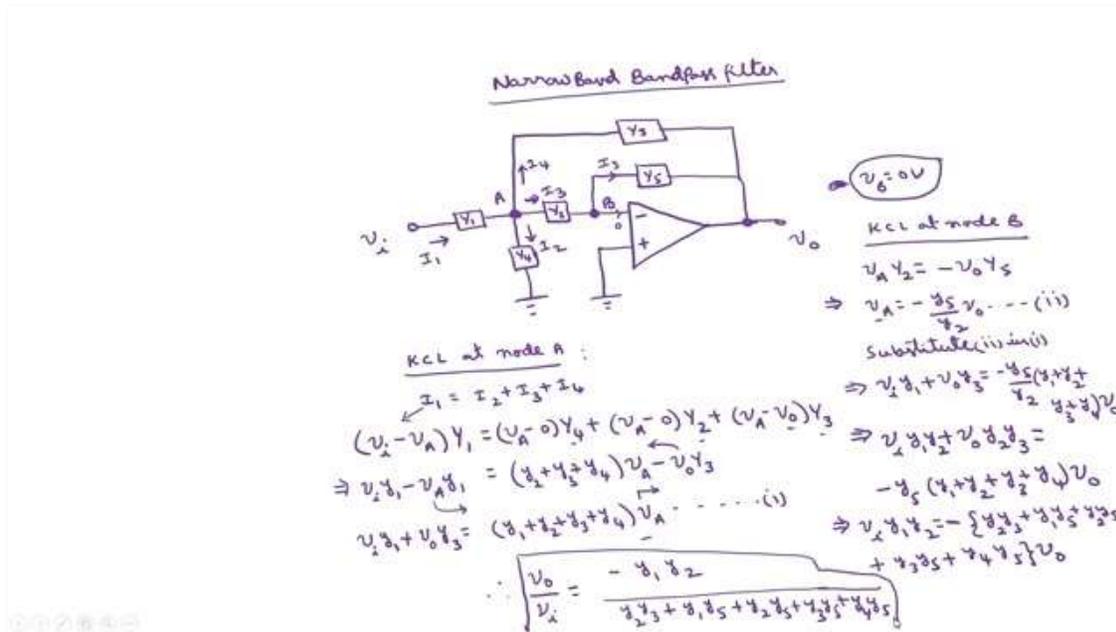


Integrated Circuits and Applications
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Active Filters II
Lecture - 19
Design of Band Pass Filter

So, in the last lecture we are discussing about the band pass filter and there are two types of band pass filters. One is narrow band band pass filter and wide band band pass filter depends upon the quality factor. Today, we will discuss the transfer function of narrow band band pass filter. First, I will derive the generalized transfer function, later we will substitute the impedances or the admittances with the corresponding the resistance and capacitances. So, this is the generalized circuit diagram of a band pass filter which will be having narrow band. So, this can be obtained by taking the two feedback paths.

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There is one feedback path here and one more feedback path from here. So, operational amplifier circuit with two feedback paths is going to behave like narrow band band pass filter. This is v_i , this is v_o , you call this one as Y_1, Y_2, Y_3, Y_4, Y_5 . First, I will derive the generalized transfer function in terms of Y_1 to Y_5 . Later, we will substitute this Y_1 to Y_5 with the corresponding resistors or capacitors.

So, in order to derive the expression for $\frac{v_o}{v_i}$, I will consider two nodes this is node A, this is node B. So, because of this ideal op amp characteristics v_B will be 0 volts, this is virtual ground. So,

if I apply the KCL at node A. So, you see the current which is entering here, if I call this one as I_1 , this is I_2 is leaving, I_3 is leaving, I_4 is also leaving.

So, what will be this? $I_1 = I_2 + I_3 + I_4$. What is I_1 in terms of admittance? This v_i is the voltage here and the voltage here if you call as $(v_i - v_A)Y_1$ is this I_1 . Now, this admittance you have to multiply, if it is resistance you have to divide by resistance is equal to I_2 , this voltage is v_A , the other voltage is 0. So, $(v_A - 0)Y_4$ and here this is $(v_A - v_B)$, but v_B is 0. So, $(v_A - 0)Y_2$ and this is v_A and this is v_o . So, this I_4 is $(v_A - v_o)$ admittance is Y_3 . If you take all v_i terms to one side and v_A and v_o terms to other side, we will get this one as this is equal to. So, only one v_o term is there remaining all v_A times. So, v_A this is Y_4, Y_2, Y_3 . So, $(Y_2 + Y_3 + Y_4)v_A - v_o Y_3$.

This will be from this side $v_i Y_1 - v_A Y_1$. Now, we will take this v_A to other side and v_o to left hand side. We will get $v_i Y_1$ minus becomes plus $v_o Y_3$, this minus becomes plus this is v_A and Y_1 . So, if I take v_A as common this will be $(Y_1 + Y_2 + Y_3 + Y_4)v_A$. This is equation (i). But in order to derive the transfer function we need $\frac{v_o}{v_i}$. So, I have to express now this v_A in terms of v_o . For that I will consider the KCL at node B.

So, this I_3 is entering into node B and here the current is 0. So, the same I_3 will flows through Y_5 also. So, I_3 in terms of v_A, v_B is $(v_A - v_B)$ is 0. So, $v_A Y_2$ is equal to this is v_B is 0, $(0 - v_o)Y_5$ implies what is v_A ? $-\frac{Y_5}{Y_2}v_o$. Now, we will substitute (ii) in (i). So, that v_A can be expressed in terms of v_o , we can derive the transfer function. (ii) in (i) implies $v_i Y_1 + v_o Y_3 = -\frac{Y_5}{Y_2}(Y_1 + Y_2 + Y_3 + Y_4)v_o$.

