Integrated Circuits and Applications

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Active Filters I
Lecture - 15
First Order Low Pass Filter

Okay! In the previous lectures, we have discussed some of the general applications of operational amplifier such as adder, subtractor, logarithmic amplifier, anti-logarithmic amplifier, instrumentation amplifier and so on. So, in this lecture now we will discuss some of the specialized applications such as filters. So, in this lecture we will discuss about the active filters.

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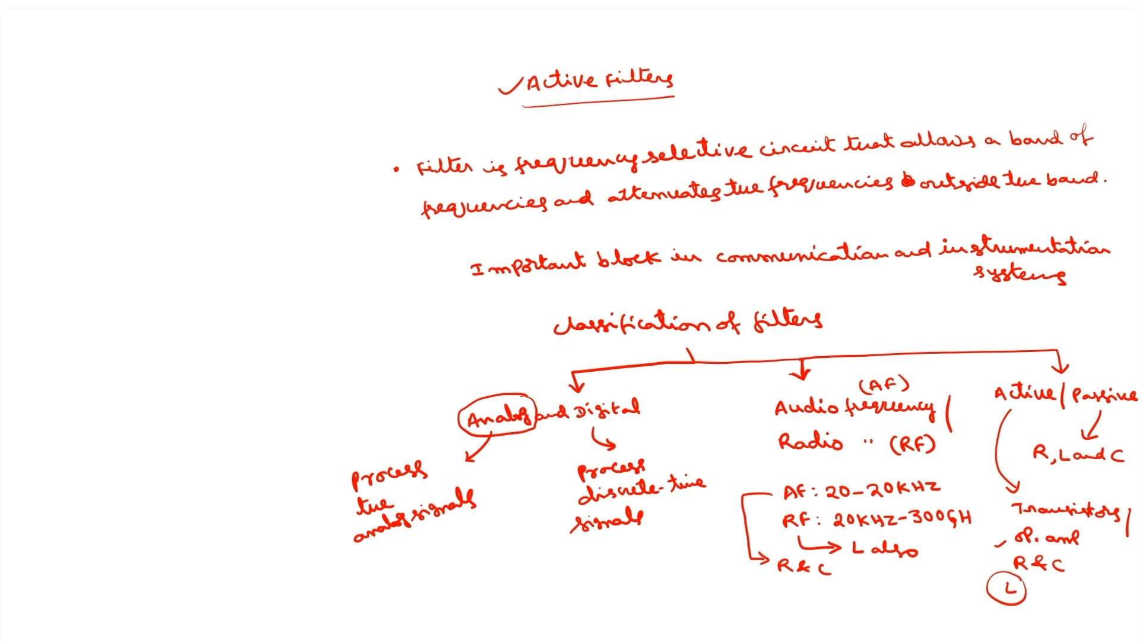

So, I will come to what is meant by active filter. So, before that I will discuss what is a filter. Filter is a electronic circuit which allows a band of frequencies and rejects the remaining frequencies. It is basically a frequency selective network. Filter is a frequency selective circuit that allows a band of frequencies and attenuates or rejects frequencies outside the band. So, this filter is one of the important block in many of the communication and instrumentation systems. This is an important block. Because in communication, so we want to recover the original desired signal from the noise corrupted signal. So, in order to remove the noise, we can use the filters. Similarly, in the instrumentation if we record some data from the sensor. So, the data consists of the desired signal as well as some undesired noises. In order to remove those noises, we can use these filters.

So, filters basically we can classify the filters in three ways, analog and digital filters. Depends upon the type of signal that we are going to process it is called as either analog filter or digital filter. Analog filter as the name implies process the analog signals. Digital filter process digital signals or discrete time signals. So, in this course we will mostly discuss about the analog filter design.

