Integrated Circuits and Applications
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Active Filters I
Lecture - 17
Design of Butterworth Low Pass Filter

Okay, In the last lecture we have discussed about the second order low pass filter and we have seen that the second order low pass filter will be having $-40dB$ per decade roll-off in the transition or stop band. And, for different damping factors we have plotted the frequency response and we found that Butterworth approximation is more suitable for many of the practical applications because of the maximally flat frequency response in the pass band. So, we will discuss now the transfer functions of Butterworth approximation or Butterworth filter.

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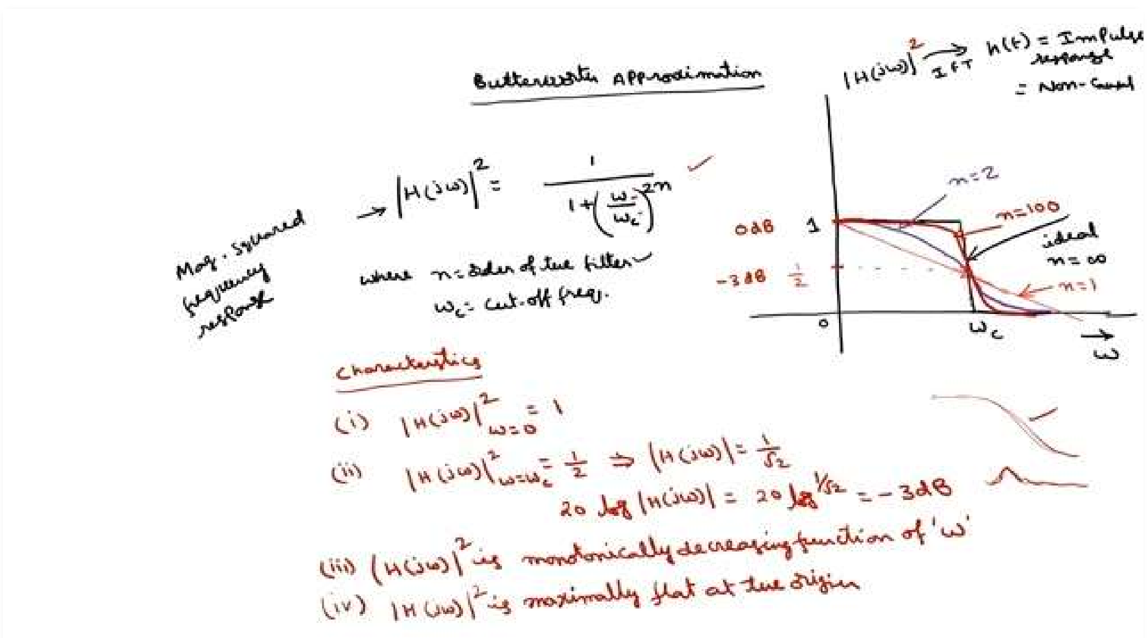

Butterworth approximation and the filter which we are going to derive using this Butterworth approximation are called as Butterworth filter. So, the generalized frequency response of n^{th} order Butterworth approximation is given by:

$$|H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}$$

where,
$|H(j\omega)|^2$ *= Magnitude squared frequency response,*
n = order of the filter
ω_c *= cut-off frequency*

Why we need this type of approximations is if I take the ideal low pass filter frequency response it will be something like this up to ω_c, it should have unity gain and beyond the ω_c, 0 gain. This is the magnitude of $H(j\omega)$ as a function of ω. This is ideal frequency response. Ideal filters are not practically realizable. Why because if I take the Inverse Fourier Transform (IFT) of this one, we will get *h(t)* which is the impulse response.

If we take the Inverse Fourier Transform (IFT) of this type of frequency response we will get *h(t)* which is non-causal. In the sense output not only depends upon the present input, but also on the previous as well as the past inputs also. So, that is practically not possible. So, only thing that you can do is we have to approximate this filter response. So, that we can get the impulse response which is causal and we can practically realize those filters.

Okay! So, for that this is ideal filter for $n = \infty$. If order is infinity then we will get this type of the filter whereas, for different orders the filter response will be different I am going to plot for three different filter values. So, this type of response you will get for say $n = 1$. So, this is half. This is for $n = 1$, lower order. So, this is far away from the ideal frequency response. Okay.

Now, if I take one more value of *n*. So, we will get something like this response this is say for example, $n = 2$. This is better than $n = 1$, but it this is also far away from the ideal frequency response. Now, if I further increase the *n* value. So, we will get something like this response this is for $n = 100$. So, as the *n* value increases the frequency response approaches the ideal frequency response.

There are some characteristics of this Butterworth filter.

One is the magnitude squared response at $\omega = 0$ is if I substitute this $\omega = 0$ here this will be 1. So, this is 1 this you can get as good.

$$(i)\quad |H(j\omega)|^2_{\omega=0} = 1$$

Second one is the magnitude squared response at $\omega = \omega_c$ is equal to if $\omega = \omega_c$, 1 by 1 ω_c ω_c get cancelled 1 to the power of *2n* which is 1. So, 1 by 2. So, here this will be 1 by 2 at $\omega = \omega_c$ this is 1 by 2

$$(ii) \quad |H(j\omega)|^2_{\omega=\omega_c} = \frac{1}{2}$$

Or implies if I take $H(j\omega)$ is 1 by root 2.

$$\Rightarrow |H(j\omega)| = \frac{1}{\sqrt{2}}$$

Normally this frequency response will be plotted in logarithm scale. So, if I take 20 logarithm of modulus of $H(j\omega)$ will be 20 logarithm of 1 by root 2 and this value will be approximately minus 3 dB.

$$20log|H(j\omega)| = 20log\frac{1}{\sqrt{2}} = -3dB$$

So, if I start this with *0 dB* because $log\ 1 = 0$. So, this value will be minus 3 dB. At cutoff frequency the frequency response will be dropped by 3 dB. So, you see the second characteristics of a Butterworth approximation or Butterworth filter.

Third one is:

$$(iii) \quad |H(j\omega)|^2 \ is\ monotonically\ decreasing\ function\ of\ '\omega'$$

That is this response will continuously decreases. It is not like sometime increases sometime decreases this is not monotonic response, this is continuously decreases. So, this is called monotonically decreasing function of omega.

And fourth important characteristics of this Butterworth filter is:

$$(iv) \quad |H(j\omega)|^2 \ is\ maximally\ flat\ at\ the\ origin$$

So, this you can easily prove by differentiating this and equating to 0 at $\omega = 0$ you will get the maximum value. Okay! So, these are the four characteristics of this Butterworth approximation.

Now, we will derive the transfer functions of the Butterworth filter for different orders because later we are going to realize these filters using operational amplifiers. First, I will derive the transfer functions of Butterworth approximation for different orders.