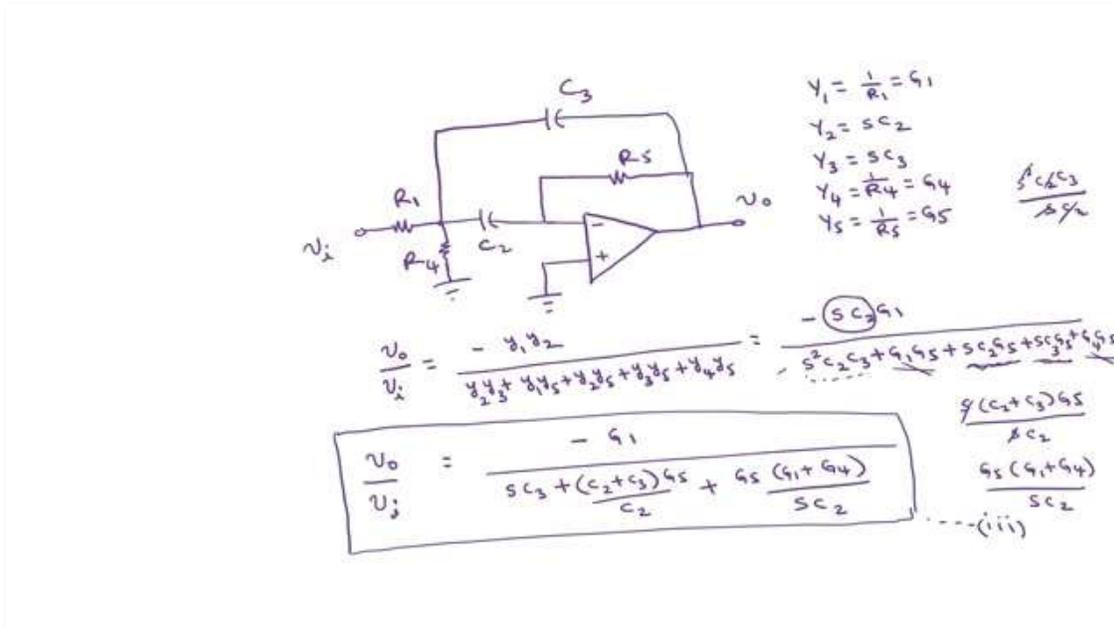
If we take v_o terms to one side. So, this will become if I take to other side this will be minus sign implies v_i into Y_2 if I take to the other side $Y_1 Y_2$ plus $v_o Y_2 Y_3$ is equal to left hand side is $-Y_5(Y_1 + Y_2 + Y_3 + Y_4)v_o$ or $v_i Y_1 Y_2$ is equal to minus of this is $Y_2 Y_3$ and these are $Y_1 Y_5$ plus $Y_2 Y_5, Y_3 Y_5$ plus $Y_4 Y_5$ this whole thing into v_o .

Therefore, what is the transfer function? $\frac{v_o}{v_i}$ is equal to Y_1 minus this minus and I will take to other side $\frac{-Y_1 Y_2}{Y_2 Y_3 + Y_1 Y_5 + Y_2 Y_5 + Y_3 Y_5 + Y_4 Y_5}$. This is generalized expression for the op-amp circuit with two feedbacks.

Now, if I want to realize this using resistance and capacitances the circuit will be like this. This is v_o this is v_i this is R R C R C. So, what is now Y_1 ? This is Y_1 you call as R_1 this is $\frac{1}{R_1}$ which is you can call as G_1 and this is Y_2 in place of Y_2 what is there? We have C_2 you can call. So, $Y_2 = sC_2$ because this is admittance and Y_3 what is Y_3 ? Y_3 this one is also capacitance. So, this is equal to s if I call this as C_3, sC_3 and Y_4 is nothing, but this and this is Y_5 . So, you call this one as R_4, R_5 . So, Y_4 is $\frac{1}{R_4}$ which is G_4 this is $\frac{1}{R_5}$ which is G_5 .

If I substitute these values in this transfer function $\frac{v_o}{v_i}$ this is equal to $\frac{-Y_1 Y_2}{Y_2 Y_3 + Y_1 Y_5 + Y_2 Y_5 + Y_3 Y_5 + Y_4 Y_5}$
 So, if you substitute these values what happen? $Y_1 Y_2, Y_1 Y_2$ becomes $sC_2 G_1 - sC_2 G_1$ divided by $Y_2 Y_3, sC_2, sC_3, s^2 C_2 C_3$ plus $Y_1 Y_5, Y_1 Y_5$ is $G_1 G_5, Y_2 Y_5, sC_2 G_5, Y_3 Y_5, sC_3 G_5, Y_4 Y_5, G_4 G_5$. Now, I will write this in the form of a transfer function of a parallel RLC circuit. So, I will discuss what is that parallel RLC circuit. That transfer function will be in the form of $-G_1$ by this will be s and $\frac{1}{s}$.