The second classification is based on the frequency range that a filter can process. It can be called as audio frequency filters or radio frequency called RF. AF or RF. So, what is the range of the audio frequencies? We know that this is normally in the range of 20 to 20 kilohertz and radio frequencies will be normally from 20 kilohertz to 300 gigahertz which is high frequencies. So, this in order to process this audio frequencies we can use the components like resistor and capacitor whereas, in order to process this radio frequencies we require inductor also.

The third classification is based on the active and passive filters. So, in case of passive filter it uses only the components like R, L and C, resistance, inductance and capacitance passive components whereas, in active filters in addition to RLC it also uses transistors or operational amplifiers. So, in active filter in addition to RLC it also uses transistors or operational amplifiers.

So, in this course we will discuss about the active filters which uses operational amplifier and then basically R and C because inductor it becomes difficult to fabricate in the IC form.

And as you have discussed in the earlier lectures that a inductor can be simulated by using a RC network connecting a resistor, capacitor in the feedback path. And there are many advantages of this active filters compared with the passive filters that I will discuss later in this lecture. This is one of the classifications of this filters and depends upon the frequency range that it will select as I have told this is frequency selective network.

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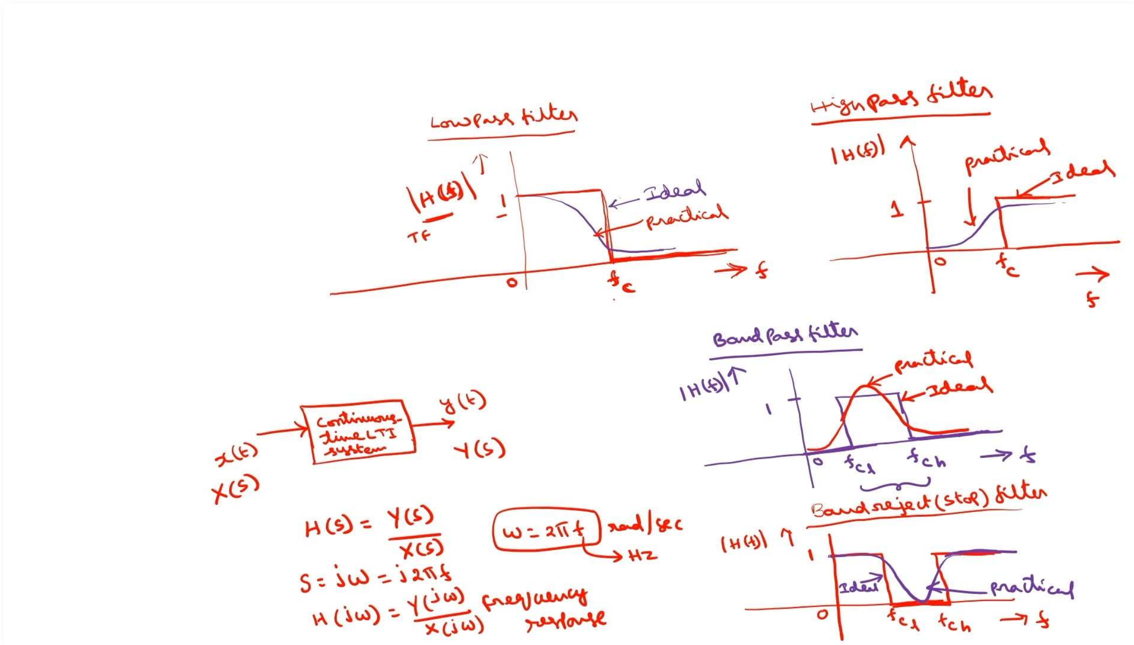

So, we can depends upon the frequency range that it is going to be select we have again four types of the filters one is called low pass filter. As the name implies it passes the low frequencies, reject the high frequencies. If I take the ideal response of the low pass filter the frequency response is something like this. Up to $f = f_c$ this is the frequency f and this is the $|H(f)|$ this is unity this is 0. And of course, this is symmetric with respect to the origin. I am not showing the other part.