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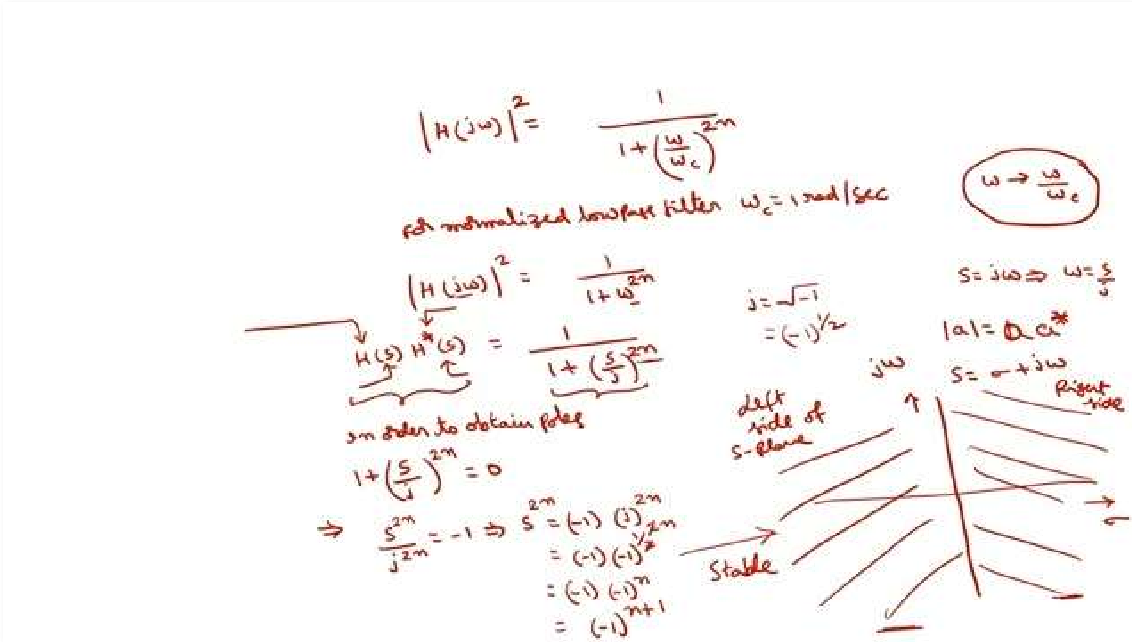

So, if we consider with:

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}$$

Now, for the sake of simplicity I will assume that $\omega_c = 1$ and that filter is called normalized filter. Later if you want the filter whose cutoff frequency is other than 1 radian per second then you have to simply substitute ω by ω_c. For normalized filter or approximation $\omega_c = 1$ radian per second. But in many cases, we want to design the filters whose cutoff frequencies other than 1 radian per second also. In that case simply we will substitute wherever ω is there replaced with ω by ω_c. This we are going to do later after deriving this normalized filter frequency responses or transfer functions.

Therefore, what will be the $|H(j\omega)|^2$ for normalized filter?

$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}$$

So, we know that s is equal to $j\omega$. So, this is nothing, but $H(s)H^*(s)$ because magnitude of a can be written as $|a| = aa^*$, a^* is the complex conjugate.

So, this $j\omega$ I have substituted as *s* and here what will be this ω because this is function of *s* this also should be function of *s*. So, from this what is ω? $\frac{s}{j}$.

$$H(s)H^{*}(s) = \frac{1}{1 + \left(\frac{s}{j}\right)^{2n}}$$

Here, this is product of two transfer functions. Later, we are going to show that one transfer function will be having poles in the left half of the s-plane another will be having poles in the right half of the s-plane. If I consider this *s* in fact, is σ (sigma) plus $j\omega$. For the simplicity we will assume $\sigma = 0$.

If I take the s-plane you might have studied in your Laplace transform this is σ axis this is $j\omega$ axis. This is left side of the s-plane and this is right side of the s-plane. And there is one important fact that in signals and systems and as well as in Laplace transform you might have studied. If poles of a transfer function lies in the left half of the s-plane the system is stable and we have to always design the filters which are stable.

So, in order to derive the transfer function of a stable transfer function here we will be having *2n* roots because this is order is *2n*, *2n* roots or *2n* poles. Out of this *2n* poles *n* poles lies on the right half of the s-plane *n* on the left half of the s-plane. If you construct a transfer function using the poles which lies on the left half of the s-plane then you will get a stable transfer function.

So, in order to get the poles, you have to equate the denominator to 0.

$$1 + \left(\frac{s}{j}\right)^{2n} = 0$$

$$\Rightarrow \frac{s^{2n}}{j^{2n}} = -1$$

$$\Rightarrow s^{2n} = (-1)j^{2n}$$

What is *j*?

$$j = \sqrt{-1} = (-1)^{\frac{1}{2}}$$

So, if I substitute that, this is equal to:

$$\Rightarrow s^{2n} = (-1)(-1)^{\frac{2n}{2}} = (-1)(-1)^{n}$$

$$\therefore s^{2n} = (-1)^{n+1}$$

So, this is the expression that we are going to use to derive the transfer functions of this normalized Butterworth filter.

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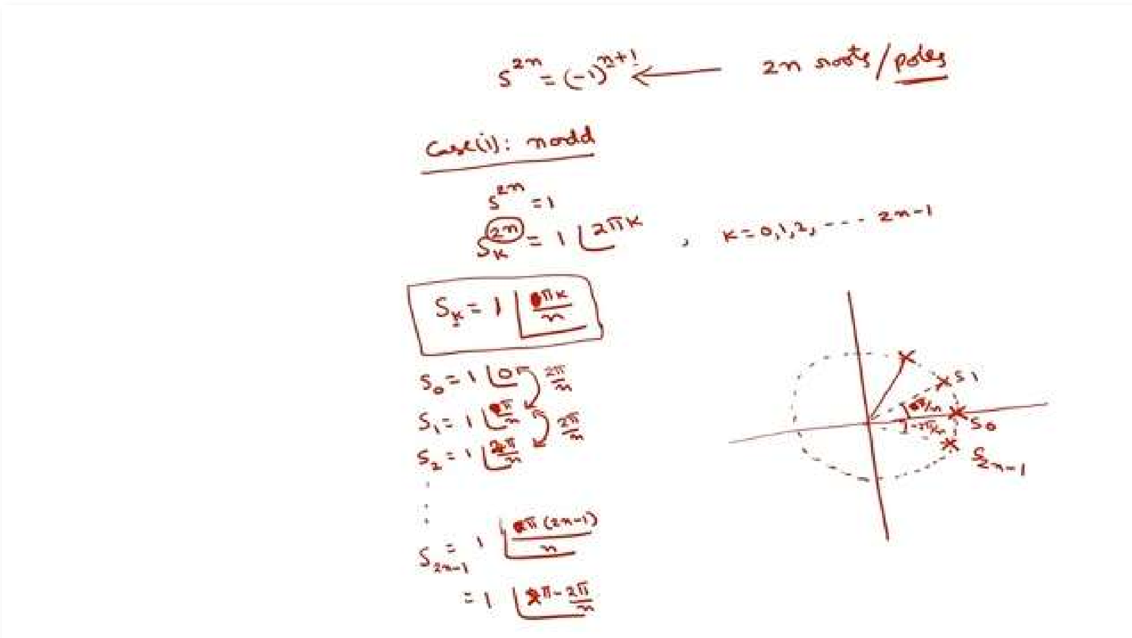

So, the expression that you have got is:

$$s^{2n} = (-1)^{n+1}$$

So, this will be having total how many roots? *2n* roots slash poles. Roots are nothing, but poles because this is denominator. So, these poles depends upon the *n* whether *n* is even or odd the poles will be having different locations. Right!

