Integrated Circuits and Applications

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Problem Solving
Lecture – 14
Examples on Instrumentation CMMR Computation

Ok. In the last few lectures, we are discussing about some of the practicing problems. Today we will discuss some more problems on finding the *CMRR* of a given circuit. So, first, I will find out the *CMRR* of a instrumentation amplifier. We have find out the *CMRR* of differential amplifier. So, that time, I told that the *CMRR* of instrumentation amplifier is greater than that of the differential amplifier, which is one of the advantage of instrumentation amplifier. Today, we will derive the expression for the *CMRR* of instrumentation amplifier.

Find the *CMRR* of the following instrumentation amplifier. See the circuit diagram given. Basically, two voltage followers followed by a differential amplifier. This is the output R_3 R_4 R_3 then this R_4.

Here we are going to apply the input signal. This is R_2 R_2 this is R_1 R_1. Then here we will be having two components one is differential signal, another is common mode signal. This is common mode signal. This can be plus or minus; also like in previous example, this was plus and minus. Now, I am taking minus plus. This can be of any type.

So, here I am going to apply $\frac{v_d}{2}$ voltage differential voltage another $\frac{v_d}{2}$ here total is v_d. So, you have to find out the common mode rejection ratio of this circuit *CMRR* is common mode rejection ratio. How efficiently this circuit rejects the common mode signal such as the noise? If I call this signal as v_{i1} input 1 this signal as v_{i2} this output as v_{01} this as v_{02}. So, what are the expressions for v_{i1}, v_{i2}, v_{01}, v_{02}? If I consider this v_{cm} this $\frac{v_d}{2}$ this v_{i1}. So, what will be the loop? This v_{cm} is minus to plus this is v_{cm} this is, of course, grounded, and then this $\frac{v_d}{2}$ is minus to plus then you have to find out this v_{i1} this is with respect to plus and minus this is v_{i1}.

So, what is KVL? This is minus to plus this is plus to minus this is plus to minus. If I take minus to plus as positive then this will be v_{cm} and this is plus to minus. So, $v_{cm} - \frac{v_d}{2} - v_{i1} = 0 \Rightarrow v_{i1} = v_{cm} - \frac{v_d}{2}$, whereas, to find out this, v_{i2}. So, what will be circuitry? This v_{cm} this v_d by 2. This v_{i2} similar type of the circuit you can have here. So, this is now v_{cm}.

This is plus-minus, and this is minus plus, and this is connected to v_{i2}. This is v_d by 2.

If I take minus to plus as positive this is plus to minus this is plus to minus this is minus to plus. You can see here minus to plus, then minus to plus, then this is plus to minus, this is also minus to plus this is minus to plus. If you go in this direction again, this is minus to plus. So, what will be this minus to plus if I take as $v_{cm} + \frac{v_d}{2} - v_{i2} = 0 \Rightarrow v_{i2} = v_{cm} + \frac{v_d}{2}$.

