_1 set v_2 to 0. If you set v_2 to 0, then what happens this will be ground, this will be ground. So, this sort will be removed from this one. So, this will be equivalent to v_1 R, this is v_{01}. So, v_2 R will be grounded, this is v_1 R, R.

This is basically a inverting circuit. So, what is v_{01}? is equal to $\frac{-R}{R}v_1$, simply $-v_1$. And similarly, if I want v_{02}, we set v_1 to ground, then output will get as $\frac{-R}{R}v_2$. So, what will be v_o is equal to $v_{01} + v_{02}$, this is equal to $-(v_1 + v_2)$. So, this will be equal to $-(v_o' + v_i)$, but we want $v_i' - v_i$, this is v_o'. $v_o' - v_i$, we are getting $v_o' - v_i$.

So, but this v_o' is negative, as we have discussed in the last lecture, what is the expression for this transfer function? This is $-A_o\omega_o\xi s$ by $s^2 + \omega_o\xi s + \omega_o^2$. So, because of this minus sign, this minus minus becomes plus. So, this is already minus. So, this minus minus become this is minus.

So, this is equal to minus of this minus, if I take the magnitude of this one plus v_i. So, minus of minus becomes plus. So, $v_o' - v_i$, this is the desired output. So, this is how we can construct a band reject filter or band stop filter by just adding the output of the band pass filter with the input. And what will be the transfer function of this band stop filter?

Transfer function of band pass filter is which you have derived $-A_o\omega_o\xi s$ divided by $s^2 + \omega_o\xi s + \omega_o^2$. This is equal to $\frac{v_o'}{v_i}$ in this case, this is $\frac{v_o'}{v_i}$. So, you have to subtract here because you are providing a gain of A_o. So, you have to use here also a gain of A_o. If it is unity, then simply you have to subtract if it is having gain of A_o, here also you have to put gain of A_o. So, you have to basically subtract A_o into vi is this output minus A_o into vi.

H band stop or band reject filter response should be equal to $A_o v_i$ minus of course, $v_i(s)$ I am not writing function of s. $A_o\omega_o\xi s$ divided by $s^2 + \omega_o\xi s + \omega_o^2$. This is equal to this is overall $\frac{v_o}{v_i}$ of s. If I take the LCM $A_o v_i[s^2 + \xi\omega_o s + \omega_o^2]$ divided by $s^2 + \omega_o\xi s + \omega_o^2$.

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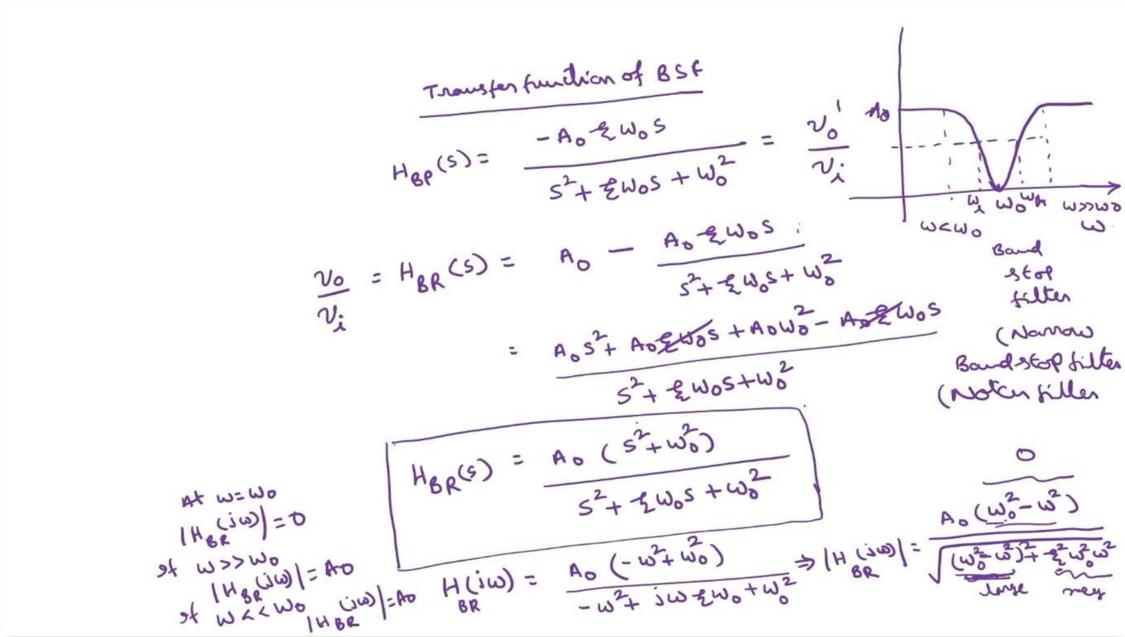

This is equal to $A_o v_i s^2$ plus this is minus also there this minus of $A_o \xi \omega_o s$ plus $A_o \xi v_i \omega_o s$.