And it is shown that the poles of this Butterworth filter lies on the unit circle. So, if I take the first case where *n* is odd then what happens?

<u>Case (i): n odd</u>

$$s^{2n} = 1$$

s to the power of 2*n*, *n* is odd. So, odd plus 1 becomes even. So, minus 1 to the power of even becomes plus 1. So, if I want to represent this one in phasor form 1 at an angle of $2\pi k$.

If I call 1 otherwise I will write another step, the kth root total it will be having $2n$ roots or *2n* poles the *kth* root with $2n$ is given by 1 at an angle of $2\pi k$ where *k* varies from 0, 1, 2 so on up to $2n$-1 totally $2n$ poles.

$$s_k^{2n} = 1\angle 2\pi k$$

Or what is s_k k^{th} pole? 1 if I take this $2n$ to the other side in angle it will be divided with $2\pi k$.

$$s_k = 1\angle\frac{\pi k}{n}$$

So, what is s_0? is 1 at angle of *k* is 0. So, this is 0, s_1 the second pole 1 at angle of $\frac{\pi}{n}$, s_2 1 at an angle of $\frac{2\pi}{n}$. So, what is the gap between these angles? This to this is $\frac{2\pi}{n}$, this to this is $\frac{2\pi}{n}$ so on. So, what is the last pole s_{2n-1} is equal to 1 at an angle of 2π *k* becomes $2n$-1 divided by *n*. This will be 1 at an angle of $4\pi n$, *n n* get cancelled.

$$s_0 = 1\angle 0$$

$$s_1 = 1\angle\frac{\pi}{n}$$

$$s_2 = 1\angle\frac{2\pi}{n}$$

$$\dots$$

$$s_{2n-1} = 1\angle\frac{\pi(2n-1)}{n} = 1\angle 2\pi - \frac{2\pi}{n}$$

If I plot these poles they lies on the unit circle. This is one of the important observation that the poles of Butterworth approximation lies on the circle whereas, if I take the Chebyshev approximation they lies on the ellipse. So, if I assume that the first pole is here s_0, s_1 will be here. What is this phase angle between these two is $\frac{2\pi}{n}$ and at another $\frac{2\pi}{n}$ we have s_2 and so on. And this is finally, other side this is $2\pi - \frac{2\pi}{n}$. This is s_{2n-1}, this is 2π actually in fact, this is $2n$ we have to take this becomes $\frac{\pi n}{n}$.

So, this is also $\frac{\pi}{n}$ because this is $2n$ we have to divide so, 2 2 get cancelled $\frac{\pi k}{n}$. So, $\frac{\pi k}{n}$ means this is π into $\frac{\pi}{n}$, this is $\frac{2\pi}{n}$ and so on. This is π by this is 2π so, these are. So, this will be angle of $(2\pi - \frac{2\pi}{n})$ is this is total is 2π up to here this is $(2\pi - \frac{2\pi}{n})$ this is $-\frac{2\pi}{n}$. So, these are the location of the pole. This is important.

So, the k^{th} pole will be having magnitude of unity angle of $\frac{\pi k}{n}$ not $\frac{2\pi k}{n}$. This is the case of n is odd.

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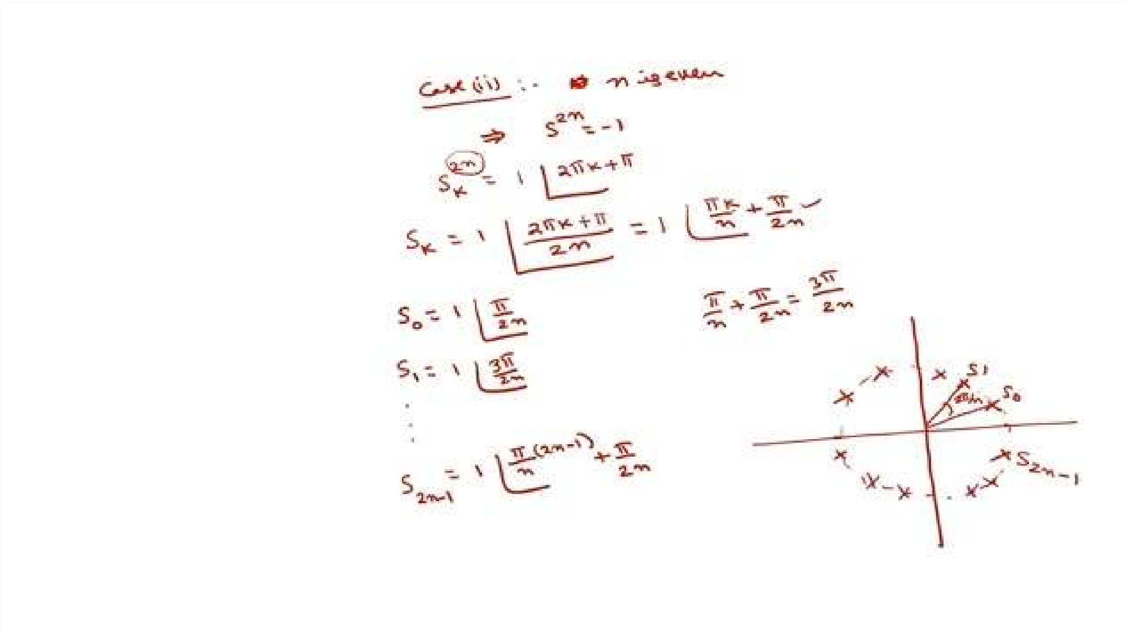

For n is even case (ii). So, what happens to this relation n is even means n plus 1 becomes odd, minus 1 whole to the power of odd means minus 1. So, s to the power of 2 of $2n$ will be minus 1.