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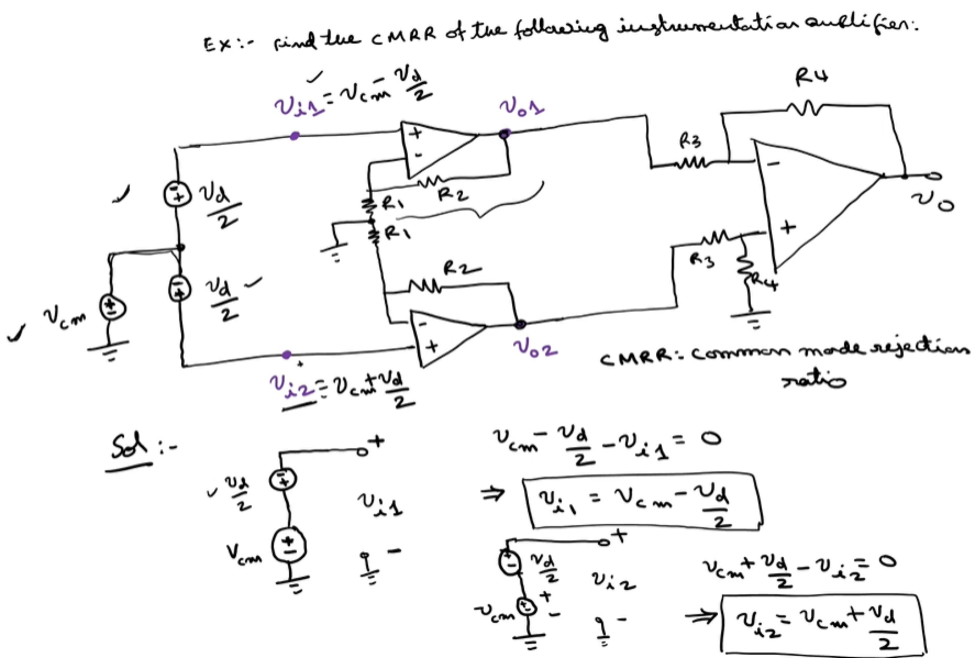

Now I have to find out this output voltages v_{01} v_{02}. So, what is v_{01}? You can see that this circuit this circuit is nothing, but a non-inverting amplifier this is grounded. So, the gain is one plus feedback resistance by input resistance. So, if I draw the equivalent circuit of this one. So, you say positive terminal negative terminal we have these two ground these two this and at positive terminal we have v_{i1} this output is v_{01}.

You can see that this is positive terminal is v_{i1}, and at negative terminal, we have feedback resistance R_2 input resistance which is grounded is R_1, and output is v_{01} this is R_1 this is R_2. This is a standard non-inverting amplifier whose gain is $(1 + \frac{R_2}{R_1})$. So, $v_{01} = (1 + \frac{R_2}{R_1})v_{i1}$, but what is v_{i1} from the previous circuit $v_{cm} - \frac{v_d}{2}$. So, this $v_{01} = (1 + \frac{R_2}{R_1})(v_{cm} - \frac{v_d}{2})$. Similarly, what about v_{02}? The circuitry for v_{02} is also similar type of circuit.

Here at positive terminal we have v_{i2} negative terminal R_1 is grounded R_2 is feedback resistance output is v_{02}. So, to find out R_2, the same circuit is there, but in place of v_{i1}, we have v_{i2}, and this is grounded. This is v_{02} R_2 R_1. So, $v_{02} = (1 + \frac{R_2}{R_1})v_{i2}$. Now, what is the difference between the v_{02} and v_{01}? Because this is differential amplifier, the input that is applied to the non-inverting terminal is v_{02}. So, you take the difference signal of $v_{02} - v_{01}$.

The differential output due to differential signal is $v_{02} - v_{01} = \left(1 + \frac{R_2}{R_1}\right) v_{i2} - \left(1 + \frac{R_2}{R_1}\right) v_{i1} = (1 + \frac{R_2}{R_1})(v_{i2} - v_{i1})$. What is v_{i2}? Is $v_{i2} = v_{cm} + \frac{v_d}{2}$ and what is v_{i1}? $v_{i1} = v_{cm} - \frac{v_d}{2}$. What is v_{i2} minus v_{i1}? v_{cm} will get cancelled this becomes $\frac{2v_d}{2}$ or simply v_d. So, you see nothing but v_d. This is $v_{02} - v_{01} = \left(1 + \frac{R_2}{R_1}\right) v_d$.

So, this will be the differential gain this is differential output differential input. So, whatever the factor of this one is differential gain. Therefore, differential gain $A_d = 1 + \frac{R_2}{R_1}$.