It allows the frequencies from 0 to f_c with unity gain from f_c to infinity the gain is 0. So, all frequencies above this f_c will be rejected. So, you see the ideal low pass filter first I will discuss what is this transfer function this is called transfer function. If I take a system say this is LTI system the linear time invariant system and this continuous time. So, this filter will acts as a continuous time LTI system with input $x(t)$ output $y(t)$.

If I take the Laplace transform of input and output and if I call these Laplace transforms as $X(s)$ and $Y(s)$ respectively. Then the ratio of this output Laplace transform to the input Laplace transform ratio of output Laplace transform to input Laplace transform is called as transfer function $H(s)$, $Y(s)$ by $X(s)$. And if I want to obtain the frequency response, s you have to substitute as s = $j\omega$ = $j2\pi f$.

So, $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$ this is called the frequency response of the system. So, we can alternatively write this as f or ω the relation is $\omega = 2\pi f$.

We are familiar with this f which is expressed in a Hertz and if it is ω it will be radians per second f will be in Hertz. So, that 2π scaling factor will be there. Here this can be $H(\omega)$ or $H(f)$. So, it allows the frequencies from 0 to f_c and rejects the frequency from f_c to infinity this is called ideal lowpass filter. This is the response of ideal one and practically it will be different from the ideal one. If you want practical lowpass filter it will be something like this.

This is ideal one and this is practical. And the second type of filter is called highpass filter as the name implies it passes the high frequencies rejects the low frequencies. The ideal and practical response of this high pass filter is like this. It rejects the frequencies from 0 to f_c this is 0 to f_c and from f_c to infinity it passes with unity gain. This is the ideal response and if you want practical response it will be something like this. This is the practical response.

And the third type of filter is called bandpass filter. It allows a band of frequencies. Now, this will be having two cutoff frequencies one is called lower cutoff frequency, another is called upper cutoff frequency. This will be 0, from here to here it this will allow. 0, f_{cl} lower cutoff frequency, f_{ch} higher cutoff frequency. This will allow with unity gain. From 0 to f_{cl} this will reject, from f_{ch} to infinity also rejects only this range of frequencies will be allowed this is called bandpass filter. And if I take the practical bandpass filter this will be something like this, this is practical bandpass filter.

And the last one is band reject filter just opposite to bandpass filter is band reject filter or also called as bandstop filter. It rejects only a band of frequencies and the remaining all frequencies will be possible. This is f_{cl}, f_{ch} it rejects only f_{cl} to f_{ch} 0 to f_{cl} unity gain f_{ch} to infinity unity gain.

This is the ideal frequency response of band reject or bandstop filter. And if you want practical one, it will be something like. This is the practical bandstop filter. Now, in this course we will discuss the design of these filters.

So, let us first start with the low pass filter. So, in order to demonstrate the advantages of this active filters over passive filters.