<u>*Case (ii): n is even*</u>

$$s^{2n} = -1$$

And, if you want to write the k^{th} pole s_k to the power of $2n$ in phasor notation:

$$s_k^{2n} = 1\angle 2\pi k + \pi$$

Or what is s_k? This $2n$ if I take to the other side, it will divide this will be:

$$s_k = 1\angle\frac{2\pi k + \pi}{2n} = 1\angle\frac{\pi k}{n} + \frac{\pi}{2n}$$

So, what is s_0?

$$s_0 = 1\angle\frac{\pi}{2n}$$

$$s_1 = 1\angle\frac{3\pi}{2n}$$

$$\dots$$

$$s_{2n-1} = 1\angle\frac{\pi(2n-1)}{n} + \frac{\pi}{2n}$$

And, if you plot these poles, they also lies on the unit circle, but the spacing is $\frac{\pi}{2n}$. In case of n is odd, spacing is $\frac{\pi}{n}$ whereas, here $\frac{\pi}{2n}$ and the poles will not lie here and here. It will start from here here there will be symmetry. This is s_0 s_1. This gap is $\frac{2\pi}{n}$, this is s_{2n-1}. So, with this background we can derive the transfer functions of normalized Butterworth low pass filter. Okay! First, I will go for the first order filter. Low pass filter.

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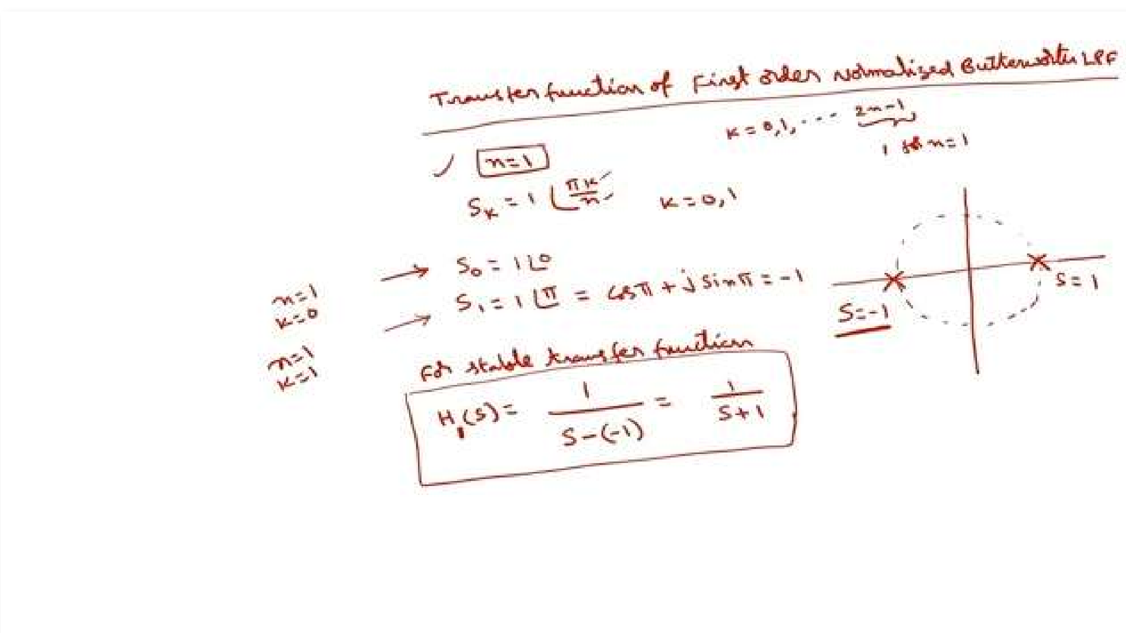

So, the previous theory actually I have derived for your understanding. So, what is important is this transfer functions only if you remember the transfer functions also it is okay. So, n is equal to 1 first order means n is equal to 1. So, which case you have to consider? this case. In this case, what is s_k? 1 at angle of $\frac{\pi k}{n}$. And k varies from in general 0, 1, 2, so on up to $2n - 1$. Here, n is equal to 1 means $2n - 1$ becomes 1 for $n = 1$ only simply 2 only $k = 0, 1$. So, what is s_0? Two poles will be there. What is s_1? s_0 is $k = 0$ means $1\angle 0$. n is equal to 1, k is equal to 1. Here n is equal to 1 in this case n is equal to 1, k is equal to 0, so $1\angle 0$. In this case n is equal to 1, k is equal to 1, so n k will get cancelled we will get $1\angle\pi$ which is nothing, but $cos\,\pi + jsin\,\pi$. $cos\,\pi$ is minus 1, $sin\,\pi$ is 0. So, it is simply minus 1. There are two poles, one at s is equal to $+1$, another at s is equal to -1 on the unit circle. So, this is one pole s is equal to $+1$, this is another pole s is equal to -1.

As I have told, half of the poles lies on the right half of the s-plane and half will lie on the left half of the s-plane. But for stable transfer function, *H(s)*, you can also call as this is first order this is first order this one suffix transfer order is given by:

$$H_1(s) = \frac{1}{s - (-1)} = \frac{1}{s + 1}$$

So, this is the transfer function of the first order filter, which is stable. Now, if you want to derive the transfer function of the second order normalized Butterworth low pass filter, you can proceed in a similar manner. So, I will derive up to third order and after that I will give the transfer functions of fourth and fifth orders.

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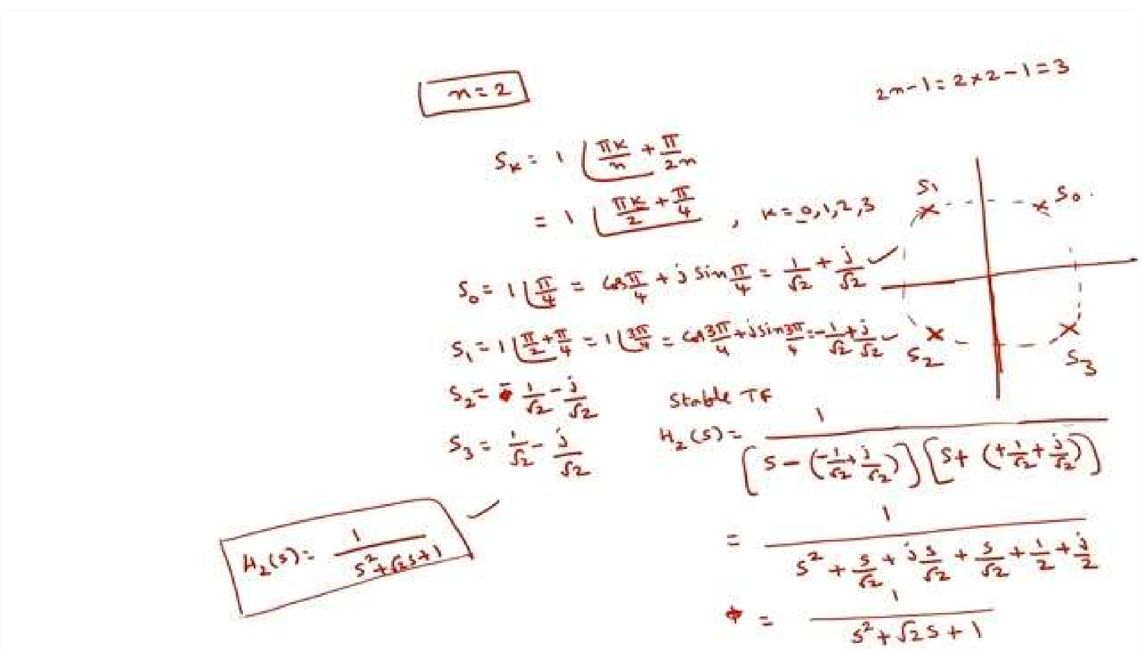

n is equal to 2. So, which formula I have to use now? This case which is s_k will be:

$$s_k = 1\angle\frac{\pi k}{n} + \frac{\pi}{2n}$$

So, n is equal to 2. So,

$$s_k = 1\angle\frac{\pi k}{2} + \frac{\pi}{4}$$

So, total we will be having $2n$ roots or $2n$ poles and k varies from 0, 1, 2, so on up to $2n-1$. What is $2n-1$? This is equal to 2 into n is 2 minus 1 is 3. So, 0, 1, 2, 3 totally four poles are there for the second order system. And two poles lies on the left half of the s-plane, two poles lies on the right half of the s-plane.

