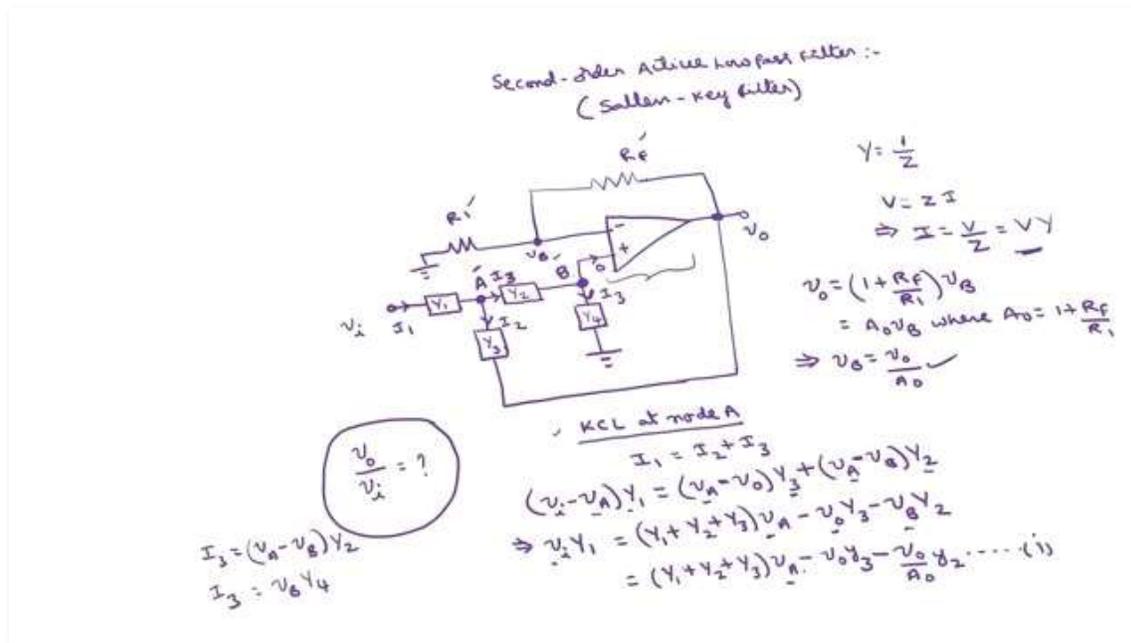


**Integrated Circuits and Applications**  
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**Active Filters I**  
**Lecture - 16**  
**Second Order Low Pass Filter**

Okay! In the last lecture we have discussed about the first order low pass active filter. So, the drawback of first order filter is, in the transition band the roll-off rate is  $-20dB$  per decade. So, that is less actually. So, if you want to have the frequency response which is near to the ideal response then the roll-off rate in this transition band should be large. So, in order to have the large roll-off rate in the transition band we have to go for the higher order filters. So, today we will discuss about the second order active low pass filter.

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This is sometimes called as Sallen-Key filter also. So, the general structure of this second order active low pass filter is this consisting of two RC sections. In case of first order one RC section is there now two RC sections. First, I will derive the generalized expression in

terms of admittances, later I will replace the admittances with resistance and capacitances.

This is  $R_F$   $R_1$  this is grounded then at the positive terminal we have the admittances. This is output  $v_o$ , this is  $v_i$ . This you call as  $Y_1$  admittance,  $Y_2$  admittance,  $Y_3$ ,  $Y_4$ .  $Y$  is the admittance which is the reciprocal of the impedance.

$$Y = \frac{1}{Z}$$

We know that

$$V = ZI$$

$$\Rightarrow I = \frac{V}{Z} = VY$$

So, if you want to write the expression for the current you have to multiply the admittance with voltage. Here I am going to derive the expression for the  $\frac{v_o}{v_i}$  initially say in terms of the Laplace transform  $s$ . So, first I will derive in terms of  $Y_1, Y_2, Y_3, Y_4$ . Later, I will replace this with resistance and capacitances. Let us call this node as node A and the voltage here is called  $v_A$  and this is node B and the voltage here is  $v_B$  and this current you call as  $I_1$ , this current you call as  $I_2$ , this current you call as  $I_3$ .

