Integrated Circuits and Applications
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Active Filters I
Lecture - 18
Design of Butterworth High Pass Filter

In the last lecture I have discussed about this design of low pass filters. We started with the first order, then second order and in order to design the higher order filters we can cascade the first order and second order sections. Today we will discuss about the second type of the filter which is high pass filter. Here also first I will start with the first order, then I will discuss the second order and the design of higher order filters.

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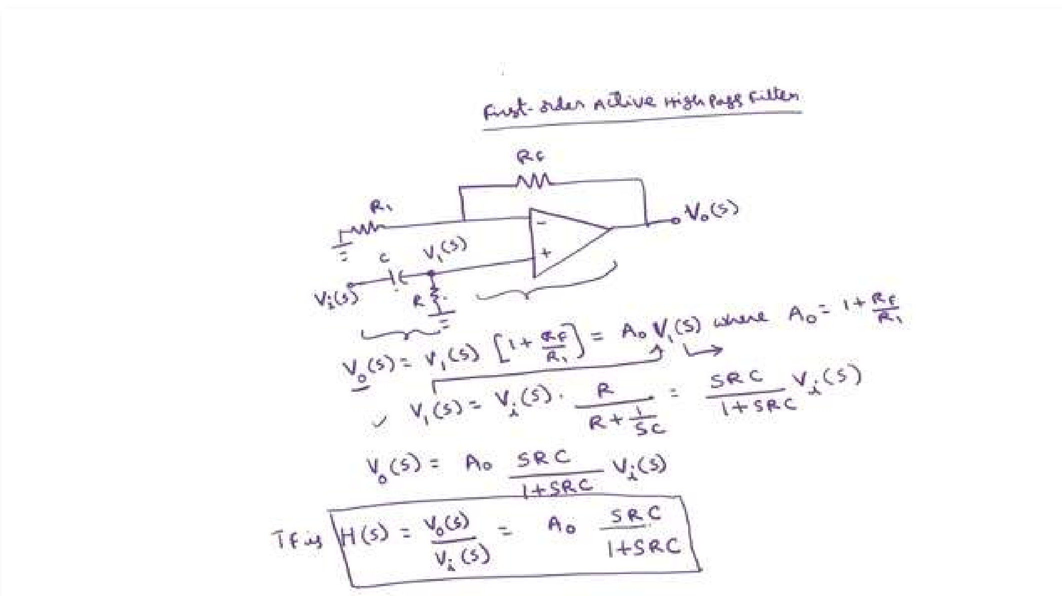

First order active high pass filter. So, the circuit diagram of this high pass filter can be obtained by just interchanging the capacitance and resistance of low pass filter. This is the feedback circuit, this is same as low pass filter, this is R_1, R_F whereas, here you have to exchange the R and C. So, for low pass filter here capacitance was there resistance was there, now this is reverse. This is the circuit diagram of the first order active high pass filter.

I will derive the transfer function I will show that this will acts as high pass filter. Let us assume that this is some $v_i(s)$, I will take all capital letters say $V_i(s)$, $V_o(s)$. If I know this $V_i(s)$, then output is simply this is non-inverting amplifier.

So, output $V_o(s)$ is given by $V_1(s)$ into 1 plus $\frac{R_F}{R_1}$. We define this 1 plus $\frac{R_F}{R_1}$ as A_o, this is A_o times $V_1(s)$, where A_o is closed loop gain $1+\frac{R_F}{R_1}$. Now, we have to express this V_1 in terms of V_i, so that we can take the ratio of V_{out} to V_{in}. So, what is $V_1(s)$ in this circuit? This is voltage divider, so $V_1(s)$ is given by total voltage which is $V_i(s)$ and we are going to take the voltage across the resistor. So, resistance divided by $R+\frac{1}{sC}$.