So, what is s_0? k is 0, so k is 0 means this is 0, 1 at an angle of $\frac{\pi}{4}$ simply. What is this value?

$$s_0 = 1\angle\frac{\pi}{4} = cos\frac{\pi}{4} + jsin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

So, this lies in the first coordinate, somewhere here on the unit circle.

$$s_1 = 1\angle\frac{\pi}{2} + \frac{\pi}{4} = 1\angle\frac{3\pi}{4} = cos\frac{3\pi}{4} + jsin\frac{3\pi}{4} = \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

Similarly, s_2 we can derive that:

$$s_2 = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

This is minus 1 by root 2 minus j by root 2 this is complex conjugate of this. This is plus this is minus complex conjugate minus minus this minus this plus becomes minus complex conjugate this will be somewhere here both axis are negative axis.

And s_3 we will get minus 1 by root 2 plus 1 by root 2 complex conjugate of this in fact, 1 by root 2 minus j by root 2 here.

$$s_3 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

So, out of these four poles, this is s_0 pole this is s_1, s_2, s_3 pole. s_1 and s_2 lies left half of the s-plane.

So, in order to have the stable transfer function TF transfer stable transfer function $H_2(s)$ this is second order is given by:

$$H_2(s) = \frac{1}{\left[s - \left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)\right]\left[s - \left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)\right]}$$

$$= \frac{1}{s^2 + \frac{s}{\sqrt{2}} + \frac{js}{\sqrt{2}} + \frac{s}{\sqrt{2}} + \frac{1}{2} + \frac{j}{2}}$$

$$= \frac{1}{s^2 + \sqrt{2}s + 1}$$

This we have already derived in the last lecture also, damping factor we have got as root 1.414.

So, the second order transfer function is $H_2(s)$ is equal to 1 by s^2 plus $\sqrt{2}s$ plus 1. This you have to remember. Previous theory I have developed to derive this transfer functions only. So, we are going to use this in the design of the filters. So, you can directly remember also otherwise you can derive also it is up to you.

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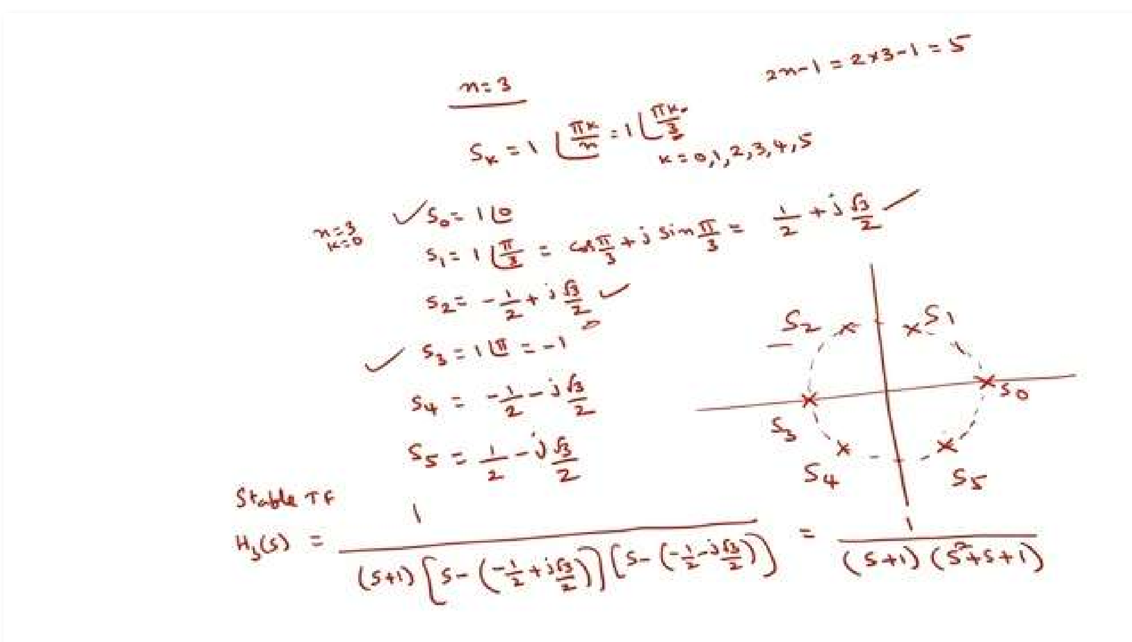

Third order filter, for third order filter again odd, odd means this expression this

$$s_k = 1\angle\frac{\pi k}{n}$$

Now, and k value varies from 0 to $2n - 1$. 0, 1, $2n - 1$ becomes 2 into 3 minus 1 = 5. 0, 1, 2, 3, 4, 5 total 6 roots. So, s_0 will be n is equal to 3, k is equal to 0. Of course, in all these cases n is equal to 3 you can replace this as 1 at angle of $\frac{\pi k}{3}$.

$$s_k = 1\angle\frac{\pi k}{n} = 1\angle\frac{\pi k}{3}$$

So, k is equal to 0 means $1\angle 0$, $s_1 = 1\angle\frac{\pi}{3}$ which is equal to $cos\frac{\pi}{3} + jsin\frac{\pi}{3}$. This is equal to $\frac{1}{2} + \frac{j\sqrt{3}}{2}$. Similarly, you will get s_2 is equal to $-\frac{1}{2} + \frac{j\sqrt{3}}{2}$. s_3 you will get this 3 3 get cancel $1\angle\pi$ which is nothing, but minus 1. s_4 you will get complex conjugate of this. This is $-\frac{1}{2} - \frac{j\sqrt{3}}{2}$. s_5 you will get complex conjugate of this. This is equal to $\frac{1}{2} - \frac{j\sqrt{3}}{2}$.

So, total we have six poles. If you plot these six poles on the unit circle, one is here which is s_0 pole, one is here which is s_3 pole and the remaining are so, here, here, here, here. These are the four poles. Out of these, which poles lies on the left half of s-plane? This is s2, s3, s4.