So, to get band stop filter H band stop or band reject filter s, this you have to subtract from $A_o - \frac{A_o \xi \omega_o s}{s^2+\xi\omega_o+\omega_o^2}$. You can see here, this is $A_o v_i$ this v_i is common okay, we are taking v_i in the ratio. So, this will be overall this one is equal to $\frac{v_o}{v_i}$. Here also one v_i was there here also v_i I have taken outside. So, what will be this? $A_o s^2 + A_o \xi \omega_o s + A_o \omega_o^2 - A_o \xi \omega_o s$ divided by $s^2 + \xi\omega_o + \omega_o^2$.

So, these two will get cancelled. This will be $A_o(s^2 + \omega_o^2)$ divided by $s^2 + \xi\omega_o + \omega_o^2$. This is the transfer function of H band reject or band stop filter. We can easily see that this will acts as a band reject filter. If I take the frequency response $j\omega$ if I take $j\omega^2$, $j^2 = -1$. So, we will get this as $(-\omega^2 + \omega_o^2)$ divided by this is $j\omega^2$ is $-\omega^2 + j\omega\xi\omega_o + \omega_o^2$. So, what will be the magnitude of this one? So, in the numerator will be having A_o there is no imaginary part.

So, you will get the same value itself is the magnitude. In the denominator we have real part and imaginary part square root of real part square real part is $(\omega_o^2 - \omega^2)^2$ this imaginary part square is $\xi^2\omega_o^2\omega^2$. This is the frequency response. So, at $\omega = \omega_o$, what happens to this one? So, this becomes 0 because $\omega = \omega_o$. So, the magnitude of $H(j\omega) = 0$. And at high frequencies if ω is much much greater than ω_o.

So, what happens to this one? This ξ is small value less than unity. So, this term will be large when compare with this term. So, if I neglect this, this $(\omega_o^2 - \omega^2)$ and this $(\omega_o^2 - \omega^2)$ will get cancelled will get A_o. This is the case if even if ω is much much less than ω_o also. Modulus of $H_{BR}(j\omega)$ is A_o because this is $(\omega_o^2 - \omega^2)^2$ whether this difference is positive or negative we will get the same relation. So, if you plot the response of this system at $\omega = 0$ is 0 at lower frequencies as well as higher frequencies it is unity.

This is ω_o ω for ω values greater than ω_o also unity A_o for ω values less than ω_o also A_o. This is the band stop filter and this is narrow band stop filter because this bandwidth is if I take $\frac{1}{\sqrt{2}}$ times this one this bandwidth is very less ω_l ω_h. Another name for the narrow band stop filter is notch filter. This has applications in many of the fields especially in biomedical in order to remove the 50 Hertz hum, we can use the notch filter.

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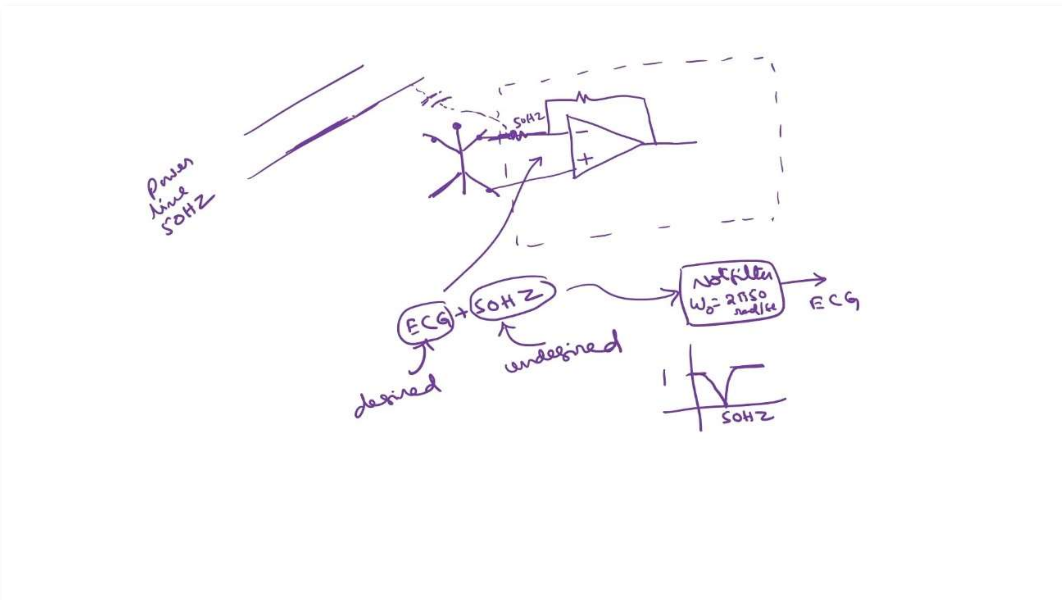