So, this is equal to $\frac{sRC}{1+sRC}$ into $V_i(s)$. This is $V_1(s)$. If you substitute this $V_1(s)$ here, then what is $V_o(s)$ is:

$$V_o(s) = A_o \frac{sRC}{1+sRC} V_i(s)$$

Then the transfer function is given by $H(s)$ which is the transfer function output Laplace transform by input Laplace transform. This is given by A_o times sRC divided by 1+sRC. So, you see the transfer function of the first order high pass filter and if you want to study the frequency characteristics, we can use the frequency response.

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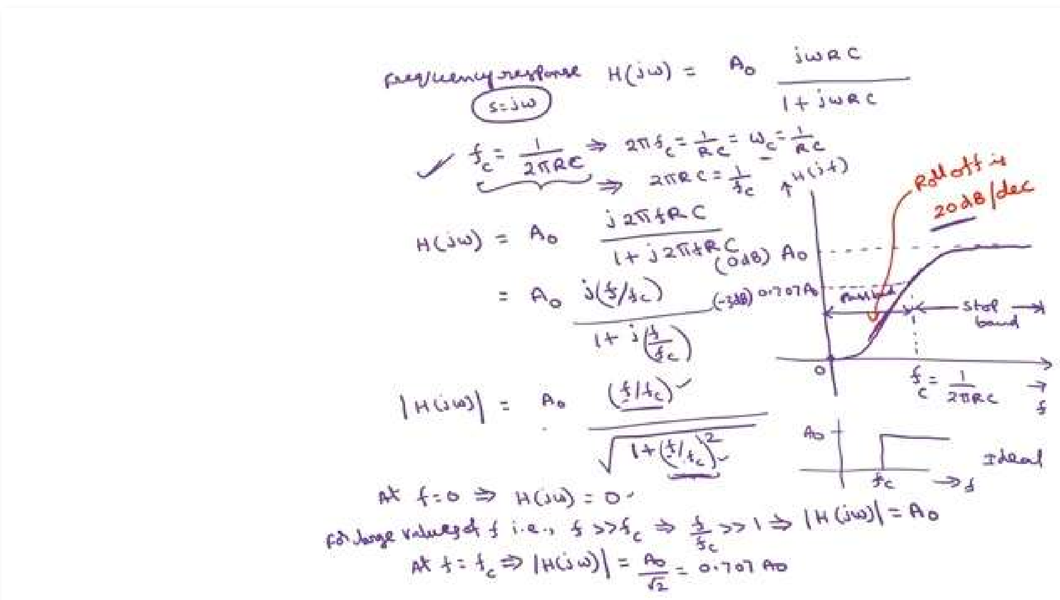

So, what is the frequency response? $H(s)$, s you have to replace with $j\omega$. So, right hand side also you have to replace s with $j\omega$. So, you will get sRC, s is $j\omega RC$ divided by $1+sRC, j\omega RC$. Now here I am going to define say f_c is the cutoff frequency, this is equal to $\frac{1}{2\pi RC}$. I am going to prove that f_c is $\frac{1}{2\pi RC}$ later.

If I assume that f_c is $2\pi RC$, then what is ω_c? 2π into f_c is equal to $\frac{1}{RC}$. This is 2π into f_c is nothing, but ω_c. So, either you can express in terms of the ω_c or f_c it is up to you. If I want to express in terms of f_c, what will be H of $H(j\omega)$? This RC becomes $\frac{1}{f_c}$. So, $H(j\omega)$ is equal to:

$$H(j\omega) = A_o \frac{j2\pi fRC}{1 + j2\pi fRC}$$

What is $2\pi RC$ from this? $2\pi RC$ is equal to $\frac{1}{f_c}$. So, if you substitute this here, this is:

$$H(j\omega) = A_o \frac{j\left(\frac{f}{f_c}\right)}{1 + j\left(\frac{f}{f_c}\right)}$$

This is the frequency response and if you take the magnitude of this frequency response. So, A_o numerator is $\frac{f}{f_c}$, magnitude of j is unity and the denominator is square root of 1 plus $\left(\frac{f}{f_c}\right)^2$.