So, in order to have a stable transfer function. $H_3(s)$ is equal to:

$$H_3(s) = \frac{1}{(s+1)\left[s - \left(\frac{-1}{2} + \frac{j\sqrt{3}}{2}\right)\right]\left[s - \left(\frac{-1}{2} - \frac{j\sqrt{3}}{2}\right)\right]}$$

This if you simplify, finally you will get this:

$$H_3(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Similarly, we can derive for the fourth and fifth order. So, here I will derive I will write down all the transfer functions up to fifth order, okay.

$H_1(s)$ the first order which you have derived as 1 over $s + 1$. This is transfer functions of normalized Butterworth low pass filter. Normalization means you have taken ω_c as 1 radian per second. This is first order. This also you have derived $s^2 + \sqrt{2}s + 1$.

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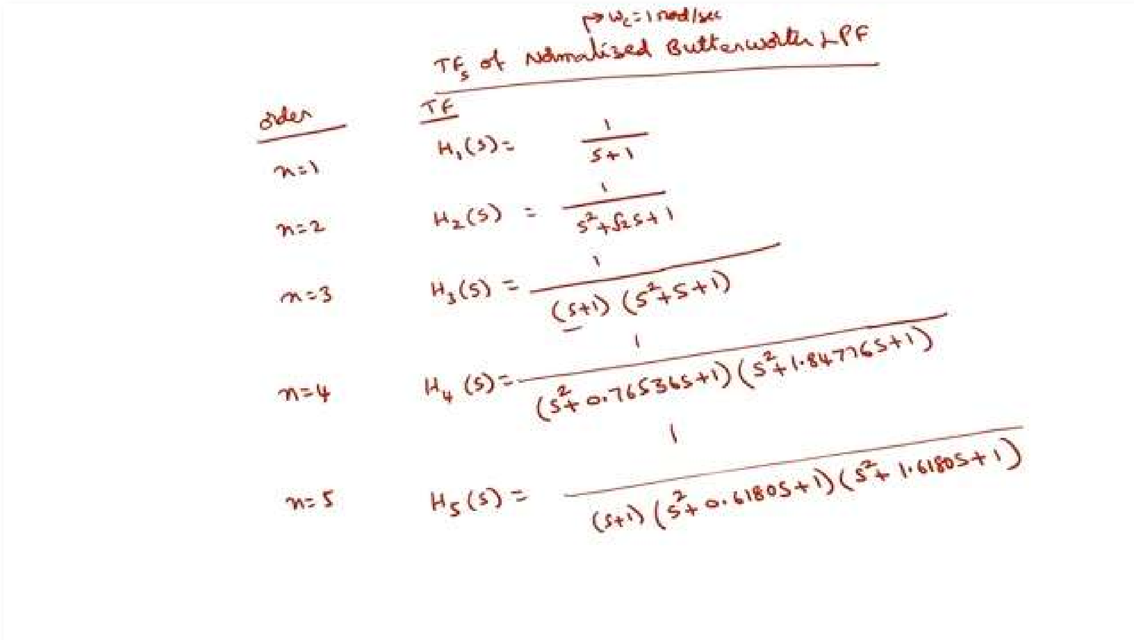

This is 1 by $s + 1$ into $s^2 + s + 1$. Fourth order if you derive $s^2 + 0.76536s + 1$ into $s^2 + 1.84776s + 1$. And if I take *n* is equal to 5, $H_5(s)$ there will be $+1$ factor for all the odd order filters. Then $s^2 + 0.6180s + 1$ into $s^2 + 1.6180s + 1$.

These are these transfer functions of the first five orders, okay. If you want, the larger order also you can derive in a similar manner. So, I will just take one example of designing the filter using opamp.

Design an active second order Butterworth low pass filter with a cutoff frequency of 1 kilohertz. I will first forget about this 1 kilohertz cutoff frequency. First, I will derive for the transfer function of normalized filter whose cutoff frequency is 1 radian per second.

So, you have derived the expression for the second order directly if you want. This is second order:

$$H_2(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

This is without amplification.

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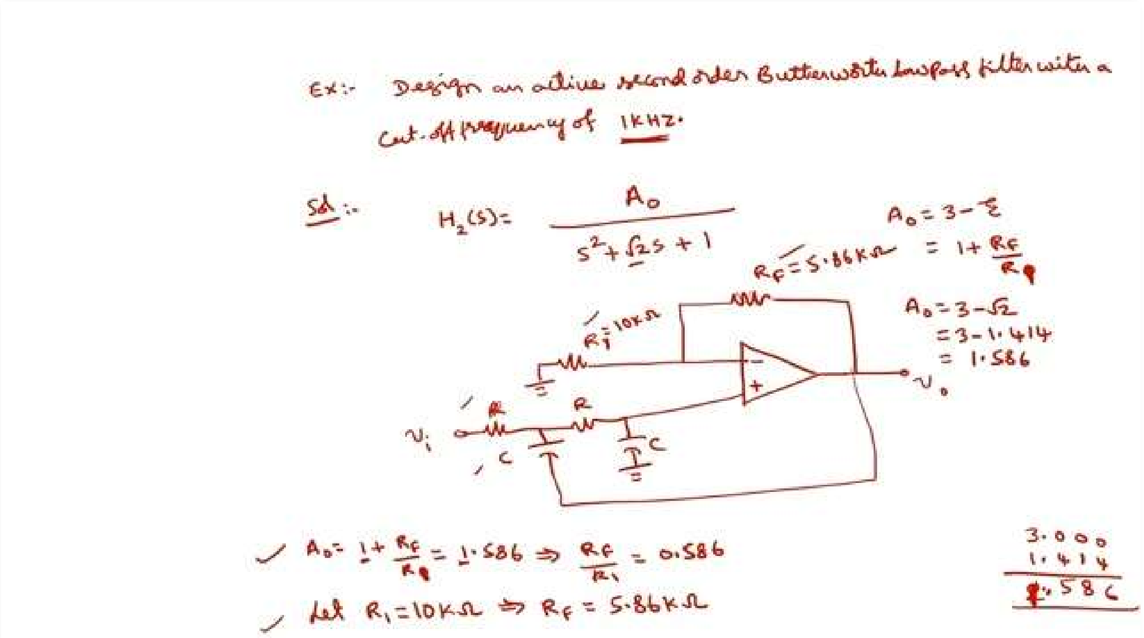

If I take the circuitry here. This is circuit diagram that we have discussed in the last lecture v_i v_o. This is two RC sections will be there and this is R_1 R_f. So, because of this R_1 R_f there will be some amplification factor A_o. This A_o is given by 3 minus ξ (zeta) which is damping factor. This is also equal to $1 + \frac{R_f}{R_1}$ which we have discussed in the earlier lecture. So, in case of second order filter, what is ξ (zeta)? is $\sqrt{2}$.