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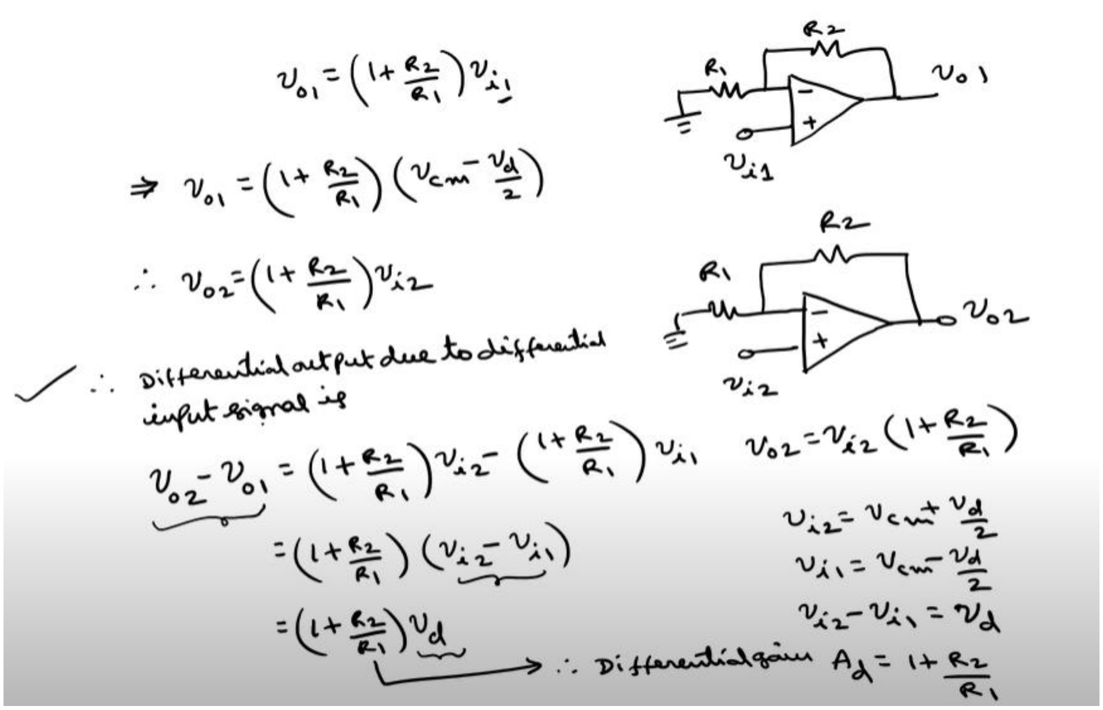

Then you have to find out the common mode gain to find out the *CMRR*. So, what is common mode gain? This is differential output due to the differential input signal. Now, what is the output due to the common mode signal? Is nothing, but $\frac{v_{01}+v_{02}}{2}$, this is the output. This is equal to what is v_{01} is this v_{02} is this $\frac{\left(1+\frac{R_2}{R_1}\right)v_{i1}+\left(1+\frac{R_2}{R_1}\right)v_{i2}}{2}$. This is equal to $\frac{1}{2}\left(1+\frac{R_2}{R_1}\right)(v_{i1}+v_{i2}) = \frac{1}{2}\left(1+\frac{R_2}{R_1}\right)\left(v_{cm}-\frac{v_d}{2}+v_{cm}+\frac{v_d}{2}\right) = \left(1+\frac{R_2}{R_1}\right)v_{cm}$.

So, you will get this is simply $\left(1+\frac{R_2}{R_1}\right)v_{cm}$, v_{cm} is the common mode signal. So, this is common mode gain. The output due to the common mode signal is equal to the common mode signal into whatever the factor that factor is common mode gain. Therefore, the common mode gain $A_{cm} = 1+\frac{R_2}{R_1}$, and what is the differential gain we have obtained is also same. Therefore, the common mode rejection ratio is defined as $CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right|$ 20 logarithm of modulus of AD by ACM. This is equal to $20\log\left|\frac{A_d}{A_{cm}}\right|$. Both are the same. So, $20\log 1$. So, this is simply 0dB. So, for the given circuit the *CMRR* is 0dB. So, this is not the good circuit. So, if *CMRR* is high, then it is a good circuit ok. So, this circuit is not a good circuit for the instrumentation amplifier.