KCL at node A:

$$I_1 = I_2 + I_3$$

$$\Rightarrow (v_i - v_A)Y_1 = (v_A - v_o)Y_3 + (v_A - v_B)Y_2$$

So, the final expression that we want is  $\frac{v_o}{v_i}$ . I have to express everything in terms of  $v_o$  and  $v_i$ . Basically, I have to express this  $v_A$  and  $v_B$  in terms of  $v_i$  and  $v_o$  so that all the terms will be having either  $v_i$  or  $v_o$ , you can take the ratio of  $v_o$  to  $v_i$ . So, from this

$$\Rightarrow v_i Y_1 = (Y_1 + Y_2 + Y_3)v_A - v_o Y_3 - v_B Y_2$$

Here we have the two variables  $v_A$  and  $v_B$  which you have to express in terms of  $v_i$  and  $v_o$ . This is of course,  $v_i$  this is  $v_o$ . This is no problem, we can keep these two terms. But these two terms we have to express in terms of  $v_i$  and  $v_o$ . So, because the op-amp is ideal, if this is  $v_B$ , this is also  $v_B$ . And if this is  $v_B$ , the remaining amplifier is this is non-inverting amplifier.

So, output  $V_o$  is given by:

$$v_o = \left(1 + \frac{R_F}{R_1}\right)v_B$$

If I forget about this entire circuit before this, so this will be  $v_B$ . The remaining circuit is here from the  $v_B$  to this one, this is a non-inverting amplifier whose gain is 1 plus feedback resistance divided by  $R_1$ . This we have already derived in the earlier lectures.

$$v_o = A_o v_B, \text{ where } A_o = \left(1 + \frac{R_F}{R_1}\right)$$

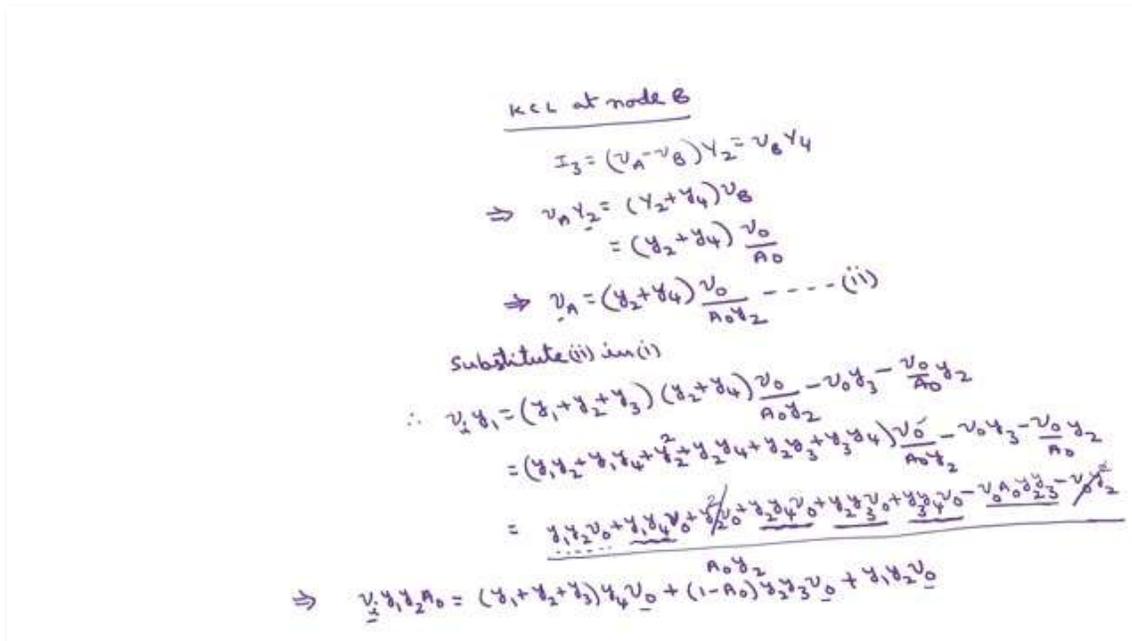
$$\Rightarrow v_B = \frac{v_o}{A_o}$$

This I will substitute here.

$$\Rightarrow v_i Y_1 = (Y_1 + Y_2 + Y_3)v_A - v_o Y_3 - \frac{v_o}{A_o} Y_2 \dots\dots\dots (i)$$

Now, only thing is this  $v_A$  I have to express in terms of  $v_i$  or  $v_o$ . I will call this as expression (i). Now, I have to replace  $v_A$  by either  $v_o$  or  $v_i$ .