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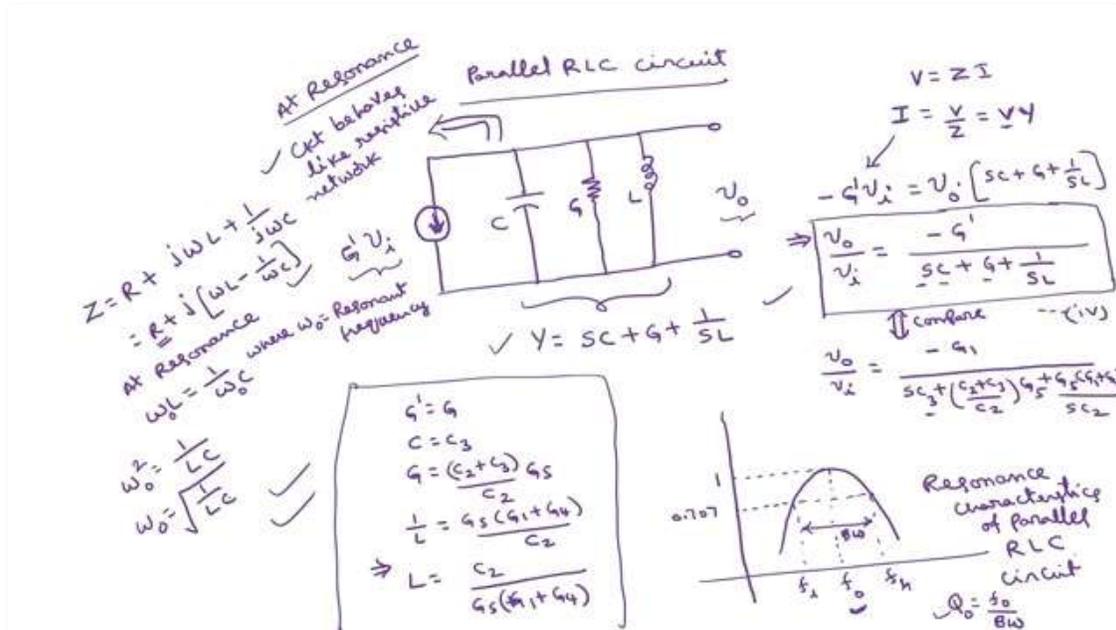
So, in order to get this transfer function in the form of the transfer function of parallel RLC circuit I will take sC_2 common. So, if I take sC_2 common in the numerator we will get $-G_1$ divided by we divide each and every term in the denominator by sC_2 . So, this one s and one C_2 will get cancelled what is left here? s times C_3 . This is nothing but $s^2 C_2 C_3$ we are going to divide with sC_2 . So, $C_2 C_2$ get cancelled one s get cancelled sC_3 plus what is the coefficient of s here in these two terms we have s .

So, if I take s as common here and of course, G_5 also common. So, $s(C_2 + C_3)G_5$ this if I divide with sC_2 , $s s$ get cancelled we will get simply $(C_2 + C_3)G_5$ divided by C_2 . So, this term is over these two terms are over. Now, there are two terms which are left this one this one. So, that is G_5 is common and $(G_1 + G_4)$ this you have to divide with sC_2 .

So, we will get this plus $G_5(G_1 + G_4)$ by sC_2 . This is the expression for $\frac{v_o}{v_i}$. Let us call this as equation (iii). I will come to this expression later after discussing the parallel RLC circuit. So, I will compare these two circuits and accordingly we will derive the expressions for the center frequency, Q factor and bandwidth. These are the three important parameters of a band pass filter.

This is a parallel RLC circuit you might have studied this in your circuit theory course. So, you call this one as $G'v_i$ is the current source and we have capacitance C and resistance in parallel circuit is G which is reciprocal of resistance $G = \frac{1}{R}$ then we have inductor L and you are taking the output across this. So, what is the relation between this input current $G'v_i$ output voltage v_o and this admittance. So, what is this overall admittance Y ? is equal to $sC + G + \frac{1}{sL}$. So, we know that $V = ZI$ in terms of impedance, but what is I ? is equal to $\frac{V}{Z}$, $\frac{1}{Z}$ is nothing, but VY .

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So, the current is equal to voltage into admittance. Here this current is opposite direction. So, I will take minus of this one. So, this current I will take here as $-G'v_i$ this is the current. What is V ? v_o this is equal to v_o . What is Y ? is this $(sC + G + \frac{1}{sL})$. This is the I mean relation of this parallel RLC circuit. So, from here what is $\frac{v_o}{v_i}$? $\frac{-G'}{sC + G + \frac{1}{sL}}$. This we call equation (iv). Now the transfer function of band pass filter is in a similar manner to that of parallel RLC circuit. So, by comparing these two. So, I will rewrite this expression of band pass filter here.

This is $\frac{-G_1}{sC_3 + (\frac{C_2 + C_3}{C_2})G_5 + \frac{G_5(G_1 + G_4)}{sC_2}}$. So, we compare these two now. By comparing this G' is equal to G numerator and the factor of s is C here C_3 here. So, C is equal to C_3 and G will be $\frac{(C_2 + C_3)}{C_2} G_5$ and $\frac{1}{s}$ is this one $\frac{1}{L}$ is $\frac{G_5(G_1 + G_4)}{C_2}$ implies what is L ? $\frac{C_2}{G_5(G_1 + G_4)}$. So, by comparing these are the relation between parallel RLC circuit and the narrow band band pass filter.

Now there are some important parameters of this parallel RLC circuit. This parallel RLC circuit resonates under some conditions. So, if I plot this current through this parallel RLC circuit this will be resonates something like this. So, this occurs at some frequency say f_0 this is unity and at 0.707 if the frequencies are f_l and f_h .

This is higher cutoff frequency, lower cutoff frequency, this is central frequency and this is bandwidth. This is the resonance characteristics of parallel RLC circuit. Now in band pass filter also we require similar type of the characteristics that is why we are comparing these two. First I will derive the expressions for this f_0 bandwidth and quality factor which is defined as $\frac{f_0}{\text{bandwidth}}$. If I call this as Q_0 . So, I will derive the first these three important parameters one is the resonant frequency or this is corresponding to central frequency in case of band pass filter and then quality factor and then bandwidth.

First, I will derive the expressions for this parallel RLC circuit. Now by using this substitution I will find out the corresponding parameters for the narrow band pass filter. So, at resonance what happens is the circuit becomes resistive. This circuit becomes resistive at resonance. So, when does the circuit becomes resistive network? the imaginary term has to be become 0.