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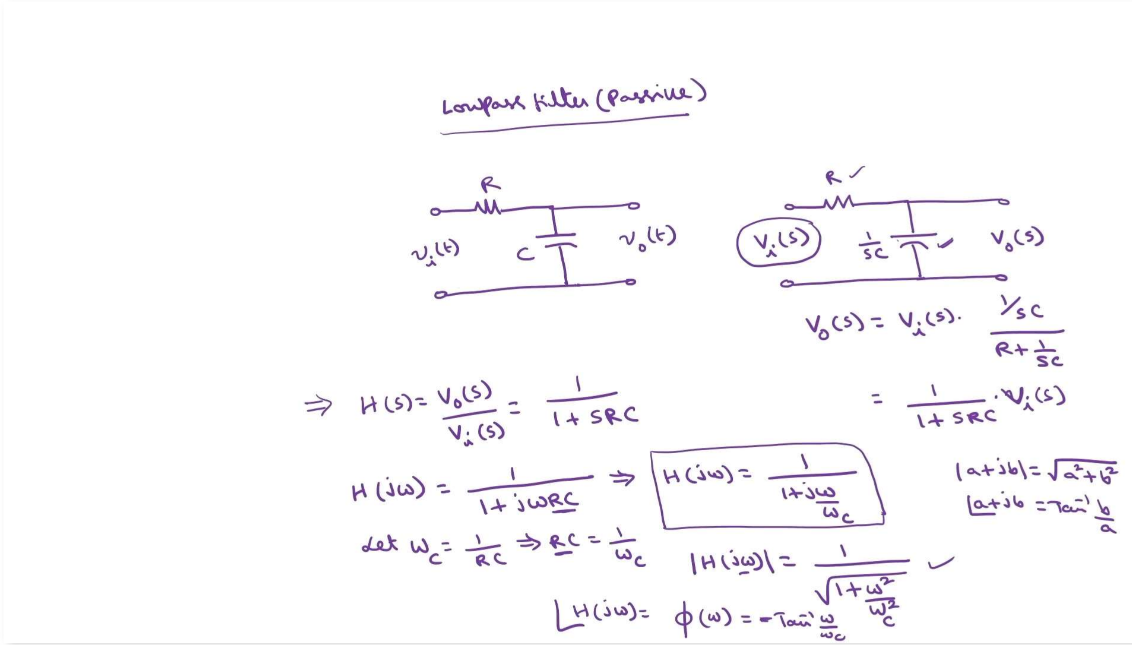

I will first consider the passive low pass filter and after that I will consider the active low pass filter then I will compare these two and I will discuss the advantages of passive filters over active filters. If I take a lowpass filter (passive) means it has to allow the low frequencies up to a some frequency called cutoff frequency after that it has to reject the remaining frequencies.

So, simply a resistor and capacitor will form a lowpass filter. This is passive because this does not contain the transistor or operational amplifier. This is $V_i(t)$, this is $V_o(t)$, this is *R*, this is *C*, this is the input output. So, if you want in s domain. So, this circuit in s domain will be, instead of t will be having s and capacitance becomes $\frac{1}{sC}$, $V_i(t)$, $V_o(t)$.

The ratio of this output Laplace transform to input Laplace transform is called as transfer function. And if you replace *s* with $j\omega$ you will get frequency response. Basically, here we are interested in frequency response of the filters. So, from this what is $V_o(t)$? This is voltage division. This $V_i(s)$ is going to distribute between R and $\frac{1}{sC}$. I want the voltage across $\frac{1}{sC}$. So,

$$V_o(s) = V_i(s)\frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}V_i(s)$$

In place, what is the transfer function?

$$\Rightarrow H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC}$$

This is the transfer function of this circuit.

And what is the frequency response? Replace *s* with $j\omega$.

$$\Rightarrow H(j\omega) = \frac{1}{1 + j\omega RC}$$

If I define, let $\omega_c = \frac{1}{RC} \Rightarrow RC = \frac{1}{\omega_c}$. So, what happens to this $H(j\omega)$?

$$\Rightarrow H(j\omega) = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

So, this is the transfer function of the given circuit. Now, this will be having two parts, one is magnitude part and the phase part.

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{{\omega_c}^2}}}$$

As you know that for a complex number, the magnitude is square root of real parts plus imaginary part square.

$$|a + jb| = \sqrt{a^2 + b^2}$$

And phase angle is let us call $\phi(\omega)$ is phase angle of $H(j\omega)$.

$$\angle H(j\omega) = \phi(\omega) = -\tan^{-1}(\frac{\omega}{\omega_c})$$

Because this is in the denominator if I take to the numerator it will be minus sign and *tan* inverse of phase angle of *a* plus *jb* is equal to *tan* inverse *b* by *a*.

$$\angle a + jb = \tan^{-1}(\frac{b}{a})$$

Because of this denominator term we have to keep minus sign. So, we basically interested in the magnitude response. Here I will plot as a function of omega you can plot as a function of *f* also the relation is only $\omega = 2\pi f$.