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So, for that I will take the KCL at node B. So, this KCL at node B. This is the current  $I_3$  and here the current is 0 because op-amp is ideal, the entire  $I_3$  will flows through  $Y_4$  also. So, if I write the expression for  $I_3$  in terms of A and B, I will write here:

$$I_3 = (v_A - v_B)Y_2$$

$I_3$  also another expression in terms of this  $v_B$  and this ground is  $v_B$  times  $Y_4$ .

$$I_3 = v_B Y_4$$

So, these two are same:

$$I_3 = (v_A - v_B)Y_2 = v_B Y_4$$

$$\Rightarrow v_A Y_2 = (Y_2 + Y_4)v_B$$

But what is  $v_B$ ? I have just derived  $v_B$  is  $\frac{v_o}{A_o}$ . This  $v_B$  is  $\frac{v_o}{A_o}$ .

$$\Rightarrow v_A Y_2 = (Y_2 + Y_4) \frac{v_o}{A_o}$$

Therefore, what is  $v_A$ ?

$$\Rightarrow v_A = (Y_2 + Y_4) \frac{v_o}{A_o Y_2} \dots \dots \dots (ii)$$

Now, we will substitute equation (ii) in equation (i). Here  $v_A$  we have expressed in terms of  $v_o$ . Here I will replace this. So, what is expression (i) now?

$$\therefore v_i Y_1 = (Y_1 + Y_2 + Y_3)(Y_2 + Y_4) \frac{v_o}{A_o Y_2} - v_o Y_3 - \frac{v_o}{A_o} Y_2$$

Now, all the terms are either in terms of  $v_i$  or  $v_o$ . So, you have to basically take the ratio of  $v_o$  to  $v_i$ . This is equal to after simplification:

$$\Rightarrow v_i Y_1 = (Y_1 Y_2 + Y_1 Y_4 + Y_2^2 + Y_2 Y_4 + Y_2 Y_3 + Y_3 Y_4) \frac{v_o}{A_o Y_2} - v_o Y_3 - \frac{v_o}{A_o} Y_2$$

So, what is the LCM here?  $A_o Y_2$

$$\Rightarrow v_i Y_1 = \frac{Y_1 Y_2 v_o + Y_1 Y_4 v_o + Y_2^2 v_o + Y_2 Y_4 v_o + Y_2 Y_3 v_o + Y_3 Y_4 v_o - v_o A_o Y_2 Y_3 - v_o Y_2^2}{A_o Y_2}$$

$$\Rightarrow v_i Y_1 Y_2 A_o = (Y_1 + Y_2 + Y_3) Y_4 v_o + (1 - A_o) Y_2 Y_3 v_o + Y_1 Y_2 v_o$$

So, the gain is  $\frac{v_o}{v_i}$ . So, what will be expression for  $v_o$  by  $v_i$ ?

$$\Rightarrow \text{Gain} = \frac{v_o}{v_i} = \frac{Y_1 Y_2 A_o}{(Y_1 + Y_2 + Y_3) Y_4 + (1 - A_o) Y_2 Y_3 + Y_1 Y_2}$$

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By comparing we get

$$\omega_c^2 = \frac{1}{R^2 C^2} \Rightarrow \omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi RC}$$

Cut-off freq =  $\omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi RC}$

Damping factor  $\zeta = \frac{3-A_0}{2} \Rightarrow \zeta = \frac{3-A_0}{2}$

Gain =  $\frac{V_o}{V_i} = \frac{Y_1 Y_2 A_0}{(Y_1 + Y_2 + Y_3) Y_4 + (1 - A_0) Y_2 Y_3 + Y_1 Y_2}$

$Y_1 = Y_2 = \frac{1}{R}$   
 $Y_3 = Y_4 = sC$

$\frac{V_o(s)}{V_i(s)} = \frac{A_0 \left(\frac{1}{R^2}\right)}{\left(\frac{2}{R} + sC\right) sC + (1 - A_0) \frac{sC}{R} + \frac{1}{R^2}}$