So, the general expression for the impedance Z is given by $R + j\omega L + \frac{1}{j\omega C}$ or this is equal to R plus j times if I take this j to the numerator this becomes minus ωL minus $\frac{1}{\omega C}$. So, in order to behave this as a resistance the imaginary part has to be 0. So, at resonance what happens ωL is equal to $\frac{1}{\omega C}$. If I call this resonant frequency as ω_0 then this is ω_0 where ω_0 is the resonant frequency. So, what is the expression for the resonant frequency? $\omega_0^2 = \frac{1}{LC}$ or $\omega_0 = \sqrt{\frac{1}{LC}}$. This is the resonant frequency of parallel RLC circuit.

Now, what is the resonant frequency? The resonant frequency in case of parallel RLC circuit is similar to the central frequency of band pass filter. Therefore, the central frequency of band pass filter call as band pass filter is given by this LC is parameters of this parallel RLC circuit. Whereas, what are the parameters of the band pass filter? These R_1, C_2, C_3, R_4, R_5 . So, how to express this central frequency in terms of these parameters? Resonant frequency you can express in terms of these parameters.

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∴ The central frequency of BPF is given by $\omega_0 = \frac{1}{\sqrt{LC}}$

Quality factor $Q_0 = \frac{\omega_0 R C}{1} = \frac{\omega_0 C}{G}$

Bandwidth Parallel RLC $Q_0 = \frac{f_0}{BW} \Rightarrow BW = \frac{f_0}{Q_0} = \frac{\omega_0}{2\pi Q_0} \Rightarrow BW = \frac{\omega_0 G}{2\pi \omega_0 C} = \frac{G}{2\pi C}$

BPF $Q_0 = \frac{\omega_0 C_3}{(C_2 + C_3) G_5} \Rightarrow Q_0 = \frac{\omega_0 C_2 C_3}{(C_2 + C_3) G_5}$

BPF: $BW = \frac{G}{2\pi C} = \frac{(C_2 + C_3) G_5}{2\pi C_2 C_3}$

For $C_2 = C_3 = C$ $Q_0 = \frac{\omega_0 C^2}{2C G_5} = \frac{\omega_0 C}{2G_5}$
 $\omega_0 = \frac{\sqrt{G_5(G_1 + G_4)}}{C}$ $BW = \frac{2CG_5}{2\pi C^2} = \frac{G_5}{\pi C}$

So, in order to get this central frequency in terms of R's and C's. So, I will use this notation. So, what is L and what is C here? This is L and C. L is $\frac{C_2}{G_5(G_1 + G_4)}$ and C is C_3 . So, L is $\frac{C_2}{G_5(G_1 + G_4)}$. This is 1 by square root of $\frac{C_2}{G_5(G_1 + G_4)}$ and C is C_3 . So, this is equal to now $\sqrt{\frac{G_5(G_1 + G_4)}{C_2 C_3}}$. So,

$G_1 G_2 G_1$ is nothing, but $\frac{1}{R_1}$. Similarly, $G_4 \frac{1}{R_4}$, $G_5 \frac{1}{R_5}$. So, if I substitute this, we will get this expression in terms of R's and C's. This R's and C's you have expressed with the central frequency of this band pass filter. This is the one important parameter. Then I will discuss about the quality factor. First, I will discuss about the parallel RLC circuit. Then I will substitute the corresponding values in the parallel RLC circuit expression to get the quality factor of band pass filter.

So, for parallel RLC circuit, how do you define the quality factor? Q_o can be defined as $\omega_o RC$. This is one of the standard expression which you might have used in your circuit theory. Otherwise also you can remember that the quality factor of parallel RLC circuit is ω_o times RC. Okay. Now, what will be this for the band pass filter? ω_o , what is R and what is C? This you can also express as because R is not there, G is there.

So, $\frac{C}{G}$, $\frac{1}{R}$ is G. So, this is ω_o , C becomes C_3 and what is G? C is C_3 . G is $\frac{(C_2+C_3)}{C_2} G_5$. This is your G. So, if I substitute this, what will be quality factor? $\frac{\omega_o C_2 C_3}{(C_2+C_3) G_5}$. This is the quality factor of band pass filter. This is the central frequency of band pass filter. Then the third parameter is bandwidth or parallel RLC circuit, what is bandwidth and what is corresponding to band pass filter? So, relation between the Q factor and bandwidth is this is central frequency divided by bandwidth implies what is bandwidth $\frac{f_o}{Q_o}$. That what is f and ω is $2\pi f_o$ implies what is f_o ? $\frac{\omega_o}{2\pi}$. This is equal to $\frac{\omega_o}{2\pi Q_o}$. What is Q_o ? is this If I substitute this one implies bandwidth of parallel RLC circuit is $\frac{\omega_o}{2\pi}$ and Q_o is $\frac{\omega_o C}{G}$. So, $\omega_o \omega_o$ will get cancelled. This will get $\frac{G}{2\pi C}$. Now what is corresponding to band pass filter bandwidth? $\frac{G}{2\pi C}$ but what is G? G is this, C is C_3 , $\frac{(C_2+C_3)}{C_2} G_5$ this is G divided by 2π into C becomes C_3 . So, implies bandwidth is equal to $\frac{(C_2+C_3)}{2\pi C_2 C_3} G_5$. So, here the three important parameters of a band pass filter whose response which we have discussed in the last lecture that will be something like this. This is exactly similar to that of parallel RLC circuit.