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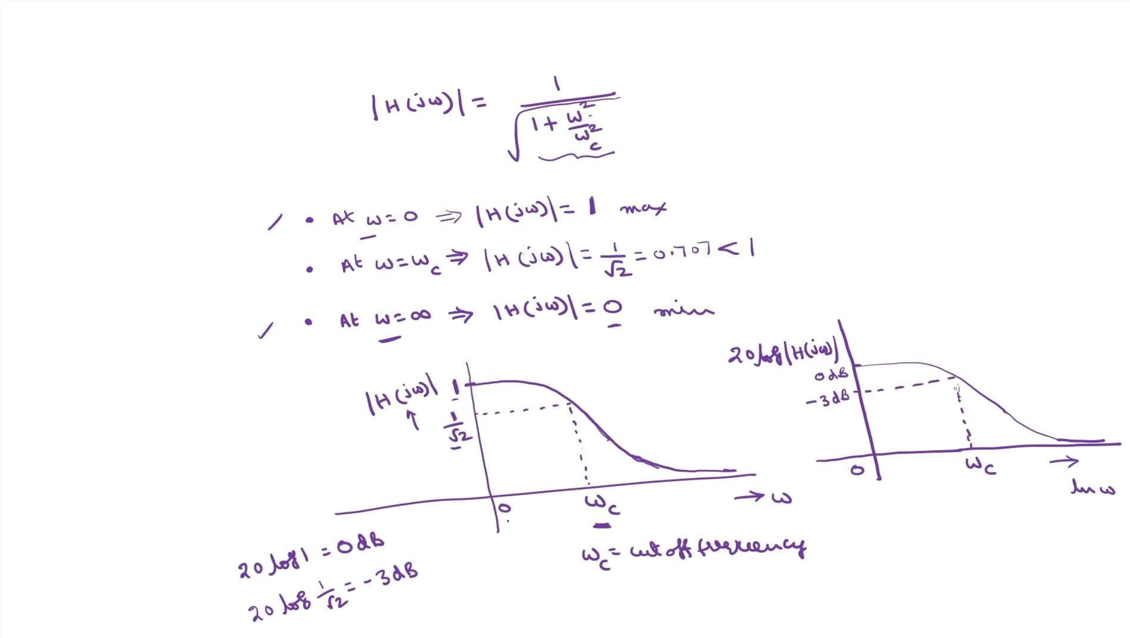

So, what is magnitude of H of $j\omega$?

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{{\omega_c}^2}}}$$

So, we can see the properties here. At $\omega = 0$, low frequencies basically. So, what happens to magnitude of $H(j\omega)$? This is 0. So, 1 by 1 its unity.

$$|H(j\omega)| = 1$$

At $\omega = \omega_c$. What is this modulus of $H(j\omega)$ is equal to this is ω, ω_c becomes 1. So, 1 by root 2 which is 0.707.

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = 0.707 < 1$$

Means as the frequency increases, the gain is going to be decreased. At $\omega = 0$ unity gain at another frequency at ω_c the gain is 0.707 which is less than 1. And if I think take the extreme end this is the beginning and this is the end omega is equal to infinity.

At $\omega = \infty$, what is $H(j\omega)$?

$$|H(j\omega)| = 0\text{, this is minimum value at maximum frequency}$$

And at $\omega = 0$,

$$|H(j\omega)| = 1\text{, maximum value at minimum frequency.}$$

So, if I plot for different values of the ω you can find out that this response will be something like this. At ∞, it will go to 0. This is I am plotting with respect to ω.

You can plot with respect to *f* also because the only difference is 2π. This is magnitude of frequency response with ω. At $\omega = 0$, what is the value here? This is 1, this is maximum value. At $\omega = \infty$, it will go to 0 and at a value which is called as $\omega = \omega_c$, this is $\frac{1}{\sqrt{2}}$. So, the frequency at which the magnitude is $\frac{1}{\sqrt{2}}$ times the maximum value is called as cutoff frequency.