Suppose you are recording the ECG of a patient. So, you will be having electrodes. The signal will be connected to this amplifier. Here what happens is if you have ECG machine we are going to record this ECG through ECG machine. So, if you have nearby power lines which is of 50 Hertz signal. So, this signal that is recorded here will be ECG plus 50 Hertz signal.

This is desired one, this is undesired. Why because wherever the conductor is there this conductor and this conductor there will be some capacitance exist. This is parasitic capacitance because of this 50 Hertz signal will be coupled through this capacitor and that 50 Hertz signal also available here. So, in order to remove this only 50 Hertz signal remaining ECG signal I have to output. So, we can pass this through a notch filter whose notch frequency is $2\pi 50\ rad/sec$. So, the response that I have seen is like this.

So, this rejects only this 50 Hertz signal whereas, the all the frequencies above this 50 Hertz below 50 Hertz will be passed through this one. So, output will be almost pure ECG signal. This is one of the application of this notch filter. Like that we have several other applications.

So, this is one way to design this notch filter is we can subtract the output of the band pass filter from the input signal.

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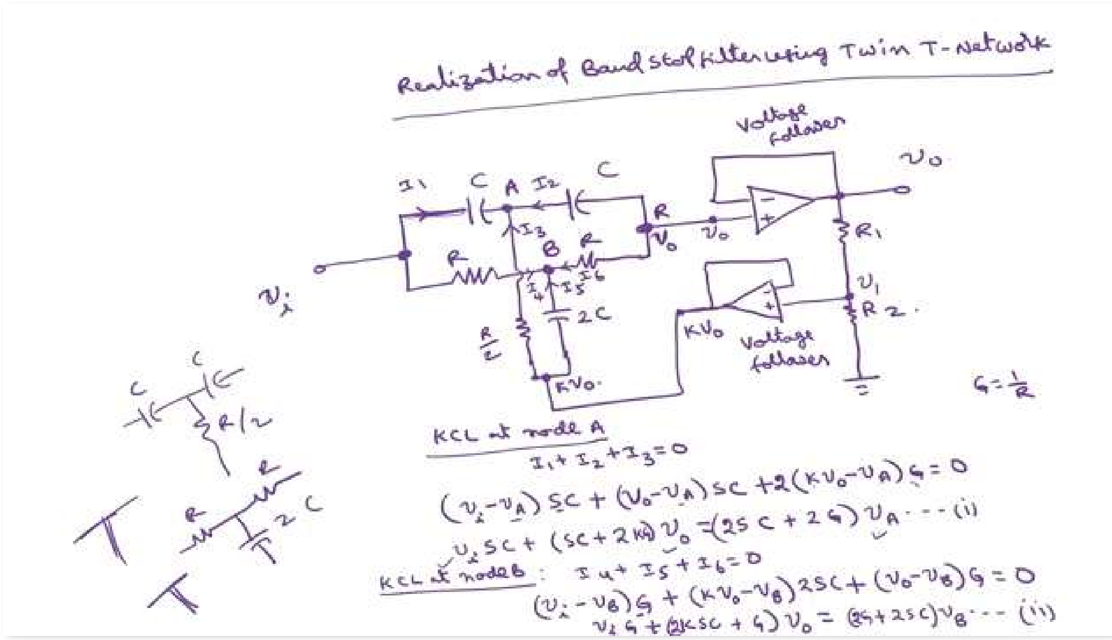

There is another way using a Twin T Network. So, as the name implies there will be two T networks. There are capacitances here and here resistance, here the input v_i is applied. Then we take one connection from here, this is $\frac{R}{2}$, this is R, this is R, C C. So, this will form one T network this capacitor, this capacitor and this resistor this is in the form of T shape.

So, this is one T network. Another T network is between these resistors then capacitance this will be 2C this is another T network is R R and this is 2C this is another T network that is why the name twin T network. Then we have to connect this output of this one to two voltage followers. This is the final output v_o and there will be voltage divider R1 R2. This will be connected to this voltage follower this is another voltage follower. This is the complete circuit diagram of band stop filter using twin T network.