This is your center of frequency. This is unity. This is f_l . This is f_h and bandwidth is $f_h - f_l$. This is 0.707. So, the expression for this f_o or $\omega_o 2\pi f_o$ is this and bandwidth is this and the quality factor which is given by f_o by bandwidth is this. For the sake of simplicity if I assume that this $C_3 = C_2$ then what happens to these three expressions? For a special case of $C_2 = C_3 = C$ what is ω_o ? $C_2 = C_3$ means C^2 . So, simply this is square root of $G_5(G_1 + G_4)$ divided by C^2 becomes C outside the root and what happens to quality factor? $\omega_o C$ becomes C^2 divided by $C_2 + C_3$ becomes $2C G_5$. So, this is equal to one C, one C get cancelled, $\frac{\omega_o C}{2G_5}$ and what is bandwidth? This is $\frac{2C G_5}{2\pi C^2}$. This becomes C to C^2 . This is $2C$. This is $2C$, $2C$ get cancelled. You will get $\frac{G_5}{\pi C}$. So, these are the important expression that you have to use to design the narrow band band pass filter.

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$$H(s) = \frac{-A_o \left(\frac{\omega_o}{Q}\right) s}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2}$$

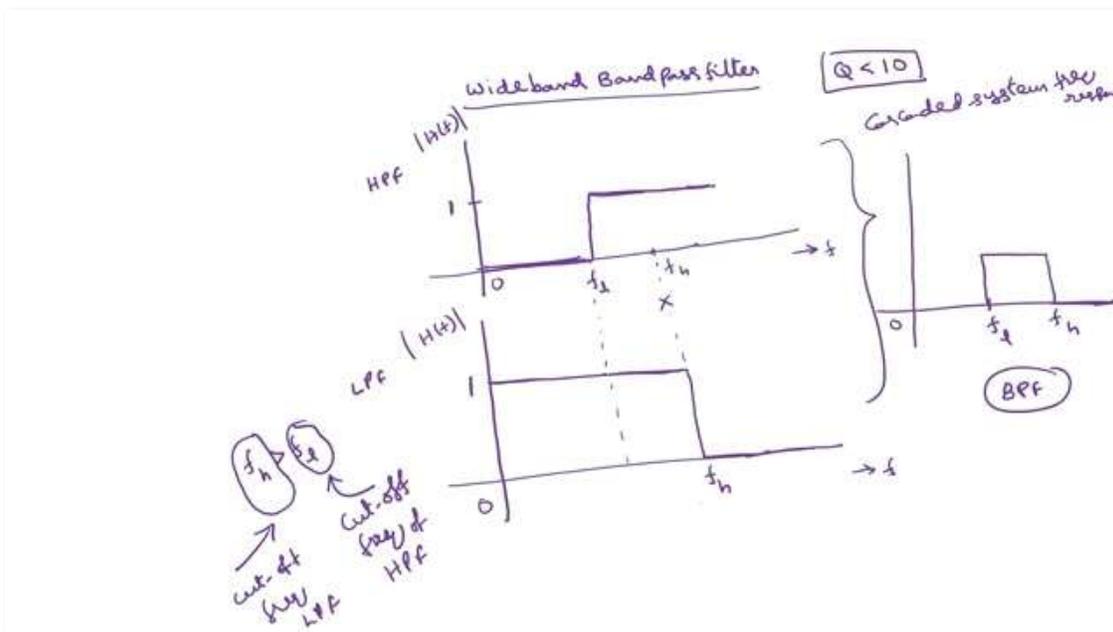
Generalized
expression
for TF of narrow band
BPF

$$= \frac{-A_o \omega_o \zeta s}{s^2 + \omega_o \zeta s + \omega_o^2}$$

$$\zeta = \frac{1}{Q}$$

And the generalized expression for this narrow band pass filter is $H(s)$ will be in the form of $-A_o$, A_o is the open loop gain which is $1 + \frac{R_F}{R_1}$, $\left(\frac{\omega_o}{Q}\right) s$ divided by $s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2$. This is generalized expression for transfer function of narrow band band pass filter where the damping factor zeta is $\frac{1}{Q}$. So, if you want to express this in terms of the damping factor minus $A_o \omega_o$ zeta s divided by s^2 plus ω_o zeta s plus ω_o^2 . This is about this narrow band band pass filter.

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Similarly, we can derive wide band band pass filter. This Q is less than 10. For narrow band Q is greater than 10. So, the design of wide band band pass filter is relatively easy when compared with the narrow band band pass filter. A wide band band pass filter can be obtained by cascading a low pass filter with high pass filter. So, what is the transfer function of ideal low pass filter? And what is the transfer function of ideal high pass filter? If I take this as high pass filter, this as low pass filter. High pass means it passes the high frequencies. So, up to some frequency 0 gain and a particular frequency onwards unity gain.

Low pass means it passes up to only some low frequencies. Then high frequencies will be rejected. Now, if I choose this value as f_h is the cutoff frequency of low pass filter, f_l is the cutoff frequency of high pass filter and we will take $f_h > f_l$. We know that if I cascade two systems, the resultant transfer function is product of those two transfer functions. So, if I cascade this low pass and high pass, what is the resultant frequency response? You simply have to multiply this, this by this. So, if I multiply this from 0 to f_l , this is 0, this is 1. So, 0 into 1 becomes 0. What will be overall response? From 0 up to f_l , this is 0, this is 1, 0 into 1 becomes 0. From f_l to f_h , this is also 1, f_l to f_h is somewhere here, this is also 1, this is also 1. So, f_l to f_h will be having unity. From f_h to infinity, this is 0, whereas this is 1 up to infinity, 1 into 0 becomes 0.