You can easily see that at f is equal to 0, what will be the magnitude of $H(j\omega)$? f is 0 means this is 0, this is 0. So, 0 by 1 which is equal to 0. For large values of the f that is f is much much greater than f_c. So, what happens? This $\left(\frac{f}{f_c}\right)$ is much much greater than 1. So, I can neglect this one when compare with this $\left(\frac{f}{f_c}\right)$.

So, this is square root of $\left(\frac{f}{f_c}\right)^2$. So, this $\left(\frac{f}{f_c}\right)$, $\left(\frac{f}{f_c}\right)$ will get cancelled. So, implies you will get modulus of $H(j\omega)$ is simply A_o. You have this $\left(\frac{f}{f_c}\right)$ this $\left(\frac{f}{f_c}\right)$ square root of square will get cancelled.

So, now if I plot this response, this is $H(j\omega)$ or $j2\pi f$ in fact, is a function of f. So, you can also call as f also 2π is scaling. So, at f is equal to 0, this is having 0 value. So, this starts with 0. For larger values this becomes A_o somewhere here this is A_o. So, this will move in this fashion, it will finally reach at high frequencies A_o.

And also, you can see that at f is equal to f_c. So, this will be modulus of $H(j\omega)$ this $\left(\frac{f}{f_c}\right)$ becomes unity, this becomes unity, 1 plus 1 is 2, $\frac{1}{\sqrt{2}}$. This is A_o by $\sqrt{2}$, this is 0.707 times A_o.

So, this is 0.70 times A_o this will be f_c whose expression is you have taken this one as $\frac{1}{2\pi RC}$. So, if I take this f_c then you will get 0.707 times A_o value at this particular cutoff frequency. So, if I take in terms of the dB this will be 0 dB, if I start with 0 dB this will be $-3\ dB$. So, the cutoff frequency will be normally defined as the frequency at which the gain drops by 3 dB.

And if I take this roll-off ratio of this normally this will call as pass band and this will call as stop band. So, in the pass band if I take the roll-off ratio. So, this roll-off ratio of this slope is 20 dB per decade. If I take the ideal frequency response it should be something like this. So, in order to get this response near to the ideal response you have to increase the roll-off in this pass band.

So, in order to get the double roll-off we have to go for the second order filter this is first order filter. So, we have to go for the second order filter.

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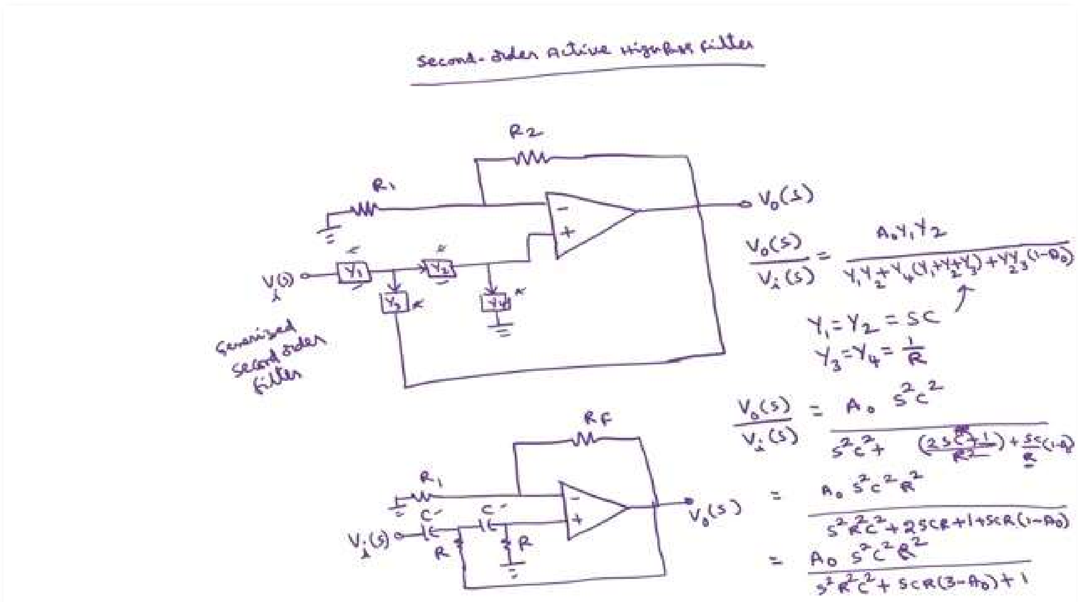