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output due to common mode signal

$$\frac{V_{01}+V_{02}}{2} = \frac{\left(1+\frac{R_2}{R_1}\right)V_{i1} + \left(1+\frac{R_2}{R_1}\right)V_{i2}}{2}$$

$$= \frac{1}{2}\left(1+\frac{R_2}{R_1}\right)(V_{i1}+V_{i2})$$

$$= \frac{1}{2}\left(1+\frac{R_2}{R_1}\right)\left(V_{cm} - \cancel{\frac{V_d}{2}} + V_{cm} + \cancel{\frac{V_d}{2}}\right)$$

$$= \underbrace{\left(1+\frac{R_2}{R_1}\right)}V_{cm}$$

Common mode gain $A_{cm} = 1+\frac{R_2}{R_1}$

$$\therefore \quad CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log 1 = 0\text{ dB}$$

Now, I will consider the second circuit and I will find out the *CMRR* and I am going to prove that the second circuit is having good *CMRR*. So, most of the practical applications uses the second type of the instrumentation amplifier.

So, the next example is find the *CMRR* of the following instrumentation amplifier. So, almost similar circuit with slight modification. So, this part is exactly same. This is output, this is R_3 R_3 R_4 R_4. Here you are going to take v_{i1}, this is v_{i2}, and here, we are going to connect a 2R resistor, $2R_1$ resistor.

Instead of having $2R_1$ resistors in series and in between we have a ground. Now, we have $2R_1$ resistor without any ground. The remaining connections are same. This is R_2, this is also R_2, and here this differential signal, this is another differential signal, and this is common mode signal. This is plus minus v_{cm}, this is minus plus minus plus, this is $\frac{v_d}{2}$, this is $\frac{v_d}{2}$.

Then we have already derived that this is $v_{01} = v_{cm} - \frac{v_d}{2}$. This you have derived that $v_{02} = v_{cm} + \frac{v_d}{2}$. But the output v_{01} and v_{02} will be different now because here the connection has been changed. If I assume that this current is I, the same current flows through. If this current is I, the same current flows through this, the same current flows to this because the operational amplifiers are ideal.

So, the current through this terminal is 0, current through this terminal is 0. So, whatever the current coming from here, so this will go to this, this will goes through this $2R_1$, then the same current will flows through this R_2 resistor, here current is 0. So, now, what will be expression for v_{01}, v_{02}? For that first, I will find out current *i*. This is the direction of the *i*. So, this voltage is nothing but v_{01}.

The second assumption of the ideal operational amplifier, the voltage at non-inverting terminal is equal to voltage at the inverting terminal. So, this is v_{01} means this is also v_{01}. So, the voltage at non-inverting terminal of second operational amplifier is v_{02}, means the voltage at inverting terminal also v_{02}. Now, this is the current, this is v_{01}, this is v_{02}, direction of the current is from v_{01} to v_{02}. So, what is expression for $i = \frac{v_{01} - v_{02}}{2R_1}$.

Now, knowing this v_{01}, how to find out? This is v_{i1}, v_{i2}, this is v_0, this is input currents. So, this is $v_{i1} - v_{i2}$. So, to find out v_{01} from v_{i1}, so what is the KVL? This is v_{i1}, and this is plus to minus, and this v_{i1} is with respect to this, this is plus with respect to ground, this is with respect to ground. So, this is something like we have this v_{i1} with respect to ground, this is v_{i1} plus minus, then we have the resistor, then you are going to take the output here with respect to ground, this is plus minus and the direction of this current here is this means this is plus, this is minus, this is R_2, this is v_{01}, this is v_{i1}. So, what is the relation between the v_{i1}, v_{01} and R_2? Minus to plus if I take as positive, so v_{i1} and this is also minus to plus. So, the current is $v_{i1} + iR_2 - v_{01} = 0 \Rightarrow v_{01} = v_{i1} + iR_2$. But what is i? $i = \frac{v_{01}-v_{02}}{2R_1}$. So, $v_{01} = v_{i1} + (\frac{v_{01}-v_{02}}{2R_1})R_2$, what is v_{i1}? $v_{cm} - \frac{v_d}{2}$. What is v_{i1} minus v_{i2}? v_{i1} is this one. So, $v_{i1} - v_{i2} = v_{cm} - \frac{v_d}{2} - v_{cm} - \frac{v_d}{2} = -v_d$. So, implies what is v_{01} final expression? $v_{01} = v_{cm} - \frac{v_d}{2} + \frac{R_2}{2R_1}(-v_d)$.