So, this is the characteristics of a band pass filter. But this technique is valid only for the wide band band pass filters okay. So, in order to obtain the wide band band pass filter, what you have to do is you have to cascade a high pass filter with low pass filter such that the cutoff frequency of low pass filter is greater than cutoff frequency of high pass filter. This is the cutoff frequency of low pass filter, this is cutoff frequency of high pass filter. So, in order to obtain the transfer function of first order band pass filter with wide band characteristics. So, we will take two first order low pass filter, one first order low pass filter and one first order high pass filter.

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Consider first order HPF

Let $f_c = \frac{1}{2\pi R_1 C_1} \Rightarrow 2\pi f_c R_1 C_1 = \frac{1}{R_1 C_1} \Rightarrow R_1 C_1 = \frac{1}{2\pi f_c} = \frac{1}{\omega_c}$

$\frac{V_o(s)}{V_i(s)} = H(s) = A_{ol} \frac{SR_1 C_1}{1 + SR_1 C_1}$

$H(j\omega) = A_{ol} \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}$

$= A_{ol} \frac{j(\omega/\omega_c)}{1 + j(\omega/\omega_c)}$

$|H(j\omega)| = A_{ol} \frac{(\omega/\omega_c)}{\sqrt{1 + (\omega/\omega_c)^2}} \Rightarrow$

$|H(j\omega)| = \frac{A_{ol} f/f_c}{\sqrt{1 + f^2/f_c^2}}$ where $A_{ol} = 1 + \frac{R_f}{R_1}$
 $f_c = \frac{1}{2\pi R_1 C_1}$

$\omega = 2\pi f$
 $\omega_c = 2\pi f_c$
 $\frac{\omega}{\omega_c} = f/f_c$

$V_o = V_i \left[1 + \frac{R_f}{R_1} \right] = V_i A_{ol}$

$V_i = V_o \frac{R_1}{R_1 + S C_1}$

$= V_o \frac{S R_1 C_1}{1 + S R_1 C_1}$

$V_o = A_{ol} \frac{S R_1 C_1}{1 + S R_1 C_1} V_o$

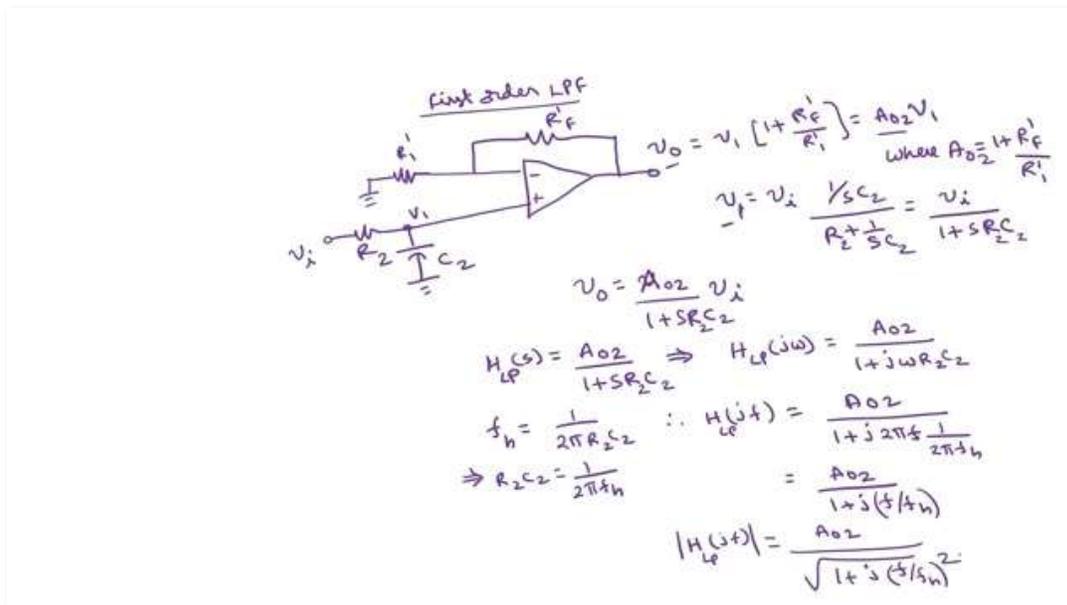
$\frac{V_o}{V_i} = A_{ol} \frac{S R_1 C_1}{1 + S R_1 C_1}$

So, what is the transfer function of the first order high pass filter which we have derived in the earlier lectures also. So, this will be something like this is resistance, this will be of course, multiplied with A_o . So, if I take the complete circuit this one, this is the circuit diagram of first order high pass filter. And what was the transfer function here whatever the voltage if I call as v_1 , v_o is equal to $v_1(1 + \frac{R_F}{R_1})$ which we will call as A_{o1} , we call as this as A_{o1} this is for high pass filter. So, what is v_1 in terms of v_i is equal to $v_i \frac{R}{R + \frac{1}{sC}}$, this is equal to $v_i \frac{sRC}{1+sRC}$. If I call these values as $R_1 C_1$ because I am going to cascade with low pass filter for that I will call as sC_2 and R_2 . So, this will be $R_1 R_1 C_1$ therefore, what is v_o is equal to A_{o1} into sR_1C_1 divided by $1 + sR_1C_1$ into v_o . So, $\frac{v_o}{v_i}$ is $A_{o1} \frac{sR_1C_1}{1+sR_1C_1}$. And we are defining let the cutoff frequency of high pass filter which will be lower value $\frac{1}{2\pi R_1C_1}$. Then what happens to this $\frac{v_o}{v_i}$ as a function of $j\omega$ is equal to $H(s)$ this is equal to $A_{o1} \frac{sR_1C_1}{1+sR_1C_1}$.