So, if I consider the second order high pass filter, we will be having two sections of RC. So, in general I have taken the diagram of second order filter in terms of Y_1, Y_2, Y_3. This is the diagram. This is $V_i(s)$, this is $V_o(s)$ this we called as Y_1, Y_2, Y_3, Y_4. For this we have derived the expression for $\frac{V_o(s)}{V_i(s)}$ in the previous lectures. But the values of these Y_1, Y_2, Y_3, Y_4 will be different for low pass and high pass filters. Here this will be having C or C or in case of high pass filter and this will be reverse in case of low pass filter. So, what is derivation? This is the one which we have derived. This is the Y_1, Y_2, Y_3, Y_4. I will take this same as it is A0 into Y_1Y_2 divided by Y_1Y_2 plus Y_4 into Y_1 plus Y_2 plus Y_3 plus Y_2Y_3, 1 minus A_o right.

$$\frac{V_o(s)}{V_i(s)} = \frac{A_oY_1Y_2}{Y_1Y_2 + Y_4(Y_1 + Y_2 + Y_3) + Y_2Y_3(1 - A_o)}$$

So, you saw the expression that we have derived already. Now, here for high pass filter, this is the generalized one. For high pass filter this circuit becomes.

So, here we will be having capacitance, here we will be having resistance, here capacitance, here resistance. So, what is Y_1? is Y_2 also because this is Y_1, Y_2 both are capacitances, this is simply sC and Y_2, Y_3 is equal to Y_4 which is equal to $\frac{1}{R}$. So, if I substitute these values here, what will be the transfer function $\frac{V_o(s)}{V_i(s)}$? is equal to A_o into sC into sC, s^2C^2 divided by again s^2C^2 plus Y_4 is $\frac{1}{R}$, Y_1 plus Y_2 plus Y_3 is $2sC$ plus $\frac{1}{R}$ plus Y_2Y_3, Y_2Y_3 is $\frac{sC}{R}$ into 1 minus A_o.

$$\frac{V_o(s)}{V_i(s)} = \frac{A_o s^2 C^2}{s^2C^2 + \frac{1}{R}(2sC + \frac{1}{R}) + \frac{sC}{R}(1 - A_o)}$$

So, if I further simplify this, what will be LCM in the denominator $A_o s^2 C^2$. So, this R you can take here sCR and this you can take overall divided by R, this is sCR. Now, this R becomes R^2. So, $\frac{1}{R^2}$ is the LCM in the denominator that will go to the numerator. So, you will get here $s^2\ R^2\ C^2$ plus $2sCR$ plus 1 and here you will get R is there. So, we will get one R in the numerator sCR into 1 minus A_o.

$$\frac{V_o(s)}{V_i(s)} = \frac{A_o s^2 C^2 R^2}{s^2R^2C^2 + 2sCR + 1 + sCR(1 - A_o)}$$

So, it will be coefficient of *s* in the denominator, this is $2sCR$, this is sCR becomes $3sCR$. So, sCR if I take as a common, we will get this one as $A_o s^2 C^2 R^2$ divided by $s^2R^2C^2$ plus sCR if I take as common, we will get $(3 - A_o)$ plus 1.