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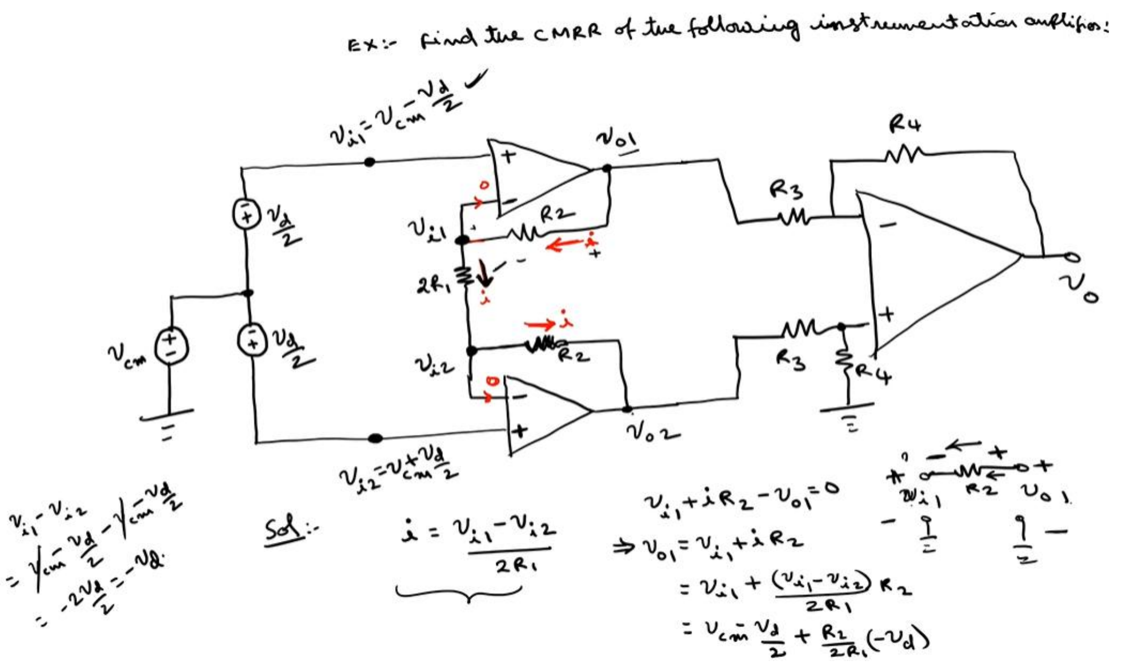

And if you take this $\frac{v_d}{2}$ as common, this is $v_{01} = v_{cm} - \frac{v_d}{2}(1 + \frac{R_2}{R_1})$. This is one expression for v_{01}. Similarly, how to find out the expression for v_{02}? So, you can consider now this path, this resistance, this current, this is v_{i2} and this is v_{02}. So, if I write a similar type of the circuit here, v_{i2} R_2 v_{02}, v_{02}, this is v_{i2} passing through the circuit R_2, and here we are taking the output with respect to ground plus-minus this is v_{02}.