$H(s) = \frac{A_0 \omega_c^2}{s^2 + 2\zeta \omega_c s + 1}$   
 Generalized Second-order transfer function

$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{A_0}{(2 + sCR) sCR + (1 - A_0) sCR + 1}$

$= \frac{A_0}{s^2 R^2 C^2 + (2 + 1 - A_0) sCR + 1}$

$= \frac{A_0}{s^2 R^2 C^2 + (3 - A_0) \frac{s}{RC} + \frac{1}{R^2 C^2}}$

This is the expression for the gain of a generalized Sallen-Key second order active low pass filter. Now, we replace these admittances with the resistance and capacitances. So, the first order filter have one  $RC$  section now we will be having two  $RC$  sections. So, in terms of  $R$  and  $C$ , this circuit will be this is  $v_i$  this is one  $RC$  section this is another  $RC$  section and this will be connected to the output this is input.

We will take the identical  $RC$  sections  $RC$   $RC$  this will be grounded and positive side we have resistance  $R_1$   $R_F$ . So, this circuit is similar to the generalized circuit that I have given with what are the values of  $Y_1$   $Y_2$  this was  $Y_1$  this was  $Y_2$  this was  $Y_3$  this was  $Y_4$ . So,  $Y_1$   $Y_2$  are  $\frac{1}{R}$  because the admittances and  $Y_3$   $Y_4$  is  $sC$ . If it is  $Z$ , 1 by  $sC$   $YsC$  if it is  $Z$ ,  $R$  simply because of  $Y$  it is  $\frac{1}{R}$ .

$$Y_1 = Y_2 = \frac{1}{R}$$

And

$$Y_3 = Y_4 = sC$$

Now, what will be the transfer function of this in terms of Laplace transforms?  $\frac{v_o(s)}{v_i(s)}$  So, in this expression you substitute the values of  $Y_1 Y_2 Y_3 Y_4$

$$\frac{v_o(s)}{v_i(s)} = \frac{\frac{A_o}{R^2}}{\left(\frac{2}{R} + sC\right) sC + (1 - A_o) \frac{sC}{R} + \frac{1}{R^2}}$$

So, if you simplify this:

$$\Rightarrow \frac{v_o(s)}{v_i(s)} = \frac{A_o}{(2 + sCR)sCR + (1 - A_o)sCR + 1}$$

$$\Rightarrow \frac{v_o(s)}{v_i(s)} = \frac{A_o}{s^2 R^2 C^2 + (2 + 1 - A_o)sCR + 1}$$

$$\Rightarrow \frac{v_o(s)}{v_i(s)} = \frac{A_o}{s^2 R^2 C^2 + (3 - A_o)sCR + 1}$$

But we know that the second order system will be having a frequency response which is of the form of:

$$H(s) = \frac{A_o \omega_c^2}{s^2 + \xi \omega_c s + 1}$$

Here,  $\xi$  (zeta) which is dumping factor. This is the generalized second-order transfer function. So, in order to compare these two, we have to express this in the form of  $s$  square coefficient has to be make 1. So, if I make  $s$  square coefficient as 1  $A_o R$  square  $C$  square you have to take as common. So,  $s$  square plus 3 minus  $A_o sCR$  by  $R$  square  $C$  square. So, one  $R$  one  $C$  will get cancel  $s$  by  $R C$  plus 1 by  $R$  square  $C$  square.

$$\Rightarrow \frac{v_o(s)}{v_i(s)} = \frac{\frac{A_o}{R^2 C^2}}{s^2 + (3 - A_o) \frac{s}{RC} + \frac{1}{R^2 C^2}}$$

Here this  $\omega_c$  is cutoff frequency. So, by comparing these two comparing with standard and this one by comparing you will get:

$$\omega_c^2 = \frac{1}{R^2 C^2}$$

cutoff frequency:

$$\Rightarrow \omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi RC}$$

The same expression we have obtained for the first order low pass filter also. This  $R$  and  $C$  is going to decide the cutoff frequency.