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$$\Rightarrow H(s) = \frac{V_o(s)}{V_i(s)} = A_o \frac{s^2}{s^2 + \frac{(3-A_o)}{RC}s + \frac{1}{R^2C^2}}$$

$$\omega_c = \frac{1}{RC} \text{ (or) } f_c = \frac{1}{2\pi RC}$$

$$H(j\omega) = \frac{A_o (j\omega)^2}{(j\omega)^2 + \frac{3-A_o}{RC}(j\omega) + \frac{1}{R^2C^2}} = \frac{-A_o\omega^2}{-\omega^2 + (3-A_o)j\left(\frac{\omega}{\omega_c}\right) + \omega_c^2}$$

$$|H(j\omega)| = \frac{A_o\omega^2}{\sqrt{[\omega_c^2 - \omega^2]^2 + (3-A_o)^2\left(\frac{\omega}{\omega_c}\right)^2}} = \frac{A_o}{\sqrt{\left(\frac{\omega_c^2}{\omega^2} - 1\right)^2 + (3-A_o)\frac{1}{\omega\omega_c}}}$$

$$\sqrt{(0-1)^2 + 0} = \sqrt{1} = 1$$

At $\omega = 0 \Rightarrow |H(j\omega)| = 0$

At $\omega = \infty \Rightarrow |H(j\omega)| = A_o$

$\Rightarrow$ Highpass filter

For Butterworth filter $\xi = 3 - A_o = \sqrt{2}$

If I take R^2C^2 also common, $H(s)$ which is equal to $\frac{V_o(s)}{V_i(s)}$ is equal to if I take R^2C^2 as common this R^2C^2 get cancelled, this becomes $\frac{1}{RC}$, this becomes $\frac{1}{R^2C^2}$. So, this will be $A_o\ s^2\ A_o$ into s^2 divided by s^2 plus then sRC it becomes $\frac{(3-A_o)}{RC}$ times s plus $\frac{1}{R^2C^2}$ and this will be $\frac{1}{RC}$. So, $\frac{(3-A_o)}{RC}$ into s plus $\frac{1}{R^2C^2}$.

$$H(s) = \frac{V_o(s)}{V_i(s)} = A_o \frac{s^2}{s^2 + \frac{(3 - A_o)}{RC}s + \frac{1}{R^2C^2}}$$

Here also the cutoff frequency ω_c is equal to $\frac{1}{RC}$ or f_c is equal to $\frac{1}{2\pi RC}$. We can easily check that at this particular frequency this gain $\frac{V_o(s)}{V_i(s)}$ will be $\frac{1}{\sqrt{2}}$ times that of the gain at ω is equal to 0. Then what will be this $H(s)$? A_o s square is nothing, but $j\omega$ square if I take $j\omega$ here the frequency response divided by $j\omega$ square plus $\frac{(3-A_o)}{RC}$ $j\omega$ plus $\frac{1}{R^2C^2}$ $.-A_o$ j^2 is -1 ω^2divided by $-\omega^2$ plus $(3-A_o)$ into j, $\frac{1}{RC}$ nothing, but ω_c ω by ω_c plus $\frac{1}{R^2C^2}$ is nothing, but ω^2.

$\Rightarrow H(j\omega) = \frac{-A_o\omega^2}{-\omega^2+(3-A_o)j\left(\frac{\omega}{\omega_c}\right)+\omega_c^2}$. So, what will be the magnitude of this $H(j\omega)$? is equal to $A_o\omega^2$ divided by square root of real part is ω_c^2 minus ω^2 whole square plus $3-A_o$ into $\frac{\omega}{\omega_c}$ whole square.

$$|H(j\omega)| = \frac{A_o\omega^2}{\sqrt{(\omega_c^2-\omega^2)^2 + (3-A_o)\left(\frac{\omega}{\omega_c}\right)^2}}$$

We can easily see that at ω is equal to 0 we will get A_o. So, if this ω is equal to 0 regardless of this denominator modulus of $H(j\omega)$ is 0. At ω is equal to infinity you can see that this ω^2 if I take as common this can be written as A_o divided by this ω^2 whole square if I take outside this will be omega square this omega square omega square will get cancel this omega square will get cancel.