And what is the direction of the current in this R_2? This is the direction of the current from v_{i2} to v_{02}, this is I means this is plus this is minus. So, what is the relation between v_{i2} v_{02} and the voltage drop across R_2? Minus to plus if I take as positive v_{i2}, the voltage drop across R_2 is plus to minus. So, $v_{i2} - iR_2 - v_{02} = 0 \Rightarrow v_{02} = v_{i2} - iR_2$. This is equal to v_{i2} is nothing, but $v_{cm} + \frac{v_d}{2}$ minus I have derived as this $i = \frac{v_{i1}-v_{i2}}{2R_1}$.

So, $v_{i1} - v_{i2} = v_d$. So, $v_{02} = v_{cm} + \frac{v_d}{2} + \frac{R_2}{2R_1}(v_d) = v_{cm} + \frac{v_d}{2}(1 + \frac{R_2}{R_1})$. This is equal to v_{cm}. If we take as common, this will be $1 + \frac{R_2}{R_1}$. So, this is the expression for v_{02} you can write as 2. This is equation for v_{01}. Now we have to take $v_{02} - v_{01}$. This is differential output due to differential signal, and the factor of v_d is differential gain.

So, this is v_{02} minus v_{01}. So, the factor of v_d is the differential gain. So, what is $v_{02} - v_{01}$? $v_{02} - v_{01} = v_{cm} + \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right) - v_{cm} + \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right) = v_d(1 + \frac{R_2}{R_1})$, implies this will be the common mode gain $A_d = (1 + \frac{R_2}{R_1})$.

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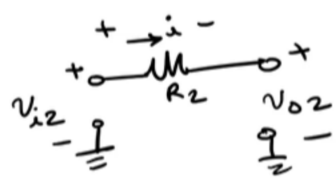
$$\therefore\ v_{01} = v_{cm} - \frac{v_d}{2} - \frac{R_2}{2R_1} v_d$$

$$= v_{cm} - \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right) \text{ - - - - (i)}$$



$$v_{i2} - iR_2 - v_{02} = 0$$

$$\Rightarrow v_{02} = v_{i2} - iR_2$$

$$= v_{cm} + \frac{v_d}{2} - \left(\frac{v_{i1} - v_{i2}}{2R_1}\right) R_2$$

$$\Rightarrow v_{02} = v_{cm} + \frac{v_d}{2} - \frac{R_2}{2R_1}(-v_d)$$

$$= v_{cm} + \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right) \text{ - - - (ii)}$$

$$v_{02} - v_{01} = \cancel{v_{cm}} + \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right) - \cancel{v_{cm}} + \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right)$$

$$= \left(1 + \frac{R_2}{R_1}\right) v_d \longrightarrow A_d = 1 + \frac{R_2}{R_1}$$

Similarly, in order to obtain the common mode gain you have to take the sum of this by 2, therefore, $\frac{v_{01} + v_{02}}{2} = \frac{\left\{v_{cm} - \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right)\right\} + \left\{v_{cm} + \frac{v_d}{2}\left(1 + \frac{R_2}{R_1}\right)\right\}}{2} = v_{cm}$.

Here what is the factor of v_{cm} which is the common mode gain is unity. So, we got $A_d = \left(1 + \frac{R_2}{R_1}\right)$ and $A_{cm} = 1$ ACM as unity. Therefore, what is $CMRR = 20 \log \left|\frac{A_d}{A_{cm}}\right| =$

$20\log\left(1+\frac{R_2}{R_1}\right)$. So, there are two circuits. For the first circuit, we have got *CMRR* as 0; for the second circuit, we have got as $20\log\left(1+\frac{R_2}{R_1}\right)$. So, which one is better circuit because this *CMRR* is greater than 0dB this is better circuit this is better circuit for instrumentation amplifier.