A_o is equal to 3 minus ξ (zeta) is $\sqrt{2}$ which is 1.414. So, what will be value of A_o? 3 if I subtract 1.414, 2.586, 1.586 this is 1.586.

$$A_o = 3 - \xi = 1 + \frac{R_f}{R_1}$$

$$A_o = 3 - \sqrt{2} = 3 - 1.414 = 1.586$$

Now, I have to find out design in the sense we have to find out the values of $R_1, R_f, R \; and \; C$ to get this frequency of 1 kilo Hertz cutoff frequency. So, A_o which is equal to $1 + \frac{R_f}{R_1}$ this is equal to 1.586 implies what is $\frac{R_f}{R_1}$? R_f by this R_1 this is R_1, $\frac{R_f}{R_1}$ is equal to 0.586 this 1 will be subtracted.

Now, let $R_1 = 10k\Omega$ you can choose any value which is available in your laboratory implies what is R_f? R_f should be 5.86 $k\Omega$. So, this R_f should be 5.86 $k\Omega$ and this R_1 is $10k\Omega$. So, to meet this second order requirement R_1 and R_f are these values. Now, what is the value of *R* and *C* to meet this frequency cutoff frequency?

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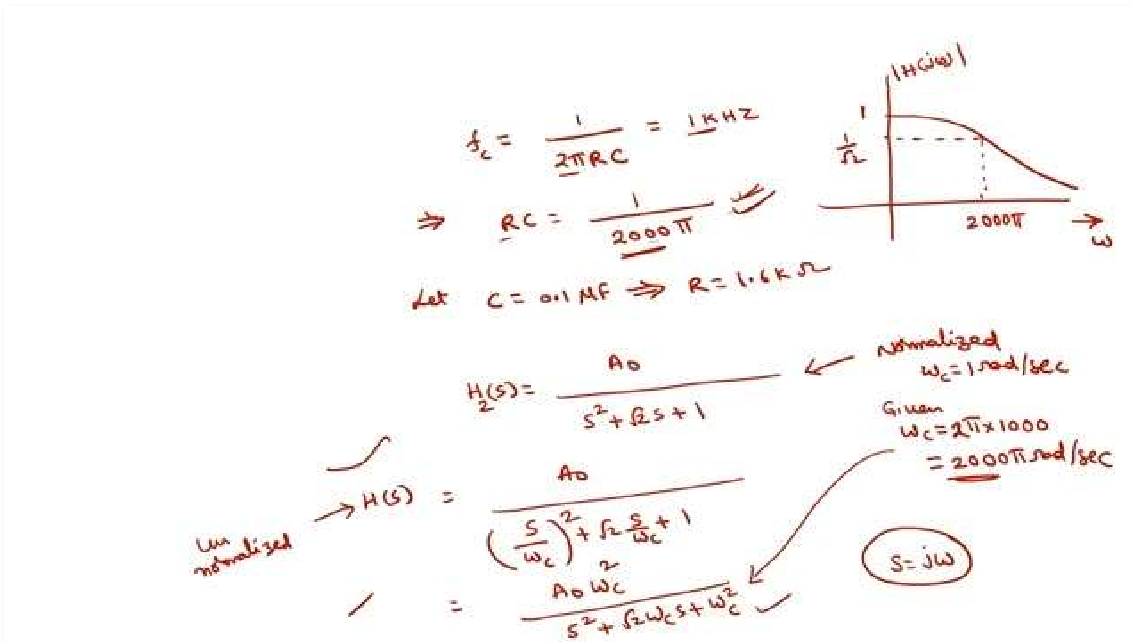

This also we have derived f_c cutoff frequency we have derived as $\frac{1}{2\pi RC}$ this should be equal to 1 kilo Hertz, implies *R C* is equal to $\frac{1}{2000\pi}$. This is thousand, this is 2π, 2000π. Now, let you have to choose the value of *C* because this is a random value of *C* if you get it becomes difficult to pick up the capacitance values. So, that is why in all these designs wherever *R C* product we have to satisfy for some numerical value we have to first choose *C* say $0.1\mu F$. Then from this you can derive that *R* is equal to 1.6 $k\Omega$. You can see this, *R* is equal to once if you know *C*, $\frac{1}{2000\pi C}$ that comes around 1.6 $k\Omega$.

Now, this *R* is 1.6 $k\Omega$ and *C* is $0.1\mu F$ this R is 1.6 $k\Omega$ this C is $0.1\mu F$. This is the design of this one, but what about the transfer function? Transfer function will be something like this.

So, transfer function of this filter is:

$$H_2(s) = \frac{A_o}{s^2 + \sqrt{2}s + 1}$$

This is for normalized whereas, for a cutoff frequency of $\omega_c = 1$. But in the problem what is ω_c? Given ω_c is $1\ kHz$ means $2\pi \times 1000$ this is equal to 2000π radians per second.

Now, to get this normalized this one what you have to do this is normalized this is un-normalized can be obtained by simply substituting s by ω_c, s by ω_c. This is equal to $A_o\omega_c^2$ divided by $s^2 + \sqrt{2}\omega_c s + \omega_c^2$, where ω_c is this. If you substitute this here you will get the transfer function and if you replace s with $j\omega$ you can plot the frequency response.

You can see that there will be 20 dB roll-off. It will be something like this. At cutoff frequency this will get 2000π radians per second ω this is unity at ω is equal to 0 here we will get $\frac{1}{\sqrt{2}}$. So, this actually in fact, we have derived in the previous lectures also this. This is the one which we have derived which is same as this. This is for un-normalized, normalized means$\omega_c = 1$. So, we will get simply $s^2 + \xi s + 1$. ξ is in case of second order it is $\sqrt{2}$. This is how you can design the low pass Butterworth filters.

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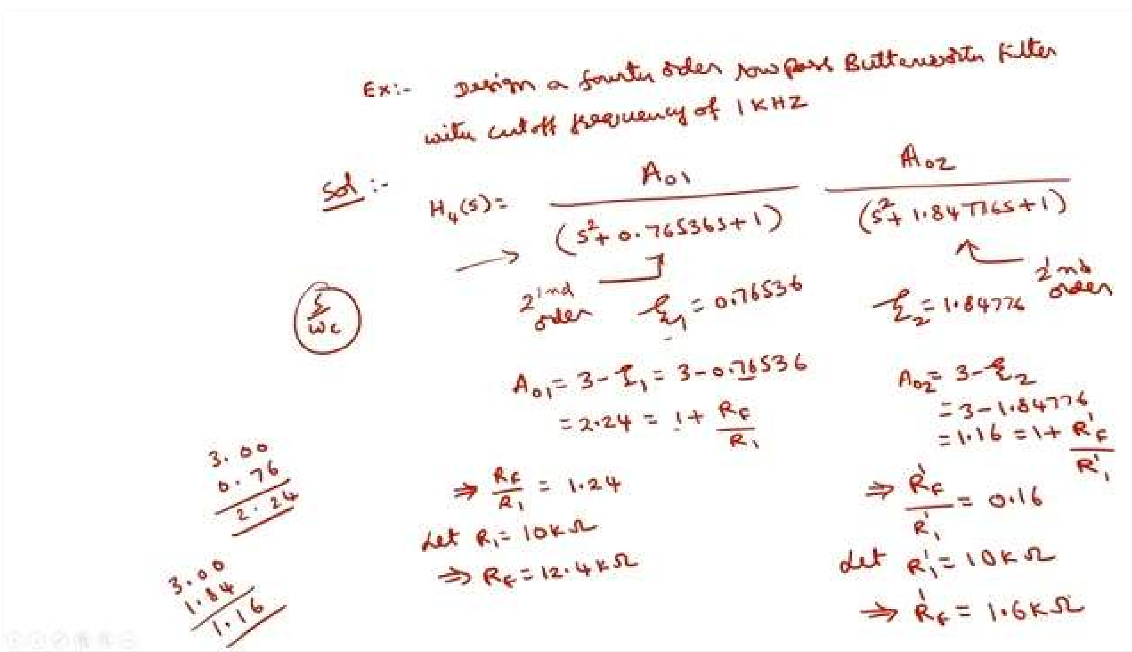