This is the transfer function and frequency response is $j\omega$ is equal to $A_{o1} \frac{j\omega R_1C_1}{1+j\omega R_1C_1}$. From here what is ω_l ? $2\pi f_l$ is equal to $\frac{1}{R_1C_1}$ or implies R_1C_1 is equal to $\frac{1}{2\pi f_l}$ we got this as $\frac{1}{\omega_l}$. So, this R_1C_1 is equal to $\frac{1}{\omega_l}$ divided by $1 + j(\frac{\omega}{\omega_l})$. So, what is the magnitude of $H(j\omega)$? is equal to $\frac{A_{o1}(\frac{\omega}{\omega_l})}{\sqrt{1+(\frac{\omega}{\omega_l})^2}}$ or 2π 2π get cancelled ω is $2\pi f$, ω_l is $2\pi f_l$.

So, what is $\frac{\omega}{\omega_l}$? is $\frac{f}{f_l}$. So, this implies magnitude of $H(j\omega)$ or jf we can call as now we are expressing in terms of the f . So, this is the expression for the low pass filter. We can call this one as LP otherwise where A_{o1} is $1 + \frac{R_F}{R_1}$ and f_l is $\frac{1}{2\pi R_1C_1}$. This is about the high pass filter.

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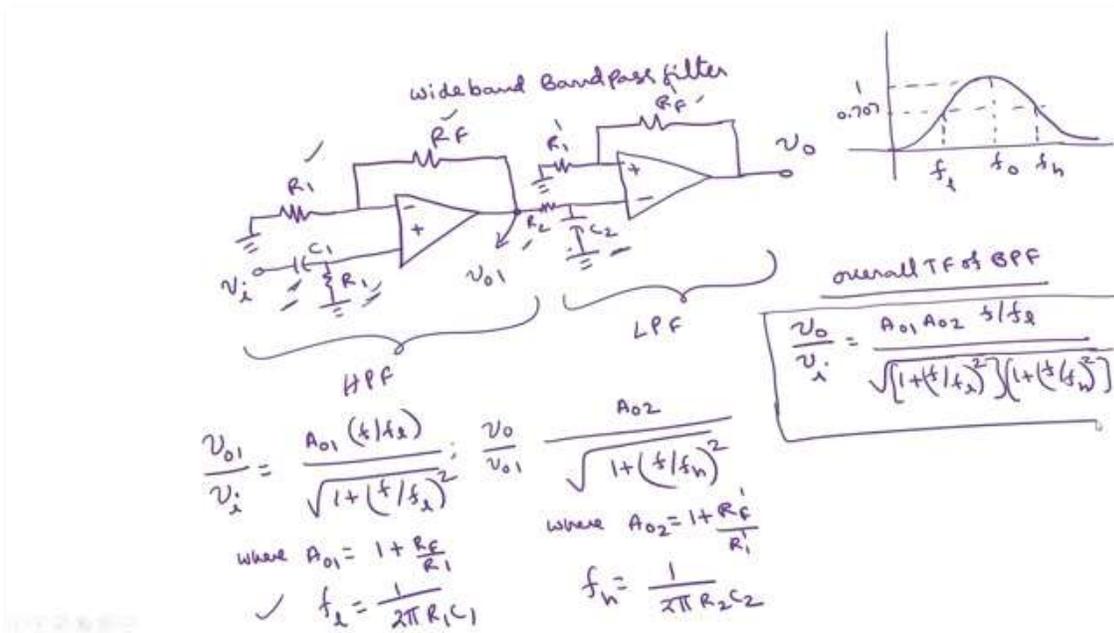
And similarly, if you take the low pass filter this actually I am repeating this I have already discussed in the earlier lectures. We have the continuity I am just repeating this. This is v_o we will call this as R'_F and this is R'_1 because for high pass filter we called as R_1 . For low pass filter you have to take voltage across capacitor. So, this is v_1 . So, $v_o = v_1 \left[1 + \frac{R'_F}{R'_1} \right]$ this you call as $A_{o2} v_1$ where A_{o2} is $\left[1 + \frac{R'_F}{R'_1} \right]$.

And what is v_1 ? is equal to $v_i \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$. So, sC sC will get cancelled is equal to $\frac{v_i}{1 + sRC}$. This is v_1 .

So, what is v_o ? A_{o2} times v_1 $A_{o2} v_1$ which is v_i by. So, what is the transfer function H I will call as HP sorry LP. And what is frequency response and you call this one as 2 2 because 1 1 we called as the RC section parameter values of high pass filter.

So, we will call this as $2's.1 + j\omega R_2 C_2$. Now we will define the cutoff frequency of low pass filter which is f_h is $\frac{1}{R_2 C_2}$. Therefore, what is $H(jf)$? I call as this f is equal to A_{o2} divided by $1 + j2\pi f$ and $R_2 C_2$ will be $\frac{1}{2\pi f_h}$ and this $R_2 C_2$ is. So, this $2\pi 2\pi$ will get cancelled will get $\frac{A_{o2}}{1 + j(\frac{f}{f_h})^2}$ or magnitude of low pass jf is equal to $\sqrt{\frac{A_{o2}}{1 + j(\frac{f}{f_h})^2}}$.

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Now, what will be the circuit diagram of wide band band pass filter? First order by cascading two first order systems. So, we will get this wide band band pass filter. So, which is first high pass filter? This is the high pass filter R_1, R_F this we called as $R_1 C_1$ this is input. This will be now given as input for first order low pass filter. This is the overall final output of band pass filter. We are calling this one as $R'_1 R'_F$ and this we are going to connect to RC section R here C here this we are calling as $R_2 C_2$.