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$$\frac{v_{o1}+v_{o2}}{2} = \frac{\left\{v_{cm}-\frac{v_d}{2}\left(1+\frac{R_2}{R_1}\right)\right\} + \left\{v_{cm}+\frac{v_d}{2}\left(1+\frac{R_2}{R_1}\right)\right\}}{2}$$

$$= v_{cm}$$

$$A_{cm} = 1$$

$$A_d = 1+\frac{R_2}{R_1}$$

$$\therefore \boxed{\begin{aligned} CMRR &= 20\log\left|\frac{A_d}{A_{cm}}\right| \\ &= 20\log\left(1+\frac{R_2}{R_1}\right) \end{aligned}}$$

So, both the circuits are instrumentation amplifiers only, but the second circuit is a better circuit in terms of the common mode rejection ratio because it can reject the common mode signals in efficient manner. What is the reason for that one? You can see this from this if I take v_{01} what is the gain for this v_{cm} is $1+\frac{R_2}{R_1}$ even in this v_{02} also if I substitute this $v_{cm}+\frac{v_d}{2}$ the gain is $1+\frac{R_2}{R_1}$. So, the gain for the common mode signals in first circuit is $1+\frac{R_2}{R_1}$ that is undesired common mode signal is normally the noise. So, for which we should not provide any gain, but the first circuit is providing a gain of $1+\frac{R_2}{R_1}$ for the common mode signal.

Interestingly if I take the second circuit. So, this is the output for the v_{01}. So, what is the gain for v_{cm}? Unity. So, whatever this common mode signal here the same v_{cm} is obtained at the output also without any gain whereas, in case of the first circuit this v_{cm} at the input after the first stage this is appearing as $1+\frac{R_2}{R_1}$ times v_{cm}. So, because this circuit amplifies the common mode signals also. So, the common mode rejection ratio is less whereas, the second circuit does not amplify this v_{cm} here the output is also v_{cm}.

So, this second circuit will be having better *CMRR*. So, this is about this two instrumentation amplifier circuits and the corresponding *CMRR*. I will take one problem on the slew rate. Suppose a sinusoidal signal is having a slew rate of $60V/\mu sec$. What is the highest frequency that attains a $20V(P-P)$ output without distortion? *20V* peak to peak signal means peak value will be *10V*. So, what is the type of the input signal? The output signal that we have to obtain is *20V* peak to peak signal $v_0 = 10sin\omega t$.

What will be the highest frequency of this one? That will obtain this output voltage with *10V* peak signal without any distortion ok. For that we have to find out a $\frac{dv_0}{dt}$. This is, $\frac{dv_0}{dt} = 10\omega cos\omega t$, and what is the maximum value of this $\frac{dv_0}{dt}$? Of course, minus and I am not taking this I want the magnitude only maximum value maximum value of cos is unity. So, $|\frac{dv_0}{dt}|_{max} = 10\omega$, is defined as slew rate. This is equal to 60×10^6. So, it $10\omega_{max}$, you have to find out the maximum value of the frequency which can give *10V* peak value of the output sinusoidal signal is equal to $60 \times 10^6 \Rightarrow 2\pi f_{max} = 60 \times 10^6 \Rightarrow f_{max} = \frac{3}{\pi} \times 10^6 Hz \approx 45.5kHz$.

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Ex:- Let sinusoidal signal has a slew rate of 60 V/μsec
What is the highest frequency that attains a $\underline{20}$V(P-P) output without distortion?

Sol:- $v_o = 10 \sin \omega t$

$\left|\frac{dv_o}{dt}\right| = 10\omega \underbrace{\cos\omega t}_{1}$

$\left|\frac{dv_o}{dt}\right|_{max} = 10\omega$ = slew rate = 60×10^6

$1\cancel{0}\,\omega_{max} = 6\cancel{0}\times10^6$

$\Rightarrow 2\pi f_{max} = 6\times10^6$

$\Rightarrow f_{max} = \frac{3}{\pi}\times10^6$ Hz $\approx$ 45.5KHZ

So, we know that the slew rate for a given output without any distortion, we can find out the maximum input frequency, or we can do in another way also. For a given maximum frequency, what is the maximum output that can be obtained without any distortion? This we have already discussed in the previous lectures.

So, these are some problems. Next, we will discuss about some specialized applications of the operational amplifiers, such as filters, oscillators, etcetera. Thank you.