So, we can derive the expression for the transfer function and we can show that this will acts as a band stop filter. So, in order to derive the transfer function of this one I will consider three nodes A B and then R node. So, what is the voltage here? This is voltage follower output voltage is v_o means input voltage is v_o. So, this is also v_o this is voltage follower and this voltage is actually this is if I call as v_1 this is also v_1, but what is that v_1? So, v_1 is nothing, but $v_o \frac{R_2}{R_1+R_2}$ this is equal to kv_o where k is $\frac{R_2}{R_1+R_2}$. So, this you can call as kv_o this is v_o this is kv_o.

Now, we can apply the KCL at nodes A B and R to derive the overall transfer function. Let us apply KCL at node A. So, what are the currents entering? If a current call this as I_1 this as I_2 this as I_3. So, at node A what happens all the three currents are entering.

So, $I_1 + I_2 + I_3 = 0$. So, what is I_1? This side it is v_i this side it is v_A. So, $(v_i - v_A)sC$, sC is the admittance plus I_2, I_2 is this way this voltage is v_o this voltage is v_o. So, $(v_o - v_A)sC$ again this is sC plus. What about I_3? This voltage is this voltage is kv_o this is v_A. So, this is $(kv_o - v_A)$ this is resistance is there. In fact, this resistance you have to divide $\frac{R}{2}$ is equal to 0, but G is equal to $\frac{1}{R}$.

So, this 2 will go to the numerator and this $\frac{1}{R}$ becomes G this into G you can call as 0. So, if I take all v_i and v_o terms to one side and v_A term to other side, what will be this one? This is $v_i sC + (sC + 2k)v_o$ is equal to this sC if I take to the other side this is sC and this sCv_i if I take to the other side another sC two sC and this v_A term is twice G into v_A. This is $2k$ G also there into v_o this is G also this is one expression (i).

Basically, to find out the transfer function you have to express in terms of v_i v_o, v_A you have to eliminate. Now, if I apply the KCL at node B if I call this current as I_4 this current as I_5 this current as I_6 at node B $I_4 + I_5 + I_6 = 0$.

So, what is I_4? this side is v_i this side is v_B. So, $(v_i - v_B)$ divided by R R and multiply with G, $\frac{1}{R}$ is G and what is I_5? this time is kv_o this is v_B plus $(kv_o - v_B)2sC$ plus what is I_6? this is v_o this is v_B. So, $(v_o - v_B)$ divided by R. So, this will be into G is equal to 0. If I take v_i and v_o terms to one side and v_B term to other side what will be this one this is $v_i G$ and v_o term is k into $2sC$ k $2ksC$ is this term and here this is simply G into v_o is equal to what is v_B term here we have G here you have $2sC$ G plus $2sC$ plus again G 2G into v_B, this is equation (ii). So, in equation (i) and (ii) left hand side we have v_i v_o whereas, right hand side we have v_A v_B. So, another condition we have to derive in terms of v_A and v_B.

For that consider the KCL at node R this is node R I_2 and at node R I_2 is leaving and no current here the current is 0 because of ideal op-amp the current in the input terminals is 0 there are two currents which is leaving I_2 and I_6. So, what happens $I_2 + I_6$ should be equal to 0. What is I_2? I_2 is this side v_o this side v_A and C $(v_o - v_A)sC$ plus what is I_6? this is v_o this is v_B divided by R into G $(v_o - v_B)G$. This is another expression in terms of v_o v_B. If I take this v_A sC to the other side $v_A sC$ is equal to this v_o v_o $(sC + G)v_o - Gv_B$. So, by substituting this value of v_A into this (i) and (ii) and finally, solving for this one we will get the transfer function as using (i) and (iii) and after some manipulation.

This is simple algebraic manipulation only this I am leaving you to as exercise. So, we will get the transfer function as $\frac{v_o}{v_i}$ as final expression for this one is $s^2 + \left(\frac{G}{C}\right)^2$ whole divided by $s^2 + \left(\frac{G}{C}\right)^2 + 4(1-k)(\frac{G}{C})s$, k we know which is $\frac{R_2}{R_1+R_2}$, into $(\frac{G}{C})s$. This is the transfer function that we are going to get for notch filter this is exactly same as that of the transfer function which we have derived in the last slide. This is the transfer function of band stop or band reject filter.