So, this is high pass filter and this is low pass filter. So, what is the transfer function of this one which we have derived is this. This is high pass filter $\frac{A_{o1} \left(\frac{f}{f_l}\right)}{\sqrt{1 + \left(\frac{f}{f_l}\right)^2}}$ where A_{o1} is $1 + \frac{R_F}{R_1}$ this R_F this

R_1 and f_l is this $R_1 C_1 \frac{1}{2\pi R_1 C_1}$. Whereas, for low pass filter which we have derived here $\frac{A_{o2}}{\sqrt{1 + \left(\frac{f}{f_h}\right)^2}}$

where A_{o2} is also 1 by R_F , but here this is $\frac{R'_F}{R'_1}$ and f_h is $\frac{1}{2\pi R_2 C_2}$. So, this is going to decide the higher cutoff frequency of wide band pass filter. This is going to decide the lower cutoff frequency of wide band pass filter. Now, this will be wide band pass filter response. This is wide, this is center frequency, this is 1, this is 0.07, this f_l is going to be decide by these values and this f_h is going to decide by R_2 and C_2 . So, then what will be overall transfer function? Transfer function of band pass filter means simply you have to cascade these two.

So, overall is $\frac{v_o}{v_i}$. If I call this intermediate output as v_{o1} , then this will be $\frac{v_{o1}}{v_i}$ and this will be $\frac{v_o}{v_{o1}}$. So, to get $\frac{v_o}{v_i}$, this $v_{o1} v_{o1}$ will get cancelled if I multiply these two. This is equal to $A_{o1} A_{o2} \frac{f}{f_l}$ divided by square root of $\left[1 + \left(\frac{f}{f_l}\right)^2\right]$ into square root of or you can multiply here inside this here itself $\left[1 + \left(\frac{f}{f_h}\right)^2\right]$. This is how you can design the wide band band pass filter.

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Ex:- Design a bandpass filter with a passband gain of 4,
 lower cut-off frequency of 400Hz and higher cut-off
 frequency of 2KHz. Determine quality factor of this filter

Sol

$$A_{o1} A_{o2} = 4 \Rightarrow A_{o1} = 2 = 1 + \frac{R_F}{R_1}$$

$$\Rightarrow \frac{R_F}{R_1} = 1 \Rightarrow R_F = R_1 = 10K\Omega$$

$$A_{o2} = 2 = 1 + \frac{R'_F}{R'_1} \Rightarrow R'_F = R'_1 = 10K\Omega$$

$$f_l = 400 = \frac{1}{2\pi R_1 C_1}$$

$$\Rightarrow R_1 C_1 = \frac{1}{800\pi}$$

Let $C_1 = 0.001 \mu F$

$$\Rightarrow R_1 = \frac{1}{800\pi C_1} = \frac{1}{800\pi \times 0.001 \times 10^{-6}}$$

$$f_h = 2000 = \frac{1}{2\pi R_2 C_2}$$

$$\Rightarrow R_2 C_2 = \frac{1}{4000\pi}$$

Let $C_2 = 0.01 \mu F$

$$\Rightarrow R_2 = \frac{1}{4000\pi \times 0.01 \times 10^{-6}}$$

I will take one example. Design a band pass filter with a pass band gain of 4, lower cutoff frequency of 400 Hertz and higher cutoff frequency of 2 kiloHertz and also determine the

quality factor also. So, overall gain is 4. So, in this we have A_{o1} some gain, A_{o2} some gain. It is up to you can distribute equally or 1 and 4, it is up to you, but here I want to distribute this gain equally.

Two gain here in the low pass filter gain of 2, high pass filter with the gain of 2. So, A_{o1} is $A_{o1}A_{o2}$ should be 4. Let A_{o1} is equal to 2 which is equal to $1 + \frac{R_F}{R_1}$ implies $R_F = 1, \frac{R_F}{R_1}$ is equal to 1 implies R_F is equal to R_1 , we call this as 10 kilo Ohms and A_{o2} is also 2, this is equal to $1 + \frac{R'_F}{R'_1}$ implies again R'_F is equal to R'_1 this also if we choose 10 kilo Ohms. So, all the four values will be having same resistance 10 kilo Ohms each. Now lower cutoff frequency is going to be decide by $\frac{f_l}{R_1 C_1}, \frac{1}{2\pi R_1 C_1}$ equal to 400 Hertz is given $\frac{1}{2\pi R_1 C_1}$.

So, implies $R_1 C_1$ is equal to $\frac{1}{800\pi}$. As I have told in the last lecture also we have to choose some value of C_1 say 0.01 or 0.01 micro Farads, then we can find out the R_1 value from here.

R_1 is equal to $\frac{1}{800\pi C_1}$ that is equal to $\frac{1}{800\pi \times 0.01 \times 10^{-6}}$. So, we will get some value. Similarly, f_h is given as 2000 2 kilo Hertz $\frac{1}{2\pi R_2 C_2}$ implies $R_2 C_2$ is 4000π .

Let $R_2 C_2$ is equal to 0.01 or 0.001 it is up to you. You can choose any available value for this C_1 and C_2 , then you can compute $R_2 \frac{1}{4000\pi \times 0.01 \times 10^{-6}}$. So, whatever the value you will get you fix these values here for $R_1 C_1 R_2 C_2 R_1 R_F R'_1 R'_F$ you will get the design of band pass filter. So, this is about this band pass filters.

So, we have two types of the band pass filters wide band band pass filter narrow band band pass filter. For narrow band band pass filter you have to take two feedback paths and here also you can design in a similar manner. By properly choosing these R's and C's here we can design this filter with any cutoff frequencies whereas in case of wide band pass filter you have to cascade two first order sections, two filters one is low pass filter and other is high pass filter. In this example we have taken one first order low pass filter and one first order high pass filter. Similarly, you can cascade one second order high pass filter, one second order low pass filter to get the higher order band pass filters. So, next type of the filter is band reject filter that we will discuss in the next lecture. Thank you.