You can say that $\frac{1}{\sqrt{2}}$ attenuation is allowed. So, ω_c is called cutoff frequency means this allows from 0 to ω_c and ω_c to ∞ it attenuates this is the attenuation. So, normally this frequency response will be plotted in *dB* scale because it covers lot of wide range of the frequencies. So, normally this will be plotted in *dB* scale. If I plot in the *dB* scale this is logarithm of ω, this is 20 logarithm of $|H(j\omega)|$.

So, what happens to this plot? Plot remains same, this value becomes *0 dB* because 20 logarithm of 1, logarithm of 1 is 0. So, this is *0 dB*.

What is 20log$\frac{1}{\sqrt{2}}$?

You can see that this is $-\ 3\ dB, 3.01$ something.

So, this is called $-\ 3\ dB$. So, normally the cutoff frequency will be defined at $-\ 3\ dB$ the frequency at $-\ 3\ dB$ is called cutoff frequency. So, this will acts as a lowpass filter as you have discussed in the previous slides that this is the ideal response, this is the practical response.

We are getting this practical response using a simple *RC* circuit. So, we can implement this lowpass filter using *RC* components which is passive elements. Then why you have to go for the active filter? In active filter what happens is we have to add an operational amplifier.

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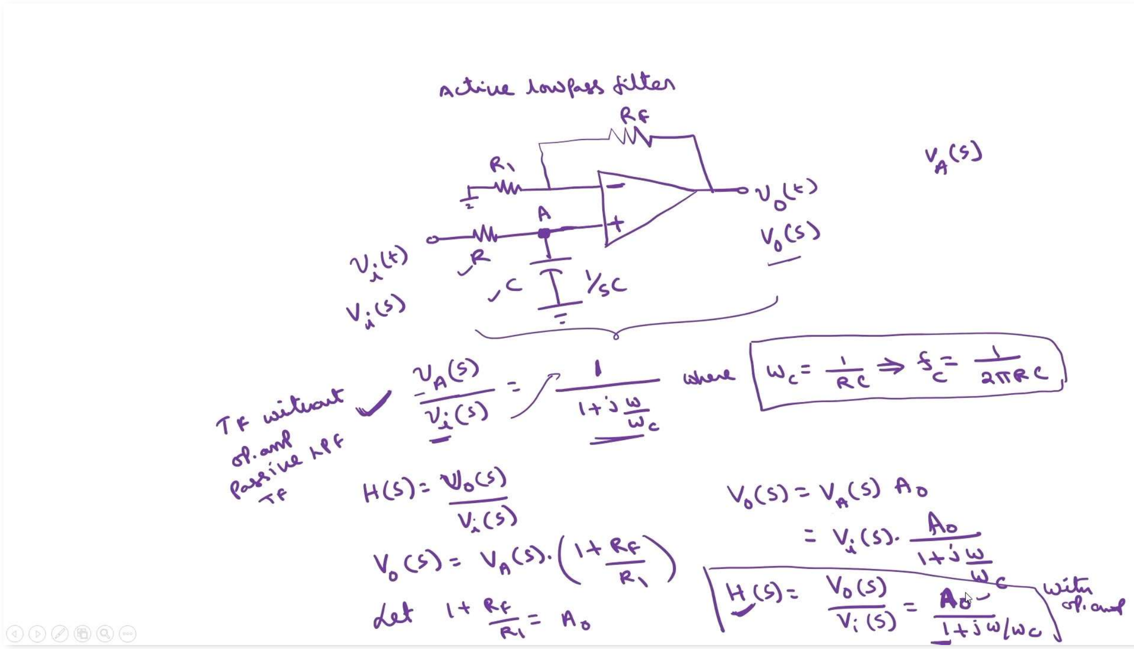

If I take the active low pass filter, the network that we have considered is the resistance and then capacitance. So, the same circuit will be here also, but in addition to this we have to use operational amplifier. Instead of taking the output from here, we are taking output after the operational amplifier. This is R_1, R_F, this is simply R and C. So, if I take this V_i in terms of Laplace transform, then this C will be taken in terms of $\frac{1}{sC}$.