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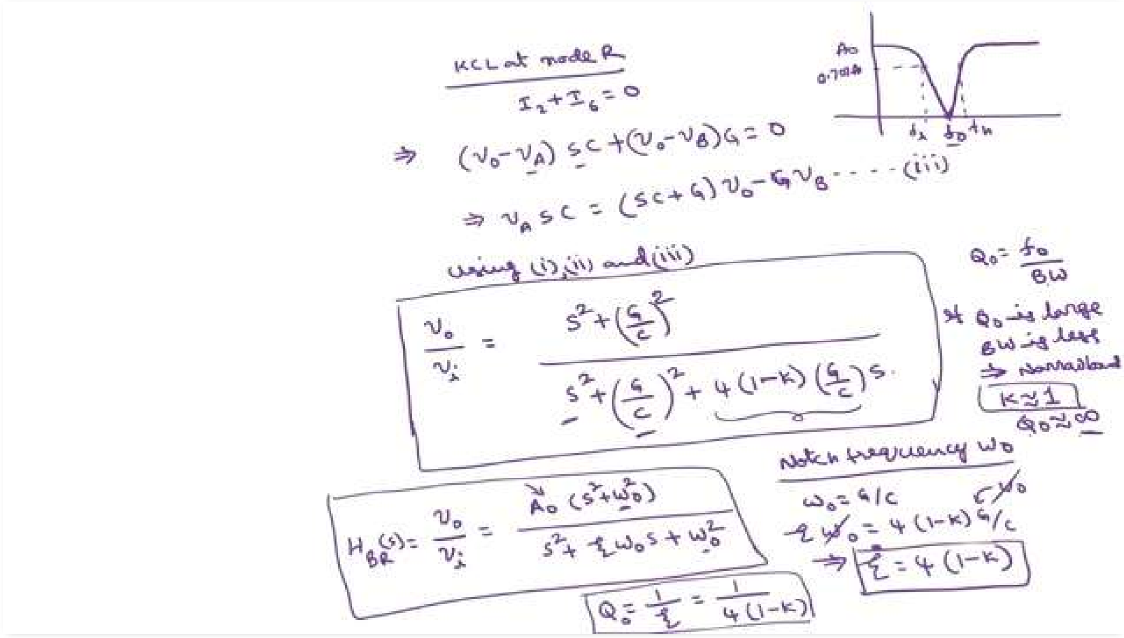

If I compare these two, this we have derived this one as $\frac{A_o(s^2+\omega_o^2)}{s^2+\xi\omega_o s+\omega_o^2}$.

So, these two are now similar. So, by comparing these two what will be this notch frequency ω_o the expression for the notch frequency? This A_o is gain here also if you have the gain you can have A_o. So, let us take A_o as 1 for the sake of simplicity then in place of ω_o^2 we have $\left(\frac{G}{C}\right)^2$. So, therefore, ω_o is equal to $\frac{G}{C}$ and this ω_o^2 is this s^2 plus this one. Now this is nothing, but $\xi\omega_o = 4(1-k)\frac{G}{C}$.

$\frac{G}{C}$ is nothing but ω_o. So, from this is ω_o, this ω_o, this ω_o get cancelled. What is the damping factor ξ ? or what is the quality factor as we have discussed in the last lecture 1 by damping factor. So, this is $\frac{1}{4(1-k)}$. So, in order to have a better band stop filter this should have a very narrow band. So, narrow band in the sense what will be this bandwidth Q is equal to f_o by bandwidth.

This if we call as ω_o or f_o relation is 2π into f_o is ω_o . So, in order to have this small value of Q_o, bandwidth should be less. So, Q_o is small bandwidth is less implies narrow band. So, how to make Q_o small? k we have to make approximately equal to 1. If k is approximately equal to 1, Q is Q_o is large because reverse.

Q_o is large means bandwidth is less. So, if k is approximately equal to 1, Q_o approximately equal to infinity. So, in order to make the large quality factor you have to make k is equal to 1 in the sense. So, this k is nothing, but here $\frac{R_2}{R_1+R_2}$. This you have to make equal to equal to approximately equal to 1 so that you will get good quality band stop filter.

So, Q_o is equal to infinity means bandwidth almost 0. So, only it rejects only a single frequency this bandwidth if I take 0.70 times this one this is the bandwidth. So, this will be very small means it is almost it will be equal to 0 it rejects only single frequency f_o okay. This is what is desired for a good band reject filter or a good notch filter it has to reject only a single frequency whereas, the remaining frequency it should not attenuate okay. So, in order to get that good notch filter what you have to make here? Q you have to make as large.

So, in order to make Q large k we have to design which is approximately equal to 1. So, this is about the circuit diagram of a band reject filter okay. So, we will discuss some examples of this design of a band stop filters in the next lecture. Thank you.