Now, if you want to design the same low pass filter, but fourth order with cutoff frequency of same 1 kilohertz. Then what you have to do is you have to cascade a two second order filters. So, if I take the normalize this one this is $H_4(s)$ is equal to which have given in the previous slides 1 by $(s^2 + 0.76536s + 1)$ into 1 by $(s^2 + 184776s + 1)$. This is one second order, this is another second order. Here what is ξ_1 is 0.76536, here what is ξ_2 is 1.84776.

Of course, in these two cases you require some A_{01} gain A_{02} gain. And if you want other than this cutoff frequency 1 you have to replace s with ω_c that I will do at the end. So, with this ξ_1 what is A_{01} relation 3 minus ξ_1 this is equal to 3 minus 0.76536. This is equal to if I take up to only two digits 2.24, but what is this 1 plus $\frac{R_F}{R_1}$. $\frac{R_F}{R_1}$ is 1 you have to subtract so, 1.24.

Here let R_1 is equal to 10 $k\Omega$ implies R_F is equal to 12.4 $k\Omega$. And here A_{02} is equal to 3 minus ξ_2 that is equal to 3 minus 1.84776. This is approximately if I take the two digits 1.16. This is equal to 1 plus if I call the feedback destination R_F' for the second stage and R_1' for the second stage implies $\frac{R_F'}{R_1'}$ is equal to 0.16.

Here also let $R_1'$is equal to 10 $k\Omega$ implies R_F' is equal to 1.6 $k\Omega$. So, the circuit will be cascading of two such circuits which you have derived in the previous example.

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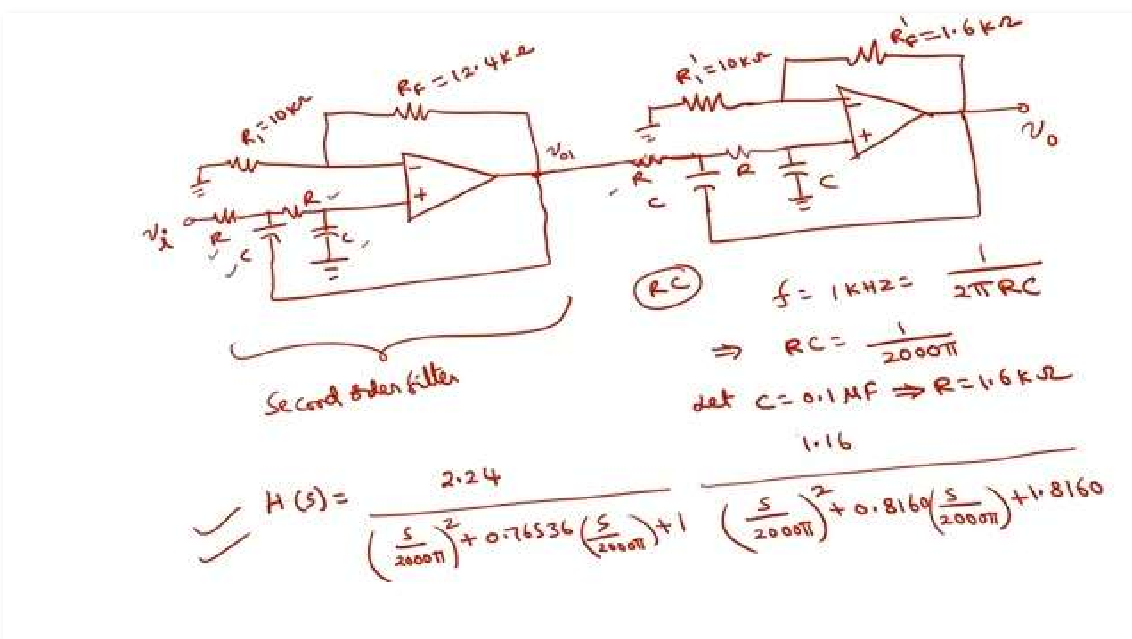

This is one second order filter. This is R_1 R_F this is R C v_i this is v_i input voltage. This is not the final output this is after the first stage you can call as v_{01}.

The final output will be this you have to apply to another second order filter. This is second order filter. Another two sections of this R C this is grounded this will be applied to the non-inverting terminal.

This is the final output. This I am calling as R_1', R_F'. This is again R C, R C. So, this R_1 you have taken as 10 $k\Omega$, R_1' also you have taken as 10 $k\Omega$ correspondingly you got this R_F as 12.4 $k\Omega$ and this R_F' as 1.6 $k\Omega$. And what about this R and C here this R and this C all are same.

R C is same again to get this R C, f is given as 1 kHz this is equal to $\frac{1}{2\pi RC}$. Regardless of the whether the order is second order, fourth order the cutoff frequency is same $\frac{1}{2\pi RC}$ implies R C is equal to $\frac{1}{2000\pi}$. So, in the previous example also you have taken C as $0.1\mu F$ correspondingly you got R as 1.6 $k\Omega$. This is the complete design whether the transfer function will be overall transfer function of $H(s)$ is equal to this A_{01}. What is A_{01} is 2.24. So, 2.24 divided by s^2 now s becomes $\frac{s}{2000\pi}$ because this is not the normalized this is with cutoff frequency of 1 kHz means 2000π radians plus 0.7636 into $\frac{s}{2000}$ plus 1 is the first transfer function A_{01}.

A_{02} is 1.16, s becomes $\left(\frac{s}{2000\pi}\right)^2$ plus $0.8160\left(\frac{s}{2000\pi}\right)$ plus 1.8160. This is the overall transfer function of this fourth order Butterworth filter whose cutoff frequency is 1 kHz or 2000π radians per second. This is also similar to the expression that we have derived. This in the form of this one only, but we have two such cascade sections. So, this is how we can design this low pass Butterworth non-opposite filters and then the filters with any cutoff frequency. Next, we will discuss about the high pass filter in the next lecture. Thank you.