And what is damping factor?  $\xi$  (zeta) is called damping factor.

$$\omega_c \xi = \frac{3 - A_o}{RC}$$

But  $\omega_c RC = 1$

$$\Rightarrow \xi = (3 - A_o)$$

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Handwritten derivation of the transfer function and its magnitude:

$$H(s) = \frac{A_o \omega_c^2}{s^2 + \xi \omega_c s + 1} \quad \text{where } \omega_c = \frac{1}{RC} \text{ and } \xi = (3 - A_o)$$

$$A_o = \left(1 + \frac{R_2}{R_1}\right)$$

$$H(j\omega) = \frac{A_o \omega_c^2}{(j\omega)^2 + j\omega \omega_c + 1} = \frac{A_o}{\left(\frac{j\omega}{\omega_c}\right)^2 + j\xi \frac{\omega}{\omega_c} + \frac{1}{\omega_c^2}} = \frac{A_o}{\frac{-\omega^2}{\omega_c^2} + j\xi \frac{\omega}{\omega_c} + \frac{1}{\omega_c^2}}$$

$$|H(j\omega)| = \frac{A_o}{\sqrt{\left[\frac{1}{\omega_c^2} - \frac{\omega^2}{\omega_c^2}\right]^2 + \xi^2 \frac{\omega^2}{\omega_c^2}}}$$

$$= \frac{A_o}{\sqrt{\left[\frac{1 - \omega^2}{\omega_c^2}\right]^2 + \xi^2 \frac{\omega^2}{\omega_c^2}}}$$

At  $\omega = 0 \Rightarrow |H(j\omega)| = A_o$   
 At  $\omega = \infty \Rightarrow |H(j\omega)| = 0$

So, this is second-order system with the standard expression

$$H(s) = \frac{A_o \omega_c^2}{s^2 + \xi \omega_c s + 1} \text{ where } \omega_c = \frac{1}{RC} \text{ and } \xi = (3 - A_o)$$

Here,

$$A_o = \left(1 + \frac{R_F}{R_1}\right)$$

So, we can have some discussion on this transfer function. So, how does this frequency response varies? Frequency response if you want you have to replace  $s$  with  $j\omega$ .

$$H(j\omega) = \frac{A_o \omega_c^2}{(j\omega)^2 + j\xi \omega \omega_c + 1}$$

If you want to write in terms of  $\frac{\omega}{\omega_c}$ , if I take this  $\omega_c^2$  common you divide everything with this  $\omega_c^2$ . So, I want to make in the numerator only  $A_o$   $\omega_c^2$  I am taking as common.

$$\Rightarrow H(j\omega) = \frac{A_o}{\left(j \frac{\omega}{\omega_c}\right)^2 + j\xi \frac{\omega}{\omega_c} + \frac{1}{\omega_c^2}}$$

$$\Rightarrow H(j\omega) = \frac{A_o}{-\frac{\omega^2}{\omega_c^2} + j\xi \frac{\omega}{\omega_c} + \frac{1}{\omega_c^2}}$$

So, what will be magnitude response?

$$|H(j\omega)| = \frac{A_o}{\sqrt{\left[\frac{1}{\omega_c^2} - \frac{\omega^2}{\omega_c^2}\right]^2 + \xi^2 \frac{\omega^2}{\omega_c^2}}}$$

$$\Rightarrow |H(j\omega)| = \frac{A_o}{\sqrt{\left[\frac{1 - \omega^2}{\omega_c^2}\right]^2 + \xi^2 \frac{\omega^2}{\omega_c^2}}}$$

First, I will show that this is a low pass filter I will take two extreme points. At  $\omega = 0$  what will be this one, what is magnitude of  $H(j\omega)$  is equal to this is 0. So, 1 square is 1 this is 0. So, simply  $A_o$