If I call this point as point A, so what is the expression for $\frac{V_A}{V_i}$? Either it can be t or s function of s, is the one that we have derived.

$$\frac{V_A(s)}{V_i(s)} = \frac{1}{1+\frac{j\omega}{\omega_C}}, \text{ where } \omega_C = \frac{1}{RC} \Longrightarrow f_c = \frac{1}{2\pi RC}, \text{ this is called cutoff frequency.}$$

This $R\ and\ C$ is going to decide the cutoff frequency of the filter. Now, for this entire circuit what is the transfer function?

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

If I forget about this circuit, if I know that this voltage is V_A, $V_A(t)$ or $V_A(s)$, this voltage here is $V_A(s)$.

What is the remaining circuit? This is non-inverting amplifier whose gain is equal to 1 plus R_F by R_1. So, what is the expression for $V_o(s)$ in terms of $V_A(s)$?

$V_o(s) \ = V_A(s)(1+\frac{R_F}{R_1})$, because this is non-inverting amplifier.

Let $1+\frac{R_F}{R_1} = A_o$

Therefore, what is $V_o(s)$?

$$V_o(s) \ = V_A(s)A_o$$

So, we want finally, $\frac{Vo}{Vi}$. This is equal to from this what is $V_A(s)$?

$$\Rightarrow V_o(s) = V_i(s)\frac{A_o}{1\ +\frac{j\omega}{\omega_c}}$$

What is $H(s)$ transfer function?

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{A_o}{1\ +\frac{j\omega}{\omega_c}}$$

This is exactly same as this transfer function without op-amp except for that only A_o is coming here. Here A_o is not there, unity is there, here A_o is there and the denominator is exactly same. This is the transfer function using op-amp, this is the transfer function without op-amp.

$\frac{V_A(s)}{V_i(s)} \ = \frac{1}{1+\frac{j\omega}{\omega_c}}$, this is the transfer function without op-amp

which you can call as passive lowpass filter transfer function.
And

$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{A_o}{1+\frac{j\omega}{\omega_c}}$, this is with op-amp

This you can call as the transfer function with operational amplifier. So, if I plot the response only this magnitude becomes A_o here.

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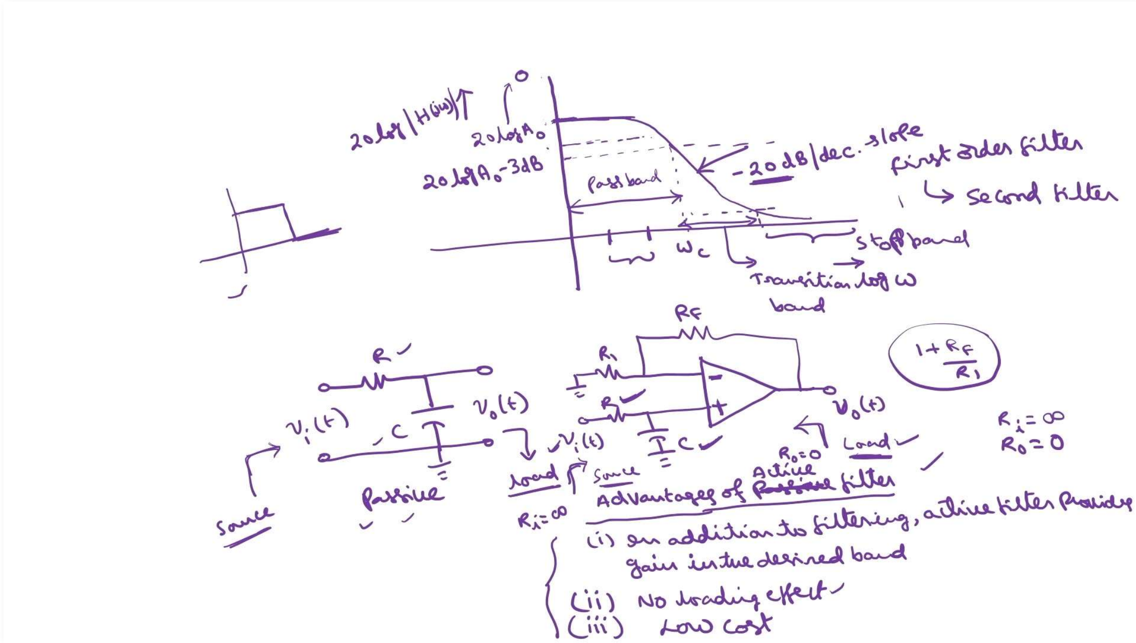