So, we will get this one as $\sqrt{\left(\frac{\omega_c^2}{\omega^2}-1\right)^2 + (3-A_o)\frac{1}{\omega\omega_c}}$. So, at ω is equal to infinity this is 0, $(0-1)^2$ is 1, this is 0. So, this is simply 1. So, this is equal to A_o. This is 0, this is 1. So, this square root becomes this is $(0-1)^2+0$ which is nothing, but square root of 1 which is 1. So, only numerator A_o. So, this will acts as a high pass filter.

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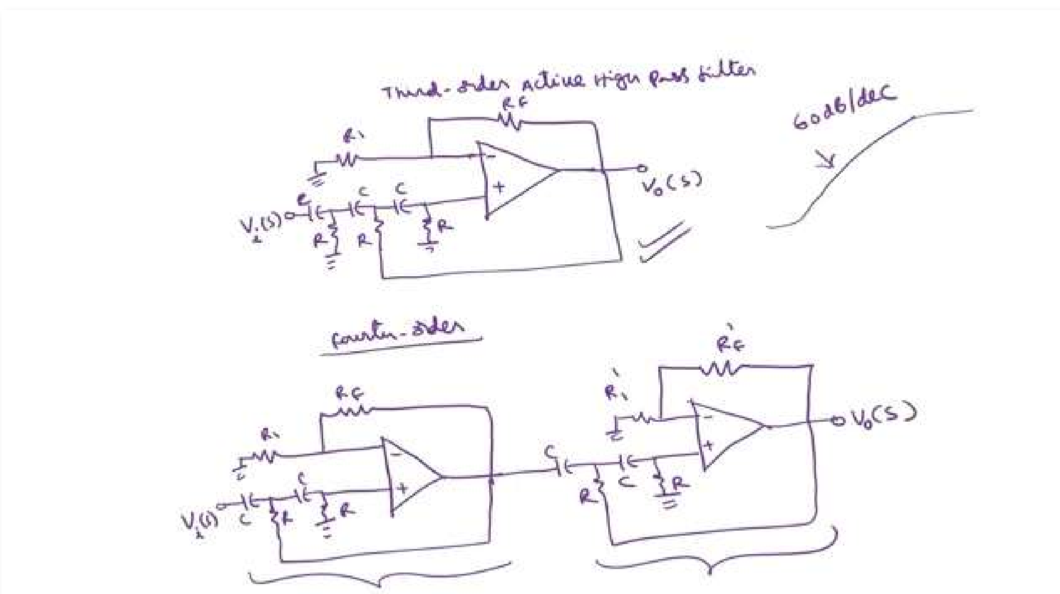

For Butterworth filter as you have derived in the previous lectures, this is the damping factor zeta is given by $3 - A_o$ this is equal to $\sqrt{2}$ for the second order filter. Similarly, if I want to design the higher order we can cascade the lower order filters.

If I take the third order active high pass filter there will be 3 RC sections this is the circuit diagram. I am not going to derive the transfer function, but this circuit diagram will be of this form.

This is capacitance resistance capacitance resistance capacitance resistance this will be grounded this will be grounded this will be connected to the output. So, the advantage of this one is the roll-off rate will here will be in the pass band this roll-off rate will be minus 60 dB per decade and second order it is 40 dB per decade.

And if I want the fourth order you can cascade two second order sections. So, this is one second order section then you cascade to another second order section. This is $R_1{}'$ $R_F{}'$ the gains may be different this is overall input this is overall output this is one second order this is another second order overall fourth order. This even if you want to design you can design using cascading of one second order and first order. So, like that higher order filters can be realized by cascading of the lower order filters. So, you can design these filters in a similar manner I will take one small example here.