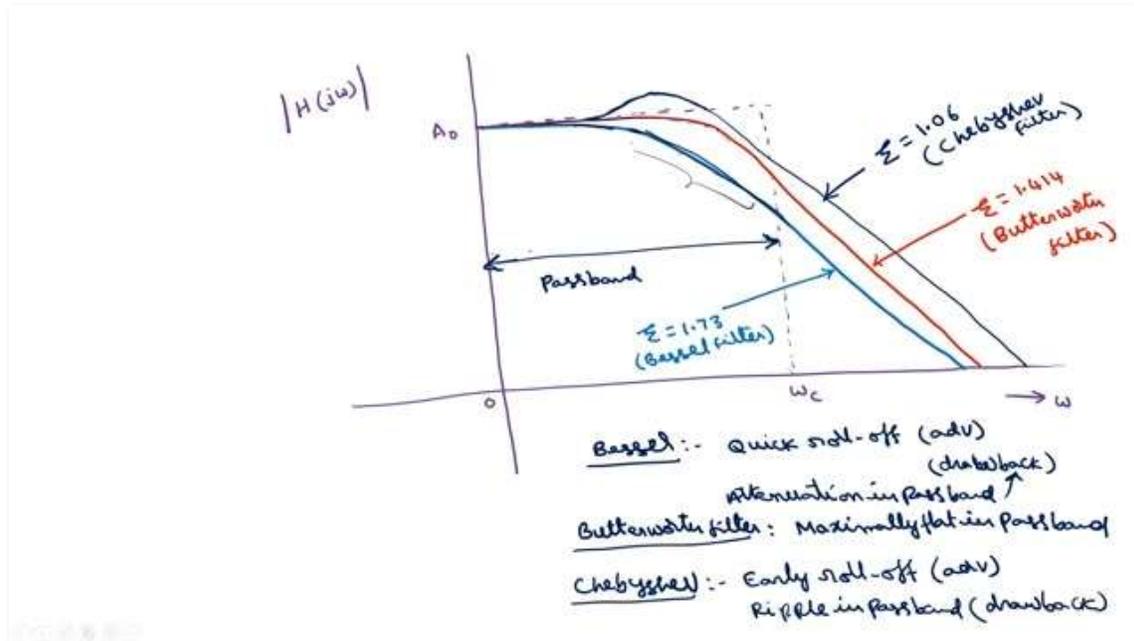
$$\text{At } \omega = 0 \Rightarrow |H(j\omega)| = A_o$$

If I take at  $\omega_c$ ,  $\omega$  is equal to  $\omega_c$ . So, what happens this becomes unity zeta square this is equal to  $\omega$  is equal to  $\omega_c$  this is equal to 1. So, you will get a value. So, at  $\omega$  is equal to infinity the second extreme point what is this denominator is infinity 1 by infinity is 0.

$$\text{At } \omega = \omega_c \Rightarrow |H(j\omega)| = 0$$

So, this will act as a low pass filter.

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So, if I plot this response for different values of the damping factor. I will take three different cases. This is the ideal response. It has to allow 0 to  $\omega_c$  and it has to reject  $\omega_c$  to infinity. If I take three different cases of  $\xi$  (zeta).

So, we will get three different curves like this. This is one curve. This is another curve, this is another curve. So, this is for  $\xi$  (zeta) is equal to 1.73, this is called Bessel filter. This is for  $\xi$  (zeta) 1.414, which is root 2. This is called Butterworth filter. This is for  $\xi$  (zeta) is equal to 1.06. This is called Chebyshev filter.

Now, we have a comparison between these three types of the filters. So, if I take this Bessel filter where damping factor is high. The advantage of this one is this will be having quick roll-off. So, the roll-off starts from here itself, but the drawback is this is advantage, but drawback is.

So, this is the pass band this is the pass band of course, a part of stop this transition band is also there. So, we want the flat response in the pass band that is the ideal this one. This is the ideal if I take the dotted line it should be flat throughout the pass band, but here in

the pass band there will be some attenuation. So, this part will indicate that this part will indicate that there is attenuation in the pass band also. This is the drawback whereas, if we come for the Butterworth filter this is called maximally flat in pass band.

Compared to this this and this this is maximally flat. So, in the pass band if you consider over this from here to here the maximally flat thing is this red curve which is Butterworth filter. That is why mostly we will use Butterworth type of the filters. So, in this course we will consider only the Butterworth filter. Of course, the roll-off will start a little late than this Bessel function, but it is maximally flat. Coming for the Chebyshev so, this have roll-off even before this Bessel roll-off starts here itself even better than Bessel, but there will be ripple in the pass band.

This is advantage drawback is ripple in the pass band. So, among the three actually Butterworth is having the moderate response. So, mostly we will use Butterworth filter for the design of active filters. So, in this course we will discuss only the Butterworth filters. So, what are the transfer functions of these Butterworth filters and how to derive this normalized Butterworth filter and all that we'll discuss in the next lecture. Thank you.