So, what will be the frequency response with and without this one? This is 20 logarithm of this is logarithm of ω modulus of $H(j\omega)$ will be same response you will get, but now here this will be instead of 0 dB this will be $20logA_o$ and this will be minus 3 dB less than this $20logA_o - 3$ dB. Initially this was 0 dB and this is -3 dB.

So, the response is almost same. Then what are the advantages of this passive and active lowpass filter? This is passive lowpass filter with input $V_i(t), V_o(t)$, this is passive and active is the same circuit with op-amp. This is of course, with respect to ground only. This R and this R, this C and this C same. So, the response is almost same except for that there will be some gain here. So, this was initially $0\ dB$, now this will be $20logA_o$.

So, these advantages of the passive filters are in addition to filtering, active filter provides gain in the desired band. So, this desired band of the frequencies 0 to f_c is called as pass band.

The frequencies from here to here, these are stop band, here to infinity stop band. Of course, you can divide this also into again two values, one is up to some particular frequency we will call this one as transition band and this one as stop band. From here to here stop band and from here to here this is called as transition band. So, in the pass band this is allowing some gain without operational amplifier passive this will be having something like here this is the gain this much is the gain.

Now, with active we are getting some gain that gain factor is you can see that this $1 + \frac{R_F}{R_1}$. This was the gain provided in the pass band whereas, here this will not provide the gain. This is one advantage of passive filter both will perform the filtering of the high pass signals, low pass signals will be passed, but here low pass signals will be passed with gain here without gain this is one of the advantage.

Second advantage is active filters will be having no loading effect. Here, this you have to give from the source, this you have to give to the destination or load. So, here because of this high input resistance this will load the source and this will load the load whereas, here this operational amplifier by its construction this will be having high input impedance ideally this is infinity output impedance is ideally 0 because of these values. So, this will not load this will not load this load device and it will not load the source also from where you are taking $V_i(t)$.

This $V_i(t)$, will be normally taken from some sensor or some circuit and this will be delivered to some load. So, this is output resistance because of this output resistance is low this will not affect this load and this input resistance is large it will not load this source this is no loading effect in case of active filter sorry this is active filter.

And the third one is low cost because the operational amplifier does not contains any inductors and all, here inductor can be simulated by using *R* and *C* elements. So, the cost of this active filter is less than that of passive filter. So, because of these three reasons this active filter are more popular. Nowadays, in almost all the electronic appliances these active filters are present. Now this is actually called as first order filter. Because we have only one *R*, one *C*. So, this number of *R* and *C* is going to decide the order of the filter.

The drawback of this one is ideally we want this type of the response this is the response that we are interested ideal low pass filter. So, in order to get the ideal lowpass filter this slope in the transition band has to be more.

For the first order filter if you see this one minus $20dB$ per decade. For ten-fold increment in this frequency ten-fold increment and if you see this corresponding gain it will be reduces by $20\ dB$. So, this is called $-20\ dB$ per decade slope. So, if the slope is more, then these characteristics approaches the ideal characteristics.

So, in order to increase this slope in the transition band you have to go for the higher orders. So, the next one is second order filter. So, this second order filter we will discuss in the next lecture. Thank you.