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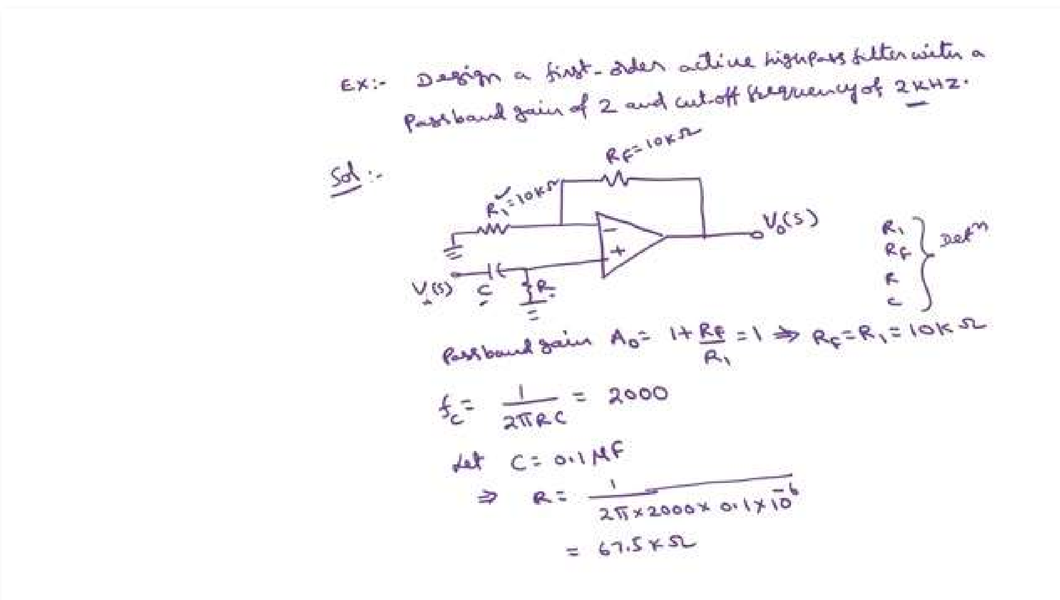

Design a first order active high pass filter with a pass band gain of 2 and cutoff frequency of 2 kilo Hertz. So, designing in the sense you have to find out all the component values. This is circuit diagram.

You have to find out $R_1, R_F, R\ and\ C$. Finding these values is nothing, but design. Determining these values is nothing, but design. So, what are the values specifications that are given pass band gain is nothing, but $A_o = 1 + \frac{R_F}{R_1}$ this is given as 1 implies $R_F = R_1 = 10k\Omega$. You can take any value which is available in your laboratory normally 10 kilo ohms. Now, take the large value because this is going to affect the input resistance of the circuit. Okay! So, R_1 I am setting to this 10 K even you can take larger value also this is also 10 k.

Then to find out the R and C we have the second specification which is cutoff frequency. So, we know that the cutoff frequency of either low pass or high pass filter is same expression $\frac{1}{2\pi RC}$. So, this is given as 2000. Let $C = 0.1\,\mu F$ in this design first you have to assume the capacitance values because if I get any other values which are not like 0.1 and 0.01 and all. So, it becomes difficult to design the filter that is why you have to start with the available capacitance values then you find out the resistance we can find out the resistance value by series and parallel combinations of the available resistors. So, implies what is R? $\frac{1}{2\pi \times 2000 \times 0.1 \times 10^{-6}}$ this I think you will get $67.5k\Omega$. So, similarly you can design the second order filter also, fourth order filter also. For the second order filter we have to take damping factor as $\sqrt{2}$. We have discussed about the design of second order and higher order low pass filter here also the design procedure is exactly same.

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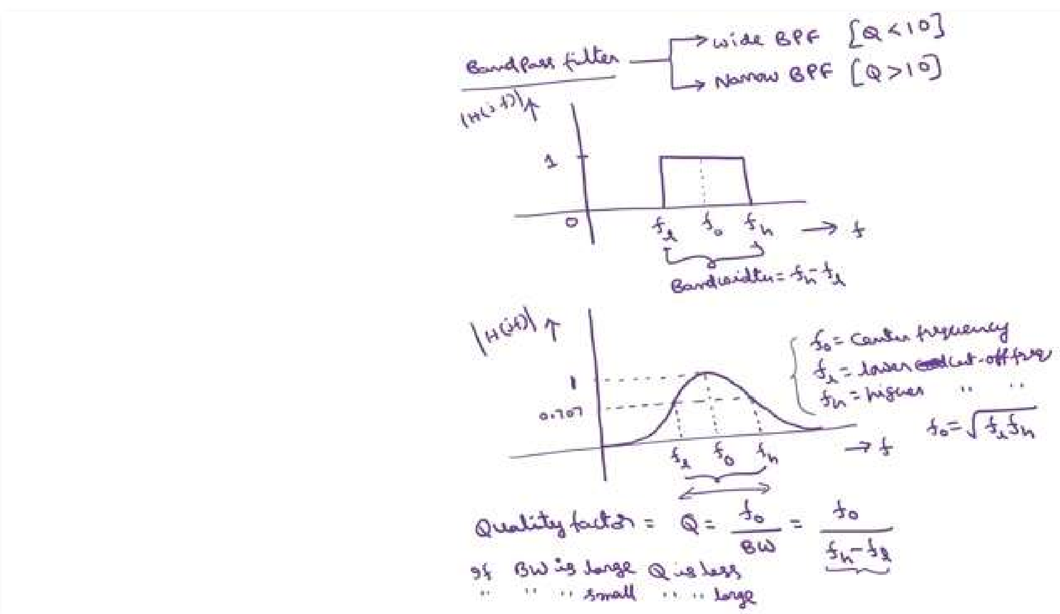

So, next the third type of filter is called band pass filter. So, in many of the communication applications so this band pass filter is useful. If I take the ideal band pass filter it allows a band of frequencies. If I take the frequency response magnitude of H of j omega jf versus frequency this is unity this is 0 this is f_l and this is f_h. f_l is called lower cutoff frequency, f_h is called higher cutoff frequency and the middle value we can call as center frequency this is center of these two. So, this difference is called as bandwidth.

f_h is larger so, $f_h - f_l$. This is the ideal one, practically it will be something like this. With $f = f_o$ center frequency this is maximum normally unity or it can be A_o and at $\frac{1}{\sqrt{2}}$, 0.707 times of this peak value this particular frequency is called lower cutoff frequency this is called higher cutoff frequency. So, f_o is called center frequency, f_l is lower cutoff frequency and f_h is called higher cutoff frequency. And there is one important term like quality factor is going to decide how effective is this bandpass filter also can be called as a figure of merit.

This Q is given by f_o divided by bandwidth BW, $\frac{f_o}{BW} = \frac{f_o}{f_h - f_l}$. And the relation between the these three are $f_o = \sqrt{f_l f_h}$. So, depends upon whether this bandwidth is large or small we have wide bandpass filter and narrow bandpass filter. BPF is bandpass filter. If bandwidth is large what happens Q? denominator is large means Q is less.

Bandwidth is large means this is wide bandpass filter. For a wide bandpass filter as the name implies bandwidth will be large, but Q will be less. So, normally Q less than 10 is called as wide bandpass filter. If bandwidth is small means Q is large. So, Q greater than 10 is called as narrow bandwidth. For narrow bandwidth filter Q is greater than 10 and wide bandpass filter Q is less than 10.

So, we will discuss the design of the narrow band and wide bandpass filters in the next lecture. Thank you.